

Second Edition

OUANTITATIVE APTITUDE AND REASONING



R.V. Praveen

Quantitative Aptitude and Reasoning

Second Edition

R.V. Praveen

2013

QUANTITATIVE APTITUDE AND REASONING, Second Edition R.V. Praveen
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ISBN-978-81-203-4777-9
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Seventh Printing (Second Edition) October, 2013
Published by Asoke K. Ghosh, PHI Learning Private Limited, Rimjhim House, 111, Patparganj Industrial Estate, Delhi-110092 and Printed by Rajkamal Electric Press, Plot No. 2, Phase IV, HSIDC, Kundli-131028, Sonepat, Haryana.

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Preface

This well-received book, now in its second edition, is revised as per the feedback received from students and readers.

This new edition contains five new chapters:

- Simplification
- Races and Games
- Stocks and Shares
- Discount
- Logarithm

Each chapter includes several new solved examples and exercises. Besides, it also includes previous year's questions that are asked in various competitive examinations such as Civil Services, Bank PO, CDS, MAT and so on. Many short-cut methods are used to solve objective type questions within a fraction of a minute.

The book is recommended to all engineering colleges in Kerala by the Director of Technical Education, Kerala. This volume is useful for aspirants of all graduate level competitive examinations in which speed and accuracy are the main factors to get success in it.

I have done this revision to the maximum extent possible within the limited time. I hope that this book will be a valuable resource for the students preparing for graduate level competitive examinations and campus recruitment test as well.

Constructive feedback from the readers for the improvement of this book is highly appreciated.

R.V. Praveen

Preface to the First Edition

Almost all competitive and job recruitment examinations have sections on quantitative aptitude and reasoning. Keeping its importance and the current pattern of examination in mind, this book caters to the needs of the students preparing for competitive examinations.

The book is organized in two parts: Part I (Quantitative Aptitude) and Part II (Reasoning). The reasoning section treats both verbal and nonverbal reasoning. The main aim of this text is to guide the students to solve the problems within the stipulated time and that too with a higher degree of accuracy. For ease of understanding, numerous notes and short-cut methods are given. Each chapter contains theory, illustrative examples, worked-out examples and exercises followed by necessary explanations. It contains numerous solved previous years' questions from various competitive examinations. Model test papers are also included to sharpen the analytical and logical thinking of the students.

This book is mainly intended exclusively for the aspirants of graduate level competitive examinations such as Civil Services, LIC—Assistant Administrative Officer (AAO), GIC—Assistant Manager, Common Admission Test (CAT), Management Aptitude Test (MAT), State Eligibility Test (SET), National Eligibility Test (NET), Bank Probationary Officer (PO), Staff Selection Commission (SSC), National Defence Academy (NDA), Combined Defence Services (CDS), Income Tax and Central Excise, etc. Besides, it would be equally useful for campus recruitment screening tests, career competency development courses as well as undergraduate students of Mathematics for their course in Aptitude Examination.

I extend my thanks to all those who took pains and interest in the publication of this book. Any constructive suggestion or feedback from the reader about the contents of the book or further improvement of the book is always welcome.

Finally, I thank PHI Learning for publishing this book.

R.V. Praveen

Part I

QUANTITATIVE APTITUDE

HCF and LCM

HIGHEST COMMON FACTOR (HCF): The highest common factor (HCF) of two or more numbers is the greatest number that divides each of them completely. There are two methods for finding the HCF of two or more numbers.

Split Method: In the split method, we split all the given numbers into prime factors and then take the product of all prime factors common to all the given numbers. The product is the HCF.

EXAMPLE 1 Find the HCF of 84 and 90.

$$84 = 2 \times 2 \times 3 \times 7$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$HCF = 2^1 \times 3^1 = 6$$

EXAMPLE 2 Find the HCF of 1365, 1584 and 1872.

Solution
$$1365 = 3 \times 5 \times 7 \times 13$$

$$1584 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11$$

$$1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13$$

$$HCF = 3^1 = 3$$

EXAMPLE 3 Find the HCF of 792, 1155 and 1716.

Solution

$$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$$

$$1155 = 3 \times 5 \times 7 \times 11$$

$$1716 = 2 \times 2 \times 3 \times 11 \times 13$$

$$HCF = 3^1 \times 11^1 = 33$$



Note: Find out the prime factors of one of the given numbers and check which of those factors, by which the other numbers, are completely divisible. The product of these numbers is the HCF.

EXAMPLE 4 Find the HCF of 72 and 1452.

Solution The prime factors of 72 are 2, 2, 3 and 3

i.e.
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

of these, 1452 is divisible by $2 \times 2 \times 3$

∴ HCF is 12

EXAMPLE 5 Find the HCF of 84 and 1584.

Solution
$$84 = 2 \times 2 \times 3 \times 7$$

$$1584 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11$$

$$\therefore$$
 HCF = $2^2 \times 3^1 = 12$

EXAMPLE 6 Find the HCF of 396 and 612.

Solution
$$396 = 2 \times 2 \times 3 \times 3 \times 11$$

$$612 = 2 \times 2 \times 3 \times 3 \times 17$$

:.
$$HCF = 2^2 \times 3^2 = 36$$

Division Method: In the division method, we divide the greater number by the smaller number; then divide the divisor by the remainder; then divide the remainder with the next reminder and continue the process until no remainder is left. The last divisor in this process is the required HCF.

EXAMPLE 7 Find the HCF of 84 and 96.

Solution

$$\begin{array}{r}
84) 96 (1 \\
 \underline{84} \\
 12) 84 (7 \\
 \underline{84} \\
 \underline{0}
\end{array}$$

$$\therefore$$
 HCF = 12

This method is based on the following two principles.

- 1. A number that divides another number also divides any multiple of that number. For example, 6 divides 24. Therefore, 6 divides any multiple of 24.
- 2. A number that divides each of the given two numbers, also divides their sum, difference and the sum and difference of any multiple of that numbers.

HCF of 84 and 210

 \therefore The HCF of 84 and 210 = 42

EXAMPLE 8 Find the HCF of 1672 and 374.

 \therefore HCF = 4

EXAMPLE 9 Find the HCF of 3672 and 2496.

 \therefore HCF = 24

EXAMPLE 10 Find the HCF of 2648 and 3972.

HCF OF MORE THAN TWO NUMBERS: Find the HCF of any two of the given numbers and then find the HCF of this HCF and the third number. Continue this process and the last HCF will be the required HCF.

EXAMPLE 11 Find the HCF of 1266, 1461 and 1863.

∴ 3 is the HCF of 1266 and 1461.

3)
$$1863 (621 \frac{1863}{0}$$

... The required HCF is 3.

RULE

Method of finding the HCF may be simplified by the following rules:

- 1. Prime factors that are not common to the given numbers are rejected.
- 2. Common factors may be removed before the rule is applied. This factor should be multiplied with the HCF of the quotients.
- 3. Any factor of the divisor not contained in the dividend may be rejected.

EXAMPLE 12 Find the HCF of 84474 and 75582.

Solution
$$84474 = 2 \times 9 \times 4693$$
 $75582 = 2 \times 9 \times 4199$

Here, 2 and 9 are common factors.

.. Find the HCF of 4693 and 4199 and then multiply the common factors with the HCF obtained.

494 is divisible by 2, but 4199 is not.

... Divide 494 by 2 and then proceed with 247 and 4199.

- .: The HCF of 4199 and 4693 is 247.
- \therefore The HCF of the given numbers is $247 \times 9 \times 2 = 4446$.

EXAMPLE 13 Find the HCF of 372, 954 and 1728.

:. HCF of 1728 and 954 = 18.

Now, HCF of 372, 954 and 1728 = HCF of 372 and 18.

 \therefore HCF of 372, 954 and 1728 = 6

EXAMPLE 14 Find the HCF of 896, 1084 and 2372.

$$\begin{array}{r}
2168 \\
\hline
204) 1084 (5 \\
1020 \\
\hline
64) 204 (3 \\
\underline{192} \\
12) 64 (5 \\
\underline{60} \\
4) 12 (3 \\
\underline{12} \\
0
\end{array}$$

- :. HCF of 2372 and 1084 = 3.
- : HCF of 896, 1084 and 2372.

= HCF of 896 and 3.

Hence, the HCF of 896, 1084 and 2372 = 1.

HCF OF DECIMALS: Make the same number of decimal places in all the given numbers; then find their HCF as if they are integers and mark off in the result as many decimal places as there are in each of the given numbers.

EXAMPLE 15 Find the HCF of 25.5, 42.5 and 170.

Solution The given numbers are equivalent to 255, 425 and 1700.

- (i) First, find the HCF of 255, 425 and 1700 which comes to 85 and then divide by 10.
- (ii) The required HCF is 8.5.

EXAMPLE 16 Find the HCF of 36.54, 48.42 and 58.41.

Solution

- (i) The given numbers are equivalent to 3654, 4842 and 5841.
- (ii) Find the HCF of 3654, 4842 and 5841 which comes to about 9 and then divide the result so got by 100.
- (iii) Therefore, the required HCF is

$$4842) 5841 (1) \\ \underline{4842} \\ \overline{999) 4842 (4)} \\ \underline{3996} \\ \overline{846) 999 (1)} \\ \underline{846} \\ \overline{153) 846 (5)} \\ \underline{765} \\ \overline{81) 153 (1)} \\ \underline{81} \\ \overline{72) 81 (1)} \\ \underline{72} \\ \overline{9) 72 (8)} \\ \underline{72} \\ \overline{0}$$

- \therefore HCF = 9
- \therefore Required HCF = HCF of 3654 and 9.

- ∴ HCF = 9
- \therefore Required HCF = $\frac{9}{100}$ = **0.09**

EXAMPLE 17 Find the HCF of 78.9, 165.6 and 376.2.

Solution

- (i) The given numbers are equivalent to 789, 1656 and 3762.
- (ii) First find HCF of above three numbers and then divide the result by 10.
- (iii) Therefore, required HCF is

∴ HCF = 18

Now, Required HCF = HCF of 789 and 18.

$$\therefore$$
 Required HCF = $\frac{3}{10}$ = **0.3**

EXAMPLE 18 Find the HCF of 24.3, 68.4 and 127.8

Solution

- (i) The given numbers are equivalent to 243, 684 and 1278.
- (i) Find the HCF of the above three numbers and divide the result by 10.
- (i) Then the required HCF is

$$684)1278 (1)$$

$$\underline{684}$$

$$594) 684 (1)$$

$$\underline{594}$$

$$90) 594 (6)$$

$$\underline{540}$$

$$54) 90 (1)$$

$$\underline{54}$$

$$36) 54 (1)$$

$$\underline{36}$$

$$18) 36 (2)$$

$$\underline{36}$$

$$\underline{0}$$

Hence HCF = 18

The HCF of 243 and 18 is given by

$$\therefore \text{ Required HCF} = \frac{9}{10} = \textbf{0.9}$$

HCF OF FRACTIONS: The HCF of two or more fractions is the highest fraction by which each of the fractions is completely divisible.

Express the given fractions in their lowest terms and then find the HCF given by the following formula

$$HCF = \frac{HCF \text{ of Numerator}}{LCM \text{ of Denominator}}$$

EXAMPLE 19 Find the HCF of $\frac{63}{9}$, $5\frac{6}{17}$ and $\frac{42}{51}$.

Solution
$$\frac{63}{9} = \frac{7}{1}$$
; $5\frac{6}{17} = \frac{91}{17}$ and $\frac{42}{51} = \frac{14}{17}$.

The fractions are $\frac{7}{1}$, $\frac{91}{17}$ and $\frac{14}{17}$.

$$HCF = \frac{HCF \text{ of } 7,91 \text{ and } 14}{LCM \text{ of } 1,17,17} = \frac{7}{17}$$

Each of the given numbers is completely divisible by $\frac{7}{17}$.

$$\therefore$$
 Required HCF = $\frac{7}{17}$.

EXAMPLE 20 Find the HCF of $\frac{31}{9}$, $2\frac{5}{7}$ and $3\frac{5}{11}$.

Solution
$$2\frac{5}{7} = \frac{19}{7}$$
; $3\frac{5}{11} = \frac{38}{11}$

Given fraction are $\frac{31}{9}$, $\frac{19}{7}$ and $\frac{38}{11}$.

$$\therefore \qquad \text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{\text{HCF of } 31,19 \text{ and } 38}{\text{LCM of } 9,7 \text{ and } 11} = \frac{1}{9 \times 7 \times 11} = \frac{1}{693}$$

Each of given fractions is completely divisible by $\frac{1}{693}$.

$$\therefore$$
 Required HCF = $\frac{1}{693}$

EXAMPLE 21 What is the greatest number that will divide 3600 and 2715 and leave remainders 9 and 6, respectively?

Solution On dividing 3600, 9 is left as remainder.

The required number must divide (3600 - 9) completely.

Similarly, it must divide (2715-6) or 2709 completely.

Hence, the required greatest number is the HCF of 3591 and 2709.

$$\begin{array}{c} 2709)\ 3591\ (1) \\ \underline{2709} \\ 882\)\ 2709\ (3) \\ \underline{2646} \\ \hline 63\)\ 882\ (14) \\ \underline{63} \\ \underline{252} \\ \underline{252} \\ 0 \end{array}$$

HCF of 3591 and 2709, is 63 which is the last divisor in the above process of division.

$$\therefore$$
 HCF = **63**.

EXAMPLE 22 Find the greatest number which will divide 415, 748 and 1007 so as to leave a remainder 8 in each case?

Solution Required greatest number = HCF of (415 - 8), (748 - 8) and (1007 - 8) i.e., HCF of 407, 740 and 999

HCF of 999 and 740 is 37.

Hence, the required greatest number is 37.

EXAMPLE 23 The numbers 12284 and 8655 when divided by a certain number of three digits, leave the same remainder. Find that number of three digits.

Solution The required number must be a factor of (12284 – 8655) or 3629.

$$3629 = 19 \times 191$$

:. 191 is the required number.

EXAMPLE 24 The product of two numbers is 5376 and their HCF is 16. Find the number. **Solution** The numbers are multiples of 16. Let it be 16a and 16b.

$$\therefore$$
 16 $a \times 16b = 5376$ or $ab = 21$ \therefore $a = 3$; $b = 7$.
The numbers are 16×3 , 16×7 , i.e., **48** and **112**.

COMMON MULTIPLE: A common multiple of two or more numbers is a number which is exactly divisible by all of the given numbers.

LEAST COMMON MULTIPLE (LCM): The LCM of two or more given numbers is the least number which is exactly divisible by all the given numbers. For example,

35 is a common multiple of 5 and 7.

70 is a common multiple of 5 and 7.

105 is a common multiple of 5 and 7.

But, 35 is the least common multiple (LCM) of 5 and 7.

The following are the methods to find the LCM of two or more given numbers.

Method of Prime Factors: Resolve the given numbers into prime factors and then take the product of the highest powers of all the factors and this product is the required LCM.

EXAMPLE 25 Find the LCM of 12, 16, 21 and 36.

Solution
$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

 $16 = 2 \times 2 \times 2 \times 2 = 2^4$
 $21 = 3 \times 7$
 $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

Here, the prime factors are 2, 3 and 7 and their highest powers are 2^4 , 3^2 and 7^1 .

Therefore, the required LCM =
$$2^4 \times 3^2 \times 7$$

= $16 \times 9 \times 7$
= 1008

EXAMPLE 26 Find the LCM of 27, 36, 80 and 175.

Solution

$$27 = 3 \times 3 \times 3$$
 $= 3^3$
 $36 = 2 \times 2 \times 3 \times 3$
 $= 2^2 \times 3^2$
 $80 = 2 \times 2 \times 2 \times 2 \times 5$
 $= 2^4 \times 5^1$
 $175 = 5 \times 5 \times 7$
 $= 5^2 \times 7^1$

Here, the prime factors in the highest powers occurring in the given numbers are 2^4 , 3^3 , 5^2 and 7^1 .

Therefore, the required LCM =
$$2^4 \times 3^3 \times 5^2 \times 7^1$$

= $16 \times 27 \times 25 \times 7$
= 75600

EXAMPLE 27 Find the LCM of 42, 56, 91 and 126.

Taking highest powers of all prime factors in the above expression, we get $HCF = 2^{0} \times 7^{1} \times 13^{0} = 7$

Division Method: Write down the numbers in a single line separated by comas, and then divide these numbers by a prime number which will divide at least two of the given numbers completely. Continue the division, till no two numbers are having common factors further. Those numbers which cannot be divided should be written as such during each division. Then the required LCM will be the product of all divisors and the numbers written in the last line.

EXAMPLE 28 Find the LCM of 15, 18, 54, 135 and 180.

Hence, the required LCM =
$$2 \times 3 \times 3 \times 5 \times 3 \times 2$$

= $2^2 \times 3^3 \times 5$
= 540

EXAMPLE 29 Find the LCM of 18, 32, 72 and 96.

Solution

2	18,	32,	72,	96
2	9,	16,	36,	48
3	9,	8,	18,	24
3	3,	8,	6,	8
2	1,	8,	2,	8
2	1,	4,	1,	4
2	1,	2,	1,	2
	1,	1,	1,	1

 \therefore LCM = $2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 1 \times 1 \times 1 \times 1 = 288$



Note: The product of any two numbers is equal to the product of their HCF and LCM.

LCM of Decimals: First, make the same number of decimal places in all the given numbers and find their LCM as they are integers and mark in the result as many decimal places as there are in each of the given numbers.

EXAMPLE 30 Find the LCM of 0.8, 0.9 and 0.36.

Solution The given numbers are equivalent to 0.80, 0.90 and 0.36.

The LCM of 80, 90 and 36 = 720

 \therefore The required LCM = 7.2

EXAMPLE 31 Find the LCM of 0.72, 0.64 and 0.968.

Solution The given numbers are equivalent to 0.720, 0.640 and 0.968

The LCM of 720, 640 and 968 = 696960

 \therefore The LCM of 0.720, 0.640 and 0.968 = **696.96**

EXAMPLE 32 Find the LCM of 0.96, 1.28 and 0.84.

Solution The given numbers are equivalent to 96, 128 and 84. LCM of 96, 128 and 84 is given by

$$\therefore LCM = 2 \times 2 \times 7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 1 \times 2 \times 1$$
$$= 2^{7} \times 3^{1} \times 7^{1}$$
$$= 128 \times 21$$

LCM of 96, 128 and 84 = 2688

 \therefore LCM of 0.96, 1.28 and 0.84 = **26.88**

LCM of Fractions: The LCM of two or more fractions is the least fraction or integer which is exactly divisible by each of them.

First, express the fractions in their lowest terms. Then the LCM of fraction is given by ratio of LCM of numerators to HCF of denominators of the given fraction.

i.e.
$$LCM = \frac{LCM \text{ of Numerators}}{HCF \text{ of Denominators}}$$

EXAMPLE 33 Find the LCM of

(i)
$$\frac{2}{3}, \frac{6}{7}$$
 (ii) $\frac{7}{12}, \frac{16}{27}$ (iii) $\frac{8}{17}, \frac{18}{51}$

Solution

(i) The required LCM =
$$\frac{\text{LCM of 2 and 6}}{\text{HCF of 3 and 7}} = \frac{6}{1} = 6$$

(ii) The required LCM =
$$\frac{LCM \text{ of } 7 \text{ and } 16}{HCF \text{ of } 12 \text{ and } 27} = \frac{112}{3}$$

(iii) The required LCM =
$$\frac{\text{LCM of 8 and 18}}{\text{HCF of 17 and 51}} = \frac{72}{17}$$

EXAMPLE 34 The LCM of two numbers is 5301 and their HCF is 9. If one of the numbers is 171, find the other number.

Solution The required number =
$$\frac{LCM \times HCF}{First number} = \frac{5301 \times 9}{171} = 279$$

EXAMPLE 35 What is the least number which when divided by 48, leaves 33 as the remainder and when divided by 78 leaves 63 as remainder, and when divided by 126 leaves 111 as the respective remainders?

Solution Here, 48–33 = 15; 78–63 = 15; 126–111 = 15.

The remainder in each case is less than the divisor by 15. Therefore, if 15 is added to the required number, the resulting number will become exactly divisible by 48, 78 and 126, i.e., the required number is 15 less than the LCM of 48, 78 and 126.

The LCM of 48, 78 and 126 = 13, 104.

 \therefore The required number = 13, 104 - 15 = 13,089

EXAMPLE 36 Find the greatest number of six digits which on being divided by 5, 6, 7 and 8 leaves remainders 3, 4, 5 and 6, respectively.

Solution The LCM of 5, 6, 7 and 8 = 840 The greatest number of 6 digits = 999999

On dividing 999999 by 840, the remainder will be 399.

Therefore, the greatest six digit number divisible by 840 is

$$999999 - 399 = 999600$$

Since 5-3=6-4=7-5=8-6=2, the remainder in each case is less than the divisor by 2.

... The required number = 999600 - 2 = 999598

EXAMPLE 37 What is the least multiple of 11, which when divided by 3, 4, 5, 6 and 7, leaves the remainder 2, 3, 4, 5 and 6 respectively?

Solution The LCM of 3, 4, 5, 6 and 7 = 420.

Since 3-2=4-3=5-4=6-5=7-6=1, if you add one to the required number, the resulting number will be divisible by the given divisors.

But we require the number as a multiple of 11.

Assume the number to be $420 \times n - 1$

i.e.
$$(38 \times 11 + 2)n - 1$$

 \therefore 38 × 11n is divisible by 11, the number to be checked for divisibility by 11 is 2n-1. The least value of 2n-1 to be divisible by 11 is 11

i.e.
$$2n-1=11 \implies n=\frac{1+11}{2}=6$$

 \therefore The required number = $420 \times 6 - 1$

$$= 2519$$

EXAMPLE 38 What is the least number which when divided by 12, 16 and 18 leaves 9 as remainder in each case, but when divided by 27 leaves no remainder.

Solution The number which is divisible by all the three given numbers is of the form 144k, when k is an integer.

Here, the number leaves 9 as remainder on division

 \therefore Required number = 144k + 9.

Given
$$(144k + 9) \div 27 = 0$$

If k = 5, value of number will be equal to $144 \times 5 + 9 = 729$

Hence, required number = 729

EXERCISES

- 1. The sum of two numbers is 468 and their HCF is 39. How many pairs of such numbers can be formed?
 - (a) 11
- (b) 22
- (c) 33
- (d) 36
- (e) None of these
- 2. Find the sum of the numbers between 500 and 700 such that when they are divided by 6, 8 and 12
 - (i) it leaves no remainder
 - (a) 3200
- (b) 4600
- (c) 5300
- (d) 5400
- (e) None of these

- (ii) it leaves 5 as remainder in each case
 - (a) 4600
- (b) 4447
- (c) 4744
- (d) 4848
- (e) None of these

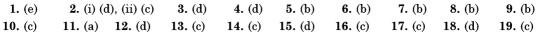
3.	Is it possible to di	ivide 1,000 into t	wo parts such	that their HCF is 15?	
	(a) Yes, it is poss	sible	(b)	No, it is sometimes j	possible
	(c) Yes, it may b	e possible	(d)	Never possible	
4.	In a school, 442 bo	ys and 374 girls h	ave been divid	ed into the largest pos	sible equal classes, so
	that each class of l	boys numbers the	same as each o	class of girls. What is t	he number of classes?
	(a) 16	(b) 18	(c) 20	(d) 24	(e) None of these
5.	Find the greatest r	number of 4 digits	and the least n	umber of 5 digits that h	ave 196 as their HCF.
	(a) 9998, 10184	(b) 9996, 10192	2 (c) 9994, 10	194 (d) 9998, 1019	6 (e) None of these
6.	The product of tw	o numbers is 421	2 and their H	CF is 9. Find the poss	ible pair of numbers.
	(a) 32, 121	(b) 36, 117	(c) 42, 114	(d) 45, 110	(e) None of these
7.	The HCF and LCM	I of two numbers as	re 36 and 756 re	spectively. Find the pos	ssible pair of numbers.
	(a) 72, 378	(b) 108, 252	(c) 144, 658	(d) 180, 622	(e) None of these
8.	Find the maximu	m possible length	n which can be	used to measure exa	ctly the lengths 6 m,
	4 m 75 cm, and 10				
	(a) 20 cm	(b) 25 cm	(c) 30 cm	(d) 35 cm	(e) None of these
9.	The LCM of two n	numbers is 1008 a	nd their HCF i	is 36. If one of the two	numbers is 144, find
	the other number	:.			
	(a) 288	(b) 252	(c) 576	(d) 504	(e) None of these
10.	Three bells ring to respectively. Afte			ervals of 6 seconds, 7 s	econds and 8 seconds
				nds (d) 164 second	s (e) None of these
11				pave the ceiling of a	
11.	and 8 m 10 cm br	_	ies required to		-
	(a) 78	(b) 64	(c) 84	(d) 92	(e) None of these
12.	Find the smallest		hich is exactly	divisible by 6, 8, 15,	18 and 24.
	(a) 120	(b) 240	(c) 740	(d) 360	(e) None of these
13.	Find the highest	common factor of	36 and 84.		[RRB]
	(a) 4	(b) 6	(c) 12	(d) 18	
14.	Find the least cor	nmon multiple of	24, 36 and 40		[RRB]
	(a) 120	(b) 240	(c) 360	(d) 480	
15.	The ratio of two n	numbers is 3 : 4 a	nd their HCF i	is 4. Their LCM is	[SSC]
	(a) 12	(b) 16	(c) 24	(d) 48	
16.	The product of tw number is	vo numbers is 41	07. If the HCF	of these numbers is	37, then the greater [SSC]
	(a) 101	(b) 107	(c) 111	(d) 185	
17.	` /	, ,	` '	respectively. If the rat	io of the two numbers
	is 1:4, then the 1				[MAT]
	(a) 12	(b) 48	(c) 84	(d) 108	
18.	The HCF of two n	numbers is 8. Whi	ich one of the f	following can never be	their LCM ? [SSC]
	(a) 24	(b) 48	(c) 56	(d) 60	
19.	` '			and LCM 4147. The s	oum of the numbers is [SSC]
	(a) 666	(b) 669	(c) 696	(d) 966	
20.	The least number divided by 9 leave			7 and 8 leaves 3 as 1	remainder, but when [LIC AAO]
	(a) 1677	(b) 1683	(c) 2523	(d) 3363	

21.	The least multipl	e of 7, which lea	ives a remainder	of 4, when divided by 6	
	(a) 74	(b) 94	(c) 184	(d) 364	[LIC AAO]
22.	` '	` '	` '	0 and 54 leaves in each	case a remainder
	of 8, is				[RRB]
	(a) 504	(b) 536	(c) 544	(d) 548	
23.	The smallest num	nber which whe	n diminished by 7	, is divisible by 12, 16,	18, 21 and 28 is
	() 1000	4	() 1000	(1) 4000	[LIC]
2.4	(a) 1008	(b) 1015	(c) 1022	(d) 1032	1 10 10 01 1
24.	What will be the I 30?	east number wh	ich when doubled	will be exactly divisibl	e by 12, 18, 21 and
	(a) 196	(b) 630	(c) 1260	(d) 2520	
25.	The greatest num is	ber which on div	iding 1657 and 20	37 leaves remainders 6	and 5 respectively [RRB]
	(a) 123	(b) 127	(c) 235	(d) 305	
26.	Let N be the gre	eatest number t	that will divide	1305, 4665 and 6905,	leaving the same
	remainder in eacl	h case. Then sur	n of the digits in	N is	[SSC]
	(a) 4	(b) 5	(c) 6	(d) 8	
27 .	Find the greatest in each case.	number that wi	ll divide 43, 91 ar	nd 183 so as to leave th	e same remainder [LIC]
	(a) 4	(b) 7	(c) 9	(d) 13	
28.				1 pens and 910 pencils number of pens and	
	(a) 91	(b) 910	(c) 1001	(d) 1911	
29.	The greatest poss 85 cm, 12 m 95 cm	-	ich can be used t	o measure exactly the	lengths 7 m, 3 m [RRB]
	(a) 15 cm	(b) 25 cm	(c) 35 cm	(d) 42 cm	
30.	The HCF and LC7 75 and 125, then		rs are 11 and 385	respectively. If one nu	mber lies between [CBI]
	(a) 77	(b) 88	(c) 99	(d) 110	
31.	The HCF of two rates are the larger of the two		nd the other two f	factors of their LCM ar	e 13 and 14. Then [SSC]
	(a) 276	(b) 299	(c) 322	(d) 345	
32.	Product of two co-	-prime numbers	is 117. Their LC	M should be	[CBI]
	(a) 1		(b) 1	117	
	(c) equal to their	$_{ m r}$ HCF	(d) (cannot be calculated	
33.				d LCM of these numb numbers is equal to	ers are 5 and 120 [CDS]
	(a) $\frac{55}{601}$	(b) $\frac{601}{55}$	(c) $\frac{11}{120}$	(d) $\frac{120}{11}$	
34.			and their HCF is	5. If the sum of the nu	mbers is 100, then [SSC]
	(a) 10	(b) 46	(c) 70	(d) 90	
35.	The HCF of two n other is	umbers is 11 an	d their LCM is 77	00. If one of the number	ers is 275, then the [Section Officer]

(a) 279 (b) 283 (c) 308 (d) 318

36.	The product of tw	o numbers is 202	28 and their	HCF is 13. Th	ne number of such pai	irs is [CBI]
	(a) 1	(b) 2	(c) 3	(d) 4	<u>[</u>	[ODI]
37 .		` '			er of pairs of numbers	satisfying
	the above condition				•	[ČBI]
	(a) 4	(b) 6	(c) 8	(d) 1	.2	
38.	Three numbers an	re in the ratio 1 :	2:3 and th	eir HCF is 12.	. The numbers are	
					[Section	Officer]
	(a) 4, 8, 12	(b) 5, 10, 15	(c) 10, 20), 30 (d) 1	2, 24, 36	
39.	The LCM of 22, 5	54, 108, 135 and 1	.98 is			[MBA]
	(a) 330	(b) 1980	(c) 5940	(d) 1	.1880	
40.	HCF of $4 \times 27 \times 3$	$8125, 8 \times 9 \times 25 \times$	7 and 16 × 8	$31 \times 5 \times 11 \times 4$	9 is	[CBI]
	(a) 180	(b) 360	(c) 540	(d) 1	.260	
	1095					
41.	$\frac{1095}{1168}$ when expre	essed in simplest	form is			[MBA]
					25	
	(a) $\frac{13}{16}$	(b) $\frac{15}{16}$	(c) $\frac{1}{26}$	(d) $\frac{2}{3}$	26	
42 .	= -	hich are co-prime	e to each oth	er are such th	at the product of the f	irst two is
	551 and that of th	_			-	[SSC]
	(a) 75	(b) 81	(c) 85	(d) 8		-
43.		numbers is 48. Th	e numbers :		o 2 : 3. The sum of the	numbers
	is					[SSC]
	(a) 28	(b) 32	(c) 40	(d) 6	34	
44.	Three numbers as	re in the ratio 3:	4:5 and th	eir LCM is 24	00. Their HCF is	[MBA]
	(a) 40	(b) 80	(c) 120	(d) 2	200	
45 .	The LCM of two p	prime numbers x	and $y (x > y)$) is 161. The v	value of $3y - x$ is	[SSC]
	(a) -2	(b) -1	(c) 1	(d) 2		
46.	The least number 130 as remainder		rided by 48,	60, 72, 108 an	nd 140 leaves 38, 50, 6	52, 98 and [CBI]
	(a) 11115	(b) 15110	(c) 15120) (d) 1	5210	[021]
47 .	, ,				6, 10 and 15, leave in	each case
	the same remaind					[SSC]
	(a) 3	(b) 4	(c) 5	(d) 6		
48.	Find the smallest	t number of five d	ligits exactly	y divisible by	12, 16, 18 and 27.	
	(a) 10432	(b) 10342	(c) 10368	3 (d) 1	.0638	
49.	Find the largest n	number of four di	gits exactly	divisible by 1	4, 24, 27 and 32.	
	(a) 9968		-			
50 .	The LCM of $2^2 \times 3$					
	(a) $2^3 \times 3^2 \times 5$,		(b) $2^5 \times 3^4 \times 5^4$		
	(c) $2^3 \times 3^2 \times 5 \times 7$	7×11		(d) $2^5 \times 3^4 \times 5^4$	$^3 \times 7^2 \times 11$	
	_					
	WERS					
	_					
	▼					





- **20**. (b) **21.** (d) 22. (d) **23.** (b) **24.** (b) **25.** (b) **26.** (a) 27. (a) 28. (a) 29. (c) **34.** (a) **30.** (a) **31.** (c) **32.** (b) **33.** (c) **35.** (c) **36.** (c) **37.** (c) 38. (d) **39.** (c) **40.** (a) **41.** (b) **42.** (c) **43.** (c) **44.** (a) **45.** (a) **46.** (b) **47.** (c) **48.** (c) **49.** (c)
- **50.** (a)



Solutions with Necessary Explanation

1. $468 \div 39 = 12$

Therefore, 11 numbers are available and the number of pair of numbers having HCF = 39 is given by $\frac{11+1}{2} = 6$

2. (i) The least number which is divisible by 6, 8 and 12 is the LCM of 6, 8 and 12 = 24.

The numbers greater than 500 and less than 700 which are divisible by 24 are

The sum of the numbers = $\frac{9}{2}$ (504 + 696)

$$= 9 \times 600 = 5400$$

(ii) The number leaves remainder as 5 in each case, the required numbers are greater by 5 than the above numbers

i.e.,
$$504 + 5$$
, $528 + 5$, ..., $696 + 5 - 24$
= 509 , 533 , ..., 677 .

The sum of the numbers = $\frac{8}{2}(509 + 677)$ = 4×1186 = 4744

- 3. Since 1000 is not divisible by 15, it is not possible to divide 1000 into two parts such that their HCF may be 15. Hence the correct answer is (d).
- 4. Number of boys = 442; Number of girls = 374

As the number of boys in the class is same as that of girls.

Strength of one class = HCF of 442 and 374 = 34

- :. Number of classes of boys = $442 \div 34 = 13$ Number of classes of girls = $374 \div 34 = 11$
- \therefore Required number of classes = 13 + 11 = 24
- 5. The greatest number of 4 digits = 9999

On dividing 9999 by 196, 3 will be left as remainder.

 \therefore The required greatest number = 9996.

Least number of 5 digits = 10000

On dividing 10000 by 196, 4 will be left as remainder.

- \therefore The required 5 digit number = 10000 4 + 196 = 10192
- **6.** Let x and y be the two numbers and their HCF = 9; product = 4212.

 \therefore $x \times y = 9^2$ (product in lower terms)

i.e. 4212 = 81 (product in lower terms)

Product in lower terms = 52

The possible values are (1, 52) or (4, 13).

- \therefore The required numbers are 4×9 and 13×9 , i.e. 36 and 117.
- 7. The given HCF is 36 and LCM is 756.
 - :. Product of two numbers = HCF × LCM

$$= 36 \times 756$$

(Product in lower terms) \times (HCF)² = HCF \times LCM

Product in lower terms =
$$\frac{LCM}{HCF} = \frac{756}{36} = 21$$

The numbers are multiples of 3 and 7.

- \therefore The required numbers are 3×36 and 7×36 , i.e. 108, 252.
- 8. Required maximum length = HCF of 6 m. 4 m 75 cm and 10 m 25 cm
 - i.e. the HCF of 600 cm, 475 cm and 1025 cm = 25 cm
- **9.** Let the second number be x.

 $x \times \text{the first number} = \text{HCF} \times \text{LCM}$

$$x \times 144 = 36 \times 1008$$

$$x = \frac{36 \times 1008}{144} = 252$$

- \therefore Required number = 252
- 10. Since the bells ring at the intervals of 6 seconds, 7 seconds and 8 seconds, respectively, next time they will ring together after an interval equal to the LCM of 6, 7 and 8 seconds.

Therefore, the required time = LCM of 6, 7 and 8 seconds

- 11. The maximum size of square tile
 - = HCF of length and breadth of hall
 - = HCF of 17 m 55 cm and 8 m 10 cm
 - = HCF of 1755 cm and 810 cm
 - = 135 cm

The number of square tiles required =
$$\frac{\text{Area of hall}}{\text{Area of one tile}} = \frac{1755 \times 810}{(135)^2} = 78$$

12. Required number = LCM of 6, 8, 15, 18 and 24

$$= 360$$

13. We have

:.

$$36 = 2 \times 2 \times 3 \times 3$$

$$84 = 2 \times 2 \times 3 \times 7$$

$$\therefore$$
 HCF = $2 \times 2 \times 3 = 12$

14. 24,36. 40 20 5

$$\therefore LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

15. Let the numbers be 3x and 4x.

HCF of above two numbers = x (given as 4)

.. Number are 12 and 16.

LCM of 12 and 16 = 48.

16. Let the numbers be 37x and 37y.

$$\therefore 37x \times 37y = 4107$$

$$\Rightarrow \qquad x \times y = \frac{4107}{37 \times 37} = 3$$

i.e.
$$x \times y = 1 \times 3 = 3$$

 \therefore Greater number = $37y = 37 \times 3 = 111$

17. Given HCF = 21; LCM = 84;

Let the numbers be 21x and 21y.

$$\therefore \frac{21x}{21y} = \frac{1}{4}$$

or
$$\frac{x}{y} = \frac{1}{4}$$

 \therefore Larger number = $21 \times 4 = 84$

18. Given HCF of two numbers = 8

LCM is completely divisible by HCF

: 24, 48 and 56 are completely divisible by 8, the value which can never be LCM = 60.

19. Let the numbers be 29x and 29y

$$\therefore$$
 LCM = 29 × $x \times y$ = 4147

$$\Rightarrow xy = \frac{4147}{29} = 143$$

Values of x and y can be (1, 143) and (11, 13).

Given that both numbers are greater than 29.

 \therefore Required numbers are 29 × 11, 29 × 13. i.e. 319, 377

 \therefore Sum of numbers = 319 + 377 = **696**

20. LCM of 5, 6, 7 and 8

$$= 2 \times 5 \times 3 \times 7 \times 4 = 840$$

 \therefore The required number = 840k + 3.

For the number to be divisible by 9, k = 2

i.e.,
$$840 \times 2 + 3 = 1683$$

21. LCM of 6, 9, 15 and 18

$$= 2 \times 3 \times 3 \times 1 \times 1 \times 5 \times 1 = 90$$

Let the number be 90k + 4.

- \therefore Number is divisible by 7, least value of k = 4.
- \therefore Required number = $90 \times 4 + 4 = 364$
- 22. LCM of 12, 15, 20 and 54

$$= 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$$

- \therefore Least number which leaves remainder 8 = 540 + 8 = 548
- 23. LCM of 12, 16, 18, 21 and 28

$$= 2 \times 2 \times 3 \times 7 \times 2 \times 2 \times 3 = 2^4 \times 3^2 \times 7 = 1008$$

- :. Smallest number diminished by 7 which is divisible by 12, 16, 18, 21 and 28 = 1008 + 7 = 1015
- 24. LCM of 12, 18, 21 and 30

$$= 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$$

$$\therefore \text{ Required number} = \frac{1260}{2} = 630$$

25. HCF of (1657 - 6) and (2057 - 5) = HCF of 1651 and 2032

∴ HCF = 127

26.
$$N = \text{HCF of } (6905 - 4665), (4665 - 1305) \text{ and } (6905 - 1305)$$

i.e. $N = \text{HCF of } 2240, 3360 \text{ and } 5600 = 1120$

$$\therefore$$
 Sum of digits of $N = 1 + 1 + 2 + 0 = 4$

27. Required number = HCF of
$$(91 - 43)$$
, $(183 - 91)$ and $(183 - 43)$
= HCF of 48 , 92 and $140 = 4$

28. HCF of
$$1001$$
 and $910 = 91$

910) 1001 (1
910
91) 910 (10
$$\frac{910}{0}$$

- .. Required number of students = 91
- 29. Required length = HCF of 7 m, 3 m 85 cm and 12 m 95 cm i.e. HCF of 700 cm, 385 cm, and 1295 cm

 $\begin{array}{r}
 35) \ 1295 \ (37) \\
 \hline
 705 \\
 \hline
 245 \\
 \hline
 0
 \end{array}$

HCF = 35

HCF of 1295 and 35 = 35

30. Given HCF = 11; LCM = 385;
$$385 = 5 \times 7 \times 11$$

$$\therefore$$
 Number lies between 75 and $125 = 7 \times 11 = 77$

- **31.** Given HCF = 23; LCM = $13 \times 14 \times 23$;
 - \therefore Larger of the two numbers = $14 \times 23 = 322$
- **32.** HCF of co-prime numbers = 1

Product of co-prime numbers = 117

$$\therefore$$
 LCM of co-prime numbers = $\frac{117}{1}$ = 117

33. Let the numbers be x and y.

Given,
$$x + y = 55 \tag{1}$$

HCF = 5; LCM = 120;

We have $HCF \times LCM = x \times y$

i.e.
$$5 \times 120 = xy$$
 (2)

(3)

i.e.
$$x + y = 55$$
 (1)

$$xy = 600 \tag{2}$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy} = \frac{55}{600} = \frac{11}{120}$$

34. Let the numbers be x and y.

Given
$$x + y = 100 \tag{1}$$

LCM = 495; HCF = 5;

 \therefore LCM \times HCF = xy

i.e.,

$$495 \times 5 = xy$$

$$xy = 495 \times 5 \tag{2}$$

$$x - y = ?$$

$$(x-y)^2 = (x+y)^2 - 4xy$$
$$= (100)^2 - 4 \times 495 \times 5$$
$$= (100)^2 - 4 \times 5^2 \times 99$$
$$= 10^2 (10^2 - 99) = 100$$

$$\therefore \qquad x - y = 10$$

: Difference between numbers = 10

35. Given HCF = 11; LCM = 7700; x = 275; y = ?

We have,

$$x \times y = HCF \times LCM$$

i.e.,

$$275 \times y = 11 \times 7700$$

$$y = \frac{11 \times 7700}{275} = \frac{7700}{25} = 77 \times 4 = 308$$

36. Given $x \times y = 2028$; HCF = 13;

$$\therefore$$
 LCM = $\frac{x \times y}{HCF} = \frac{2028}{13} = 156$

Let x = 13a and y = 13b

$$\therefore$$
 $x \times y = 13 \times 13 \ ab = 13 \times 156 = 13 \times 13 \times 12$

$$\therefore \qquad ab = 12$$

Factors of 12 are (1, 12), (2, 6) and (3, 4).

There are 3 such pairs of numbers.

37. Let the numbers be 33a and 33b.

$$\therefore$$
 33*a* + 33*b* = 528

i.e.,
$$a+b=\frac{528}{33}=16$$

- \therefore Values of (a, b) can be (1, 15), (2, 14), (3, 13), (4, 12), (5, 11), (6, 10), (7, 9) and (8, 8)
- ... Number of pairs of numbers satisfying the given condition = 8
- 38. Given ratio of numbers = 1:2:3;

Their HCF = 12

:. Numbers are 12, 24 and 36. Hence the correct answer is (d).

39. LCM =
$$2^2 \times 3^3 \times 5 \times 11 =$$
5940

2	22,	54,	108,	135,	198
2	11,	27,	54,	135,	99
3	11,	27,	27,	135,	99
3	11,	9,	9,	45,	33
3	11,	3,	3,	15,	11
5	11,	1,	1,	5,	11
11	11,	1,	1,	1,	11
	1,	1,	1,	1,	1

40.
$$4 \times 27 \times 3125 = 2^2 \times 3^3 \times 5^5$$

$$8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$$

$$16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 11 \times 7^2$$

 \therefore HCF of above three numbers = $2^2 \times 3^2 \times 5 = 180$

41.
$$\frac{1095}{1168} = \frac{15}{16}$$

42. Let co-prime numbers be x, y and z.

Given xy = 551 and yz = 1073.

$$\frac{yz}{xy} = \frac{1073}{551};$$

$$551) 1073 (1)$$

$$551$$

$$522) 551 (1)$$

$$522$$

$$29) 522 (18)$$

$$29$$

$$232$$

$$232$$

$$0$$

$$HCF = 29$$

$$\frac{z}{x} = \frac{1073}{551} = \frac{37}{19}$$

and y = HCF = 29.

$$\therefore x + y + z = 19 + 29 + 37 = 85$$

43. LCM = 48; Ratio of numbers = 2:3.

Let numbers be 2x and 3x.

 \therefore LCM of numbers = $2 \times 3 \times x = 48$

$$\therefore x = 8$$

 \therefore Sum of numbers = $2x + 3x = 5x = 5 \times 8 = 40$

44. Given LCM of three numbers = 2400;

Ratio of numbers = 3:4:5.

$$\begin{array}{r}
29 \overline{)551} (19 \\
\underline{29} \\
261 \\
\underline{261} \\
0
\end{array}$$

Let numbers be 3x, 4x and 5x.

 \therefore LCM of numbers = $3 \times 4 \times 5 \times x = 2400$

$$\therefore \qquad x = \frac{2400}{60} = 40$$

i.e. HCF of numbers = 40

45. Given LCM of two prime numbers x and y = 161

$$\therefore$$
 HCF of x and $y = 1$,

$$LCM = x \times y = 161;$$

Factorising, we get

:.

÷.

$$x \times y = 161 = 7 \times 23$$

 $x = 23; y = 7$
 $3y - x = 3 \times 7 - 23 = -2$
 $(\because x > y)$

46. \therefore Difference between divisor and remainder = 10,

Required number + 10 = LCM of 48, 60, 72, 108 and 140

$$LCM = 2^4 \times 3^3 \times 5 \times 7 = 15120$$

 \therefore Required number = LCM - 10 = 15120 - 10 = 15110

2	48,	60,	72,	108,	140
2	24,	30,	36,	54,	70
2	12,	15,	18,	27,	35
2	6,	15,	9,	27,	35
3	3,	15,	9,	27,	35
3	1,	5,	3,	9,	35
5	1,	5,	1,	3,	35
	1,	1,	1,	3,	7

47. LCM of 4, 6,10 and $15 = 2^2 \times 3^2 \times 5 = 60$

Least number of six digits = 100000

Least number of six digits divisible by above 4 numbers

$$= 100000 - 40 + 60 = 100020$$

60) 100000 (1666

$$\begin{array}{r}
 \underline{60} \\
 \underline{400} \\
 \underline{360} \\
 \underline{400} \\
 \underline{360} \\
 \underline{400} \\
 \underline{360} \\
 \underline{40} \\
 \underline{40} \\
 \underline{40} \\
 \end{array}$$

- : Number leaves a remainder 2.
- :. Required number = 100020 + 2 = 100022
- \therefore Sum of digits of number = 1 + 2 + 2 = 5
- 48. Smallest number of five digits = 10000

LCM of 12, 16, 18 and 27

$$= 2^4 \times 3^3 = 432$$

Smallest number of five digits exactly divisible by above numbers = 10000 - 64 + 432 = 10368

49. Largest number of four digits = 9999

LCM of 14, 24, 27 and 32

$$= 2^5 \times 3^3 \times 7 = 6048$$

 \therefore Largest number of four digits exactly divisible by 14, 24, 27 and 32 = 6048

50. LCM of
$$2^2 \times 3^2 \times 5 \times 11$$
, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^2 \times 5^2 \times 7^2 \times 11$
= $2^2 \times 3^2 \times 5 = 180$

Permutation and Combination

PERMUTATION: Different arrangements which can be made by taking some or all of the given things or objects at a time, is called *permutation*.

 ${}^{n}P_{r}$ denotes the number of permutations of n different things, taken r at a time. P denotes permutation.

$${}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{1 \times 2 \times 3 \times \dots \times n}{1 \times 2 \times 3 \times \dots \times (n-r)}$$

$$= \frac{1 \times 2 \times 3 \times \dots \times (n-r) (n-r+1) (n-r+2) \times \dots \times n}{1 \times 2 \times 3 \times \dots \times (n-r)}$$

$$= (n-r+1) (n-r+2) \dots \times n$$

EXAMPLE 1 Find 6P_2 .

Solution
$${}^{6}P_{2} = \frac{6!}{(6-2)!} = 6 \times 5 = 30$$

EXAMPLE 2 Find 8P_3 .

Solution
$${}^{8}P_{3} = \frac{8!}{(8-3)!} = 8 \times 7 \times 6 = 336$$

FACTORIAL NOTATION: The product of n consecutive positive integers beginning with 1 is denoted by n! or |n| and is read as factorial n.

$$:: \qquad n! = 1 \times 2 \times 3 \times \cdots \times (n-2) \times (n-1) \times n$$

For example,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$4! = 4 \times 3 \times 2 \times 1 \qquad = 24$$

If r and n are two positive integers such that r < n, then

$$\frac{n!}{r!} = \frac{n \times (n-1)(n-2)\cdots \times 2 \times 1}{r \times (r-1)(r-2)\cdots \times 2 \times 1}$$
$$= n \times (n-1) \times (n-2) \times \cdots \times (r+1)$$

EXAMPLE 3 Find 6!

Solution $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

EXAMPLE 4 Find 7!

Solution $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

COMBINATION: Different selections or groups which can be made by taking some or all of a number of given things or objects at a time, is called the *combination*.

 ${}^{n}C_{r}$ denotes the number of combinations of n different things taken r at a time.

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{1\times 2\times 3\times \cdots \times r}$$



Note: Total number of arrangements = Total number of groups or selection $\times r!$ where r is the number of objects in each group or selection.

So.

$${}^{n}P_{r} = {}^{n}C_{r} \times r!$$

For example,

$${}^{5}P_{3} = {}^{5}C_{3} \times 3!$$

ARRANGEMENTS: We have 5 different objects P. Q. R. S. and T. We have to form a group of 3 objects out of these 5 objects. We can have 10 such groups:

- (i) P, Q and R,
- (ii) P, Q and S,
- (iii) P, Q and T.

- (iv) P, R and S,
- (v) P, R and T,
- (vi) P, S and T,

- (vii) Q, R and S,
- (viii) Q, R and T,
- (ix) R, S and T,

- (x) Q, S and T.
- :. Total number of such groups = ${}^{5}C_{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \times 2!} = \frac{5 \times 4}{1 \times 2} = 10$

These three objects in each of these groups can be arranged in 6 different ways.

 \therefore Total number of arrangements = Total number of groups $\times r!$ where r is the number of objects in the group.

In the above example, total number of arrangements = $10 \times 3! = 60$ ways.

 5P_3 denotes the number of permutations or arrangements of 5 different things taken 3 at a time and

$$^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

EXAMPLE 5 You are given with four different colour pencils. You have to select two of them at a time. In how many ways, you can make such selection?

Solution Required number of selections = ${}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

EXAMPLE 6 You are supplied with six different coloured balls. You have to select three of them at a time. In how many ways, you can make such selection?

Solution Required number of selections = ${}^{6}C_{3} = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

EXAMPLE 7 In how many ways can the letters of the word 'PERFECT' be arranged? **Solution** Since the word contains 7 characters, its letters can be arranged in 7! ways. It contains two number of E. Therefore, you have to divide the above value by 2!.

 $\therefore \text{ Required number of arrangements} = \frac{7!}{2!}$

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$$

EXAMPLE 8 In how many ways can the letters of the word 'ENGINEER' be arranged? **Solution** Since the word contains 8 characters, its letters can be arranged in 8! ways.

It contains three E's and two N's. Therefore, you have to divide the above value by $(2! \times 2!)$.

$$\therefore \quad \text{Required number of arrangements} = \frac{8!}{2! \times 2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$$
$$= 2 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 2 \times 5040 = 10080$$

FUNDAMENTAL PRINCIPLES OF COUNTING

Multiplication Rule: A person wants to go from place X to place Z through place Y.

From X, the person can go to Y either by bus or by train only. From Y, the person can go to Z either by train, bus, or aeroplane. There is no direct transport facility from place X to place Z. We have to find the maximum possible number of ways by which a person can travel from X to Z.

Here the person can go from X to Y in two ways (i.e. either by bus or by train) and he can go to Z from Y in three ways (i.e. either by bus or by train, or by aeroplane). Therefore, a person can reach Z from X in $2 \times 3 = 6$ ways in all.

... Maximum number of possible ways is 6.

Addition Rule: If there are two events and each event can be completed in m and n ways (maximum possible), then the maximum number of possible ways by which both events are completed will be m + n ways.

For example, there are 20 men and 12 women in a conference. Each man and woman shakes their hands only among them. Find the maximum number of handshakes that has taken place at the conference.

Case 1: Total number of handshakes among 20 men.

$$^{20}C_2 = \frac{20!}{2!(20-2)!} = \frac{20 \times 19}{1 \times 2} = 190$$

Case 2: Total number of handshakes among 16 women

$$^{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16 \times 15}{1 \times 2} = 120$$

 \therefore Maximum number of handshakes = 190 + 120 = 310



Notes:

- 1. The number of arrangements of n different things taken all at a time = n!
- 2. The number of permutations or arrangements of *n* different things taken *r* at a time when each thing can be repeated any number of times is
 - (a) $\frac{n!}{(n-r)!}$ (when repetition is not allowed.)
 - (b) n^r (when repetition is allowed.)

EXAMPLE 9 Suppose 4 digits 2, 4, 6, and 9 are given. You have to make a four digit number. In how many ways it can be done, if

- (i) repetition of digits is allowed.
- (ii) repetition of digits is not allowed.

Solution

- (i) Required number of 4 digit numbers $n^r = 4^4 = 256$
- (ii) Required number of 4 digit numbers $=\frac{n!}{(n-r)!}=\frac{4!}{(4-4)!}=24$



- 1. If ${}^{n}C_{x} = {}^{n}C_{y}$, then either x = y or x + y = n.
- 2. The number of permutations of n things out of which p are alike and are of one type, q are alike and are of the other type and the remaining are different.

$$\frac{n!}{p!q!}$$

- 3. The number of selections of r things $(r \le n)$ out of n identical things is 1.
- 4. The total number of selections of zero or more things from n identical things = n + 1.
- 5. The total number of selections of zero or more things from n different things

$$= {}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \cdots + {}^{n}C_{n} = 2n.$$

6. The number of ways to distribute (or divide) n identical things among r persons, where any person may get any number of things

$$^{n+r-1}C_{r-1}$$

EXAMPLE 10 How many numbers of five digits can be formed with the digits 2, 4, 6, 8 and 9, provided that the repetition of digits is not allowed?

Solution Given number of digits = 5

 \therefore Required number = ${}^5P_5 = 5! = 120$



Note: If repetition of digits was allowed, then required number = 5^5 = 3125.

EXAMPLE 11 How many numbers of 4 digits can be formed with the digits 0, 1, 3, 4 and 6? **Solution** Repetition of digit is not allowed as it is not mentioned in the question.

Box 5 cannot be filled with zero as the number becomes invalid.

 \therefore Box 5 can be filled in ${}^4P_1 = 4$ ways.

Box 4 to 1 can be filled with remaining 4 numbers in 4! ways = $4 \times 3 \times 2 \times 1 = 24$ ways

 \therefore Required number of numbers = $4 \times 24 = 96$

EXAMPLE 12 How many even numbers of three digits can be formed with the digits 0, 1, 2, 3, 4 and 6?

Solution Case 1: When 0 occurs at unit place. 4 5 1

Here, Box 1 is filled with zero.

i.e. in 1 way.

Box 2 can be filled in 5P_1 ways = 5 ways

Box 3 can be filled in 4P_1 ways = 4 ways

 \therefore Required number of numbers = $1 \times 5 \times 4 = 20$

Case 2: When 0 does not occur at the unit place. 4 4 3

Here Box 1 can be filled in ${}^{3}P_{1} = 3$ ways

Box 3 can be filled in ${}^4P_1 = 4$ ways

 $(\cdot \cdot \cdot 0 \text{ cannot be filled in that box.})$

Box 2 can be filled in ${}^4P_1 = 4$ ways

(: this box can be filled with remaining numbers.)

 \therefore Required number = $4 \times 4 \times 3 = 48$

The total number of such numbers = 20 + 48 = 68

EXAMPLE 13 How many numbers greater than 2000 and less than 7,000 can be made with the digits 0, 1, 2, 4, 5, 6, 7 and 9, provided that the repetition of numbers is not allowed?

Solution Here box 1 can be filled 1 2 3 4

in ${}^4P_1 = 4$ ways [as 0, 1, 7 and 9 are to be excluded from filling in that box as per the conditions given].

Box 2 can be filled in ${}^{7}P_{1} = 7$ ways

Box 3 can be filled in ${}^6P_1 = 6$ ways

Box 4 can be filled in ${}^5P_1 = 5$ ways

 \therefore Required number of numbers = $4 \times 7 \times 6 \times 5 = 840$

EXAMPLE 14 How many words beginning with vowels can be formed with letters of the word 'EQUATOR'?

Solution There are 4 vowels in the word 'EQUATOR'

Box 1 can be filled in ${}^4P_1 = 4$ ways

1 2 3 4 5 6 7

Remaining boxes (2-7) can be filled in 6! = 720 ways.

$$\therefore$$
 Required number = $4 \times 720 = 2880$

EXAMPLE 15 In a party every person shakes hands with every other person. If there was a total of 171 handshakes in the party, find the number of persons who were present in the party.

Solution One handshake takes place between two persons. Therefore, the number of handshakes

=
$${}^{n}C_{2}$$
 where n is the number of persons

$$C_2 = 171 \text{ (given)}$$

i.e.
$$\frac{n(n-1)}{1 \times 2} = 171$$

$$n(n-1) = 171 \times 2 = 342 = 19 \times 18$$

$$\therefore$$
 $n = 19$

i.e. number of persons who was present in the party = 19.

EXAMPLE 16 How many words can be formed with the letters of the word 'ENTERPRISE'?

Solution There are 10 letters in the word 'ENTERPRISE', out of which E occurs thrice, R occurs twice and the remaining are different.

$$\therefore \text{ Required number} = \frac{10!}{3!2!} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 2 = 302400$$

EXAMPLE 17 In how many ways, 7 girls and 6 boys can be seated in a row such that they are in the alternate position?

In the above 13 positions, girls and boys can be seated in alternate positions as shown.

 \therefore 7 girls can be arranged in 7 places in $^7P_7 = 7!$ ways.

and 6 boys can be arranged in 6 places in 6P_6 = 6! ways.

:. Required number of ways =
$$7! \times 6!$$

= $5040 \times 720 = 3628800$

EXAMPLE 18 Find the number of diagonals of a polygon of 14 sides.

Solution A polygon having n sides will be having n vertices. A diagonal can be formed by joining any two vertices of the polygon.

 \therefore Number of diagonals of the polygon = ${}^{n}C_{2} - n$ where n is the number of sides of polygon.

$$\therefore$$
 Required number of diagonals = $\frac{n!}{2!(n-2)!} - n$

$$= \frac{n(n-1)}{1 \times 2} - n = \frac{n(n-1) - 2n}{2}$$

When n = 14, the required number of diagonals

$$= \frac{14(14-1)-2\times14}{2} = \frac{14(13-2)}{2} = 7\times11 = 77$$

EXAMPLE 19 Five men and four women are seated in a row, such that 4 women are always together. In how many ways they can be seated?

Solution Assume that 4 women are together.

- \therefore Total number of persons seated = 5 + 1 = 6
- \therefore Number of ways in which six persons can be seated = 6! = 720 ways.
- 4 women can be arranged among themselves in 4! ways = 24 ways.
- \therefore Total ways of arrangement = $720 \times 24 = 17280$ ways.

EXAMPLE 20

- (i) How many different arrangements can be made by using all the letters in the word 'ARITHMETIC'?
- (ii) How many of them begin with 'M' and with 'T'?
- (iii) How many words can be formed by taking three letters at a time out of the letters of the word 'ARITHMETIC'?

Solution

(i) Out of 10 letters in 'ARITHMETIC', there are two I's, two T's and other six different letters A, R, H, M, E and C.

The number of words that can be formed by taking all at a time

$$= \frac{10!}{2! \times 2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1 \times 2 \times 1 \times 2} = 907200$$

- (ii) To begin with 'M', First place can be filled in only one way.
 - $\therefore \text{ The required number of words} = \frac{1 \times 9!}{2! \times 2!} = 90720$

To being with 'T', the first place can be filled in one way and T occurs twice.

- \therefore The required number of words = $2! \times \frac{8!}{2!} = 8! = 40320$
- (iii) We have 8 different letters and out of them choose 3 at a time
 - :. Number of such words = ${}^{8}P_{3} = \frac{8!}{(8-3)!} = 8 \times 7 \times 6 = 336$

If two of the three are alike and remaining different, one pair of such letters from the available two pairs can be selected in ${}^2C_1 = 2$ ways.

The remaining one has to be selected from the remaining 7 letters in

$$^{7}C_{1} = \frac{7!}{6! \times 1!} = 7 \text{ ways}$$

 \therefore Required number of words = $2 \times 7 = 14$

But, in the three-letter group, letter can be grouped among themselves in

$$\frac{3!}{2!} = 3 \text{ ways}$$

- \therefore Total number of words = $14 \times 3 = 42$
- \therefore Total number of three letter words = 336 + 42 = 378.

EXERCISES 1. How many words can be formed out of the letters of the word 'EDUCATION' such that vowels occupy the odd positions? (a) 1440 (b) 2880 (c) 2840 (d) 2480 (e) None of these 2. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. In how many ways can he make his choice? (a) 160 (b) 180 (c) 192 (d) 200 (e) None of these 3. How many numbers of five digits can be formed with the digits 0, 1, 2, 3, 5 and 6? (b) 540 (c) 600 (d) 660 (e) None of these 4. How many odd numbers of four digits can be formed with the digits 0, 1, 2, 3, 4, 7 and 8? (a) 150 (b) 180 (c) 120 (d) 210 (e) None of these 5. How many numbers of 4 digits, divisible by 5, can be formed with the digits 0, 3, 5, 7 and 9? (a) 120 (c) 60 (d) 30 (e) None of these 6. From 8 men and 6 women, a group of seven persons is to be formed. In how many ways the group can be formed such that the group should include at least two ladies? (a) 3256 (b) 1628 (d) 4884 (e) None of these 7. How many words of 4 letters beginning with 'A' or 'E' can be formed with the letters of the word 'EQUATOR'? (a) 280 (b) 160 (c) 240 (d) 180 (e) None of these 8. In how many ways can the letters of the word 'EXCELLENT' be arranged? (a) 30024 (b) 34200 (c) 30420 (d) 30240 (e) None of these 9. How many groups can be selected for playing football out of 5 ladies and 4 gentlemen such that there should be one lady and one gentleman on each side? (a) 420 (b) 240 (e) None of these

10. In how many ways can 15 different books be divided equally among

(i) 3 sets or groups

(a) 261126

(b) 121626

(c) 122661

(d) 126126

(e) None of these

(ii) 5 sets or groups

(a) 4104100

(b) 1401400

(c) 1041040

(d) 1201200

(e) None of these

11. There are six members in a group which are to be sent abroad. The total number of members is 12. In how many ways can the selection be made such that a particular member is always

(i) included

(a) 426

(b) 462

(c) 246

(d) 264

(e) None of these

(ii) excluded

(a) 264

(b) 246

(c) 462

(d) 426

(e) None of these

12.	ring the number	oi tr	nangies formed	lby	12 points (out	oi w	ynich 4 are coi	nne	ar) in a piane:
	(a) 216	(b)	126	(c)	612	(d)	621	(e)	None of these
13.	A box contains 2 v drawn from the bo								s can 3 balls be [Bank PO]
	(a) 32	(b)	48	(c)	64	(d)	96	(e)	None of these
14.	In how many ways women?	s, ca	n a group of 5	men	and 2 women	be n	nade out of a t	otal	of 7 men and 3 [Bank PO]
	(a) 63	(b)	90	(c)	126	(d)	45	(e)	135
15.	From a group of 7 that at least 3 me								
	(a) 564	(b)	645	(c)	735	(d)	756	(e)	None of these
16.	How many three divisible by 5 and					ie di	gits 2, 3, 5, 6,	7 ar	nd 9, which are [SSC]
	(a) 5	(b)	10	(c)	15	(d)	20		
17.	How many words each letter exactly			usin	g all the letter	rs of	the word 'TR	[VA]	NDRUM' using
	(a) 10!	(b)	8!	(c)	9!	(d)	6!		
18.	In how many way	s ca	n the letters of	the	word 'LEADE	ER' b	e arranged?		[Bank PO]
	(a) 72	(b)	144	(c)	360	(d)	720	(e)	None of these
19.	In how many ways and 10 women?	s a co	ommittee, cons	isti	ng of 5 men an	d 6 v	vomen, can be	forn	ned from 8 men [Bank PO]
	(a) 266	(b)	5040	(c)	11760	(d)	86400	(e)	None of these
20.	In how many different way that the vower					rd 'A	AUCTION' be	arra	nged in such a [SBI PO]
	(a) 30	(b)	48	(c)	144	(d)	576	(e)	None of these
21.	Out of 7 consonar formed?	nts a	and 4 vowels, 1	now	many words	of 3	consonants a	nd 2	vowels can be
	(a) 210	(b)	1050	(c)	25200	(d)	21400	(e)	None of these
22.	In a group of 6 boy can they be select							nany	different ways [SBI PO]
	(a) 159	(b)	194	(c)	205	(d)	209	(e)	None of these
23.	In how many difference vowels always con		-	lett	ers of the word	d 'BA	ANKING' be a	rran	ged so that the [Bank PO]
	(a) 120	(b)	240	(c)	360	(d)	540	(e)	720
24.	In how many different that the vowels al		-		ers of the word	JU, P	JDGE' be arra	nged	l in such a way [SBI PO]
	(a) 48	(b)	120	(c)	124	(d)	160	(e)	None of these
25 .	In how many difference vowels always con			lett	ters of the wor	d 'O	PTICAL' be a	rran	ged so that the [MBA]
	(a) 120	(b)	720	(c)	4320	(d)	2160	(e)	None of these
26.	In how many way consisting of 2 me			3 m	embers can b	e sel	ected from 5	men	and 4 women
	(a) 60	(b)	72	(c)	40	(d)	32		
27 .	In how many way	s a t	eam of 11 mer	nbe	rs can be form	ed fr	om a group of	15 j	persons?
	(a) 3270	(b)	2730	(c)	1365	(d)	1635	(e)	None of these

- 28. How many 3-letter words can be formed from the letters of the word 'EXPERTISE', if repetition of letters in not allowed?
 - (a) 420
- (b) 240
- (c) 210
- (d) 120
- (e) None of these
- 29. In how many ways can 21 books on Sanskrit and 19 books on French be placed in a row on a shelf such that two books on French may not be together?
 - (a) 3990
- (b) 1540
- (c) 1995
- (d) 3672
- (e) None of these
- **30.** In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?
 - (a) 32
- (b) 48
- (c) 36
- (d) 60
- (e) 120



- 1. (b) 2. (d) **3.** (c) **4.** (a) **5.** (c) **6.** (a) 7. (c) 8. (d) 9. (b) 10. (i) (d), (ii) (b) 11. (i) (b), (ii) (c) 12. (a) 13. (c) 14. (a) 15. (d) 16. (d) 17. (c) 27. (c) **18.** (c) **19.** (c) **20.** (d) **21.** (a) **22.** (d) **23.** (e) **24.** (a) **25.** (b) **26.** (c)
- 28. (c) 29. (b) 30. (c)



Solutions with Necessary Explanation

- 1. Out of the nine letters, five are vowels and other four are consonants. Therefore, 5 vowels can be arranged in 5 odd positions in ${}^5P_5 = 5!$ ways. Similarly, 4 consonants can be arranged in 4 alternate positions (even positions) in 4! ways.
 - \therefore Total number of words = $5! \times 4! = 2880$.
- 2. There are 10 questions in total from two groups A and B each consisting of 5 questions. A candidate can attempt a maximum of 4 questions from one group and 6 questions altogether

Group A	Group		
4	2		
3	3		
2	4		

.. Number of possible combinations

$$= {}^{5}C_{4} \times {}^{5}C_{2} + {}^{5}C_{3} \times {}^{5}C_{3} + {}^{5}C_{2} \times {}^{5}C_{4}$$
$$= 5 \times 10 + 10 \times 10 + 10 \times 5$$
$$= 50 + 100 + 50 = 200$$

3. Required number of numbers = ${}^5P_1 \times {}^5P_4$

$$= 5 \times 5! = 5 \times 120 = 600$$

4. The unit place of the number can be filled in ${}^3P_1 = 3$ ways. The thousandth place of the number can be filled in ${}^5P_1 = 5$ ways. (Because 0 is excluded from selection to the thousandth position.)

The remaining two places can be filled in ${}^5P_2 = \frac{5 \times 4}{1 \times 2} = 10$ ways.

 \therefore The total number of 4 digit odd numbers $3 \times 5 \times 10 = 150$.

5. For a 4-digit number to be divisible by 5, it should end with either '5' or '0'. Therefore, the unit place of the number can be filled in ${}^{2}P_{1} = 2$ ways.

Case (a): The remaining 3 positions can be filled in 4P_3 ways = 4! ways (if 0 is filled in the units place)

 \therefore Required number of numbers = $2 \times 4! = 48$.

Case (b): The remaining 3 positions can be filled in 3P_3 ways = 3! ways (if 0 is not filled in the units place)

- \therefore Required number of numbers = $2 \times 3! = 12$.
- \therefore Total number of such numbers = 48 + 12 = 60.
- 6. A group of seven persons is to be formed from 8 men and 6 women. It is given that there should be at least two women in the group.

The possible combinations are

The number of combinations

$$= {}^{6}C_{2} \times {}^{8}C_{5} + {}^{6}C_{3} \times {}^{8}C_{4} + {}^{6}C_{4} \times {}^{8}C_{3} + {}^{6}C_{5} \times {}^{8}C_{2} + {}^{6}C_{6} \times {}^{8}C_{1}$$

$$= \frac{6 \times 5}{1 \times 2} \times \frac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} + \frac{6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4} \times \frac{8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} \times \frac{8 \times 7}{1 \times 2 \times 3} \times \frac{8 \times 7}{1 \times 2} \times \frac{8}{1 \times 2} \times \frac{8}{1 \times 2}$$

- \therefore Required number of groups = 3256
- 7. Since the 4-letter word has to begin with 'A' or 'E', the first letter can be arranged in ${}^{2}P_{1}=2$ ways.

The remaining 3 letters can be arrranged from the six letters of the word 'EQUATOR' in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways.

- \therefore Required number of words = $2 \times 120 = 240$.
- 8. The given word contains six different letters with the letter 'E' repeating thrice and 'L' repeating twice.
 - \therefore Required number of arrangements = $\frac{9!}{3! \times 2!} = 30240$.
- 9. The groups that are possible from 5 ladies and 4 gentlemen are

1 lady, 1 gentleman groups from 4 ladies and 3 gentlemen

Side A: One lady and one gentleman can be selected in ${}^5C_1 \times {}^4C_1$ ways = $5 \times 4 = 20$ ways.

Side B: For selection of one lady and one gentleman in side (B), there are only 4 ladies and 3 gentlemen.

- \therefore Required number of selection = ${}^4C_1 \times {}^3C_1 = 4 \times 3 = 12$
- \therefore Total number of ways of selection for forming the team = $20 \times 12 = 240$
- 10. (i) Required number of ways = $\frac{^{15}C_5 \times {}^{10}C_5 \times {}^{5}C_5}{3!} = 126126$
 - (ii) Required number of ways = $\frac{{}^{15}C_3 \times {}^{12}C_3 \times {}^{9}C_3 \times {}^{6}C_3 \times {}^{3}C_3}{5!} = 1401400$

11. (i) Here six members are to be selected from 12 members and a particular member is to be included.

The selection of one particular member can be done in ${}^{1}C_{1}$ or 1 way.

The selection of remaining 5 members from the 11 members can be done in $^{11}C_5$ ways.

- \therefore The required number of ways of selection = ${}^{1}C_{1} \times {}^{11}C_{5} = 462$
- (ii) When a particular member is to be excluded the required number of ways of selection = ${}^{11}C_6 = 462$.
- 12. The number of triangles which can be formed from 12 points when all of them are not collinear = ${}^{12}C_3$

 \therefore 4 points are collinear, the number of triangles that have to be formed = 4C_3 .

- :. The number of triangles formed = ${}^{12}C_3 {}^4C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} 4 = 220 4 = 216$
- 13. Required selection can be (1 black and 2 non-black) or (2 black and 1 non-black) or 3 black.
 - \therefore The required number of ways in which balls can be drawn = $({}^3C_1 \times {}^6C_2) + {}^3C_2 \times {}^6C_1 + {}^3C_3$

$$= 3 \times \frac{6 \times 5}{1 \times 2} + \frac{3 \times 2}{1 \times 2} \times 6 + 1 = 45 + 18 + 1 = 64$$

14. 5 men can be selected from 7 men in ${}^{7}C_{5}$ ways.

2 women can be selected from 3 women in 3C_2 ways.

 \therefore Total number of ways in which group can be formed = ${}^{7}C_{5} \times {}^{3}C_{2}$

$$= {}^{7}C_{2} \times {}^{3}C_{1}$$
$$= \frac{7 \times 6}{1 \times 2} \times 3 = 63 \text{ ways}$$

- 15. Required grouping can be (3 men, 2 women) or (4 men, 1 woman) or 5 men.
 - .. The number of ways in which above grouping can be made

$$= ({}^{7}C_{3} \times {}^{6}C_{2}) + ({}^{7}C_{4} \times {}^{6}C_{1}) + ({}^{7}C_{5} \times {}^{6}C_{0})$$

$$= \left(\frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{6 \times 5}{1 \times 2}\right) + \left(\frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times 6\right) + \frac{7 \times 6}{1 \times 2} \times 1$$

$$= 35 \times 15 + 35 \times 6 + 21 = 35 \times 21 + 21 = 36 \times 21 = 756$$

16. Here 6 digits are given and you have to select 3 at a time.

Since the number is divisible by 5,

Box 3 can be filled with '5' in only one way.

Box 2 can be filled in ${}^5C_1 = 5$ ways

Box 1 can be filled in ${}^4C_1 = 4$ ways

- \therefore Total number of 3 digit numbers divisible by $5 = 1 \times 5 \times 4 = 20$
- 17. 'TRIVANDRUM' consists of 10 characters in which R repeats twice.

Therefore, 9 different letters are available. By taking all characters at a time without repetition, the number of words that can be formed = 9!

- 18. 'LEADER' consists of 6 letters namely L, E, A, D and R in which E repeats twice.
 - \therefore Required number of ways = $\frac{6!}{2!} = 5! \times 3 = 360$

- 19. 5 men can be selected from 8 men in 8C_5 way. 6 women can be selected from 10 women in ${}^{10}C_6$ ways.
 - \therefore Total number of groups that can be formed = $^8C_5 \times ^{10}C_6 = ^8C_3 \times ^{10}C_4$

$$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}$$

$$= (8 \times 7) \times (10 \times 3 \times 7) = 56 \times 210 = 11760$$

20. In the word 'AUCTION', there are 7 characters in which 4 are vowels.

i.e., 4 vowels and 3 consonants.

Taking 4 vowels together, there are four characters and that can be arranged in 4! ways. Among 4 vowels, it can be arranged among them in 4! ways.

- \therefore Total number of ways = $4! \times 4! = 24 \times 24 = 576$ ways.
- 21. 3 consonants can be selected from the given 7 consonants in ${}^{7}C_{3}$ ways.

2 vowels can be selected from the given 4 vowels in 4C_2 ways.

 \therefore Total number of ways = ${}^{7}C_{3} \times {}^{7}C_{2}$

$$= \frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{4 \times 3}{1 \times 2} = (7 \times 5) \times (2 \times 3) = 210$$

22. The required selection can be (1 boy, 3 girls) or (2 boys, 2 girls) or (3 boys, 1 girl) or 4 boys.

:. Number of ways =
$$({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4 \times {}^4C_0)$$

$$= (6 \times 4) + \left(\frac{6 \times 5}{1 \times 2} \times \frac{4 \times 3}{1 \times 2}\right) + \left(\frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 4\right) + \left(\frac{6 \times 5}{1 \times 2} \times 1\right)$$

$$= 24 + (15 \times 6) + (5 \times 4 \times 4) + (3 \times 5)$$

$$= 24 + 90 + 80 + 15 = 209$$

23. In the word 'BANKING', there are 7 characters in total in which 'N' repeats twice and there are two vowels.

Letters can be grouped such that 6 characters can be arranged in 6! ways and as 'N' repeats twice

Number of grouping = $\frac{6!}{2!}$ ways

- : Two vowels, come together, it can be arranged among themselves in 2! = 2 ways.
- $\therefore \text{ Total number of ways} = \frac{6!}{2!} \times 2! = 6! \text{ ways} = 720 \text{ ways}$
- 24. Required number of arrangements of 3 consonants + (2 vowels group) = $4! \times 2 = 48$ ways
- 25. Required number of arrangements = $5! \times 3! = 120 \times 6 = 720$ ways
- **26.** 2 men can be selected from 5 men in 5C_2 ways

1 woman can be selected from 4 women in 4C_1 ways

$$\therefore \text{ Total number of ways} = {}^{5}C_{2} \times {}^{4}C_{1} = \frac{5 \times 4}{1 \times 2} \times 4 = 5 \times 2 \times 4 = 40 \text{ ways}$$

27. Required number of ways =
$${}^{15}C_{11} = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 15 \times 7 \times 13 = 1365$$

28. Here, word 'EXPERTISE' contains 9 characters in which 'E' repeats thrice.

7 6 5

 \therefore Box 1 can be filled in ${}^7C_1 = 7$ ways

Box 2 can be filled in ${}^6C_1 = 6$ ways

Box 3 can be filled in ${}^5C_1 = 5$ ways

- \therefore Required number of 3-letter words so formed = $7 \times 6 \times 5 = 210$
- 29. Here Sanskrit and French books are to be placed in alternate positions.
 - \therefore For 21 books in Sanskrit, alternately placed, there will be 22 places for French books. Taking 19 places out of 22, number of arrangements = $^{22}C_{19}$ = $^{22}C_3$

$$= \frac{22 \times 21 \times 20}{1 \times 2 \times 3} = 1540 \text{ ways}$$



- ... Vowels can be placed in positions marked 1, 3 and 5.
- \therefore No. of arrangements = ${}^{3}P_{3} = 3! = 6$
- 3 consonants can be arranged at positions marked 2, 4 and 6.
- \therefore Number of arrangements = ${}^{3}P_{3} = 3! = 6$
- \therefore Total number of ways = $6 \times 6 = 36$

Probability

Probability is the chance of happening of an event in a random experiment.

RANDOM EXPERIMENT: An experiment when conducted repeatedly under the same conditions, if it does not give the same result, it is called the *random experiment*.

Let us take an example. When a coin is thrown, it will not fall with a particular face always up. Any of the two faces marked head (H) or tail (T) come up each time.

EVENT: The outcome of a random experiment is called the *event*.

Simple or Compound Event: An event is called the *simple event*, if it corresponds to a single outcome of a random experiment.

For example, while tossing a die the chance of getting face marked 5 is a *simple event*. An event is called a *compound event* if it corresponds to more than one possible outcome. For example, in the above example, the chance of getting any face marked even is a compound event (since even numbers are more than one, i.e. 2, 4 and 6).

Exhaustive Events: All possible outcomes of an event are known as *exhaustive events*.

For example, when a coin is thrown, there are only two exhaustive events H and T (Head and Tail).

When two coins are thrown, the number of exhaustive events is 4 (2×2) , i.e. HH, HT, TH. TT.

When a die is thrown, there are six exhaustive events (i.e. the occurrence of faces marked 1, 2, 3, 4, 5 and 6).

Mutually Exclusive Events: Two or more events are said to be mutually exclusive events; if the happening of any one of them excludes the happening of all the others in a single experiment.

For example,

- (a) In throwing a coin, events H and T are mutually exclusive. Here if Head occurs, it prevents the occurrence of Tail.
- (b) When a die is thrown, the occurrence of faces marked 1, 2, 3, 4, 5 and 6 is mutually exclusive. Here the occurrence of one face prevents the occurrence of the other.

Equally Likely Events: Two or more events are said to be *equally likely events*, if the chances of their happening are equal. There is no preference to any one event to the other.

If a coin in thrown, coming up of head or tail is equally likely.

If a die is thrown, coming up of faces marked 1, 2, 3, 4, 5 and 6 are equally likely.

Independent and Dependent Events: If the happening of an event is not affected by the happening of other events, it is said to be an *independent event*.

For example, a coin is thrown twice.

The occurrence of the head on the first throw is independent of the occurrence of the head in the second throw.

If the happening of an event is affected by the happening of the other events, it is said to be *dependent event*.

For example, five red balls and four black balls are kept in a box. You have to draw 2 balls out of them successively.

Here the chance of drawing a red/black ball first is 5/9 or 4/9. If the ball is not replaced and another ball is drawn, then the chance of drawing a red or black ball is $\frac{4}{8} = \frac{1}{2}$.

SAMPLE SPACE: A set containing all possible outcomes of a random experiment is known as *sample space*.

For example, in a coin throw the number of events in the sample space is 2. i.e. $\{H, T\}$. Probability of an event A can be denoted as P(A) and can be defined as

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{Number of favourable events}}{\text{Number of events in sample space}}$$

EXAMPLE 1 Two coins are tossed. What is the probability that (i) at least one head occurs (ii) at least one tail occurs?

Solution n(S) = 4. i.e. $\{(T, T), (H, H), (H, T), (T, H)\}$

- (i) P (occurrence of at least one head) = $\frac{n(A)}{n(S)} = \frac{3}{4}$
- (ii) P (occurrence of at least one tail) = $\frac{n(A)}{n(S)} = \frac{3}{4}$

EXAMPLE 2 A positive number is selected at random and is divided by 6. What is the probability that the remainder is (i) 1; (ii) not 1?

Solution If a number is divided by 6, the remainder will be 0, 1, 2, 3, 4 or 5. n(S) = 6

(i)
$$P(A) = \frac{n(A)}{n(S)}; n(A) = 1;$$

$$\therefore P(A) = \frac{1}{6}$$

(ii)
$$P(B) = \frac{n(B)}{n(S)}; n(B) = 5;$$

$$\therefore P(\text{not } 1) = \frac{n(B)}{n(S)} = \frac{5}{6}$$

i.e.
$$P(A') = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$



Note: If an event is represented by 'A', then the event, 'not A' is represented by A' or \overline{A} or \overline{A} or \overline{A} or \overline{A} .

$$P(A) + P(A') = 1 \text{ or } P(A) = 1 - P(A')$$

 $0 \le Probability of an event \le 1$.

EXAMPLE 3 A card is drawn from a set of playing cards. What is the probability that card drawn is

- (i) a king or a queen?
- (ii) a card having the number a multiple of 2?
- (iii) a diamond card?
- (iv) a card of red suit?
- (v) a digit card of spade?
- (vi) a card having a number less than 9?
- (vii) a king of red suit?

Solution For all the above cases $n(S) = {}^{52}C_1 = 52$

(i)
$$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$$
; $P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$.

:.
$$P(\text{a king or a queen}) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

(ii) Required probability =
$$\frac{5 \times 4}{52} = \frac{5}{13}$$

(iii) Required probability =
$$\frac{13}{52} = \frac{1}{4}$$

(iv) Required probability =
$$\frac{26}{52} = \frac{1}{2}$$

(v) Required probability =
$$\frac{9}{52}$$

(vi) Required probability =
$$\frac{7 \times 4}{52} = \frac{7}{13}$$

(vii) Required probability =
$$\frac{2}{52} = \frac{1}{26}$$

EXAMPLE 4 From a pack of 52 cards, 4 cards are drawn. What is the probability that it has

- (i) all four kings?
- (ii) no ace?
- (iii) one king, one queen, one ace and one jack?
- (iv) one jack and three aces?
- (v) two digit cards and two honours card of red suit?

Solution For all the above cases,

$$n(S) = {}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4} = 13 \times 17 \times 25 \times 49 = 54145$$

(i)
$$P(\text{four kings}) = \frac{{}^{4}C_{4}}{{}^{52}C_{4}} = \frac{1}{13 \times 17 \times 25 \times 49} = \frac{1}{54145}$$

(ii)
$$P(\text{no ace}) = \frac{{}^{48}C_4}{{}^{52}C_4} = \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} \times \frac{1}{13 \times 17 \times 25 \times 49} = \frac{38912}{54145}$$

(iii) P(one king, one queen, one ace and one jack)= $\frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_4}$

$$= \frac{4 \times 4 \times 4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{256}{54145}$$

(iv)
$$P(\text{one jack and three aces}) = \frac{{}^{4}C_{1} \times {}^{4}C_{3}}{{}^{52}C_{4}} = \frac{4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{16}{54145}$$

(v) P(two digit cards and two honours cards of red suit) = $\frac{^{36}C_2 \times ^8C_2}{^{52}C_4}$

$$= \frac{\frac{36 \times 35}{1 \times 2} \times \frac{8 \times 7}{1 \times 2}}{\frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4}} = \frac{18 \times 35 \times 4 \times 7}{13 \times 17 \times 25 \times 49} = \frac{17640}{54145}$$

EXAMPLE 5 A four-digit number is formed with the given six digits 1, 2, 3, 4, 5 and 7 at random. What is the probability that the number formed is

- (i) divisible by 2?
- (ii) not divisible by 2?
- (iii) divisible by 5?

Solution A four-digit number can be formed from the given six digits in 6P_4 ways, i.e. ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ ways.

(i) For the number to be divisible by 2, either 2 or 4 should come in the unit place. Therefore, the unit place can be filled in ${}^2C_1 = 2$ ways. The remaining places can be filled in 5P_3 ways = $5 \times 4 \times 3 = 60$ ways.

$$\therefore$$
 P(number divisible by 2) = $\frac{2 \times 60}{360} = \frac{1}{3}$

- (ii) $P(\text{number not divisible by 2}) = 1 \frac{1}{3} = \frac{2}{3}$
- (iii) For the number to be divisible by 5, the unit place should be either 5 or 0. In the given set of six digits, only '5' can be used to fill in the unit place of the number to be formed. Therefore, the unit place can be filled in only one way. The remaining 3 places can be filled in 5P_3 ways.

i.e.
$${}^{5}P_{3} = 5 \times 4 \times 3 = 60$$
 ways

:.
$$P(\text{number divisible by 5}) = \frac{1 \times {}^{5}P_{3}}{{}^{6}P_{4}} = \frac{1 \times 60}{360} = \frac{1}{6}$$

EXAMPLE 6 In a box carrying one dozen of mangoes, one-third has become bad. If three mangoes are taken out from the box at random, what is the probability that at least one mango out of the three mangoes picked up is good?

Solution P(at least one good) = 1 - P(all bad)

$$P(\text{all bad}) = \frac{{}^{4}C_{3}}{{}^{12}C_{3}} = \frac{4}{\left(\frac{12 \times 11 \times 10}{1 \times 2 \times 3}\right)} = \frac{1}{55}$$

$$P(\text{at least one good}) = 1 - \frac{1}{55} = \frac{54}{55}$$

EXAMPLE 7 A bag contains 5 red and 7 green balls. Two balls are drawn from it provided that the first ball is replaced before the second ball is drawn.

What is the probability, if

- (i) both balls drawn were red;
- (ii) both balls drawn were green;
- (iii) the first ball was red and second green;
- (iv) the first ball was green and second red.

Solution

- (i) Probability that both balls drawn were red = $\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$
- (ii) Probability that both balls drawn were green = $\frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$
- (iii) Probability that the first ball drawn was red and second green $\frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$
- (iv) Probability that the first ball drawn was green and second red $\frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$

EXAMPLE 8 A box contains 5 red, 6 yellow and 7 green balls. Three balls are drawn at random. What is the probability that they are of different colour?

Solution Total number of balls = 5 + 6 + 7 = 18

3 balls can be drawn from the 18 balls in
$$^{18}C_3$$
 ways = $\frac{18 \times 17 \times 16}{1 \times 2 \times 3} = 3 \times 17 \times 16 = 816$

If all the three balls are of same colour it can be done in ${}^5C_3 + {}^6C_3 + {}^7C_3$ ways

$$= \frac{5 \times 4}{1 \times 2} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} + \frac{7 \times 6 \times 5}{1 \times 2 \times 3}$$
$$= 10 + 20 + 35 = 65 \text{ ways}$$

∴ P(all balls are of different colour)

= 1 - P(all balls are of same colour)

$$=1-\frac{65}{816}=\frac{751}{816}$$

EXAMPLE 9 A box contains 9 green and 7 yellow balls. Two balls are drawn in succession at random. What is the probability that one of them is green and the other yellow?

Solution Two balls can be drawn from 9 + 7 = 16 balls in ${}^{16}C_2$ ways.

i.e.
$$\frac{16 \times 15}{1 \times 2} = 120 \text{ ways}$$

One green ball can be drawn from the given 9 green balls in ${}^{9}C_{1}$ = 9 ways. One yellow ball can be drawn from the given 7 yellow balls in ${}^{7}C_{1}$ = 7 ways.

$$\therefore \text{ Required probability} = \frac{{}^9C_1 \times {}^7C_1}{{}^{16}C_2} = \frac{9 \times 7}{120} = \frac{21}{40}$$

EXAMPLE 10 The letter of the word 'EQUATION' is arranged in different ways randomly. What is the probability that the vowels are (i) together, (ii) not together.

Solution The eight letters of the word 'EQUATION' can be arranged in 8! ways. There are 5 vowels in the word 'EQUATION'. If they are together the word 'EQUATION' can be arranged in ${}^4P_4 \times {}^5P_5 = 4! \times 5! = 24 \times 120 = 2880$ ways

(i)
$$\therefore$$
 P(vowels occur together) = $\frac{4! \times 5!}{8!} = \frac{2880}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}$
$$= \frac{4!}{6 \times 7 \times 8} = \frac{1}{14}$$

(ii) :
$$P(\text{vowels not together}) = 1 - \frac{1}{14} = \frac{13}{14}$$

EXAMPLE 11 What is the probability that a card drawn from a pack of 52 cards will be a diamond or a queen?

Solution Out of 52 cards, 13 cards are of diamond series and 3 cards are queen

$$n(E) = 13 + 3 = 16; n(S) = 52$$

$$P(E) = \frac{16}{52} = \frac{4}{13}$$

EXAMPLE 12 A box contains 10 red and 10 green balls. What is the probability that two balls drawn from the box is of the same colour?

Solution Number of possible ways in which two red balls are drawn from given 10 balls

$$= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$$

Similarly number of possible ways in which two green balls are drawn from given 10 balls

$$= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$$

 \therefore Total possible ways = 45 + 45 = 90

Total number of possible ways in which two balls can be drawn from given 20 balls

$$^{20}C_2 = \frac{20 \times 19}{1 \times 2} = 190$$

E = event of drawing 2 balls of same colour

$$P(E) = \frac{{}^{10}C_2 + {}^{10}C_2}{{}^{20}C_2} = \frac{90}{190} = \frac{9}{19}$$

EXAMPLE 13 Two dice are thrown at a time. What is the probability that sum of results obtained is 8 or 9?

Solution When two dice are thrown, total number of results, n(S) $6 \times 6 = 36$ Event of getting total of 8 or 9

$$E = \{(2, 6), (3, 5), (4, 4), (3, 6), (4, 5)\}$$

$$E = \frac{n(E)}{n(S)} = \frac{5}{26}$$

 $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$

EXAMPLE 14 If two coins are thrown simultaneously, find the probability of getting at least one head.

Solution Total number of possible events $n(S) = \{(H, H), (H, T), (T, H), (T, T)\} = 4$ Event of getting at least one head $E = \{(H, H), (H, T), (T, H)\} = 3$

$$P(E) = \frac{3}{4}$$

EXAMPLE 15 Two cards are drawn from a pack of 52 cards. What is the probability that either both are black or both are queen?

Solution Out of 52 cards, 26 cards are of black suit. There are 4 queens in all. Let E be the event of getting both black cards or both queens.

 E_1 = Event of getting both black suits E_2 = Event of getting both queens

$$n(E_1) = {}^{26}C_2 = \frac{26 \times 25}{1 \times 2} = 325;$$

$$n(E_2) = {}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6;$$

$$n(E_1 \cap E_2) = {}^2C_2 = 1$$

$$n(E_1) = {}^{n(E_1)} = 325.$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326};$$

:.

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326};$$

$$P(E_1 \cap E_2) = \frac{1}{1326};$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} = \frac{330}{1326} = \frac{55}{221}$$

1. A bag contains 7 black and 5 white balls. A ball is drawn out and replaced in the bag. Then

a ball is drawn again. What is the probability that

? EXERCISES

	(i)	both the balls d	lrawn v	were white?						
		(a) 49/144	(b) 2	25/144	(c)	36/144	(d)	64/144	(e)	None of these
	(ii)	both balls were	black	?						
		(a) 25/144	(b) 3	86/144	(c)	49/144	(d)	64/144	(e)	None of these
	(iii)	the first ball wa	as blac	k and the se	con	d white?				
		(a) 35/144	(b) 4	15/144	(c)	65/144	(d)	55/144	(e)	None of these
	(iv)	the first ball wa	as whit	te and the se	ecor	nd black?				
		(a) 65/144	(b) 5	55/144	(c)	45/144	(d)	35/144	(e)	None of these
2.	A bo	x contains 5 red	, 4 gre	en and 3 bla	ck	balls. 3 balls	were	drawn at ra	ndo	m. What is the
	•	ability that they					(1) =	0.00		N. C.1
_	` '	,) 41/4	` '						None of these
3.		sitive integer is a ainder is	selecte	ed at random	an	id is divided b	y 9.	What is the p	rob	ability that the
		equal to 1?								
	(1)	-	(L) 2	./0	(-)	9/0	(4)	1/0	(-)	N
	(::)	(a) 4/9 not 1?	(b) 3	0/9	(c)	2/9	(u)	1/9	(e)	None of these
	(11)	(a) 4/9	(b) 2	2/0	(a)	8/9	(4)	1/0	(0)	None of these
4	Т	` '	` ′		` ′			1/9	(e)	None of these
4.		coins are tossed		t is the prob	abı.	nty of appear	ing (01		
	(1)	atmost one head		14	(-)	9/4	(4)	1	(-)	Na 64h
	(::)	(a) 1/4	(b) 2	3/4	(c)	3/4	(d)	1	(e)	None of these
	(11)	atmost two hea		./.4	<i>(</i>)	0/4	(1)	-	<i>(</i>)	NI C./1
_	<i>(</i> :)	(a) 1/4	(b) 2		` '	3/4	(d)		` ′	None of these
5.	(1)	What is the cha		= -			-			-
		(a) 1/7	(b) 2		` ′	3/7		4/7	(e)	None of these
	(11)	What is the cha		•					, ,	27 0.7
_		(a) 4/7	(b) 3			1/7	` '	2/7	(e)	None of these
б.		n two dice are tl				•	prob	ability that		
	(i)	the sum of num								
		(a) 1/4	(b) 1	./2	(c)	3/4	(d)	1	(e)	None of these

(e) None of these

	is the probability that one of them is white and the other red?						
	(a) 72/91 (b) 24/91 (c) 48/91 (d) 36/91 (e) None of these						
8.	A group of 4 students is to be formed from among 4 girls and 6 boys. What is the probability						
	that the group has less number of boys than the number of girls.						
	(a) 9/42 (b) 11/42 (c) 5/42 (d) 13/42 (e) None of these						
9.	You are given digits 2, 3, 4, 6 and 7. In how many ways you can form a 3-digit number						
	without repetition of digits which is odd and greater than 300? (a) 27 (b) 18 (c) 12 (d) 15 (e) None of these						
10	(a) 27 (b) 18 (c) 12 (d) 15 (e) None of these From a pack of 52 cards, 4 cards are drawn. What is the probability that it has						
10.	(i) no queen?						
	(a) $\frac{53}{54,145}$ (b) $\frac{52}{54,145}$ (c) $\frac{104}{54,145}$ (d) $\frac{208}{54,145}$ (e) None of these						
	(ii) one ace, one king, one queen and one jack?						
	(a) $\frac{64}{54,145}$ (b) $\frac{128}{54,145}$ (c) $\frac{256}{54,145}$ (d) $\frac{512}{54,145}$ (e) None of these						
	(iii) two digit-cards and one honours card of black and red suit?						
	(a) $\frac{1125}{7735}$ (b) $\frac{1512}{7735}$ (c) $\frac{1152}{7735}$ (d) $\frac{2511}{7735}$ (e) None of these						
11.	In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up at random. What is						
	the probability that it is neither red nor green? [Bank PO]						
	(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{7}{19}$ (d) $\frac{8}{21}$ (e) $\frac{9}{21}$						
12.	A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the						
	probability that none of the balls drawn is blue? [Bank PO]						
	(a) $\frac{10}{21}$ (b) $\frac{11}{21}$ (c) $\frac{2}{7}$						
	21 , , ,						
13.	A box contains 5 green, 4 yellow and 3 white marbles. Three marbles are drawn at random.						
	What is the probability that they are not of the same colour? [Bank PO]						
	(a) $\frac{3}{44}$ (b) $\frac{3}{55}$ (c) $\frac{52}{55}$ (d) $\frac{41}{44}$						
14.	What is the probability of getting a sum 9 from two throws of a dice? [MBA]						
_ ·							
	(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$						

(ii) the sum of numbers appeared is an even number?

(c) 1/2

(c) 6/12

(c) 3/6

7. A box contains 6 white and 8 red balls. Two balls are drawn at random in succession. What

(d) 1

(d) 1/3

(d) 7/12

(d) 1/6

(d) 4/6

(b) 3/4

(iv) the sum of numbers appeared ≤ 10 ?

(vi) numbers appeared are equal?

(iii) the sum of numbers appeared is a multiple of 3?

(a) 2/3 (b) 3/4 (c) 1

(b) 11/12

(v) the sum of numbers appeared is 6 and 7?

(b) 3/6

(b) 1/6

(a) 1/4

(a) 5/12

(a) 5/6

(a) 2/6

15 .	From a pack of 52 cards, two cards are drawn together at random.	What is the probability of
	both cards being kings?	[MBA]

(a) $\frac{1}{15}$

(c) $\frac{35}{256}$ (d) $\frac{1}{221}$

16. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is

17. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random. What is the probability that all of them are red?

(c) $\frac{2}{91}$ (d) $\frac{2}{77}$

18. In a class, 30% of students passed English, 20% passed French and 10% passed both. If a student is selected at random, what is the probability that he has passed English or French?

(d) $\frac{3}{10}$

19. Four persons are selected at random from a group of 3 men, 2 women and 4 children. The chance that exactly two of them are children is

(d) $\frac{10}{21}$

20. A box contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?

(b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{7}{15}$

21. A box contains 6 black and 4 red balls. Three balls are drawn at random. What is the probability that one ball is red and the other two are black?

(c) $\frac{3}{10}$

22. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even? [Asst. PF Commissioner's Exam]

(b) $\frac{3}{4}$ (c) $\frac{3}{8}$

(d) $\frac{5}{16}$



1. (i) (b), (ii) (c), (iii) (a), (iv) (d)

15. (d)

2. (b)

3. (i) (d), (ii) (c)

4. (i) (c), (ii) (d)

5. (i) (b), (ii) (c)

6. (i) (b), (ii) (c), (iii) (d), (iv) (b), (v) (d), (vi) (b)

7. (c) 8. (c)

10. (i) (e), (ii) (e), (iii) (c)

12. (a) 11. (a) **13.** (d)

14. (c)

9. (b) **16.** (d)

17. (c)

18. (a) 19. (d)

20. (b)

21. (a) **22**. (b)



Solutions with Necessary Explanation

(i) Both balls were white $=\frac{5}{12}\times\frac{5}{12}=\frac{25}{144}$

(ii) Both balls were black = $\frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$

- (iii) The first ball was black and the second white $=\frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$
- (iv) The first ball was white and the second black $=\frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$
- **2.** Total number of balls = 5 + 4 + 3 = 12

P(balls are not of the same colour) = ?

$$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$$

i.e. 3 balls can be drawn from the given 12 balls in 220 ways.

If all are of same colour, it can be done in ${}^5C_3 + {}^4C_3 + {}^3C_3 = 10 + 4 + 1 = 15$ ways.

P(all balls are of same colour) + P(all balls are not of same colour) = 1.

:. P(all balls are not of same colour)

= 1 - P(all balls are not of same colour)

$$=1-\frac{15}{220}=\frac{205}{220}=\frac{41}{44}.$$

3. When a positive integer is divided by 9, the remainder may be any integer from 0 to 8.

$$\therefore n(S) = 9$$

(i) $E(1) = \{1\}; n(E) = 1;$

$$\therefore P(E) = \frac{1}{9}$$

(ii) $E(\text{not } 1) = \{0, 2, 3, 4, 5, 6, 7, 8\};$

i.e.
$$n(E) = 8$$
;

$$\therefore P(E) = \frac{8}{9}$$

- **4.** $n(S) = 4 = \{(H, H), (H, T), (T, H), (T, T)\}$
 - (i) $E(appearing at most one head) = \{HT, TH, TT\}$

$$\therefore n(E) = 3$$
:

$$\therefore P(E) = \frac{3}{4}$$

(ii) E (appearing atmost two heads) = {HH, HT, TH, TT}

$$\therefore n(E) = 4;$$

$$\therefore P(E) = \frac{4}{4} = 1$$

5. (i) A leap year has 366 days and, therefore, it has 52 weeks and 2 more days. Two days can be {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}.

$$\therefore n(S) = 7.$$

Out of the above 7 cases, cases favourable for more Wednesdays are {(Tuesday, Wednesday), (Wednesday, Thursday)}

i.e.
$$n(E) = 2$$

$$\therefore P(E) = \frac{2}{7}$$

(ii) When the year is not a leap year, it has 52 complete weeks and 1 more day.

It can be {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

$$\therefore n(S) = 7$$

.. From the above 7 cases, cases favourable for one more Wednesday is {Wednesday}.

i.e.
$$n(E) = 1$$
.

$$\therefore P(E) = \frac{1}{7}$$

6. (i) The sum of numbers appeared = odd numbers from 3 to 11 (both inclusive)

So the number of events n(E) = 2 + 4 + 6 + 4 + 2 = 18

$$\therefore \text{ Sum} = 3 \to (1, 2) \text{ or } (2, 1)$$

$$= 5 \to (1, 4) \text{ or } (4, 1), (2, 3) \text{ or } (3, 2)$$

$$= 7 \to (1, 6) \text{ or } (6, 1), (2, 5) \text{ or } (5, 2), (3, 4) \text{ or } (4, 3)$$

$$= 9 \to (3, 6) \text{ or } (6, 3), (4, 5) \text{ or } (5, 4)$$

$$= 11 \to (5, 6) \text{ or } (6, 5)$$

$$n(s) = 6 \times 6 = 36$$

$$\therefore \text{ Required probability} = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

(ii) The sum of numbers appeared is even.

The required sums are 2, 4, 6, 8, 10, 12.

So
$$n(E) = 18$$

 $n(S) = 36$.

$$\therefore$$
 Required probability = $\frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

(iii) The sum of numbers is a multiple of 3.

... The required sums are 3, 6, 9 and 12.

$$\therefore$$
 Events are $(1, 2), (2, 1), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)$

$$n(E) = 12; n(S) = 36$$

So, the required probability =
$$\frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

(iv) The sum of numbers appeared ≤ 10 .

... The required sums are 2, 3, 4, 5, 6, 7, 8, 9 and 10

$$n(E) = 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3$$
$$= 33$$

$$n(S) = 36$$

∴ The required probability =
$$\frac{33}{36} = \frac{11}{12}$$

(v) The sum of numbers appeared is 6 and 7.

.. The required sums are 6 and 7.

55

$$(1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (1, 6),$$

$$(6, 1), (2, 5), (5, 2), (3, 4), (4, 3)$$

i.e. for 6,
$$n(E) = 5$$

for 7, $n(E) = 6$

$$\therefore \text{ for 6, the required probability} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

for 7, the required probability =
$$\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

- (vi) The numbers appeared are equal.
 - \therefore The required events are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

$$\therefore n(E) = 6$$

$$\therefore$$
 The required probability = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

7. The number of ways that two balls can be drawn from (6+8) or 14 balls is ${}^{14}C_2 = \frac{14 \times 13}{1 \times 2} = 91$

The number of ways in which one white ball can be drawn from the six white balls is ${}^6C_1 = 6$ The number of ways in which one red ball can be drawn from the eight red balls is ${}^8C_1 = 8$

... The number of ways in which one white and one red ball are drawn is
$$\frac{{}^{6}C_{1} \times {}^{8}C_{1}}{{}^{14}C_{2}} = \frac{6 \times 8}{91} = \frac{48}{91}$$

8. The selection of 1 boy and 3 girls can be done in ${}^6C_1 \times {}^4C_3 = 6 \times 4 = 24$ ways Selection of 4 girls and no boy can be done in ${}^6C_0 \times {}^4C_4 = 1 \times 1 = 1$ way.

$$n(E) = 24 + 1 = 25$$
 ways.

Without any restriction the group can be formed in ${}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210$ ways

$$P(E) = \frac{n(E)}{n(S)} = \frac{25}{210} = \frac{5}{42}$$

9. The given digits are 2, 3, 4, 6 and 7.

Box 1 can be filled in 4 ways, i.e. 3, 4, 6 and 7.

When box 1 is having 3,

box '3' can be filled in using 7 only and box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 1 = 3$

When box 1 is having 4,

box '3' can be filled in 2 ways (i.e. 3 or 7)

box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 2 = 6$

When box 1 is having 6,

box '3' can be filled in 2 ways (i.e. 3 or 7)

box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 2 = 6$

When box 1 is having 7,

box '3' can be filled in 1 way (i.e. 3 only)

box '2' can be filled in 3 ways.

- \therefore The number of such numbers formed = $1 \times 3 \times 1 = 3$
- \therefore The total number of such numbers = 3 + 6 + 6 + 3 = 18

10. For all cases,
$$n(S) = {}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4} = 13 \times 17 \times 25 \times 49$$

(i)
$$n(E) = {}^{48}C_4 = \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} = 2 \times 47 \times 46 \times 45$$

$$P(E) = \frac{2 \times 47 \times 46 \times 45}{13 \times 17 \times 25 \times 49} = \frac{39916}{54145}$$

(ii)
$$n(E) = {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1}$$

= $4 \times 4 \times 4 \times 4 = 256$

$$P(E) = \frac{4 \times 4 \times 4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{256}{270725}$$

(iii)
$$n(E) = {}^{36}C_2 \times {}^8C_1 \times {}^8C_1$$

= $\frac{36 \times 35}{1 \times 2} \times 8 \times 8 = 18 \times 35 \times 8 \times 8$

$$\therefore P(E) = \frac{18 \times 35 \times 8 \times 8}{13 \times 17 \times 25 \times 49} = \frac{18 \times 8 \times 8}{13 \times 17 \times 35} = \frac{1152}{7735}$$

11. Total number of favourable events n(S) = 8 + 7 + 6 = 21

Total number of possible events = 7

E = event of getting neither red nor green - event of getting blue.

$$P(E) = \frac{7}{21} = \frac{1}{3}$$

12. Total number of possible events = 2 + 3 + 2 = 7

The number of ways in which 2 balls can be drawn out of given $7 = {}^{7}C_{2} = \frac{7 \times 6}{1 \times 2} = 21$

i.e.
$$n(S) = 21$$

٠:.

E =event of getting balls not to be blue

n(E) = ways of drawing 2 balls from given 5

$$= {}^{5}C_{2} = \frac{5 \times 4}{1 \times 2} = 10$$

$$P(E) = \frac{n(E)}{n(s)} = \frac{10}{21}$$

13. The number of ways in which 3 green marble can be drawn from the given $5 = {}^5C_3$

$$= {}^{5}C_{2} = \frac{5 \times 4}{1 \times 2} = 10$$

Similarly, from 4 yellow marbles, marble can be drawn in ${}^4C_3 = {}^4C_1 = 4$ ways Similarly, from 3 white marbles, 3 white marble can be drawn in ${}^3C_3 = 1$ way

The number of ways in which 3 marbles can be drawn from
$$5 + 4 + 3 = 12$$
 marbles given $= {}^{12}C_3$

$$=\frac{12\times11\times10}{1\times2\times3}=220$$

.. Required probability = 1 - Probability (of getting all the three with same colour)

$$=1-\frac{{}^{5}C_{3}+{}^{4}C_{3}+{}^{3}C_{3}}{220}$$

$$=1-\frac{10+4+1}{220}=1-\frac{3}{44}=\frac{41}{44}$$

14. The number of possible events when a dice is thrown twice = $6 \times 6 = 36$;

Number of possible events to get a sum 9 when a dice is thrown twice

$$= \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

E =Event of getting sum 9

$$P(E) = \frac{4}{36} = \frac{1}{9}$$

15. The number of possible events when two cards are drawn together from a pack of 52 cards

$$={}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 26 \times 51 = 1326$$

The number of possible events when two cards are drawn together to get both cards of king suit

$$={}^{4}C_{2}=\frac{4\times3}{1\times2}=6$$

E =Event of getting both cards of king suit.

$$P(E) = \frac{6}{1326} = \frac{1}{221}$$

16. The number of possible events when two cards are drawn together from a pack of 52 cards

$$={}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 26 \times 51 = 1326$$

Number of possible events to draw a spade = ${}^{13}C_1$ = 13 ways.

Number of possible events to draw a heart = ${}^{13}C_1$ = 13 ways.

E =Event of getting 1 spade and 1 heart

$$n(E) = 13 \times 13 = 169$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}$$

17. Number of possible events to draw three balls from 4 white, 5 red and 6 blue balls

$$= {}^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 5 \times 7 \times 13 = 455$$

E =Event of getting 3 balls red

The number of favourable event to get 3 red balls = ${}^5C_3 = \frac{5 \times 4}{1 \times 2} = 10$

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{455} = \frac{2}{91}$$

18. Probability that student passed in English $P(E) = \frac{30}{100} = \frac{3}{10}$;

Probability that student passed in French $P(F) = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$;

Probability that student passed both English and French $P(E \cap F) = \frac{10}{100} = \frac{1}{10}$

$$P(E \text{ or } F) = P(E) + P(F) - P(E \cap F) = \frac{3}{10} + \frac{1}{5} - \frac{1}{10} = \frac{3 + 2 - 1}{10} = \frac{4}{10} = \frac{2}{5}$$

19. The number of possible ways to select 4 persons from 3 men, 2 women and 4 children

$$= {}^{9}C_{4} = \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = 9 \times 7 \times 2 = 126$$

The number of ways in which 2 children can be drawn from given 4 children = ${}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

Number of ways in which 2 person can be drawn from 3 men and 2 women = ${}^{5}C_{2} = \frac{5 \times 4}{1 \times 2} = 10$

E =Event of selecting 4 persons with exactly two children

$$P(E) = \frac{{}^{4}C_{2} \times {}^{5}C_{2}}{{}^{9}C_{4}} = \frac{6 \times 10}{126} = \frac{60}{126} = \frac{10}{21}$$

- **20.** The number of possible events to draw a ball from 4 red, 5 green and 6 white balls = ${}^{15}C_1 = 15$ The number of ways in which a red ball is drawn from given 4 red balls = ${}^{4}C_1 = 4$ The number of ways in which a green ball is drawn from 5 green balls = ${}^{5}C_1 = 5$
 - \therefore Total number of favourable events = 4 + 5 = 9

E =Event of getting either a red or a green ball

$$P(E) = \frac{n(E)}{n(S)} = \frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$$

21. Number of possible ways to draw three balls from 6 black and 4 red balls

$$= {}^{10}C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = 10 \times 3 \times 4 = 120$$

Number of favourable ways to get one red ball from given 4 red balls = 4C_1 = 4

Number of favourable ways to get two black balls from given 6 black balls = ${}^6C_2 = \frac{6 \times 5}{1 \times 2} = 15$

E = event of getting one red and two black balls

P(E) =
$$\frac{n(E)}{n(S)}$$
 = $\frac{{}^{4}C_{1} \times {}^{6}C_{2}}{{}^{10}C_{3}}$ = $\frac{4 \times 15}{120}$ = $\frac{1}{2}$

22. Number of possible events = 36

E = Event of getting product of 2 results even

Number of favourable events = $\{(1, 6), (1, 4), (1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} = 27$

$$P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$$

Ratio and Proportion

RATIO: Ratio of two quantities represents how many times a quantity contains another quantity of the same kind.

In a ratio p/q, p and q are called terms of the ratio. The number in the first term p or numerator is called the *antecedent* and the second term q or denominator is called the *consequent*.

The ratio 7:11 represents $\frac{7}{11}$ with antecedent = 7, consequent = 11.

RULE —

The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Compound Ratio: Ratios are compounded by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent.

The compounded ratio of the ratios (a:b), (c:d), (e:f) is (ace:bdf).

For example, compounded ratio of (2:3), (4:5) and (6:7) is $2 \times 4 \times 6:3 \times 5 \times 7$ = **48:105**.

EXAMPLE 1 Find the compounded ratio of 3: 4, 13: 9, 5: 26 and 15: 2

Solution The required ratio =
$$\frac{3}{4} \times \frac{13}{9} \times \frac{5}{26} \times \frac{15}{2} = \frac{25}{16}$$

EXAMPLE 2 Find the compounded ratio of 4:5,8:9 and 12:13

Solution The required ratio =
$$\frac{4}{5} \times \frac{8}{9} \times \frac{12}{13} = \frac{128}{195}$$

Important Formulae

Duplicate ratio of a:b is $a^2:b^2$

Sub-duplicate ratio of a:b is $\sqrt{a}:\sqrt{b}$

Triplicate ratio of a:b is $a^3:b^3$ Sub-triplicate ratio of a:b is $a^{1/3}:b^{1/3}$

Inverse Ratio: If a:b is the given ratio, then $\frac{1}{a}:\frac{1}{b}$ or b:a is called its *inverse* or reciprocal ratio.

EXAMPLE 3 Find three numbers in the ratio 1: 2:3, so that the sum of their squares is equal to 224.

Solution Let the numbers be x, 2x and 3x.

$$\therefore (x)^2 + (2x)^2 + (3x)^2 = 224$$

$$\therefore \qquad x^2 (1 + 4 + 9) = 224$$

$$x^2 = \frac{224}{14} = 16$$

.. The required numbers are 4, 8 and 12.

RULE -

x = 4

If the ratio between the first and the second quantities is a:b and the ratio between second and third quantities is c:d, then the ratio among the first, the second and the third quantities is given by ac:bc:bd.

Proof: First : Second = a : bSecond : Third = c : d.

To combine the two ratios, consequent (b) of the first ratio should be made equal to the antecedent (c) of the second ratio. Therefore, multiply first ratio by c and second ratio by b.

$$\therefore$$
 First : Second = $ac : bc$

Second: Third = bc : bdFirst: Second: Third = ac : bc : bd.

EXAMPLE 4 The sum of three numbers is 138. If the ratio between the first and the second is 2:3 and that between the second and the third be 5:7, find the second number.

Solution Let the numbers be a, b and c.

$$a:b=2:3;b:c=5:7$$

Combining these ratios, we get

$$a:b:c=10:15:21$$

The second number
$$b = \frac{15}{10 + 15 + 21} \times 138 = \frac{15}{46} \times 138 = 45$$

EXAMPLE 5 Ratio of money with Arun and Sunil is 3 : 4 and that with Sunil and Suresh is 3 : 7. If money with Sunil is ₹ 120, how much money does Suresh have?

 Solution
 Arun
 Sunil
 Suresh

 3
 4
 3
 7

 9
 12
 28

The ratio of money with Arun, Sunil and Suresh is 9:12:28.

Money with Sunil (12 parts) = ₹ 120

∴ Money with Suresh (28 parts) = ₹ 280

Theorem: If the first and second quantities are in the ratio a:b; the second and third are in the ratio c:d and the third and fourth quantities are in the ratio e:f, then the ratio among the first, second, third and fourth quantities is given by ace:bce:bde:bdf.

EXAMPLE 6 If A:B=3:4; B:C=8:10 and C:D=15:17, then find A:B:C:D.

Solution A:B=3:4

B:C=8:10

C: D = 15:17

 $A:B:C:D = 3 \times 8 \times 15:4 \times 8 \times 15:4 \times 10 \times 15:4 \times 10 \times 17$

= 9:12:15:17

EXAMPLE 7 ₹ 960 is divided among 4 men, 5 women and 8 boys such that the share of a man, a woman and a boy may be in the ratio of 5 : 4 : 3. What is the share of a woman? **Solution** The ratio of shares among 4 men, 5 women and 8 boys is

 $4 \times 5 : 5 \times 4 : 8 \times 3 = 20 : 20 : 24$

:. Shares of 5 women =
$$\frac{20}{20 + 20 + 24} \times 960 = 300$$

$$\therefore \qquad \text{Share of a woman} = \frac{300}{5} = 60$$

PROPORTION: Equality of ratios is called the *proportion*. The numbers a, b, c and d are said to be in proportion if a: b = c: d. Here, d is called the *fourth proportional*.

$$\therefore$$
 $ad = bc$



Notes:

- 1. If any four quantities are in proportion, the product of the extremes is equal to the product of the mean, i.e. if a: b = c: d then ad = bc.
- 2. Three quantities are said to be in *continued proportion* when the ratio of the first to the second is equal to the ratio of the second to the third.

The second quantity is called the *mean proportional* between the first and the third; and the third quantity is called the *third proportional* to the first and the second.

For example: 16, 12 and 9 are in continued proportion for 16 : 12 : : 12 : 9.

Hence, 12 is the mean proportional between 16 and 9, and 9 is third proportional to 16 and 12.

EXAMPLE 8 Find the mean proportional between 6 and 96.

Solution Let x be the required mean proportional.

Then 6:x::x:96

or $\frac{6}{x} = \frac{x}{96}$

i.e. $x^2 = 6 \times 96$

 $\therefore \qquad x = \sqrt{6 \times 96} = 24$

i.e. the mean proportional between two numbers is equal to the square root of their product.

EXAMPLE 9 Find the third proportional to 18 and 24.

Solution Let the third proportional be x.

Then 18, 24 and x are in proportion

i.e. 18:24::24:*x*

 $\therefore \qquad \qquad x = \frac{24 \times 24}{18} = 32$

EXAMPLE 10 Find the fourth proportional to 16, 28 and 36.

Solution Let *x* be the fourth proportional

 \therefore 16: 28 = 36: *x*

$$\therefore \qquad x = \frac{28 \times 36}{16} = 63$$

Direct proportion: The proportion by which the value of one variable varies directly as the value of another variable.

EXAMPLE 11 If 3 books cost ₹ 18, then what will be the cost of 21 books?

Solution If the number of books is increased n times, price of books will also increase n times where $n = 1, 2, 3, 4 \dots$

3 books : 21 books : :₹ 18 : ₹ x

where x is the required cost.

Above example illustrates the *direct proportion*.

Inverse proportion: The proportion by which the value of one variable varies inversely as another variable.

EXAMPLE 12 If 12 men can do a certain work in 25 days, in how many days, the same work will be completed by 10 men?

Solution Let the number of days required be x, so that work will be over by 10 men.

Here the inverse ratio of number of men is equal to the ratio of corresponding number of days

 $\therefore \frac{1}{12} : \frac{1}{10} :: 25 : x$

10:12::25:x

: the required number of days

or

$$x = \frac{12 \times 25}{10} = 30$$

The above example illustrates the *inverse proportion*.

To find the 4th proportion when the other three are given is called the *simple proportion*. If a, b and c are the given quantities and you have to find the fourth quantity x,

$$x = \frac{\text{Multiplication of means}}{1\text{st term}}$$

If a and b are of same kind and a > b, then $\frac{a}{b} = \frac{c}{x}$ from which x can be calculated.

EXAMPLE 13 If 96 mangoes can be bought for $\stackrel{?}{\stackrel{?}{\stackrel{?}{?}}}$ 288, how many can be bought for $\stackrel{?}{\stackrel{?}{\stackrel{?}{?}}}$ 48? **Solution** Let the required number of mangoes be x. As the required quantity is less,

$$288:48 = 96:x$$

$$\therefore \qquad \qquad x = \frac{48 \times 96}{288} = \mathbf{16}$$

EXAMPLE 14 If 8 men can do a piece of work in 25 days working 8 hours per day, how many men will do the same work for 10 days working for 5 hours?

Solution Let the required number of men be x.

As the number of days is less, the number of men will be more. Less working hours, more men.

Days:
$$10:25$$

Hours: $5:8$
Work: $1:1$

$$\therefore x = \frac{\text{Third term} \times \text{Multiplication of means}}{\text{Multiplication of first terms}}$$

or
$$x = \frac{8 \times 25 \times 8 \times 1}{10 \times 5 \times 1} = 16 \text{ men}$$

EXAMPLE 15 If 12 men can reap 80 hectares in 16 days, how many hectares 20 men can reap in 27 days?

Solution Let the required area be *x* hectares. As the number of men and the number of days increase, they will reap more hectares.

$$\therefore \text{ Area required } x = \frac{80 \times 20 \times 27}{12 \times 16} = 225 \text{ hectares}$$

Proportional division: Proportion may be applied to divide a given quantity into parts which are proportional to the given numbers. This is termed the proportional division.

EXAMPLE 16 Divide ₹ 1848 into three shares proportional to 3, 4 and 5.

Solution Ist share =
$$\frac{3}{3+4+5} \times 1848$$

= $\frac{3}{12} \times 1848 = \frac{1848}{4} = ₹ 462$
2nd share = $\frac{4}{3+4+5} \times 1848$
= $\frac{4}{12} \times 1848 = \frac{1848}{3} = ₹ 616$
3rd share = $\frac{5}{3+4+5} \times 1848$
= $\frac{5}{12} \times 1848 = 5 \times 154 = ₹ 770$

EXAMPLE 17 A certain sum of money is divided among P, Q and S such that for each rupee P has, Q has 35 paise and S has 60 paise. If Q's share is \mathbb{Z} 7, find the sum of money.

Solution
$$P: Q: S = 100: 35: 60 = 20: 7: 12$$

$$20 + 7 + 12 = 39$$

Given $\frac{7}{39}$ of sum of money = ₹ 7

∴ The sum of money = $\frac{7 \times 39}{7}$ = ₹ 39

EXAMPLE 18 Divide ₹ 1391 into three parts such that 5 times the first may be equal to 6 times the second and 7 times the third.

Solution 5 times 1st part = 6 times 2nd part = 7 times 3rd part

:. first part =
$$\frac{1}{5}$$
; second part = $\frac{1}{6}$; third part = $\frac{1}{7}$;

first part : second part : third part = $\frac{1}{5} : \frac{1}{6} : \frac{1}{7} = 42 : 35 : 30$

Now, divide 1391 in the proportion of these numbers. The numbers will be

$$\frac{42(1391)}{42+35+30}$$
, $\frac{35(1391)}{42+35+30}$ and $\frac{30(1391)}{42+35+30}$

i.e. 546, 455 and 390.

EXAMPLE 19 Divide ₹ 1540 among 8 men, 9 women and 5 boys such that each woman gets 2 times as much as a boy, and a man as much as a woman and 2 boys together. Find how much each person receives?

Solution 1 man = 1 woman + 2 boys

1 woman = 2 boys

 \therefore 1 man = 4 boys

 \therefore 8 men : 9 women : 5 boys = 32 boys : 18 boys : 5 boys = 32 : 18 : 5

Dividing $\overline{}$ 1540 in the ratio of 32:18:5, we get,

5 boy's share =
$$\frac{5 \times 1540}{32 + 18 + 5}$$
 = ₹ 140

∴ 1 boy's share =
$$\frac{140}{5} = ₹ 28$$

1 woman's share = $2 \times 28 = ₹56$

1 man's share = $4 \times 28 = ₹112$

EXAMPLE 20 One rupee coins, 50 paise coins and 25 paise coins, whose numbers are proportional to 3, $3\frac{1}{2}$ and 4 are together worth ₹ 161. Find the number of each coin.

Solution Coins are in the ratio $3:3\frac{1}{2}:4$, i.e. 6:7:8

$$\therefore \qquad \text{Their proportional value} = 6 \times 1:7 \times \frac{1}{2}:8 \times \frac{1}{4}$$

$$= 6:3\frac{1}{2}:2$$

$$= 12:7:4$$

∴ The value of one rupee coins = $\frac{12}{12+7+4} \times 161 = ₹ 84$

The value of 50 paise coins =
$$\frac{7}{12+7+4} \times 161 = ₹ 49$$

The value of 25 paise coins =
$$\frac{4}{12+7+4} \times 161 = ₹28$$

.. There are 84 one-rupee coins, 98 fifty-paise coins and 112 twenty-five-paise coins.



Note: If milk and water are in the proportion a:b in x litres mixture of milk and water, the quantity of water to be added in order to make this ratio c:d is $\frac{x(ad-bc)}{c(a+b)}$.

EXAMPLE 21 In the 45 litres mixture of milk and water, the ratio of milk and water is 5:4. Find the quantity of water required to be added so that the resultant mixture will be in the ratio 4:5.

Solution Quantity of water required to be added

$$\frac{x(ad-bc)}{c(a+b)} = \frac{45(5\times5-4\times4)}{4(5+4)} = \frac{45\times9}{4\times9} = 11.25 \text{ litres}$$



Note: A mixture contains milk and water in the ratio a:b. If x litres of water is added to the mixture, milk and water mixture will be in the ratio a:c. Then the quantity of milk in the

mixture is given by $\frac{ax}{c-b}$ and that of water is given by $\frac{bx}{c-b}$.

EXAMPLE 22 A mixture contains milk and water in the ratio 3:2. If 4 litres of water is added to the mixture, the proportion becomes 3:4. Find the quantities of milk and water in the mixture.

Solution Quantity of milk = $\frac{ax}{c-b} = \frac{3 \times 4}{4-2} = 6$ litres

Quantity of water =
$$\frac{bx}{c-b} = \frac{2 \times 4}{4-2} = 4$$
 litres

EXAMPLE 23 A mixture contains milk and water in the ratio 7:4. On adding 9 litres of water, quantity of milk and water in the mixture becomes equal. Find the quantity of milk and water in the mixture.

Solution Now, the new proportion of milk and water is 1:1 or 7:7. Therefore, proportional ratios are 7:4 and 7:7.

$$\therefore \qquad \text{Quantity of milk} = \frac{ax}{c-b} = \frac{7 \times 9}{7-4} = \frac{63}{3} = 21 \text{ litres}$$

Quantity of water =
$$\frac{bx}{c-b} = \frac{4 \times 9}{7-4} = \frac{36}{3} = 12$$
 litres

The ratio between the two numbers is a:b. If each number is increased by x, then the ratio becomes c:d. Then the two numbers are given by

$$\frac{xa(c-d)}{ad-bc}$$
 and $\frac{xb(c-d)}{ad-bc}$

EXAMPLE 24 The ratio between two numbers is 3:4. If each number is increased by 3, the ratio becomes 4:5. Find the numbers.

Solution The numbers are

(i)
$$\frac{xa(c-d)}{ad-bc} = \frac{3\times3(4-5)}{3\times5-4\times4} = \frac{9(-1)}{-1} = 9$$

(ii)
$$\frac{xb(c-d)}{ad-bc} = \frac{3\times4(4-5)}{3\times5-4\times4} = \frac{12(-1)}{-1} = 12$$

 \therefore The numbers are 9 and 12.



Note: The incomes of two persons are in the ratio a: b and their expenditures are in the ratio c: d. If each of them saves $\not\in X$, then their incomes are given by $\frac{Xa(d-c)}{(ad-bc)}$ and $\frac{Xb(d-c)}{(ad-bc)}$.

Their expenditures are given by $\frac{Xc(b-a)}{ad-bc}$ and $\frac{Xd(b-a)}{ad-bc}$

EXAMPLE 25 The incomes of Ashok and Suresh are in the ratio 4:3 and their expenditures are in the ratio 3:2. If each saves ₹800, what will be their income?

Solution The incomes are in the ratio a:b=4:3

Expenditures are in the ratio c: d = 3: 2

Savings
$$X = 800$$

∴ Ashok's income =
$$\frac{Xa(d-c)}{(ad-bc)} = \frac{800 \times 4(2-3)}{4 \times 2 - 3 \times 3} = \frac{3200(-1)}{(-1)} = ₹ 3200$$

Suresh's income =
$$\frac{Xb(d-c)}{(ad-bc)} = \frac{800 \times 3(2-3)}{4 \times 2 - 3 \times 3} = \frac{2400(-1)}{(-1)} = ₹ 2400$$

EXAMPLE 26 The incomes of Satheesh and Subhash are in the ratio 7:9 and their expenditures are in the ratio 5:7. If each of them saves ₹ 1200, what are their incomes and expenditure?

Solution The incomes are in the ratio a:b=7:9

The expenditures are in the ratio c: d = 5: 7

Satheesh's income =
$$\frac{Xa(d-c)}{(ad-bc)} = \frac{1200 \times 7(7-5)}{7 \times 7 - 9 \times 5} = \frac{8400 \times 2}{4} = ₹ 4200$$

Subhash's income =
$$\frac{Xb(d-c)}{(ad-bc)} = \frac{1200 \times 9(7-5)}{7 \times 7 - 9 \times 5} = \frac{10800 \times 2}{4} = ₹ 5400$$

Satheesh's expenditure =
$$\frac{Xc(b-a)}{(ad-bc)} = \frac{1200 \times 5(9-7)}{7 \times 7 - 9 \times 5} = \frac{6000 \times 2}{4} = ₹ 3000$$

Subhash's expenditure =
$$\frac{Xd(b-a)}{(ad-bc)} = \frac{1200 \times 7(9-7)}{7 \times 7 - 9 \times 5} = \frac{8400 \times 2}{4} = ₹ 4200$$



Note: A number which when added to the terms of the ratio a:b makes it equal to the ratio c:d is $\frac{ad-bc}{c-d}$.

EXAMPLE 27 Find the number which when added to the ratio 7:9, makes it equal to 4:5.

Solution a:b=7:9:c:d=4:5

The required number = $\frac{ad - bc}{c - d} = \frac{7 \times 5 - 9 \times 4}{4 - 5}$ = $\frac{-1}{1} = +1$

A number which when subtracted from the terms of the ratio a:b, makes it equal to the ratio c:d is $\frac{bc-ad}{c-d}$.

EXAMPLE 28 The ratio of incomes of Ajith and Sunil last year was 4:5. The ratio of their own incomes of last year and this year are 6:7 and 5:6 respectively. If the total sum of their present incomes is $\stackrel{?}{\underset{?}{|}}$ 6,400, find the present income of Ajith.

Solution The ratio of present income = $4 \times \frac{7}{6} : 5 \times \frac{6}{5}$ = $\frac{28}{6} : 6 = \frac{14}{3} : 6$ = 14 : 18 or 7 : 9

∴ Ajith's present income = $\frac{7}{7+9} \times 6400 = ₹ 2800$

Otherwise, Ratio of Ajith's incomes (last year and this year) = 6:7

$$= 6 \times \frac{4}{6} : 7 \times \frac{4}{6} = 4 : \frac{14}{3}$$

Ratio of Sunil's incomes (last year and this year) = 5:6

∴ Ratio of Ajith's income to Sunil's this year = $\frac{14}{3}$: 6 = 7:9

∴ Ajith's present income = $\frac{7 \times 6400}{7 + 9}$ = ₹ 2800

EXAMPLE 29 A bucket contains a mixture of milk and water in the proportion 2:1. If 6 litres of mixture is replaced by 6 litres of water, then the ratio of the two liquids becomes 4:5, what was the quantity of water contained in the bucket?

Solution Given 1st ratio = 2:1

2nd ratio = 4:5

 $D = \text{Difference of cross products of ratios} = 2 \times 5 - 1 \times 4 = 10 - 4 = 6$

 $\therefore \text{ Common factor of 1st ratio} = \left[\frac{\text{Quantity replaced}}{\text{Sum of terms in 1st ratio}} \right] + \left[\frac{\text{Quantity replaced} \times 4}{D} \right]$

$$= \left\lceil \frac{6}{3} \right\rceil + \left\lceil \frac{6 \times 4}{6} \right\rceil = 2 + 4 = \mathbf{6}$$

 \therefore The quantity of milk = $2 \times 6 = 12$ litres

Similarly, the quantity of water = $1 \times 6 = 6$ litres

EXAMPLE 30 A vessel contains milk and water in the ratio 5: 4. If 27 litres of mixture is removed and the same quantity of water is added, the ratio becomes 4: 5. What quantity does the vessel contain?

Solution Let the vessel contain 5x litres of milk and 4x litres of water.

The removed quantity of mixture contains $\frac{5}{5+4} \times 27 = 15$ litres of milk and 27-15=12 litres of water.

Now,
i.e.
$$5x - 15 : (4x - 12 + 27) = 4 : 5$$
i.e.
$$(5x - 15) : (4x + 15) = 4 : 5$$

$$\frac{5x - 15}{4x + 15} = \frac{4}{5}$$

$$\Rightarrow 25x - 75 = 16x + 60$$

$$\Rightarrow 25x - 16x = 75 + 60 = 135$$

$$9x = 135$$

$$\therefore x = \frac{135}{9} = 15 \text{ litres}$$

 \therefore Vessel contains $(5 + 4)x = 9 \times 15 = 135$ litres.



Short cut: When the ratio is reversed (i.e. 5:4 becomes 4:5)

Total quantity = $\frac{(5+4)^2}{5^2-4^2}$ × Quantity of first liquid in the removed mixture = $\frac{9^2}{9}$ × 15 = 135 litres

(d) 56:99:72 (e) None of these

? EXERCISES

(a) 72:56:99

1. If A : B = 7 : 9 and B : C = 8 : 11, find A : B : C.

(b) 56:72:99 (c) 99:56:72

	(a) . _	(2) 33:12:33	(6) 55:55:12	(4) 33.33.12	(6) 1.0126 01 012656
2.	The sum of two n	umbers is 25 and t	their difference is	$7\frac{1}{2}$. Find the rat	io of the numbers.
				$Hint: If \frac{a}{b}$	$- = \frac{c}{d}; \frac{a+b}{a-b} = \frac{c+d}{c-d}$
	(a) 6:13	(b) 7:13	(c) 13:7	(d) 13:6	(e) None of these
3.	₹ 120 is divided an	mong 6 men, 8 wom nuch as 3 boys. Fin	nen and 6 boys so t	hat a man gets as	much as 2 boys and
	(a) ₹3	(b) ₹6	(c) ₹ 12	(d) ₹4	(e) None of these
4.	What should be s of 3:4	subtracted from the	e terms of the rati	io 19:23 makes:	it equal to the ratio
	(a) 7	(b) 6	(c) 5	(d) 4	(e) None of these
5.	wages in the ratio	o 14:15. Find the	ratio in which the	bill of wages incr	and increases their eases or decreases.
	(a) 1/3	(b) 1/4	(c) 1/5	(d) 1/8	(e) None of these
6.		13 per kg, 75 men odays at the same co			ost. How many men
	(a) 225	(b) 250	(c) 325	(d) 175	(e) None of these
7.	-	a book of 25 sheets f a book of 32 sheet	_	of paper, how mu	ch paper is required
	(a) 160	(b) 320	(c) 480	(d) 640	(e) None of these
8.	Divide ₹ 1,000 am Then what is the		ch that P gets 2/3 of	f Q 's share and R §	gets 5/3 of <i>Q</i> 's share.
	(a) 100	(b) 200	(c) 300	(d) 400	(e) None of these
9.	If 0.6 time one nu	ımber is equal to 0	0.025 time the othe	r, what is the rat	io of two numbers?
	(a) 1/24	(b) 5/24	(c) 7/24	(d) 11/24	(e) None of these
10.	Calculate the four	rth proportional to	the numbers 286,	78, 1342	
	(a) 732	(b) 671	(c) 183	(d) 366	(e) None of these
11.		apee coins, 50-paise to 5, 7 and 12 are t			which the numbers
	(a) 50, 70, 120	(b) 60, 70, 110	(c) 70, 70, 100	(d) 70, 80, 90	(e) None of these
12.		in the ratio 3 : 5. I smaller number is		om each, the new	numbers are in the [SSC]
	(a) 27	(b) 33	(c) 49	(d) 55	
13.	Two numbers are greatest number:	is	. If 7 is added to b		nanges to 3 : 5. The otel Management]
	(a) 24	(b) 26	(c) 28	(d) 32	
14.	If $0.75:x::5:8$,	then x is equal to			[LIC]
	(a) 1.12	(b) 1.20	(c) 1.25	(d) 1.30	
15.	If $x : y = 5 : 2$, the	en (8x + 9y) : (8x + 2y)	2y) is		[SSC]
	(a) 22:29	(b) 26:61	(c) 29:22	(d) 61:26	

16	Salaries of Ravi a	nd Sumit are in t	he ratio 2 · 3 If th	e salary of each is incre	ased by ₹ 4000
10.			at is Sumit's prese	•	[Bank PO]
	(a) ₹ 17000	(b) ₹ 20000	(c) ₹ 25500	(d) None of these	
17.	If 10% of $x = 20\%$	of y , then $x : y$ is	equal to		[CDS]
	(a) 1:2	(b) 2:1	(c) 5:1	(d) 10:1	
18.				s of A increase by 50%	
	decrease by 25%,	the new ratio of t	their earnings bec	omes 8 : 7. What are A	
	(a) ₹ 21000	(b) ₹ 26000	(c) ₹ 28000	(d) Data inadequate	[Bank PO]
19	If $A: B = 8: 15$; E			_	,
10.	(a) $2:7$		(c) $8:15$	(d) 15:4	
20.	If $A: B = 2: 3, B:$			` '	[SSC]
	(a) 16:22:30:3			: 24 : 15 : 35	[SSC]
	(c) 16:24:30:3		` '	: 24 : 30 : 35	
21.	` '		` '	e increments of 15%, 10	0% and 20% are
				be the new ratio of the	
					[Bank PO]
	(a) 3:3:10	(b) 10:11:20	(c) 23:33:60	(d) cannot be determ	nined
22	If₹782 be divided	d into three narts	nroportional to	$\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$, then the first	nart is: [CRI]
			•		part is. [CBI]
	(a) ₹ 182	(b) ₹ 190	(c) ₹ 196	(d) ₹ 204	
23.	If $A: B = \frac{1}{2}: \frac{3}{8}, B$	$: C = \frac{1}{3} : \frac{5}{9} \text{ and } C$	$D = \frac{5}{6} : \frac{3}{4}$, then the	he ratio $A:B:C:D$ is	[SSC]
	(a) 4:6:8:10	(b) 6:4:8:10	(c) 6:8:9:10	(d) 8:6:10:9	
24.			f the ratio of the fix e second number i	rst to the second is 2 : 3 is:	and that of the [SSC]
	(a) 20	(b) 30	(c) 48	(d) 58	
25 .	A sum of ₹ 1,300 i	is divided among	st P, Q, R and S sı	ıch that	
	P's share Q's sh	hare R's share	2		
	$\frac{P's share}{Q's share} = \frac{Q's sh}{R's sh}$	$\frac{1}{\text{nare}} = \frac{1}{\text{S's share}} = \frac{1}{\text{S's share}}$	$=\frac{1}{3}$		
	Then P's share is				[LIC]
	(a) ₹ 140	(b) ₹ 160	(c) ₹ 240	(d) ₹ 320	
26.	A and B together	have ₹ 1210. If $\frac{1}{1}$	$\frac{4}{5}$ of A's amount is	n equal to $\frac{2}{5}$ of B's am	ount, how much
	amount does B ha			J	[AAO]
	(a) ₹ 460	(b) ₹ 484	(c) ₹ 550	(d) ₹ 664	
27.	Two numbers are numbers is	respectively 20%	and 50% more tha	an a third number. The	ratio of the two [SSC]
	(a) 2:5	(b) 3:5	(c) 4:5	(d) 6:7	
28.				s 7:8. If the percentage	

29. A sum of money is to be distributed among A, B, C and D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1000 more than D, what is B's share? [RBI]

(a) ₹ 500

(a) 8:9

(b) ₹ 1500

(b) 17:18

(c) ₹ 2000

(c) 21:22

(d) None of these

(d) cannot be determined

30.	The fourth proport	tional to 5, 8, 15	is		[RBI]
	(a) 18	(b) 24	(c) 19	(d) 20	(e) 21
31.	If 40% of a number to the second num		hirds of another nur	nber, what is the	ratio of first number [Bank PO]
	(a) 2:5	(b) 3:7	(c) 5:3	(d) 7:3	
32 .	x varies inversely equal to	as square of y. G	fiven that $y = 2$ for	x = 1. The value of	of x for $y = 6$ will be [CDS]
	(a) 3	(b) 9	(c) 1/3	(d) 1/9	
33.	The compounded r				
	(a) 2:11	(b) 11:5	(c) 4:5	(d) 5:11	
34.	The least whole nuequal to 3:4 is	umber when subt	racted from both th	e terms of the rat	tio 4 : 5 gives a ratio
	(a) 3	(b) 6	(c) 9	(d) 12	
35.	A certain sum was sum was	divided among P	and Q in the ratio	5 : 6. If Q's share v	was ₹ 2400, the total
	(a) ₹ 4800	(b) ₹ 4400	(c) ₹ 3200	(d) ₹ 2800	
36.	The sides of a triar	ngle are in the rat	$\frac{1}{2} : \frac{1}{3} : \frac{1}{4} \text{ and its } \mathbf{j}$	perimeter is 156 c	m. The length of the
	largest side is				
	(a) 54 cm	(b) 68 cm	(c) 72 cm	(d) 84 cm	
37.			3:4 and the ratio of then the income of		res is 2:3. If at the
	(a) ₹ 9000	(b) ₹8000	(c) ₹ 7000	(d) ₹ 6000	
38.	The ratio of the nu find the total num			n is 6 : 5. If there a	are 192 boys in total,
	(a) 250	(b) 325	(c) 352	(d) 425	
39.	A fraction which b	ears the same ra	tio to $\frac{1}{27}$ that $\frac{3}{11}$ do	pes to $\frac{5}{9}$ is equal to	to [SSC]
	(a) $\frac{1}{55}$	(b) $\frac{1}{11}$	(c) $\frac{3}{11}$	(d) 55	
40.	If $A : B : C = 2 : 3$: 4, then $\frac{A}{B}$: $\frac{B}{C}$:	$\frac{C}{A}$ is equal to		
	(a) 4:9:16	(b) 8:9:12	(c) 8:9:16	(d) 8:9:24	
	WERS				
-					
1.	(b) 2. (c) 3.	. (d) 4. (a)	5. (e) 6. (c)	7. (b) 8. (c)	9. (a) 10. (d)
11.	` ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '	. (c) 14. (b)	15. (c) 16. (d)	17. (b) 18. (d)	
21.	` '	* * * * * * * * * * * * * * * * * * * *	25. (b) 26. (b)	27. (c) 28. (c)	29. (c) 30. (b)
31.		. (c) 34. (a)	35. (b) 36. (c)	37. (d) 38. (c)	39. (a) 40. (d)



Solutions with Necessary Explanation

1. Given A : B = 7 : 9 ; B : C = 8 : 11

$$A : B = 7 : 9$$

$$B:C=8:11$$

Making the second proportion equal in both ratios, we get

$$A: B = 7: 9 = 7 \times 8: 9 \times 8 = 56: 72$$

B:
$$C = 8: 11 = 8 \times 9: 11 \times 9 = 72: 99$$

$$A : B : C = 56 : 72 : 99$$

2. Given a + b = 25, $a - b = 7\frac{1}{2}$; a/b = ?

$$\frac{a+b}{a-b} = \frac{25}{7\frac{1}{2}} = \frac{50}{15} = \frac{10}{3}$$

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{10+3}{10-3} = \frac{13}{7}$$

i.e.,

$$\frac{2a}{2b} = \frac{13}{7}$$
 or $\frac{a}{b} = \frac{13}{7}$

3. Given share to men, women and boys as follows.

share for 2 women = share for 3 boys

share for 1 man = share for 2 boys.

There are 6 men, 8 women and 6 boys.

∴ Total share =
$$6(2 \text{ boys}) + \frac{8}{2}(3 \text{ boys}) + 6(\text{boys}) \text{ share} = ₹120$$

i.e. Share of (12 + 12 + 6) boys = 7120

$$\therefore \qquad \text{Share of a boy} = \frac{120}{130} = 4$$

4. If a number in substracted from a:b and as a result it become c:d, then the number

substracted =
$$\frac{bc - ad}{c - d}$$

Here

$$a:b=19:23:c:d=3:4$$

 $\therefore \text{ Number subtracted} = \frac{bc - ad}{c - d} = \frac{23 \times 3 - 19 \times 4}{3 - 4}$

$$=\frac{69-76}{-1}=7$$

5. New number of employees = $\frac{7}{9}$ of old number of employees.

New wages = $\frac{15}{14} \times \text{old wages}$

 \therefore Resultant wages = $\frac{7}{9} \times \frac{15}{14} \times \text{old total wages}$

$$=\frac{5}{6}$$
 × old total wages

:. Bill of wages decrease by 1/6 or 16.17%

73

where Q_w , quantity of wheat got for $\stackrel{?}{\overline{\checkmark}}$ 13/kg.

Let x be number of men in the second case.

$$\therefore x = \frac{75 \times 15 \times 13}{45} = 25 \times 13 = 325 \text{ men.}$$

7.
$$\frac{1000 \times 25}{50} = \frac{5000 \times 32}{x}$$

$$\therefore x = \frac{5000 \times 32 \times 50}{1000 \times 25} = 320$$

:. 320 ream of paper is required.

8. Let the share of Q be x.

Then
$$\frac{2}{3}x + x + \frac{5}{3}x = 1000$$

$$\Rightarrow \frac{10}{3}x = 1000 \Rightarrow x = \frac{3 \times 1000}{10} = 300$$

∴ P's share =
$$\frac{2}{3}x = \frac{2}{3} \times 300 = ₹ 200$$

$$Q$$
's share = $x = ₹300$

R's share =
$$\frac{5}{3}x = \frac{5}{3} \times 300 = ₹ 500$$

9. Let the numbers be x and y.

$$0.6 \ x = 0.025 \ y$$

$$\therefore \frac{x}{y} = \frac{0.025}{0.6} = \frac{25}{600} = \frac{1}{24}$$

 \therefore Ratio of 2 numbers = 1 : 24.

10. Let the fourth proportional be x.

Now
$$286:78 = 1342:x$$

i.e.
$$\frac{286}{78} = \frac{1342}{x}$$

Cross multiplying, we get $286 \times x = 78 \times 1342$

or
$$x = \frac{78 \times 1342}{286} = 366$$

11.
$$(5 \times 1 + 7 \times 0.5 + 12 \times 0.25)x = 115$$

$$(5 + 3.5 + 3) = 115$$

$$11.5x = 115$$

$$\therefore$$
 $x = 10$

 \therefore Number of one-rupee coin = 5x = 50

Number of 50-paise coin 7x = 70

Number of 25-paise coin 12x = 120

12. Let the numbers be 3k and 5k.

Now,
$$\frac{3k-9}{5k-9} = \frac{12}{23}$$

$$\Rightarrow 23(3k-9) = 12(5k-9)$$

$$\Rightarrow (69-60)k = (23-12)9$$
or
$$k = 11$$

$$\therefore \text{Smaller number} = 3 \times 11 = 33$$

13. Let the numbers be k and 2k

Now,
$$\frac{k+7}{2k+7} = \frac{3}{5}$$

$$\Rightarrow \qquad 5(k+7) = 3(2k+7)$$

$$\Rightarrow \qquad (6-5)k = 7(5-3)$$

$$\therefore \qquad k = \frac{7\times 2}{1} = 14$$

- \therefore Greatest number = $2k = 14 \times 2 = 28$
- **14.** Given 0.75:x::5:8

Here,
$$0.75 \times 8 = x \times 5$$

or $x = \frac{0.75 \times 8}{5} = \frac{6}{5} = 1.2$

15. Given $x : y = 5 : 2 \implies x = 5k$ and y = 2k

$$\therefore \frac{8x + 9y}{8x + 2y} = \frac{8 \times 5k + 9 \times 2k}{8 \times 5k + 2 \times 2k} = \frac{58k}{44k} = \frac{29}{22}$$

16. Let the salary of Sumit be x.

Given salary of Ravi to Sumit = $\frac{2}{3}$

$$\therefore \qquad \text{Salary of Ravi} = \frac{2}{3} \times x = \frac{2x}{3}$$

On increasing salary of both of the above by ₹ 4000, new ratio will be

$$\left(\frac{2}{3}x + 4000\right) : (x + 4000) = 40 : 57$$
Solving,
$$57\left(\frac{2}{3}x + 4000\right) = 40(x + 4000)$$

$$\Rightarrow x\left(\frac{57 \times 2}{3} - 40\right) = 4000(40 - 57)$$

$$x\left(\frac{37 \times 2}{3} - 40\right) = 4000(40 - 57)$$
$$x(38 - 40) = 4000(-17)$$
$$x = \frac{4000(-17)}{-2} = 34000$$

Sumit's present salary = 34000 + 4000 = ₹38000

17. Given
$$\frac{10}{100} \times x = \frac{20}{100} \times y$$

$$\therefore \frac{x}{y} = \frac{20/100}{10/100} = 2:1$$

18. Given Earnings of A: Earnings of B = 4:7

Let earnings of A = 4k and that of B = 7k

Now,
$$\frac{4k(1.5)}{7k(0.75)} = \frac{8}{7}$$

$$\frac{6k}{5.25k} = \frac{8}{7}$$

Value of k cannot be determined.

Therefore data inadequate.

19. Given A: B = 8: 15; B: C = 5: 8; C: D = 4: 5

$$A: B = 8:15$$

$$B: C = 15: 24$$

$$C: D = 24:30$$

$$A: D = 8: 30 \text{ or } 4: 15$$

20. Given A: B = 2: 3; B: C = 4: 5 and C: D = 6: 7;

$$A:B:C:D=?$$

$$A: B=2: 3 \rightarrow 2 \times 4: 3 \times 4 \rightarrow 2 \times 2 \times 4: 2 \times 3 \times 4$$

$$B:C=4:5\rightarrow 4\times 3:5\times 3\rightarrow 2\times 4\times 3:2\times 5\times 3$$

$$C: D = 6: 7 \rightarrow 6 \times 5: 7 \times 5 \rightarrow 6 \times 5: 7 \times 5$$

$$A: B: C: D = 16: 24: 30: 35$$

21. Salary of A: Salary of B: Salary of C = 2:3:5

Let their salaries be 2k, 3k and 5k. New Salaries are respectively 2k(1.15), 3k(1.10) and 5k(1.20)

- :. Ratio of new salaries = 2.3k : 3.3k : 6k = 23 : 33 : 60
- **22.** Given ratio is $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$

$$= 12 \times \frac{1}{2} : 12 \times \frac{2}{3} : 12 \times \frac{3}{4}$$

$$=6:8:9$$

∴ First part =
$$\frac{6}{6+8+9}$$
 × ₹ 782 = 6 × 34 = ₹ 204

23. Given,

$$A: B = \frac{1}{2}: \frac{3}{8} = 4:3$$

$$B: C = \frac{1}{3}: \frac{5}{9} = 3:5$$

$$C: D = \frac{5}{6}: \frac{3}{4} = 10:9$$

 $A:B:C:D=2\times 4:2\times 3:2\times 5:9=8:6:10:9$

24. Let the number be x, y and z

Then $x + y + z = 98 \tag{1}$

$$\frac{x}{y} = \frac{2}{3} \tag{2}$$

$$\frac{y}{z} = \frac{5}{8} \tag{3}$$

$$x: y = 2:3 y: z = 5:8 \Rightarrow x: y: z = 2 \times 5: 3 \times 5: 3 \times 8 = 10: 15: 24 \therefore \qquad y = \frac{15}{10 + 15 + 24} \times 98 = 30$$

25. We have $P: Q = 2: 3 \to 2 \times 2: 3 \times 2 \to 2 \times 2 \times 2: 2 \times 3 \times 2$ $Q: R = 2: 3 \to 2 \times 3: 3 \times 3 \to 2 \times 2 \times 3: 2 \times 3 \times 3$

$$R: S = 2: 3 \rightarrow 2 \times 3 \times 3: 3 \times 3 \times 3$$

P: Q: R: S = 8: 12: 18: 27

∴ P's share =
$$\frac{8}{8+12+18+27} \times 1300 = ₹ 160$$

26. Let the amount with B be x and amount with A be y.

$$\therefore \qquad \qquad x + y = 1210 \tag{1}$$

$$\frac{4}{15}y = \frac{2}{5}x\tag{2}$$

$$\Rightarrow \qquad 4y = 6x \tag{3}$$

Substituting in (1), we get

$$x + \frac{6x}{4} = 1210$$

$$\Rightarrow \frac{10x}{4} = 1210$$

$$\Rightarrow \qquad x = 1210 \times \frac{4}{10} = 7484$$

27. Let the third number be x.

Then first number = 1.2x and second be = 1.5x

- \therefore First number : Second number = 1.2x : 1.5x = 4 : 5
- **28.** Ratio of boys : girls = 7 : 8

Let number of boys be 7k and girls be 8k.

:. New ratio =
$$\frac{7k(1.2)}{8k(7.1)} = \frac{8.4k}{8.8k} = \frac{21}{22}$$

29. Amount of money distributed between A, B, C and D is in the ratio 5:2:4:3.

∴ Difference in money between C and $D = \left(\frac{4}{5+2+4+3} - \frac{3}{5+2+4+3}\right)$ of total amount $= \frac{4-3}{14} \times \text{total amount} = ₹1000$

∴ Total amount =
$$14 \times 1000 = ₹ 14000$$

∴ B's share =
$$\frac{2}{5+2+4+3} \times 14000 = ₹ 2000$$

30. Fourth proportional to 5, 8, 15 is = $\frac{8 \times 15}{5} = 24$

31. Let the number be x and y

Now,
$$x(0.4) = \frac{2}{3} \times y$$

$$\therefore \frac{x}{y} = \frac{2}{3 \times 0.4} = \frac{10}{6} = \frac{5}{3}$$

 \therefore Ratio of first number to second number = $\frac{5}{3}$

32. Given
$$x \propto \frac{1}{y^2}$$
 or $x = k \times \frac{1}{y^2}$ (1)

Given, y = 2 for x = 1, for y = 6, x = ?

From (1),
$$1 = k \times \frac{1}{2^2}$$

or $k = 1 \times 2^2 = 4$
 $\therefore x = 4 \times \frac{1}{6^2} = \frac{1}{9}$

- 33. Compounded ratio of (2:5), (6:11) and (11:3) = $\frac{2}{5} \times \frac{6}{11} \times \frac{11}{3} = \frac{4}{5}$ or 4:5
- **34.** Given ratio = 4:5

Let terms of ratio be 4k and 5k.

If a number x is subtracted from both terms, we get

$$\frac{6-x}{7-x} = \frac{3}{4}$$

$$\Rightarrow \qquad (6-x)4 = 3(7-x)$$

$$\Rightarrow \qquad 6 \times 4 - 7 \times 3 = x(4-3)$$

$$24 - 21 = x$$
or
$$x = 3$$

:. Least whole number = 3

35.
$$P: Q = 5:6$$

Given $\frac{6}{5+6}$ of total sum = 2400

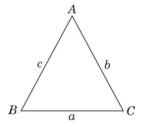
∴ Total sum =
$$2400 \times \frac{11}{6} = ₹ 4400$$

36.
$$a:b:c=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$$
 or $6:4:3$

Given perimeter =
$$(a + b + c) = 156$$

... Length of largest side =
$$\frac{6}{6+4+3} \times 156$$

= $\frac{6}{13} \times 156 = 72$ cm



37. Given ratio of income of P and Q as 3:4

i.e.
$$p_i: q_i = 3: 4 \text{ or } 3x: 4x$$

Ratio of expenditures = 2:3.

i.e.,
$$p_e: q_e = 2: 3 \text{ or } 2x: 3x$$

Amount of savings of
$$P = p_i - p_e = 3x - 2x = x$$

Amount of savings of
$$Q = q_i - q_e = 4x - 3x = x$$

Given that x = 7 2000

$$p_i = ?$$

$$p_i = 3x = 6000$$

38. Let number of boys and girls be 6k and 5k respectively.

$$\therefore$$
 Total number of students = $6k + 5k = 11k$

Given number of boys = 6k = 192

or
$$k = 32$$

 \therefore Total number of students = $11k = 11 \times 32 = 352$

39. Let the fraction be $\frac{p}{a}$.

$$\frac{p}{a} \div \frac{1}{27} = \frac{3}{11} \div \frac{5}{9}$$

$$\mathbf{or}$$

$$\frac{p}{a} \times 27 = \frac{3}{11} \times \frac{9}{5}$$

$$\mathbf{or}$$

$$\frac{p}{q} = \frac{1}{27} \times \frac{3}{11} \times \frac{9}{5} = \frac{1}{55}$$

40. Given A:B:C=2:3:4

Then

$$\frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{4}{2}$$
$$= \frac{2}{3} : \frac{3}{4} : 2 = 8 : 9 : 24$$

Percentage

A fraction whose denominator is expressed as 100 is called the *percentage* and the numerator of the fraction is called the *rate percentage*.

For example, a man spends 10% of his monthly income on dress materials means every ten rupee out of hundred rupees he gets, goes to the purchase of dress materials, i.e. if the monthly income of the man is $\stackrel{?}{\underset{?}{$\sim}}$ 5000, then the spending towards dress materials comes

about
$$\frac{10}{100}$$
 × 5000 = ₹ **500**.

Important Prime Fractions and their Equivalent Percentage						
$\frac{1}{32} = 3\frac{1}{8}\%$	$\frac{1}{20} = 5\%$	$\frac{1}{16} = 6 \frac{1}{4}\%$	$\frac{2}{25} = 8\%$	$\frac{1}{12} = 8\frac{1}{3}\%$	$\frac{1}{10} = 10\%$	
$\frac{3}{25} = 12\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{2}{15} = 13\frac{1}{3}\%$	$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{3}{20} = 15\%$	$\frac{4}{25} = 16\%$	
$\frac{1}{4} = 25\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{2}{5} = 40\%$	$\frac{3}{5} = 60\%$	$\frac{5}{8} = 62\%$	
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{4} = 75\%$	$\frac{7}{8} = 87 \frac{1}{2}\%$				

EXAMPLE 1 In an election, there are two candidates. The candidate who gets 46% is rejected by a majority of 2688 votes. Find the total number of votes polled.

Solution Out of the total votes polled (100%)

The winning candidate got (100-46) = 54% votes and the other candidate got 46% votes.

$$\therefore \qquad \text{Majority} = 54 - 46 = 8\% \text{ of total votes}$$
$$= 2688$$

:. Votes polled =
$$\frac{2688}{8/100} = \frac{2688 \times 100}{8} = 33600$$

EXAMPLE 2 What percentage is equivalent to $\frac{5}{8}$?

Solution
$$\frac{5}{8} \times 100 = \frac{125}{2} = 62\frac{1}{2}\%$$

EXAMPLE 3 The salary of Kannan is increased from ₹ 3500 to ₹ 4025. Find the increase per cent.

Solution Increase in salary = ₹4025 - ₹3500 = ₹525

Percentage increase in salary = $\frac{525}{3500} \times 100 = 15\%$

EXAMPLE 4 The price of sugar in the market rises by 25%. A person whose income is fixed, has to reduce the consumption of sugar so that the expenditure remains the same. By what percentage, he should reduce the consumption of sugar?

Solution The percentage reduction to be made in his consumption = $25 \times \frac{100}{100 + 25} = 20\%$.

Theorem 1: If the price of a certain item is increased by x% and then increased by y%, then the resultant increase in its price is given by

$$\left(x+y+\frac{xy}{100}\right)\%$$

Choose sign of *x* and *y* appropriately (i.e. +ve for increase and –ve for decrease).

Theorem 2: If two values are respectively x% and y% more than the third value, then the first is $\frac{100 + x}{100 + y} \times 100\%$ of the second.

EXAMPLE 5 Two numbers are 25% and 50% more than a third number. What percentage is the first of the second?

Solution By Theorem 2,

The required value = $\frac{100 + 25}{100 + 50} \times 100 = \frac{5}{6} \times 100 = 83\frac{1}{3}\%$

Theorem 3: If P is x% of R and Q is y% of R, then P is $\frac{x}{y} \times 100\%$ of Q.

EXAMPLE 6 Two numbers are 24% and 30% of a third number. What percentage is the first of the second?

Solution By Theorem 3,

The required value $=\frac{24}{30} \times 100 = \frac{4}{5} \times 100 = 80\%$

EXAMPLE 7 Two numbers are 20% and 40% less than a third number. What per cent is the second of the first?

Solution The per cent of the second number with respect to the first number is given by

$$\frac{100 - 40}{100 - 20} \times 100 = \frac{60}{80} \times 100 = 75\%$$

PERCENTAGE EXPENDITURE AND SAVINGS: If a person spends x% of his money on lodging and y% of the remaining on food and cloth expenses and $\not\in m$ is the amount remaining with him, then the amount he had at the beginning is given by

$$\frac{m \times 100 \times 100}{(100 - x)(100 - y)}$$

EXAMPLE 8 A person spends 5% of his income on income tax and 12.5% of the remaining is saved. The amount spent was ₹ 29925. What was his income?

Solution Income =
$$\frac{29,925 \times 100 \times 100}{(100 - 5)(100 - 12.5)}$$
 = ₹ 36000

Theorem 3: If x% of a quantity is taken by the first, y% of the remaining is taken by the second and z% of the remaining is taken by a third person and if the amount left is $\overline{\xi}$ P, then the

amount at the beginning is given by

$$\frac{P \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)}$$

Theorem 3: If x% of a quantity is added, y% of the increased quantity is added and z% of the first increased quantity is added and the amount becomes P, then the initial amount is given by

$$\frac{P \times 100 \times 100 \times 100}{(100 + x)(100 + y)(100 + z)}$$

EXAMPLE 9 A certain sum was first increased by 10% and then by 20%, and it become finally $\stackrel{?}{\sim}$ 5280. Find the initial sum.

Solution The required initial amount = $\frac{5280 \times 100 \times 100}{110 \times 120} =$ ₹ **4000**

POPULATION FORMULA: If the original population of a town is P and the annual

increase in population is r%, population after n years will be $P\left(1 + \frac{r}{100}\right)^n$.

EXAMPLE 10 Annual increase in population of a town is 4%, and the present population is 40000. What will be the population of town after 2 years?

Solution Population after 2 years =
$$40000 \left(1 + \frac{4}{100}\right)^2$$

= $\frac{40000 \times 104 \times 104}{100 \times 100} = 43264$

Theorem 6: The population of a town is P. It increases by x% during the first year, y% during the second year and z% during the third year. Population after 3 years will be

$$\frac{P(100+x)(100+y)(100+z)}{100\times100\times100}$$

EXAMPLE 11 The population of a town is 15000. It increases by 10% for the first year, 20% for the second year and 40% for the third year. What will be the population after 3 years?

Solution The required population =
$$\frac{15000 \times 110 \times 120 \times 140}{100 \times 100 \times 100} = 27720$$

EXAMPLE 12 The population of a town increases at the rate of 10% during the first year and it decreases at the rate of 5% during the second year. If it has 20900 inhabitants now, what was the population of town 2 years ago?

Solution The required population =
$$\frac{20900 \times 100 \times 100}{110 \times 95} = 20000$$

Theorem 7: Let the present value of a machine be P. Let the depreciation of the machine be at the rate of r% per annum.

Then

- 1. Value of machine after n years = $P\left(1 \frac{r}{100}\right)^n$
- 2. Value of machine n years ago = $\frac{P}{\left(1 \frac{r}{100}\right)^n}$

EXAMPLE 13 A machine costs ₹ 320000 at present. If the value of the machine depreciates at the rate of 5% per annum. What will be its value 6 years hence?

Solution Present value of machine = ₹ 320000

6 years hence, value of machine =
$$320000 \left(1 - \frac{5}{100}\right)^6$$

= $320000 \times \left(\frac{19}{20}\right)^2 \times \left(\frac{19}{20}\right)^2 \times \left(\frac{19}{20}\right)^2$
= $32 \times \frac{19^2}{4} \times \frac{19^2}{4} \times \frac{19^2}{400} = \frac{19^2 \times 19^2 \times 19^2}{200}$
= $\frac{19^6}{200} = \frac{47045881}{200} = ₹235229.41$

Percentage Relationship: If A is x% more than B, then B is $\left(\frac{x}{100+x} \times 100\right)\%$ less than A.

If A is x% less than B, then B is $\left(\frac{x}{100-x} \times 100\right)$ % m more than A.



Note: If *P* is *r*% more than *Q*, then *Q* is less than *P* by $\left[\frac{r}{100+r} \times 100\right]$ %.

If P is r% less than Q, then Q is more than P by $\left[\frac{r}{100-r} \times 100\right]$ %.

Theorem 8: If the price of a commodity increases by r%, then decrease in consumption, so

as not to increase the expenditure on this item is $\left(\frac{r}{100+r} \times 100\right)\%$.

If the price of a commodity decreases by r%, then increase in consumption, so as not to decrease the expenditure on this item is $\left(\frac{r}{100-r}\times 100\right)\%$.

EXAMPLE 14 The salary of A is 20% more than that of B. Find how much per cent is B's salary less than that of A.

Solution Let B's salary be ₹ 100 per month.

Then A's salary = 100 + 20% of 100 = 120

- ∴ B's salary is ₹ 20 less than that of A when A's salary is ₹ 120.
- ∴ When *A*'s salary is ₹ 100, *B*'s salary is $\frac{20}{120} \times 100 = \frac{100}{6} = ₹ \frac{50}{3}$ less than that of *A*, i.e.

B's salary is $\frac{50}{3}$ % less than that of A.

Theorem 9: If a number is first increased by x% and then decreased by x%, the net change is always a decrease which is equal to x% of x or $\frac{x^2}{100}$ percentage.

EXAMPLE 15 The salary of a worker is ₹ 2000. It is first increased by 10% and then decreased by 10%. What is the change in his salary?

Solution The salary of the worker = $\stackrel{?}{=} 2000$

After 10% increase, salary = $\frac{110}{100}$ × 2000 = ₹ 2200

After 10% decrease, salary = $\frac{90}{100}$ × 2200 = ₹ 1980

∴ The net reduction in salary = ₹ 20

i.e. the percentage reduction in salary = $\frac{20}{2000} \times 100 = 1\%$

i.e.
$$\%$$
 reduction = $\frac{x^2}{100}\% = \frac{10^2}{100}\% = 1\%$

The pass mark in an examination is x%. If a candidate who secures y marks

fails by z marks, then the maximum mark is given by $M = \frac{100(y+z)}{x}$.

EXAMPLE 16 In an examination, a candidate requires 35% marks to pass. If he gets 90 marks and fails by 50 marks, find the maximum marks.

Solution Maximum marks,
$$M = \frac{100(y+z)}{x} = \frac{100(90+50)}{35} = 400$$

EXAMPLE 17 In an examination, a candidate got 110 marks and failed by 70 marks. If the required pass percentage is 40, find the maximum marks.

Solution Maximum marks,
$$M = \frac{100(110 + 70)}{40} = 450$$

Theorem 11: A candidate scoring x% marks in an examination fails by a marks and another candidate scoring y% marks gets b marks more than the minimum required pass marks. Then the

maximum mark for that examination is
$$M = \frac{100(a+b)}{(y-x)} = 100 \times \frac{\text{Difference of their scores}}{\text{Difference of their } \% \text{ marks}}$$
.

EXAMPLE 18 A candidate scores 20% and fails by 45 marks and another candidate scores 45% marks gets 30 marks more than the minimum required to pass the examination. Find the maximum mark for the examination.

Solution Maximum marks =
$$\frac{100(45+30)}{(45-20)} = \frac{100\times75}{25} = 300$$

Theorem 12: In measuring the sides of a rectangle, one side is taken x% in excess and the other y% in deficit. The error per cent in the area calculated from the measurement is $x - y - \frac{xy}{100}$ in excess or deficit.

EXAMPLE 19 If the sides of a square are increased by 20%, find the percentage increase in its area.

Solution The percentage increase in area =
$$20 + 20 + \frac{20 \times 20}{100} = 40 + 4 = 44\%$$

Theorem 13: If the sides of a triangle, rectangle, square, circle, rhombus are increased by

$$x\%$$
, its area is increased by $\frac{x(x+200)}{100}\%$ or $\left(2x+\frac{x^2}{100}\right)\%$.

EXAMPLE 20 If the radius of a circle is increased by 10%, find the percentage increase in its area.

Solution Let the area of circle be πr^2 .

After 10% increase in radius, new area = $\pi \left(\frac{110r}{100}\right)^2 = 1.21 \pi r^2$

 $\therefore \qquad \text{Percentage increase in area} = \frac{(1.21 - 1)\pi r^2}{\pi r^2} \times 100 = 21\%$

Theorem 14: In an examination, x% candidates failed in English and y% candidates failed in Maths. If z% of candidates failed in both the subjects, the percentage of students who passed in both the subjects is 100 - (x + y - z).

EXAMPLE 21 In an examination, 40% of the candidates failed in Maths, 30% failed in English and 20% failed in both. Find the percentage of candidates who passed in both the subjects.

Solution Required percentage = 100 - (40 + 30 - 20) = 50%

Percentage Increase in Savings: If income of a person increases by y% and his expenditure increases by z% and his present expenditure is x% of his income, find the percentage increase in his savings.

Percentage increase in savings = $\frac{y \times 100 - z \times x}{100 - x}$

EXAMPLE 22 A person spends 60% of his income. His income increases by 15% and his expenditure increases by 5%, find the percentage increase in his savings.

Solution Percentage increase in savings = $\frac{15 \times 100 - 5 \times 60}{100 - 60} = \frac{1500 - 300}{40} = 30\%$

If the income is first increased by x% and then decreased by y%, then the effective

increase or decrease is given by $\left(x - y - \frac{xy}{100}\right)\%$.

EXAMPLE 23 The salary of a worker is first increased by 10% and then decreased by 20%. What is the percentage effect in his salary?

Solution Percentage effect = % Increase - % Decrease - $\frac{\% \text{ Increase} \times \% \text{ Decrease}}{100}$

$$= 10 - 20 - \frac{10 \times 20}{100} = 10 - 20 - 2 = -12\%$$

i.e. 12% decrease is the resultant effect.

Theorem 15: If a value is first increased by x% and then by y% the final increase is given by $\left(x+y+\frac{xy}{100}\right)\%$.

EXAMPLE 24 A trader sells his articles at 10% higher than their original price. Due to higher demand, he again increases the price by 15%. What will be the percentage profit the trader make?

Solution

% Profit = % First increase + % Second increase +
$$\frac{\text{% First increase} \times \text{% Second increase}}{100}$$

= $10 + 15 + \frac{10 \times 15}{100} = 10 + 15 + 1.5 = 26.5\%$

Effect on Revenue: If the price of a commodity in increased by x% and its consumption is decreased by y%, then the effect on revenue

= Increase % value – Decrease % value –
$$\frac{\text{Increase\% value} \times \text{Decrease \% value}}{100}$$

i.e. $\left(x - y - \frac{xy}{100}\right)$ %

EXAMPLE 25 The price of an item is increased by 15% and its sale gets decreased by 10%. What will be the percentage effect on the income?

Solution % Effect =
$$15 - 10 - \frac{15 \times 10}{100} = 5 - 1.5 = 3.5\%$$

i.e. a profit of 3.5%

:. His income is increased by 3.5%.

EXAMPLE 26 The price of an item is decreased by 10% and the sale is increased by 25%. What will be the percentage effect on income?

Solution % Effect =
$$25 - 10 - \frac{25 \times 10}{100} = 15 - 2.5 = 12.5\%$$

∴ His income is increased by 12.5%

Theorem 16: When a certain quantity of goods B is added to change the percentage of goods A in a mixture of A and B, then the quantity of B to be added is

$$\left(\frac{\text{Previous \% value of } A}{\text{Changed \% value of } A} \times \text{Mixture quantity}\right)$$
 – Mixture quantity

EXAMPLE 27 In 1 kg mixture of sand and cement, 20% is cement. How much sand should be added so that the proportion of cement becomes 10%?

Solution Quantity of cement in 1 kg mixture =
$$\frac{20}{100} \times 1000 = 200$$
 g

:. Sand =
$$1000 - 200 = 800 \text{ g}$$



Required quantity =
$$\left(\frac{\text{Previous \% value of } A}{\text{Changed \% value of } A} \times \text{Mixture quantity}\right)$$
 - Mixture quantity
= $\left(\frac{20}{10} \times 1\right) - 1 = 2 - 1 = 1 \text{ kg}$

EXAMPLE 28 Express the following as a decimal.

(i) 12% (ii) 16% (iii) 0.3% (iv) 0.06%

Solution

(i)
$$12\% = \frac{12}{100} = 0.12$$
 (ii) $16\% = \frac{16}{100} = 0.16$

(iii)
$$0.3\% = \frac{0.3}{100} = 0.003$$
 (iv) $0.06\% = \frac{0.06}{100} = 0.0006$

Express the following as rate percent: (i) $\frac{13}{25}$ (ii) $5\frac{3}{7}$ (iii) 0.007

Solution

(i)
$$\frac{13}{25} = \left(\frac{13}{25} \times 100\right)\% = 52\%$$
 (ii) $5\frac{3}{7} = \frac{38}{7} = \left(\frac{38}{7} \times 100\right)\% = 542.85\%$

(iii)
$$0.007 = \frac{7}{1000} = \left(\frac{7}{1000} \times 100\right)\% = 0.7\%$$

EXAMPLE 30 Express the following as a fraction

(i) 84% (ii) 6% (iii) 0.9% (iv) 0.08%

Solution

(i)
$$84\% = \frac{84}{100} = \frac{21}{25}$$
 (ii) $6\% = \frac{6}{100} = 0.06$

(iii)
$$0.9\% = \frac{0.9}{100} = \frac{9}{1000}$$
 (iv) $0.08\% = \frac{0.08}{100} = \frac{8}{10000} = \frac{1}{1250}$



EXERCISES

- 1. An empty fuel tank of a car was filled with ordinary type of petrol. When the tank was onethird empty, it was filled with high speed petrol. Again when the tank was one-third empty, it was filled with an ordinary type of petrol. Again, when the tank was one-third empty it was filled with high speed petrol. At this time what was the percentage of high speed petrol in the tank?
 - (a) $51\frac{23}{27}\%$ (b) $48\frac{4}{27}\%$ (c) $49\frac{4}{27}\%$ (d) $50\frac{23}{27}\%$

- (e) None of these

the rectangle will increase by

	(a) 56%	(b)	25%		(c)	$22\frac{2}{3}\%$	(d)	Data inadequate	(e)	N	one of	these
3.	In a company the workers and 20% is 126, what is the	of ι	unskilled v	vor]	kers	are permane		-				
	(a) 480	(b)	510		(c)	360	(d)	377	(e)	N	one of	these
4.	If the height of a the effect on its a		-	rea	sed	by 40% and i	ts bas	se is increased by	409	%, '	what v	will be
	(a) No change	(b)	8% decrea	se	(c)	16% decreas	e (d)	16% increase	(e)	8	% incr	ease
5.	The population during the seconduring the three	d yea	ar and 30%					-			-	
	(a) 20%	(b)	$23\frac{11}{15}\%$		(c)	$23\frac{13}{15}\%$	(d)	$26\frac{2}{3}\%$	(e)	N	one of	these
6.	The monthly in respectively. In the by 7%. The percentage of the p	ne ne	ext year his	sino	come	is increased						
	(a) 7%	(b)	21%		(c)	28%	(d)	35%	(e)	N	one of	these
7.	60 = ? % of 400											
	(a) 6	(b)	12		(c)	20	(d)	15	(e)	N	one of	these
8.	40% of ? = 240											
	(a) 60	(b)	6000		(c)	960	(d)	600	(e)	N	one of	these
9.	23% of 8040 + 42	2% o:	f 545 = ?%	of 3	3000)						
9.	23% of 8040 + 42 (a) 56.17		f 545 = ?% 63.54	of 3		71.04	(d)	69.27	(e)	N	one of	these
		(b) don	63.54 ate 5% of l	his	(c) sala	71.04 ry. On the d	ay of	donation he char	igeo	l h	is min	d and
	(a) 56.17 Sunil decided to donated ₹ 1687.8	(b) don	63.54 ate 5% of l which was	his 75'	(c) sala % of	71.04 ry. On the d	ay of	donation he char	igeo	l h	is min	d and
	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary?	(b) don 50, v	63.54 ate 5% of l which was	his 75 (b)	(c) sala % of	71.04 ry. On the da f what he ha	ay of	donation he char cided earlier. Ho	igeo	l h	is min	d and
10.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be donument of the remaining What is the amount of the sun of th	(b) don 50, v etern of h g am unt	63.54 ate 5% of leading to the second to the	his 75 (b) (e) to h	(c) sala % of ₹ 45 Nor nis w t 609 wif	71.04 ry. On the def what he had 5000 ne of these rife. He gave % on educations?	ay of d dec	donation he char cided earlier. Ho (c) ₹33750 of the remaining	nged w n	d h	is minch is S	d and Sunil's
10. 11.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be donument of the remaining What is the amount of the control	(b) don 50, v etern of h g am unt (b)	63.54 ate 5% of l which was mined his income ount he sp he gave to ₹ 5000	his 75' (b) (e) to hoent his	(c) sala % of ₹ 45 Nor is w t 609 wif (c)	71.04 ry. On the def what he had 5000 ne of these rife. He gave % on education e? ₹8000	ay of d dec 25% (on. No	donation he chan cided earlier. Ho (c) ₹33750 of the remaining ow the amount w	amo	d h nuc our hii	is min ch is S nt to h m in ₹	d and Sunil's is son. 2700.
10. 11.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be donument of the remaining What is the amount of the sun of th	(b) don 50, v etern of h g am unt (b) on 7	63.54 ate 5% of l which was mined his income ount he sp he gave to ₹5000 0% of the	his 75 (b) (e) to his	(c) sala % of ₹ 45 Nor nis w t 609 wif (c)	71.04 ry. On the da f what he ha 5000 ne of these wife. He gave % on education e? ₹ 8000 ates passed i	ay of decorated	donation he chan cided earlier. Ho (c) ₹33750 of the remaining ow the amount w ₹6000 glish, 80% passes	amo	d h nuc our hii	is min ch is \$ nt to h m in ₹	d and Sunil's is son. 2700.
10. 11.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be do Suresh gave 40% Of the remaining What is the amo (a) ₹ 4000 In an examination 10% failed in both sureshield in both salary.	(b) don 50, veter of h of h (b) on 7 oth 1	63.54 ate 5% of l which was mined his income ount he sp he gave to ₹5000 0% of the	his 75 (b) (e) to his	(c) sala % of ₹ 45 Nor is wif (c) dida If 1	71.04 ry. On the da f what he ha 5000 ne of these wife. He gave % on education e? ₹ 8000 ates passed i	ay of decorded decord	donation he chan cided earlier. Ho (c) ₹33750 of the remaining ow the amount w ₹6000 glish, 80% passes	amo	d h nuc our hii	is min ch is \$ nt to h m in ₹	d and Sunil's is son. 2700.
10. 11.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be done of the remaining what is the amount of the same of	(b) don 550, vector of h	ate 5% of lawhich was mined his income to the gave to ₹ 5000 0% of the the subject winner by number of	his 75 (b) (e) to his can ts.	(c) sala % of ₹ 45 Non is wif (c) dida If 1 (c) on di tain: oters	71.04 ry. On the def what he had 5000 ne of these wife. He gave % on education ? ₹ 8000 ates passed in 44 candidate 240 d not cast the sing 48% of the sin the election.	25% (d) (d) (d) (es paid) (in the total control with the control contr	donation he chanced donation he chanced earlier. Ho (c) ₹33750 of the remaining ow the amount w ₹6000 glish, 80% passes assed in both, the chanced in both and the chanced as the chanced donated	amorith	our hin Motal	is min ch is S nt to h m in ₹ Iathen l num	d and Sunil's is son. 2700.
10. 11. 12.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be do Suresh gave 40% Of the remaining What is the amo (a) ₹ 4000 In an examination 10% failed in bocandidates was (a) 125 About 8% of the x two candidates. 1100 votes. The 10 (a) 21000	(b) don to the done of he done of	63.54 ate 5% of leading to the subject the subject to the subject	his 75 (b) (e) to h bent his can ts.	(c) salas s	71.04 ry. On the def what he had 5000 ne of these wife. He gave % on education ? ₹ 8000 ates passed if 44 candidate 240 d not cast the ing 48% of the in the election 22000	25% (d) (d) (expression with the context of the con	donation he chanchided earlier. Ho (c) ₹33750 of the remaining ow the amount w ₹6000 glish, 80% passesed in both, the 375 tes. In this election as 27500	amo	our him him tal	is min ch is S nt to h m in ₹ Iathen l num	d and Sunil's is son. 2700.
10. 11. 12.	(a) 56.17 Sunil decided to donated ₹ 1687.8 salary? (a) ₹ 37500 (d) Cannot be done of the remaining what is the amount of the same of	(b) don to the done of he done of	63.54 ate 5% of leading to the subject the subject to the subject	his 75 (b) (e) to h bent his can ts.	(c) sala % of Normis with 60% wiff (c) dida If 1 (c) on didatain: oters (c) sed	71.04 ry. On the def what he had 5000 ne of these wife. He gave % on education ? ₹ 8000 ates passed if 44 candidate 240 d not cast the ing 48% of the in the election 22000	25% (d) (d) (expression with the context of the con	donation he chanchided earlier. Ho (c) ₹33750 of the remaining ow the amount w ₹6000 glish, 80% passesed in both, the 375 tes. In this election as 27500	amo	d h nuc our him otal	is min ch is S nt to h m in ₹ Iathen l num	is son. 2700. natics, ber of

2. The length of a rectangle increases by 20% and the breadth by 30%. Then the perimeter of

15.	Manoj's. If the		een the monthly inc		come is 10% less than noj is ₹ 1360, what is
	(a) ₹8100	(b) ₹ 10430	(c) ₹ 9000	(d) Data inadequ	ate (e) None of these
16.	The 75% of a nu	umber is 380 mo	re than 35% of the sa	ame number. What is	s 20% of that number?
	(a) 190	(b) 195.5	(c) 189.5	(d) 180	(e) None of these
17.	The length of a	rectangle is inc	reased by 30% and t	he breadth by 20%.	Then the perimeter of
	the rectangle v	vill increase by	what per cent.		
	(a) 56%	(b) 25%	(c) 26%	(d) Data inadequ	ate (e) None of these
18.		ater evaporate fi emaining solution		ution containing 10%	6 salt. The percentage
	(a) 12%	(b) 12.5%	(c) 8%	(d) 8.5%	(e) None of these
19.	_		ses and B in 90% of bing the same facts		percentage when they
	(a) 25%	(b) 26%	(c) 30%	(d) 35%	(e) None of these
20.	125% of 320 +	?% of 125 = 440			
	(a) 46	(b) 42	(c) 50	(d) 32	(e) None of these
21.					s and 20% of the girls
		ndi. Then what	percentage of boys v		
	(a) 16%		(b) 22%	(c) 55%	
	(d) 48%		(e) Can't be determ		
22 .	If the height of the effect on th				by 30%, what will be
	(a) Decrease b	y 22%	(b) Increase by 229	% (c) Decre	ase by 2%
	(d) Increase b	y 2%	(e) None of these		
23.	Evaluate 28%	of $450 + 45\%$ of	280.		[Bank PO]
	(a) 225	(b) 252	(c) 525	(d) 522	
24.		of a product wa on the total reve		the number sold inc	creased by 30%. What [RBI]
	(a) 20%	(b) 18%	(c) 17%	(d) -10%	
25 .		_	luced by 10%. By wl with his original sal	_	his reduced salary be [SSC]
	(a) 10%	(b) 9%	(c) 11.11%	(d) 9.99%	
26.	population dec	reased by 5%. I the population s	f the total population ize in the beginning	on is 9975 at the en of the first year? [F	ing the next year, the d of the second year, Hotel Management]
	(a) 8500	(b) 9000	(c) 9500	(d) 10000	
27 .			of percentage as:		[SSC]
	(a) 0.35%	(b) 3.5%	(c) 35%	(d) 350%	
28.		4% of 210 – ? =			[Bank PO]
	(a) 256	(b) 258	(c) 268	(d) 358	
29 .	860% of 50 + 5	0% of 860 = ?			[RBI]
					limi
	(a) 430	(b) 516	(c) 860	(d) 960	
30.	(a) 43045% of 750 - 2(a) 216	(b) 516	(c) 860(c) 236.50	(d) 960 (d) 245	[Bank PO]

31.	40%	of 1640 + ? =	35%	of 980 + 150%	% of	850 .		[SBI PO]
	(a) 3	372	(b)	842	(c)	962	(d)	1052
32.	60%	of 264 is the	sam	e as:				[Hotel Management]
	(a)	10% of 44	(b)	15% of 1056	(c)	30% of 132	(d)	None of these
33.	0.01	is what per co	ent o	of 0.1?				[SSC]
	<i>(</i>)	1	<i>a</i> >	1	<i>(</i>)	10	<i>(</i> 1)	100
	(a)	100	(p)	$\frac{1}{10}$	(c)	10	(a)	100
34.	Wha	t per cent of ₹	265	50 is ₹ 1987.50)?			[Hotel Management]
	(a) (60%	(b)	75%	(c)	80%	(d)	90%
35.	Raje	ev buys goods	s wo	rth ₹ 6650. He	e ge	ts a rebate of 6	3% o	on it. After getting the rebate, he
	pays	sales tax @ 1	0%.	Find the amo	unt	he will have to	pay	y for the goods. [MAT]
	(a) ₹	6876.10	(b)	₹ 6999.20	(c)	₹ 6654	(d)	₹ 7000
36.	4598	is 95% of?						[Bank PO]
	(a) 4	1800	(b)	4840	(c)	4850	(d)	4880
37.	?% o	f 360 = 129.6						[Bank PO]
	(a) 3	36	(b)	64	(c)	72	(d)	77
38.	65%	of $? = 20\%$ of	422.	.50				[Bank PO]
	(a) 8	34.5	(b)	130	(c)	139.425	(d)	200
39 .	?% o	f 932 + 30 = 3	09.6	30				[Bank PO]
	(a) 2		(b)			35	(d)	
40.				1700 = ?% of 3				[Bank PO]
	(a) 3	30	(b)	35	(c)	45	(d)	None of these
41.	If₹2	2800 is $\frac{2}{7}$ per	cen	t of the value	of a	house, the wor	rth c	of the house (in ₹) is [LIC]
						10,00,000		
42 .	If 12					f that number		
	(a) 2			120				
43.	Two-	fifths of one-tl						hat is 40 per cent of that number?
								[Bank PO]
	(a) '	72	(b)	84	(c)	136	(d)	140 (e) None of these
44.	If 35	% of a numbe	r is	12 less than 5	0%	of that number	r, th	en the number is [CBI]
	(a) 4	10	(b)	50	(c)	60	(d)	80
45.	The :	number whicl	h ex	ceeds 16% of i	t by	42 is:		[CBI]
	(a) {	50	(b)	52	(c)	58	(d)	60
46.	If a n	$\mathbf{umber} \ x \ \mathbf{is} \ 10$)% le	ess than anoth	er n			0% more than 125, then x is equal
	to							[SSC]
	, ,	123.75	` ′	140.55		143		150
47.	A nu		5 is	subtracted fro	m it	, reduces to its	8 0 p	percent. What is four-fifths of that [BSRB]
	(a) '		(b)			120		140
48.		sum of two nu numbers are	ımb	ers is 2490. If	6.5	% of one numb	er is	s equal to 8.5% of the other, than [IGNOU]
	(a) §	989, 1501	(b)	1011, 1479	(c)	1401, 1089	(d)	1411, 1079

[Hotel Management]

(d) None of these

49.	. A student multiplied a number	by $\frac{3}{5}$ instead of $\frac{5}{3}$.	What is the per-	centage error in the
	calculation?			[SSC]
	(a) 34% (b) 44%	(c) 54%	(d) 64%	
50.	 The population of a town increa cent increase of population per y 		2,62,500 in a deca	de. The average per [CBI]
	(a) 4.37% (b) 5%	(c) 6%	(d) 8.75%	
51.	. Two numbers A and B are such of 6% of A and 8% of B. Find the		A and 4% of B is	two-third of the sum [MBA]
	(a) 2:3 (b) 1:1	(c) 3:4	(d) 4:3	
52.	 The difference of two numbers is the larger number is 	s 20% of the larger nur	nber. If the smalle	r number is 20, then [SSC]
	(a) 25 (b) 45	(c) 50	(d) 80	
53.	8. 8% of the people eligible to vote those eligible to vote, who wer number of persons between 18 a to vote?	e between 18 and 21	, actually voted. I	In that election, the
	(a) 4.2 (b) 6.4	(c) 6.8	(d) 8	
54.	. In a certain school, 20% of stude	ents are below 8 years	of age. The numb	er of students above
	8 years of age is $\frac{2}{3}$ of the number number of students in the school		ears age which is 4	48. What is the total [Bank PO]
	(a) 72 (b) 80	(c) 120	(d) 150	(e) None of these
55.	 Two tailors X and Y are paid a to cent of the sum paid to Y, how r 			. If X is paid 120 per [NIFT]
	(a) $\stackrel{?}{\sim} 200$ (b) $\stackrel{?}{\sim} 250$	(c) ₹ 300	(d) None of the	
56 .	6. If $x\%$ of y is 100 and $y\%$ of z is 2	200, then find a relation	on between x and	z. [SSC]
	(a) $z = \frac{x}{2}$ (b) $z = 2x$	(c) $z = \frac{x}{4}$	(d) $z=4x$	
57.	7. If $A = x\%$ of y and $B = y\%$ of x, the	hen which of the follo	wing is true?	[Bank PO]
	(a) A is smaller than B			
	(b) A is greater than B			
	(c) Relationship between A and	d B cannot be determi	ined	
	(d) If x is smaller than y , then	A is greater than B		
	(e) None of these			
58.	3. If x is 80% of y , then what per constant x	ent of $2x$ is y ?		[CBI]
	_	(c) $66\frac{2}{3}\%$	(d) 80%	
59 .	2. If x is 90% of y , then what per constant x	ent of x is y ?		[SSC]
	(a) 90% (b) $101\frac{1}{9}\%$	(c) $111\frac{1}{9}\%$	(d) 190%	
60.	. A student secures 90%, 60% and	54% marks in test par	pers with 100, 150	and 200 respectively

as maximum marks. The percentage of his aggregate is

(c) 70

(b) 68

(a) 64

61.	In an examination, 5% of the applicants were found ineligible and 85% of the eligible candidates belonged to the general category. If 4275 eligible candidates belonged to other categories, then how many candidates applied for the examination? [Hotel Management] (a) 30000 (b) 35000 (c) 37000 (d) None of these
62.	405 sweets were distributed equally among children in such a way that the number of sweets received by each child is 20% of the total number of children. How many sweets did each child receive? [Bank PO]
	(a) 9 (b) 15 (c) 18 (d) 45
63.	From the salary of an officer, 10% is deducted as house rent, 20% of the rest, he spends on conveyance, 20% of the rest he pays as income tax and 10% of the balance, he spends on clothes. Then, he is left with ₹ 15552. Find his total salary. [LIC AAO]
	(a) $\stackrel{?}{\underset{?}{\sim}} 25000$ (b) $\stackrel{?}{\underset{?}{\sim}} 30000$ (c) $\stackrel{?}{\underset{?}{\sim}} 35000$ (d) $\stackrel{?}{\underset{?}{\sim}} 40000$
64.	Sameer spends 24% of his monthly income on food and 15% on the education of his children. Of the remaining salary, he spends 25% on entertainment and 20% on conveyance. He is now left with ₹ 10736. What is the monthly salary of Sameer? [Bank PO] (a) ₹ 27600 (b) ₹ 28000 (c) ₹ 31200 (d) ₹ 32000
65.	Rohit spends 40% of his salary on food, 20% on house rent, 10% on entertainment and 10% on conveyance. If his savings at the end of a month are ₹ 1500, then his monthly salary is [SSC]
	(a) $\stackrel{?}{\stackrel{?}{$\sim}} 6000$ (b) $\stackrel{?}{\stackrel{?}{$\sim}} 7500$ (c) $\stackrel{?}{\stackrel{?}{$\sim}} 8000$ (d) $\stackrel{?}{\stackrel{?}{$\sim}} 10000$
66.	Peter could save 10% of his income. But two years later when his income was increased by 20%, he could save the same amount only as before. By how much per cent has his expenditure increased? [RRB]
	(a) 22% (b) $22\frac{2}{9}\%$ (c) $23\frac{1}{3}\%$ (d) 24%
67.	The price of an article was increased by $r\%$. Later the new price was decreased by $r\%$. If the latest price was $thm:thm:thm:thm:thm:thm:thm:thm:thm:thm:$
	(a) $\not\equiv 1$ (b) $\not\equiv \left(\frac{1-r^2}{100}\right)$ (c) $\not\equiv \frac{\sqrt{1-r^2}}{100}$ (d) $\not\equiv \left(\frac{10000}{10000-r^2}\right)$
68.	A number is decreased by 10% and then increased by 10%. The number so obtained is 10 less than the original number what was the original number? $[SSC]$
	(a) 1000 (b) 1050 (c) 1500 (d) 2000
69.	If the price of a book is first decreased by 25% and then increased by 20%, then the net change in the price will be [SSC]
	(a) No change (b) 5% increase (c) 5% decrease (d) 10% decrease
70.	$37\frac{1}{2}\%$ of the candidates in an examination were girls, 75% of the boys and $62\frac{1}{2}\%$ of the
	girls passed and 342 girls failed. The number of boys failed was [SSC]
	(a) 350 (b) 360 (c) 370 (d) 380
71.	The price of a shirt is increased by 15% and then reduced by 15%. The final price of the shirt [Hotel Management]
	(a) does not change (b) increases by 2.25%
	(c) decreases by 2.25% (d) None of these

[DMRC]

	(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
73.	One type of liquid contains 20% water and the second type of liquid contains 35% of water. A glass is filled with 10 parts of first liquid and 4 parts of second liquid. The percentage of water in the new mixture in the glass is [CBI]
	(a) 20% (b) $24\frac{2}{7}\%$ (c) 37% (d) 40%
74.	Milk contains 5% water. What quantity of pure milk should be added to 10 litres of milk to reduce this to 2%? [Bank PO]
	(a) 5 litres (b) 7 litres (c) 15 litres
	(d) cannot be determined
75 .	How much pure alcohol has to be added to 400 ml of a solution containing 15% alcohol to change the concentration of alcohol in the mixture to 32%? [SSC]
	(a) 60 ml (b) 68 ml (c) 100 ml (d) 128 ml
76.	Fresh fruits contain 68% water and dry fruits contain 20% water. How much dry fruit can be obtained from 100 kg of fresh fruits? [SSC]
	(a) 32 kg (b) 40 kg (c) 52 kg (d) 80 kg
77.	In an examination, the percentage of students qualified to the number of students appeared from school A is 70%. In school B, the number of students appeared is 20% more than the students appeared from school A and the number of students qualified from school B is 50% more than the students qualified from school A. What is the percentage of students qualified to the number of students appeared from school B? [Bank PO] (a) 30% (b) 70% (c) 78.5% (d) 87.5%
78.	In an examination, 65% students passed in Civics and 60% in History, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students? [RRB]
	(a) 600 (b) 650 (c) 700 (d) 750
79.	In a hotel, 60% had vegetarian lunch while 30% had non-vegetarian lunch and 15% had both types of lunch. If 96 people were present, how many did not each either type of lunch? [SSC]
	(a) 20 (b) 24 (c) 26 (d) 28
80.	In an examination, 34% of students failed in Mathematics and 42% failed in English. If 20%
	of the students failed in both the subjects, then the percentage of students who passed in both the subject was [SSC]
81.	both the subject was [SSC]
	both the subject was [SSC] (a) 44 (b) 50 (c) 54 (d) 56 In an examination in which full marks were 800, A gets 20% more than B, B gets 20% more than C and C gets 15% less than D. If A got 576, what percentage of full marks did D get (approximately)? [RRB] (a) 45.7 (b) 51.2 (c) 58.8 (d) 61.7
	both the subject was [SSC] (a) 44 (b) 50 (c) 54 (d) 56 In an examination in which full marks were 800, A gets 20% more than B, B gets 20% more than C and C gets 15% less than D. If A got 576, what percentage of full marks did D get (approximately)? [RRB] (a) 45.7 (b) 51.2 (c) 58.8 (d) 61.7 Amit's monthly income is 30% more than that of Raunaq. Raunaq's monthly income is 20% less than that of Deepak. If the difference between the monthly incomes of Amit and Deepak
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72. The weight of a container alone is 25% of the container filled with a certain fluid. When some fluid is removed, the weight of the container and remaining fluid is 50% of the original

total weight. What fractional part of the liquid has been removed?

83.				ll, 175 play cricket and 50 p football and cricket?	neither play [SSC]
	(a) 50	(b) 75	(c) 100	(d) 225	
84.	The price of rice	is reduced by 2%.	How many kilo	ograms of rice can now be bo	ught for the
	money which was	s sufficient to buy	49 kg of rice ea	rlier?	[SSC]
	(a) 48 kg	(b) 49 kg	(c) 50 kg	(d) 51 kg	
85.	An empty fuel ta	nk of a car was fil	led with A type	petrol. When the tank was h	alf-empty, it
				was half-empty, it was filled	
				as filled with B-type petrol.	
	(a) 33.5%	type petrol at pres (b) 37.5%	(c) 40%	(d) 50%	[Bank PO]
9.6		` '		` '	is repeated
00.		r naving pure min d of the third oper		ed by water and the process	[SSC]
	(a) 40% pure		(c) 51.2% pu		[SSC]
87.		_	_	B eggs less are available for	₹ 7.80 The
• • • •	present rate of eg		price or eggs,	oggs loss are available for	[NIFT]
	(a) ₹ 8.64	(b) ₹ 8.88	(c) ₹ 9.36	(d) ₹ 10.40	
88.	The ratio 5: 4 ex	pressed in percen		, ,	[SSC]
	(a) 12.5%	(b) 40%	(c) 80%	(d) 125%	
89.		t written as a deci	mal is	, ,	[SSC]
	(a) 0.005	(b) 0.05	(c) 0.02	(d) 0.2	
90.	What is 15 per ce		` '	· ,	[IMT]
	(a) ₹ 3.40	(b) ₹ 3.75	(c) ₹ 4.50	(d) ₹ 5.10	
91.	What per cent of	7.2 kg is 18 gm?	• •	, ,	[SSC]
	(a) 0.025%	(b) 0.25%	(c) 2.5%	(d) 25%	
92.		a day is 3 hours?			[RRB]
			2	1	
	(a) $12\frac{1}{2}\%$	(b) $16\frac{2}{3}\%$	(c) $18\frac{2}{3}\%$	(d) $22\frac{1}{2}\%$	
93.	-	_	_	n sale. If she spent ₹ 25 fo	or the item,
		ow much per cent			n Officer's]
	(a) 8%	(b) 9%	(c) 10%	(d) 11%	
94.	5% of (25% of ₹ 1	.600) is			
	(a) ₹5	(b) ₹ 17.50	(c) ₹ 20	(d) ₹ 25	
95.	0.15% of $33\frac{1}{3}\%$	of₹ 10000 is			[SSC]
		(b) ₹5	(a) ₹ 10E	(J) 7 150	[000]
0.C				(d) ₹ 150 s 510. What is 10% of that nu	h o.u?
90.	The difference be			s 510. what is 10% of that he	[Bank PO]
	(a) 12.75	(b) 85	(c) 204	(d) None of these	
97.	What percentage	of numbers from	1 to 70 has squ	ares that end in the digit 1?	[MBA]
	(a) 1	(b) 14	(c) 20	(d) 21	
98.	The sum of two n	numbers is $\frac{28}{25}$ of	the first number	r. The second number is wha	t per cent of
	the first?	23			nagement]
	(a) 12%	(b) 14%	(c) 16%	(d) 18%	<i>5</i>

om the total appeared nd 7% candidates got number of candidates [SBI PO]	ites appeared a	ber of candid	d an equal num candidates got s	idates. State B h	c s
equate	(d) Data inade	8400	3000 (c)	7600 (b)	(
The car was damaged insurance. What was [Bank PO]	paid 90% of the	ance company	nt and the insu	_	c
	(d) ₹81250	₹ 76375	₹ 48750 (c)	₹ 32500 (b)	(
628 votes respectively. [IMT]				t percentage of th	V
	(d) 90%	65%	60% (c)	57% (b)	(
ares is 656, then the Hotel Management]		4 times the	of the other and	e number is 80% bers are	
ese	(d) None of th	16, 20	3, 10 (c)	4, 5 (b)	(
the other number. By [Bank PO]				n any number is d much per cent fir	
equate	(d) Data inad	300	200 (c)	150 (b)	(
number reduces to its r? [SBI PO]				% of a number is sixths. What is th	
equate	(d) Data inade	3:2	2:3 (c)	1:3 (b)	(
nich 30 paise went on the cost of the tax free [MAT]		_			s
	(d) ₹ 20	₹ 19.70	₹ 15.70 (c)	₹ 15 (b)	(
. What per cent of his [SSC]				tsman scored 110 score did he mal	
	(d) 55%	$54\frac{6}{11}\%$	$45\frac{5}{11}\%$ (c)	45% (b)	(
apples. Originally, he [SSC]	nd still has 420	40% apples a	apples. He sells	uit seller had som	107. A
}	(d) 700 apples	672 apples	600 apples (c)	588 apples (b)	(
arks and failed by 40 [CBI]	ss. He got 125 m	l marks to pa	_	ident has to obta ks. The maximum	
	(d) 1000	800	500 (c)	300 (b)	(
andidates. 10% of the ne valid votes and won er's list was [SSC]	ate got 54% of th olled on the vot	cessful candid r of voters en	invalid. The suc otes. The numbe	s polled were foun majority of 1620	v b
rotes, 20% of the votes	(d) 40000	35000	` ,	` ,	,
id votes that the other [RRB]					v
	(d) 3100	3000	2900 (c)		
ny students appeared [SSC]	5 failed. How ma	passed and 45	of the students	n examination, 35 he examination?	
	(d) 1300	845	700 (c)	490 (b)	(

(a) 40

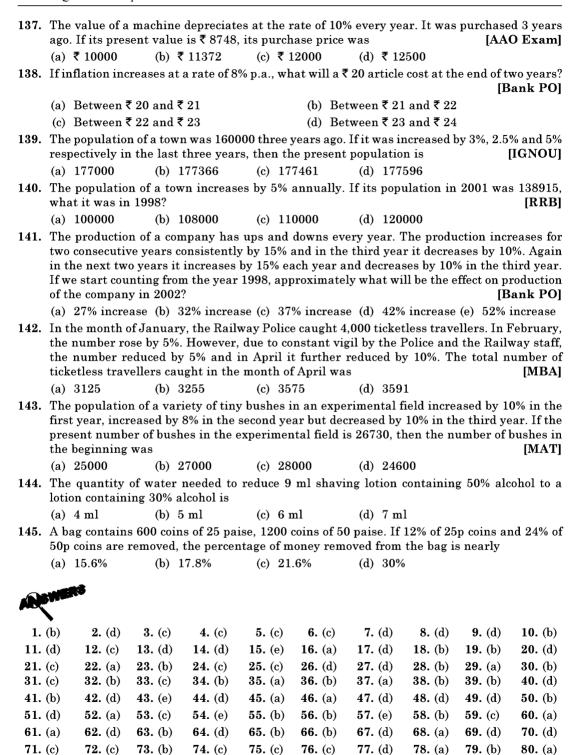
(b) 50

112.	A person who spen	ids $66\frac{2}{3}\%$ of his	s income is able to	save ₹ 1200 per n	nonth. His monthly			
	expenses (in ₹) are	3			[SSC]			
	(a) ₹ 1200	(b) ₹ 2400	(c) ₹ 3000	(d) ₹ 3200				
113.	In an election a car What is the total n				ajority of 476 votes. otel Management]			
	(a) 672	(b) 700	(c) 749	(d) 848				
114.	If P% of P is 36, the (a) 15	en <i>P</i> is equal to (b) 60	(c) 600	(d) 3600	[SSC]			
115.	If $x\%$ of y is equal t	` '	(-)	(a) 3000	[SSC]			
	· -			100^{2}	Ç-:1			
	(a) $\frac{y^2}{100}$	(b) $\frac{y}{100^2}$	(c) $\frac{100}{100}$	(d) $\frac{100}{v}$				
116	Aman gave 40% of		•	•	a faunth of what ha			
110.	f the amount he got Aman have? [Bank PO]							
	(a) ₹ 4000	(b) ₹8000	(c) ₹ 12000	(d) Data inadeq				
117.	A spider climbed 62	_						
	A spider climbed $62\frac{1}{2}\%$ of the height of the pole in one hour and in the next hour it covered							
	$12\frac{1}{2}\%$ of the rema	ining height. If t	the height of the po	ole is 192 m, then	distance climbed in			
	second hour is]	Section Officer's]			
	(a) 3 m	(b) 5 m	(c) 7 m	(d) 9 m				
118.	Of the 1000 inhabi inhabitants, 25% as							
	(a) 22.5	(b) 27.5	(c) 32.5	(d) 37.5	. ,			
119.	In a recent survey, only one person, 25 contain exactly one	5% were having or e female and no i	only a male. What males?	is the percentage	houses containing of all houses, which [SBI PO]			
	(a) 15	(b) 4	40	(c) 75				
	(d) Cannot be dete	ermined (e)	None of these					
120.	A scored 30% mark more than that req	quired to pass. Th			obtained 35 marks [SSC]			
	` '	(b) 38%	(c) 43%	(d) 46%				
121.	students of the class	nd 20% scored 90 ss?	0 marks, what was	the average mar	10% of the students as of the remaining [DMRC]			
	` '	(b) 72.5	(c) 75	(d) 85				
122.	In an examination, pass. In one paper, get, out of 180, in t	he gets 62 out of	f 150 and in the sec	ond 35 out of 150.	35% of the total to How much must he [RRB]			
	(a) 60.5	(b) 68	(c) 70	(d) 71				
123.	The sum of the nur the number of girls							

(c) 60

(d) 90

124.	A salesman is allowed $5\frac{1}{2}\%$ discount on the total sales made by him plus a bonus of $\frac{1}{2}\%$ on						
				90, then his total sales (in ₹)			
	(a) 30000	(b) 32000	(c) 34000	(d) 35000	[CBI]		
125	` '	, ,		earned 20% less than Michae	l Peter		
120.	earned more tha	n Michael by			[SSC]		
100	(a) 10%		(c) 20%	(d) 25%	,		
126.	A's salary is 40% salary?		. 18 25% of Us salar	y. What percentage of C's sala	ry is A's [MBA]		
	(a) 5%	(b) 10%	(c) 15%	(d) 20%			
127.				is vanaspati. If 10 kg of pure 20%. The original quantity wa [Hotel Manag	s		
	(a) 10 kg	(b) 15 kg	(c) 20 kg	(d) 25 kg			
128.	Two numbers are	e respectively $12\frac{1}{2}$	% and 25% more t	han a third number. The first	number		
	as a percentage of	of the second numb	er is		[CBI]		
	(a) 50	(b) 60	(c) 75	(d) 90			
129.		e less than a third n mber less than the		37% respectively. How much	per cent [SSC]		
	(a) 3%	(b) 4%	(c) 7%	(d) 10%			
130.				cent B's height is more than th	at of A? [SSC]		
	(a) $33\frac{1}{3}\%$	(b) 40%	(c) 60%	(d) $66\frac{2}{3}\%$			
131.	A's salary is 50%	6 more than B's. Ho	ow much per cent i	s B's salary less than A's?	[SSC]		
	(a) 33%	(b) $33\frac{1}{4}\%$	(c) $33\frac{1}{3}\%$	(d) $33\frac{1}{2}\%$			
132.	The population of a town increases 4% annually but is decreased by emigration annually the extent of (1/2)%. What will be the increase per cent in 3 years?						
	(a) 9.8	(b) 10	(c) 10.5	(d) 10.8			
133.	and 15% respect		ently the population	•			
	(a) 2000	(b) 2500	(c) 3000	(d) 4000			
134.	Depreciation approximation now will be less l		oment is 20%. The	value of the equipment 3 year	rs from [MBA]		
	(a) 45%	(b) 48.8%	(c) 51.2%	(d) 60 %			
135.		of a town 2 years ag e rate of 4%. The pr		to migration to big cities, it de of the town is:	ecreases [SSC]		
	(a) 56700	(b) 57600	(c) 58800	(d) 60000			
136.	A district has 640	000 inhabitants. If	the population inc	reases at the rate of $2\frac{1}{2}\%$ per	annum,		
		r of inhabitants at t		<u> </u>	[SSC]		
	(a) 68911	(b) 68921	(c) 69200	(d) 70000	_		



(1)



${f S}$ olutions with Necessary Explanation

1. Let the capacity of tank be 27 litres.

	Ordinary	${\it High\ speed}$
I	27	Nil
II	18	9
III	12 + 9 = 21	6
IV	14	4 + 9 = 13

Hence, the required per cent of high speed petrol in the tank

$$\frac{13}{14+13} \times 100 = \frac{13}{27} \times 100 = 48 \frac{4}{27} \%$$

2. Let length and breadth of rectangle be 'l' and 'b' units and perimeter be 'p' units. Then p = 2 (l + b)

Here l is increased to 1.2l and b is increased to 1.3b

 \therefore New value of perimeter = 2 (1.2l + 1.3b)

$$= 2.4l + 2.6b$$

Hence answer is data inadequate.

- 3. Suppose the total number of workers in the company is 1000. Then the number of
 - (i) Skilled workers = 75% of 1000 = 750
 - (ii) Unskilled workers = 25% of 1000 = 250
 - (iii) Permanent workers = $\frac{80}{100} \times 750 + \frac{20}{100} \times 250 = 600 + 50 = 650$
 - (iv) Temporary workers = 1000 650 = 350

Again, since the number of temporary workers is 126, the total number of workers in the company = $\frac{1000}{350} \times 126 = 360$

- **4.** Required percentage in the area = $-40 + 40 + \frac{(-40) \times 40}{100} = -16\%$ (16% decrease)
- 5. The population after the three years will be

$$123456789 \times \frac{110}{100} \times \frac{120}{100} \times \frac{130}{100} = 123456789 \times \frac{171.6}{100}$$

.. Overall percentage increase in the previous population during the three years = 71.6%

Average per cent increase during the three years $=\frac{71.6}{3}=\frac{716}{30}=23\frac{13}{15}\%$

6. Monthly income of the person = ₹ 13500

Monthly expenditure of the person = ₹ 9000

Savings of the person = ₹ 4500

Next year his monthly income = $\frac{114}{100} \times 13500 = 15,390$

His monthly expenditure = $\frac{107}{100} \times 9000 = ₹9630$

His savings =
$$114 \times 135 - 107 \times 90$$

= $45(342 - 214)$
= 45×128

:. Percentage increase is savings =
$$\frac{45 \times 128 - 45 \times 100}{45 \times 100} = \frac{45 \times 28}{45 \times 100} = 28 \%$$

7.
$$60 = ?\% \text{ of } 400$$

$$\Rightarrow \qquad 60 = ? \times \frac{400}{100}$$

$$\Rightarrow$$
 60 = ? × 4

$$\Rightarrow \qquad ? = \frac{60}{4} = 15$$

$$\Rightarrow \frac{40 \times ?}{100} = 240$$

$$\Rightarrow \qquad ? = \frac{240 \times 100}{40} = 600$$

9.
$$23\%$$
 of $8040 + 42\%$ of $545 = ?\%$ of 3000

$$\Rightarrow \qquad 23 \times \frac{8040}{100} + 42 \times \frac{545}{100} = \frac{3000}{100} \times ?$$

$$\Rightarrow$$
 3000 × ? = 23 × 8040 + 42 × 545

$$\Rightarrow$$
 3000 × ? = 184920 + 22890

$$\Rightarrow 3000 \times ? = 207810$$

$$\Rightarrow ? = \frac{207810}{3000} = 69.27$$

10. Let Sunil's salary be \overline{x}

$$5\% \text{ of } x = \frac{75x}{100} = \frac{7x}{20}$$

i.e.
$$1687.50 = \frac{75}{100} \times \frac{x}{20} = \frac{3x}{80}$$

$$\Rightarrow 3x = 1687.50 \times 80$$

⇒
$$x = \frac{1,687.50 \times 80}{3} = ₹ 45000$$

11. The amount given to wife $\rightarrow 40\%$ (remaining 60%)

The amount given to son \rightarrow 25% of 60% = 15% (remaining 45%)

The amount spent on education \rightarrow 60% of 45% = 27% (remaining 18%)

∴ 18% → ₹ 2700

∴
$$40\% \to \frac{2700}{18} \times 40 = ₹ 6000$$

12. Let the total number of candidates be 100

Number of candidates passed in English = 70

- :. Failed candidates in English = 30
- \therefore Number of failed candidates in English alone = 30 10 = 20

Number of candidates passed in Maths = 80

Number of candidates failed in Maths = 20

Number of candidates failed in Maths alone = 20 - 10 = 10

Number of candidates failed in both = 10

- \therefore Total number of failures = 20 + 10 + 10 = 40
- \therefore Total number of candidates passed = 100 40 = 60
- :. Strength of candidates attended the examination for a pass of 144 candidates

$$=\frac{100}{60}\times144=240$$

- 13. Let the total number of votes be 100.
 - .. Number of uncast votes = 8

Votes polled = 100 - 8 = 92

Number of votes obtained by the winner = 48

Number of votes obtained by the loser = 92 - 48 = 44

- \therefore The difference in votes obtained by the winner and loser = 48 44 = 4
- \therefore Given difference in votes = 1100

$$= \frac{100}{4} \times 1100 = 27500$$

- 14. The required percentage increase = $7 + 7 + \frac{7 \times 7}{100} = 14.49\%$
- 15. Let the monthly income of Manoj be 100.

∴ Ashok's income =
$$\frac{90}{100} \times 100 = ₹90$$

Monthly income of Anil =
$$\frac{130}{100}$$
 × 90 = ₹ 117

Difference between income of Anil and Manoj = 117 – 100 = ₹ 17

∴ Monthly income of Ashok for a difference in salary of ₹ 1360

$$=\frac{90}{17} \times 1360 = ₹ 7200$$

16. Let the number be x.

$$\therefore$$
 (75 – 35)% of $x = 380$

i.e.
$$\frac{40}{100} \times x = 380$$

$$\therefore \frac{20}{100} \times x = \frac{380}{2} = 190$$

17. Require more information like ratio of the length of the rectangle to its breadth.

18. Required per cent =
$$\frac{10}{(10-2)} \times 10 = 12.5 \%$$

19. Required percentage =
$$\frac{8}{10} \times \frac{1}{10} + \frac{9}{10} \times \frac{2}{10}$$

= $\frac{8}{100} + \frac{18}{100} = \frac{26}{100} = 26\%$

20.
$$\frac{125}{100} \times 320 + \frac{?}{100} \times 125 = 440$$

$$\Rightarrow \frac{?}{100} \times 125 = 440 - \frac{125}{100} \times 320$$

$$= 440 - 400 = 40$$

$$\therefore ? = \frac{40 \times 100}{125} = 32$$

21. The students who do not read Hindi = 100 - 70 = 30

Number of boys = 40; Number of girls = 60

Number of girls who does not read Hindi = 20% of 60 = 12

Number of boys who does not read Hindi = 30 - 12 = 18

$$\therefore$$
 No. of boys who reads Hindi = $40 - 18 = 22$

:. Percentage of boys who read Hindi =
$$\frac{22}{40} \times 100 = 55\%$$

22. Net effect on area =
$$\left(-40 + 30 - \frac{40 \times 30}{100}\right)\% = -22\%$$

i.e. decrease by 22%

23. 28% of 450 + 45% of 280 =
$$\frac{28}{100} \times 450 + \frac{45}{100} \times 280 = 126 + 126 = 252$$

24. Let the original price of commodity be \overline{x} .

Due to decrease in price by 10%, new price = 0.9x.

Selling quantity increased by 30%

$$\therefore$$
 Effect on revenue = $0.9x \times 1.3 = 1.17x$

.: Selling of commodity increased by 17%.

Effect on revenue = (-10 + 30 - 3)% = 17%

25. Let x be the per cent by which reduced salary be raised so as to bring it to the original

Then
$$\left(-10 + x - \frac{10x}{100}\right)\% = 0$$
i.e.
$$x - \frac{x}{10} - 10 = 0$$
or
$$\frac{9x}{10} = 10$$
or
$$x = \frac{10 \times 10}{9} = 11.11\%$$

26. Let the population at the beginning of a year be x

Then
$$x\left(1+\frac{5}{100}\right)\left(1-\frac{5}{100}\right) = 9975$$

$$\therefore \qquad x = \frac{9975 \times 100 \times 100}{105 \times 95} = 10000$$

27.
$$3.5 = \frac{350}{100} = 350\%$$

28.
$$88\%$$
 of $370 + 24\%$ of $210 - ? = 118$

$$\therefore ? = \frac{88}{100} \times 370 + \frac{24}{100} \times 210 - 118$$
$$= 325.6 + 50.4 - 118 = 258$$

29. 860% of 50 + 50% of 860 =
$$\frac{860}{100} \times 50 + \frac{50}{100} \times 860 = 430 + 430 = 860$$

30. 45% of 750 – 25% of 480 =
$$\frac{45}{100} \times 750 - \frac{25}{100} \times 480 = 337.50 - 120 = 217.50$$

$$\therefore$$
 ? = 35% of 980 + 150% of 850 - 40% of 1640

$$= \frac{35}{100} \times 980 + \frac{150}{100} \times 850 - \frac{40}{100} \times 1640$$

$$=343+1275-656=962$$

33.
$$0.01 = x\% \text{ of } 0.1 = \frac{x}{100} \times 0.1$$

$$\therefore \qquad x = \frac{0.01 \times 100}{0.1} = 10$$

34.
$$x\%$$
 of $2650 = 1987.50$

i.e.
$$\frac{x}{100} \times 2650 = 1987.50$$

$$\therefore \qquad x = \frac{1987.5}{2650} \times 100 = 75$$

35. Rajeev got 6% rebate as ₹ 6650

∴ Net price =
$$6650 \times 0.94 = ₹ 6251$$

Tax on net price @
$$10\% = \frac{10}{100} \times 6251 = ₹ 625.10$$

∴ Total price =
$$6251 + 625.10 = ₹ 6876.10$$



Total effect on the price due to rebate and sales tax in percentage

$$\left(-6 + 10 - \frac{6 \times 10}{100}\right)\% = 3.4\%$$

i.e.
$$6650 \times \frac{3.4}{100} = 226.10$$

$$\therefore$$
 Total price = 6650 + 226.10 = ₹ 6876.10

36. Let
$$4598 = \frac{95}{100}$$
 of $x = \frac{95x}{100}$ or $x = \frac{4598 \times 100}{95} = 242 \times 20 = 4840$

37.
$$x\%$$
 of $360 = 129.6$

i.e.
$$\frac{x}{100} \times 360 = 129.6$$

or
$$x = \frac{129.6 \times 100}{360} = 36$$

38.
$$65\%$$
 of $x = 20\%$ of 422.50

i.e.
$$\frac{65}{100} \times x = \frac{20}{100} \times 422.50$$

or $x = \frac{20}{100} \times 422.50 \times \frac{100}{65}$
 $= \frac{4 \times 422.5}{13} = 130$

39. Let
$$x\%$$
 of $932 + 30 = 309.6$

i.e.
$$\frac{x}{100} \times 932 + 30 = 309.6$$

or
$$x = (309.6 - 30) \times \frac{100}{932}$$

= $\frac{27960}{932} = 30$

40. Let
$$45\%$$
 of $1500 + 35\%$ of $1700 = x\%$ of 3175

i.e.
$$\frac{45}{100} \times 1500 + \frac{35}{100} \times 1700 = \frac{x}{100} \times 3175$$

i.e.
$$x = (45 \times 15 + 35 \times 17) \frac{100}{3175}$$
$$= (675 + 595) \frac{100}{3175} = 1270 \times \frac{100}{3175} = 40$$

41. Let the value of house be $\not\in x$.

Given
$$2800 = \frac{2}{700} \times x$$

or $x = \frac{2800 \times 700}{2} = ₹980000$

42. Let the number be x.

$$120 = \frac{20}{100} \text{ of } x$$

$$\therefore \quad \frac{120}{100} \text{ of } x = 6 \times 120 = 720$$

43. Let the number be x.

$$\frac{2}{5}$$
 of $\frac{1}{3}$ of $\frac{3}{7}$ of $x = 15$

$$\therefore \qquad x = 15 \times \frac{7}{3} \times 3 \times \frac{5}{2}$$

$$\therefore \frac{40}{100} \text{ of } x = \frac{40}{100} \times 15 \times \frac{7}{3} \times 3 \times \frac{5}{2}$$
$$= 5 \times 7 \times 3 = 105$$

44. Let the number be x.

$$\therefore \frac{35}{100} \times x = \frac{50}{100} \times x - 12$$

i.e.
$$\left(\frac{50-35}{100}\right)x = 12$$

$$\mathbf{or}$$

$$x = 12 \times \frac{100}{15} = 80$$

45. Let the number be x.

$$\therefore \qquad x - \frac{16}{100} x = 42$$

$$x = 42 \times \frac{100}{84} = 50$$

46. Given x = 0.9y

$$y = 125 \times 1.1$$

$$\therefore x = 0.9 \times 125 \times 1.1$$
$$= 125 \times 0.99 = 123.75$$

47. Let the number be x.

$$x - 35 = 0.8x$$

or
$$x(1-0.8) = 35$$

$$x = \frac{35}{0.2}$$

$$\therefore \frac{4}{5}x = \frac{4}{5} \times \frac{35}{0.2} = 140$$

48. Let x and y be the numbers.

$$x + y = 2490 \tag{1}$$

$$\frac{6.5}{100}x = \frac{8.5}{100} \times y \tag{2}$$

(2)
$$\Rightarrow$$
 $x = \frac{8.5}{6.5}y = \frac{17}{13}y$ (3)

Substituting value of x from (3) in (1),

$$\frac{17}{13}y + y = 2490$$

$$\Rightarrow \frac{30}{13}y = 2490$$

⇒
$$y = \frac{2490 \times 13}{30}$$

= 83 × 13 = **1079**
∴ (1) ⇒ $x = 2490 - 1079 = 1411$

49. % error =
$$\frac{3/5 - 5/3}{5/3} \times 100 = \frac{(9 - 25)/15}{5/3} \times 100$$

= $\frac{-16}{15} \times \frac{3}{5} \times 100 = \frac{-16}{25} \times 100 = -64\%$

50. Average percentage increase of population/year

$$= \frac{262500 - 175000}{175000 \times 10} \times 100$$
$$= \frac{87500}{175000} \times \frac{100}{10} = 5\%$$

51. 5% of A + 4% of B =
$$\frac{2}{3}$$
 of (6% of A + 8% of B)

i.e.
$$\frac{5}{100} A + \frac{4}{100} B = \frac{2}{3} \left(\frac{6}{100} A + \frac{8}{100} B \right)$$

i.e.
$$A\left(\frac{5}{100} - \frac{2}{3} \times \frac{6}{100}\right) = B\left(\frac{8}{100} \times \frac{2}{3} - \frac{4}{100}\right)$$
$$A\left(\frac{5-4}{100}\right) = B\left(\frac{16-12}{300}\right)$$

$$\begin{array}{c} A & A & 100 \end{array}$$

$$\therefore \frac{A}{B} = \frac{4}{300} \times \frac{100}{1} = \frac{4}{3}$$

52. Let the numbers be x and y.

Given Then,

$$x - y = \frac{20}{100} \times x$$

$$y = 20;$$
(1)

(1)
$$\Rightarrow$$
 $x\left(1 - \frac{20}{100}\right) = y$
 \Rightarrow $x \times \frac{80}{100} = 20$

$$x = \frac{100 \times 20}{80} = 25$$

53.
$$\frac{85}{100} \times \frac{8}{100} \times x = \frac{680}{10000} x$$
$$= \frac{6.8}{100} \times x$$

i.e. **6.8**%

54. Let the total number of students be x.

Number of students below 8 years = $\frac{20}{100} \times x$

Number of students of 8 years age = 48

$$\therefore \text{ Number of students above 8 years} = x - \frac{20}{100}x - 48 = 0.8x - 48$$

Given

$$0.8x - 48 = \frac{2}{3}(48) = 32$$

or

$$0.8x = 32 + 48 = 80$$

or

$$x = \frac{80}{0.8} = 100$$

55. Let amount paid be x and y.

$$x + y = 550 \tag{1}$$

Given

$$x = \frac{120}{100} y \tag{2}$$

$$\therefore (1) \Rightarrow 1.2y + y = 550$$

$$\Rightarrow$$

$$y = \frac{550}{2.2} = 250$$

56. Given

$$\frac{x}{100} \times y = 100 \tag{1}$$

$$\frac{y}{100} \times z = 200 \tag{2}$$

$$\frac{(1)}{(2)}$$
 \Rightarrow $\frac{x}{z} = \frac{1}{2}$

$$\frac{x}{z} = \frac{1}{2}$$

$$z = 2x$$

57. Given $A = \frac{x}{100} \times y$; $B = \frac{y}{100} \times x$;

$$\therefore$$
 $A = B$

58. Given
$$x = \frac{80}{100} \times y$$

(1)

Let P% of 2x = y

$$\frac{P}{100} \times 2x = y$$

$$P = \frac{100 \times y}{2x} \tag{2}$$

Solving (1) and (2),

$$P = \frac{100}{2} \times \frac{100}{80}$$

$$=\frac{100\times10}{16}=\frac{25\times5}{2}=62.5$$

59. Given
$$x = \frac{90}{100} \times y$$
 (1)

Let p% of x = y

i.e.
$$\frac{p}{100} \times x = y \tag{2}$$

Solving (1) and (2)

(2)
$$\Rightarrow$$
 $p = 100 \times \frac{y}{x}$
= $100 \times \frac{100}{90} = 111 \frac{1}{9} \%$

60. Percentage of aggregate =
$$\frac{90 + 90 + 108}{450} \times 100\%$$

= $\frac{288}{450} \times 100\%$
= $\frac{32}{50} \times 100\% = 64\%$

61. Number of general category candidates = 85% of 95% =
$$\frac{85}{100} \times \frac{95}{100}$$

Number of other category candidates = 15% of 95% = $\frac{15}{100} \times \frac{95}{100}$

Given
$$\frac{15}{100} \times \frac{95}{100} \times x = 4275$$

$$\therefore x = \frac{4275 \times 100 \times 100}{95 \times 15} = \frac{225 \times 20 \times 100}{15} = 75 \times 4 \times 100 = 30000$$

 \therefore Total number of candidates applied for the examination = 30000

62. Let the number of children be x.

Number of sweets/children =
$$\frac{20}{100} \times x = \frac{x}{5}$$

$$x \times \frac{x}{5} = 405$$
or
$$x^2 = 2025$$

63. Let the total salary of officer be x.

$$\therefore \qquad \text{House rent} = \frac{10}{100} x$$

$$\text{Conveyance} = \frac{20}{100} \left(x - \frac{10}{100} x \right)$$

$$= \frac{20}{100} \times \frac{90}{100} x = \frac{18}{100} x$$

Income tax =
$$\frac{20}{100} \left(x - \frac{10x}{100} - \frac{18x}{100} \right)$$

= $\frac{20}{100} \times \frac{72x}{100} = \frac{14.4x}{100}$
Clothes = $\frac{10}{100} \left(x - \frac{18x}{100} - \frac{10x}{100} - \frac{14.4x}{100} \right)$
= $\frac{10x}{100} \times \frac{57.6}{100} = \frac{576x}{10000}$
Amount left = $\frac{57.6x}{100} - \frac{5.76x}{100}$
= $\frac{51.84x}{100} = 15552$
 $x = \frac{15552 \times 100}{51.84} = \frac{15552 \times 100}{5184} \times 100 = ₹30000$

64. Let the monthly salary of Sameer be x.

Food =
$$24\%$$
; education = 15%

From
$$= 24\%$$
; education $= 15\%$

For entertainment $= 25\%$ of $(100 - 24 - 15)\% = 25\% \times 61\%$

Conveyance $= 20\%$ of $(100 - 24 - 15)\% = 20\% \times 61\%$

Amount left $= \left(100 - 24 - 15 - \frac{61}{4} - \frac{61}{5}\right)\%$ of $x = 10736$

i.e. $61\left(1 - \frac{1}{4} - \frac{1}{5}\right) \times \frac{x}{100} = 10736$

or $x\left(\frac{20 - 5 - 4}{20}\right) = \frac{10736 \times 100}{61}$

or $x = \frac{10736 \times 100 \times 20}{61 \times 11}$
 $= \frac{976 \times 100 \times 20}{61} = 16 \times 100 \times 20 = ₹ 32000$

∴.

for food =
$$40\%$$

$$rent = 20\%$$

entertainment = 10%

conveyance = 10%

Total = 80%

:. Savings =
$$100 - 80 = 20\%$$
 of his salary = 1500

∴ His monthly salary =
$$\frac{1500}{\frac{20}{100}}$$

= $1500 \times \frac{100}{20} = ₹7500$

66. Let the income of Peter be x.

$$Savings = \frac{10}{100} \times x = 0.1x$$

After increase, new income, $x + \frac{20}{100}x = 1.2x$

Savings = 0.1x

% change in expenditure =
$$\frac{(1.2 - 0.1)x - (1 - 0.1)x}{(1 - 0.1)} \times 100$$
$$= \frac{(1.1 - 0.9)x}{0.9x} \times 100 = \frac{0.2}{0.9} \times 100 = 22\frac{2}{9}\%$$

67. Let the original price be P

$$P\left(1 + \frac{r}{100}\right)\left(1 - \frac{r}{100}\right) = 1$$
i.e.
$$P = \frac{1 \times 100 \times 100}{(100 + r)(100 - r)} = \frac{10000}{100^2 - r^2}$$

68. Let the number be n

$$n + n \left(-10 + 10 - \frac{100}{100}\right)\% = n - 10$$

i.e. $n - n\% = n - 10$
or $n\% = 10$
or $\frac{n}{100} = 10$
or $n = 10 \times 100 = 1000$

69. Net change in price = $\left(-25 + 20 - \frac{500}{100}\right)\% = -10\%$ or **10% decrease**

70. Girls =
$$37\frac{1}{2}\%$$
 of total; Boys = $62\frac{1}{2}\%$ of total $62\frac{1}{2}\%$ passed 75% passed $37\frac{1}{2}\%$ failed 25% failed

i.e. Number of girls failed = $\frac{37\frac{1}{2}}{100} \times \frac{37\frac{1}{2}}{100} \times \text{total} = 342;$

Similarly $\frac{25}{100} \times \frac{62\frac{1}{2}}{100} \times \text{total} = \text{Number of boys failed}$

i.e.
$$\frac{25}{100} \times \frac{62\frac{1}{2}}{100} \times \frac{342 \times 100 \times 100}{37\frac{1}{2} \times 37\frac{1}{2}} = \text{number of boys failed}$$

i.e. Number of boys failed =
$$\frac{25 \times 62 \frac{1}{2} \times 342}{37 \frac{1}{2} \times 37 \frac{1}{2}}$$

$$= \frac{25 \times 125 \times 342}{75 \times 37\frac{1}{2}} = \frac{125 \times 114}{37\frac{1}{2}}$$
$$= \frac{125 \times 228}{75} = \frac{5 \times 228}{3} = 380$$

- 71. Net effect on price of shirt = $\left(+15 15 \frac{225}{100}\right)\% = -2.25\%$ or 2.25% decrease
- 72. Weight of container = 25% of total weight of container with fluid After remaining part of fluid,

= Weight of container + Fluid = 50% of total weight of container + Fluid

$$\therefore \qquad \text{Quantity of fluid removed} = \frac{50\%}{75\%} = \frac{2}{3}$$

$$\frac{\text{Quantity displaced}}{\text{Original capacity of tank}} = \frac{50\% \text{ of total weight}}{75\% \text{ of total weight}}$$
$$= \frac{2}{3}$$

73.
$$\frac{1}{14} \left(10 \times \frac{20}{100} + 4 \times \frac{35}{100} \right) = (2 + 1.4) \frac{1}{14} = \frac{3.4}{14} = \frac{1.7}{7}$$

Expressed as percentage =
$$\frac{1.7}{7} \times 100 = 24 \frac{2}{7} \%$$

74. Quantity of pure milk in 10 litres of milk = 9.5 litres

Quantity of milk to be added to:

10 litres of milk of 95% strength to convert it of 98% strength =?

Quantity of water in the milk = 0.5 litre

i.e.
$$2\% = 0.5$$
 litre

.. Required quantity of milk to be added = 15 litres

- 75. Quantity of pure alocohol in 400 ml sold = $\frac{15}{100} \times 400 = 60$ ml
 - :. Quantity of water in the alcohol = 340 ml

In the new solution,
$$68\% = 340 \text{ ml}$$

 \therefore Quantity of alcohol to be added to make concentration of 32% = 500 - 400 = 100 ml

76. Fresh fruit contains = 68% water

Dry fruit contains = 20% water

From 100 kg of fresh fruit =
$$100 \left(1 - \frac{48}{100}\right)$$

= 52 kg dry fruits can be obtained

- 77. Let x be the number of students appeared for the examination in school A.
 - \therefore Number of students qualified in school A = 0.7x

Number of students appeared in school B = 1.2x

Number of students qualified in school B = $1.5 \times 0.7x = 1.05x$

∴% of students qualified to students appeared from school B = $\frac{1.05x}{1.2x} \times 100$

$$= \frac{3.5}{4} \times 100 = 87.5\%$$

78. Students failed in Civics = 100 - 65 = 35%

Students failed in History = 100 - 60 = 40%

Students failed in Civics or History = 100 - 40 = 60%

 \therefore Students failed in both =35 + 40 - 60 = 15%

i.e. 15% = 90

$$100\% = \frac{90}{15} \times 100 = 600$$

79. The percentage of people did not eat either type of lunch =?

Number of persons took vegetarian food = $\frac{60}{100} \times 96$

$$V_n = \frac{288}{5}$$

Number of persons took non-vegetarian = $\frac{30}{100} \times 96$

$$N_v = \frac{144}{5}$$

$$V_n \cap N_v = \frac{15}{100} \times 96 = \frac{72}{5}$$

$$V_n \cup N_v = V_n + N_v - (V_n \cap N_v)$$
$$= \frac{288}{5} + \frac{144}{5} - \frac{72}{5} = \frac{360}{5} = 72$$

 \therefore People did not took any type of food = 96 - 72 = 24

80. % of students failed in Mathematics $F_M = 34\%$

% of students passed in Mathematics $P_M = 66\%$

% of students failed in English F_E = 42%

% of students passed in English $P_E = 58\%$

% of students failed in both subjects $F_B = 20\%$

% of students passed in both subjects $P_B = ?$

i.e.
$$P_B = P_M + P_E - (100 - F_B) = 66 + 58 - (100 - 20) \\ = 66 + 58 - 80 = 44\%$$

81. Let the marks got for D be x.

Marks for C = 0.85x

Marks for B = $1.2 \times 0.85 x$

Marks for A = $1.2 \times 1.2 \times 0.85x$

Given Marks for A = 576

$$\therefore$$
 576 = 1.2 × 1.2 × 0.85 x

$$x = \frac{576}{1.2 \times 1.2 \times 0.85} = \frac{400}{0.85}$$

$$= 400 \times \frac{100}{85} = 400 \times \frac{20}{17} = \frac{8000}{17}$$

$$\therefore \qquad \% \text{ of marks} = \frac{8000}{17} \times \frac{1}{800} \times 100 = \frac{1000}{17} \% = 58.82\%$$

- 82. Let the monthly income of Deepak be x.
 - \therefore Monthly income of Raunag = 0.8x

Monthly income of Amit = $1.3 \times (0.8x) = 1.04x$

Given
$$(1.04x - x) = 800$$

or $0.04x = 800$
or $x = \frac{800}{0.04} = 20000$

- \therefore Monthly income of Raunaq = $0.8 \times 20000 = 16000$
- 83. Number of students in the school = 450

Number of students play neither football nor cricket = 50

 \therefore Number of students who play either football or cricket = $n (F \cup C) = 450 - 50 = 400$

Number of students who play football n(F) = 325

Number of students who play cricket n(C) = 175

Number of students who play both $n(F \cap C) = n(F) + n(C) - n(F \cup C)$

$$=325+175-400=100$$

84. Let the price of rice be \overline{x} and quantity purchased be y kg.

$$\therefore \qquad x \times y = 0.98x \times y_1$$

Given y = 49

 $\therefore \qquad \qquad x \times 49 = 0.98x \times y_1$

or
$$y_1 = \frac{49}{0.98} = 50 \text{ kg}$$

85. At the time of first filling,

quantity of A-type petrol in the tank = 100%

At the time of second filling,

quantity of A-type petrol in the tank = 50%

quantity of B-type petrol in the tank = 50%

At the time of third filling,

quantity of A-type petrol in the tank = 25 + 50 = 75%

quantity of B-type petrol in the tank = 25%

At the time of fourth filling,

quantity of A-type petrol in the tank = $\frac{75}{2}$ % = 37.5%

quantity of B-type petrol in the tank = $\frac{25}{2} + 50 = 62.5\%$

86. Quantity of milk in the container after third operation = $0.8 \times 0.8 \times 0.8 = 0.512 = 51.2\%$ pure

- 87. Let the price of egg = x
 - \therefore Increased price = 1.3x
 - ∴ For ₹ 7.8, number of eggs available before increase in price = $\frac{7.8}{x}$
 - $\therefore \qquad \text{Number of eggs available after increase in price} = \frac{7.8}{1.3x}$

$$\therefore \frac{7.8}{x} - \frac{7.8}{1.3x} = 3$$

i.e.
$$\frac{7.8}{x} - \frac{6}{x} = 3$$

$$\Rightarrow \qquad (7.8 - 6) = 3 \times x$$

or
$$x = \frac{7.8 - 6}{3} = 0.6$$

 $\therefore \qquad \text{New rate/dozen} = 12 \times 0.6 \times 1.3 = \text{ } \text{? } \textbf{9.36}$

88. 5:
$$4 = \frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$$

89.
$$\frac{1}{2} \times 1\% = \frac{1}{2}\% = \frac{1}{200} = \mathbf{0.005}$$

90. 15% of ₹ 34 =
$$\frac{15}{100}$$
 × 34 = ₹ **5.1**

91. Let x be the required percentage.

$$x\%$$
 of 7.2 kg = 18 g

i.e.
$$\frac{x}{100} \times 7.2 \text{ kg} = 18 \text{ g}$$

or
$$\frac{x}{100} \times 7200 \text{ g} = 18 \text{ g}$$

or
$$x = \frac{18 \times 100}{7200} = \frac{1}{4}$$
$$= \frac{1}{4} \times 100\% = 25\%$$

92. Required percentage =
$$\frac{3}{24} \times 100\% = 12 \frac{1}{2}\%$$

93. Actual price of the item =
$$25 + 2.50 = 27.50$$

Savings on selling the item = $\frac{27.50 - 25}{25} \times 100\% = 10\%$

94. 5% of (25% of ₹ 1600) =
$$\frac{5}{100} \times \frac{25}{100} \times ₹ 1600 = ₹ 20$$

95. 0.15% of 33
$$\frac{1}{3}$$
% of ₹10,000

$$= \frac{0.15}{100} \times \frac{33\frac{1}{3}}{100} \times ₹10000$$

$$= \frac{15}{10000} \times \frac{100}{300} \times ₹10000$$
$$= \frac{15}{3} = ₹5$$

96. Let the number be x

$$x - \frac{2}{5}x = 510$$
or
$$x\left(1 - \frac{2}{5}\right) = 510$$
or
$$x = \frac{510 \times 5}{(5 - 2)}$$

 \therefore 10% of number = 0.1x

i.e.
$$0.1x = 0.1 \times \frac{510 \times 5}{3} = \frac{51 \times 5}{3} = 85$$

97. Number that ends with '1' or '9' when squared we get the result ending with digit '1'.

Therefore, every 10 consecutive numbers consists of two such numbers whose square ends with digit '1'.

Here 1, 9, 11, 19, 21, 29, 31, 39, 41, 49, 51, 59, 61, 69 are these numbers.

$$\therefore$$
 Percentage of such numbers = $\frac{14}{70} \times 100 = 20\%$

98. Let the first number be x and second number be y.

Then
$$x + y = \frac{28}{25} \times x$$

$$y = \left(\frac{28}{25} - 1\right)x$$

$$= \left(\frac{28 - 25}{25}\right)x = \frac{3x}{25}$$

$$\therefore \qquad y = x \times \frac{3}{25} = x \times \frac{12}{100}$$
(1)

i.e. second number is 12% of the first.

99. Let x be the total number of candidates appeared from each state A and B.

Number of candidates selected from state
$$A = \frac{6}{100}x$$
 (1)

Number of candidates selected from state
$$B = \frac{7}{100}x$$
 (2)

Given that
$$\frac{7x}{100} - \frac{6x}{100} = 80$$

 \therefore $(7-6)x = 80 \times 100$
or $x = 8000$

100. Given price of car = ₹ 325000

Insured amount = $0.85 \times ₹ 325000 = ₹ 276250$

Insurance claim amount = $0.90 \times 276250 = ₹248625$

∴ Difference between price and claim amount = 325000 - 248625 = ₹ 76375

101. Percentage of total votes winning candidate got = $\frac{11628}{1136 + 7636 + 11628} \times 100$

$$= \frac{11628}{20400} \times 100 = \frac{11628}{204} = 57\%$$

102. Let one number be x

:. Second number =
$$0.8x$$

Then $4[x^2 + (0.8x)^2] = 656$

or
$$x^2 + (0.8x)^2 = \frac{656}{4}$$

or
$$x^2(1+0.64) = 164$$

or $x^2 = 100$

.. The numbers are 8 and 10.

103. Let x and y be the numbers.

$$\frac{x}{12} = \frac{1}{4}y$$

 \mathbf{or}

or

$$x = 3y$$

$$\therefore \text{ Required percentage} = \left(\frac{x-y}{y}\right) \times 100\% = \frac{\left(3y-y\right)}{y} \times 100\% = 200\%$$

104. Let the numbers be x and y.

Then
$$y - \frac{25}{100}x = \frac{5}{6}y$$

i.e. $y - \frac{5}{6}y = \frac{25}{100}x$
 $\frac{1}{6}y = \frac{1}{4}x$
or $\frac{x}{y} = \frac{6}{4} = \frac{3}{2}$

 \therefore First number : Second number = x : y = 2 : 3

105. Let the cost of taxable item be \overline{x} .

$$\therefore \qquad \text{Tax paid} = \frac{6}{100} \times x = \frac{6x}{100} = 30 \text{ paise}$$
or
$$\frac{6x}{100} = \frac{30}{100}$$
or
$$x = \frac{30}{6} = \frac{30}{5}$$

.. Cost of tax free item = (25 - 5 - 0.30) = ₹ 19.70

106. Total score of batsman = 110 runs

Score by running between wickets = 110 - 3 boundaries - 8 sixes = $110 - 3 \times 4 - 8 \times 6 = 50$

$$\therefore \qquad \text{Required percentage} = \frac{50}{110} \times 100 = 45 \frac{5}{11} \%$$

107. Let the number of apples originally with fruit seller be x.

i.e.
$$x - \frac{40}{100} \times x = 420$$
$$x \times \frac{60}{100} = 420$$
or
$$x = \frac{420 \times 100}{60} = 700$$

108. Let maximum marks be x.

Pass mark required = $\frac{33x}{100}$ = 125 + 40 = 165

i.e.
$$\frac{33x}{100} = 165$$
or
$$x = \frac{165 \times 100}{33} = 500$$

109. Out of 100% voters, 10% voters did not cast votes. 10% votes were invalid

Net votes polled = 90%

$$\therefore$$
 Net valid votes polled = 90% of 90% = 81%

Vote polled for successful candidate = $\frac{54}{100} \times \frac{81}{100} = 43.74\%$

Majority by which candidate won =
$$\frac{81}{100} \times (54 - 46)\% = \frac{81}{100} \times \frac{8}{100} = 6.48\%$$

Given majority = 1620 = 6.48% of total voters

$$\therefore \qquad \text{Total number of voters} = 1620 \times \frac{100}{6.48} = 25000$$

110. Let total polled votes be x.

Invalid votes = 20%x

Votes successful candidate got =
$$\frac{55}{100} \times (x - 0.2x)$$

Given x = 7500

.. Valid votes scored by failed candidate =
$$\frac{45}{100}(x - 0.2x) = \frac{45}{100} \times 0.8x$$

= $\frac{45}{100} \times 0.8 \times 7500 = 45 \times 60 = 2700$

111. In the examination, let total appeared candidates be x.

Number of failed candidate = 455

$$\therefore \frac{65}{100}x = 455$$
or
$$x = \frac{455 \times 100}{65} = 700$$

112.
$$\left(100 - 66\frac{2}{3}\right)\%$$
 of person's income = 1200

i.e.
$$33\frac{1}{3}\%$$
 of person's income = 1200

∴ Expense =
$$66\frac{2}{3}\%$$
 of person's income = $2 \times 1200 = ₹ 2400$

113. Let total number of votes polled be x.

Winning candidate got $\frac{84}{100} \times x$ votes

Failed candidate got $\frac{16}{100} \times x$ votes

Given majority =
$$\frac{84-16}{100} \times x = 476$$

or
$$\frac{68}{100}x = 476$$

or
$$x = \frac{476 \times 100}{68} = 700$$

114.
$$\frac{P}{100}$$
 of $P = 36$

or
$$\frac{P}{100} \times P = 36$$

or
$$P^2 = 36 \times 100$$

or
$$P = \sqrt{36 \times 100} = 60$$

115. Given
$$\frac{x}{100} \times y = z$$

or
$$x = \frac{z}{y} \times 100 = \frac{100}{y} \times z = \frac{100^2}{y} \%$$
 of z

116. Let the amount with Aman be x.

Amount paid to Rohan = 0.4x

Amount paid to Sahil by Rohan =
$$\frac{1}{4}(0.4x)$$

Amount left with Sahil after paying taxi fare of ₹ 200 = 600

i.e.
$$\frac{1}{4}(0.4x) - 200 = 600$$

i.e.
$$0.1x = 600 + 200 = 800$$

or
$$x = 7 8000$$

117. Given height of pole = 192 m

Distance climbed in second hours =
$$12\frac{1}{2}\%$$
 of $\left(100 - 62\frac{1}{2}\right)\%$ of 192
= $\frac{12.5}{100} \times \frac{37.5}{100} \times 192$

$$= \frac{25}{200} \times \frac{75}{200} \times 192$$

$$= \frac{1}{8} \times \frac{3}{8} \times 192 = 9 \text{ m}$$

118. Number of literate inhabitant male =
$$1000 \times \frac{60}{100} \times \frac{20}{100} = 120$$

Number of literate inhabitants in total = $1000 \times \frac{25}{100} = 250$

$$\therefore$$
 Number of literate female inhabitants = $250 - 120 = 130$

: Percentage of females who are literate =
$$\frac{130}{\frac{40}{100} \times 1000} \times 100$$

i.e. Required percentage =
$$\frac{130}{400} \times 100 = 32 \frac{1}{2} \%$$

119. A house with two or more people

B houses with only a male

C house with only one female and no male

Let number of houses in total be x.

$$n(A) = 40\%$$

 $n(B) = 25\% \times 60\% = 15\%$
 $n(C) = ?$
 $n(A) + n(B) + n(C) = 100$
 $= 100 - 40 - 15 = 45\%$



Short cut: Number of houses with only one female

$$= (100 - 25)\% \text{ of } (100 - 40)\% \text{ of } x$$

$$= 75\% \times 60\% \times x$$

$$= \frac{75}{100} \times \frac{60}{100} \times x = \frac{3}{4} \times \frac{3}{5}x = \frac{9}{20}x$$

Required maximum percentage = $\left(\frac{9}{20}x \times \frac{1}{x} \times 100\right)\% = 45\%$

120. Let x be maximum marks.

:. Pass mark required =
$$\frac{30}{100}x + 15 = \frac{40}{100}x - 35$$

Solving, we get,

$$\left(\frac{40-30}{100}\right)x = 15+35=50$$

 \mathbf{or}

$$x = \frac{50 \times 100}{10} = 500$$

∴ Required pass percentage =
$$\frac{\frac{30}{100}(500) + 15}{500} \times 100$$

= $\frac{150 + 15}{100} \times 100 = 33\%$

121. Average score in the class = 80 marks

Let x be the required average

Average =
$$\frac{10}{100} \times 95 + \frac{20}{100} \times 90 + \frac{70}{100} \times x = 80$$

i.e. $\frac{70x}{100} = 80 - 18 - 9.5 = 52.5$
or $x = \frac{52.5 \times 100}{70} = \frac{3 \times 100}{4} = 75 \text{ marks}$

122. For pass required percentage = 35%

Let x be the required marks in third paper to pass.

Given, score for 1st paper =
$$\frac{62}{150}$$

Score for 2nd paper = $\frac{35}{150}$
Score for 3rd paper = $\frac{x}{180}$
 $\therefore \frac{62 + 35 + x}{150 + 150 + 180} \times 100 = 35\%$
or $\frac{97 + x}{480} \times 100 = 35\%$
or $97 + x = \frac{480 \times 35}{100}$
or $x = \frac{480 \times 35}{100} - 97$
 $= 24 \times 7 - 97 = 168 - 97 = 71$

123. Let the number of girls be y.

$$x + y = 150$$
Then
$$y = \frac{x}{100} \times 150 = 1.5x$$

$$\therefore (1) \Rightarrow x + 1.5x = 150$$
or
$$2.5x = 150$$
or
$$x = \frac{150}{2.5} = 60$$

i.e. Number of boys = 60

124. Let total sales be for ₹ x.

$$\therefore \text{ Total earnings of salesmen} = \frac{5\frac{1}{2}}{100} \times x + \frac{1}{2} \times (x - 10000) = 1990$$

i.e.
$$\frac{5.5x}{100} + \frac{x - 10000}{200} = 1990$$

$$\Rightarrow \frac{11x + x - 10000}{200} = 1990$$

$$\Rightarrow 12x - 10000 = 1990 \times 200$$

$$\Rightarrow 12x = 398000 + 10000 = 408000$$

$$\Rightarrow x = \frac{408000}{12} = 34000$$

- 125. Let earning by Michael be x.
 - :. Earning of Albert = 0.8xEarning of Peter = $1.4 \times 0.8x = 1.12x$
 - .. Peter earned more than Michael by 12%
- **126.** Let C's salary be x.

$$\therefore \text{ B's salary} = \frac{25}{100} \times x$$

$$\therefore \text{ A's salary} = \frac{40}{100} \times \frac{25}{100} \times x$$

$$=\frac{x}{10} = 10\%$$
 of C's salary

127. Let the original quantity of ghee be x kg.

$$\therefore \qquad \text{Pure ghee} = 0.6x$$

$$\text{Vanspati} = 0.4x$$

After addition of 10 kg of pure ghee

New quantity of pure ghee = 0.6x + 10

New quantity of Vanaspati = 0.4x

Now,
$$\frac{0.4x}{x+10} \times 100 = 20$$
or
$$\frac{0.4x \times 5}{x+10} = 1$$
or
$$2x = x + 10$$
or
$$x = 10 \text{ kg}$$

128. Let the third number be x.

$$\therefore \qquad \text{First number} = \frac{12.5}{100}x$$

$$\text{Second number} = \frac{25}{100}x$$

$$\therefore \quad \text{Required percentage} = \frac{12.5x/100}{25x/100} \times 100 = 50\%$$

129. Let x be the third number.

First number =
$$(1 - 0.3)x = 0.7x$$

Second number = $(1 - 0.37)x = 0.63x$

∴ Required percentage =
$$\frac{(0.7 - 0.63)x}{0.7x} \times 100$$

= $\frac{0.07}{0.7} \times 100 = 10\%$

130. Given A's height =
$$\left(1 - \frac{40}{100}\right)B$$
's height

$$\therefore \qquad \text{B's height} = \frac{100}{60} \times \text{A's height}$$

$$\therefore \text{ Required percentage} = \frac{100 - 60}{60} \times 100 = 66 \frac{2}{3} \%$$

131. A's salary is 50% more than B's

:. B's salary in less than A's by
$$\left[\frac{50}{100 + 50} \times 100 \right] \% = \frac{100}{3} = 33\frac{1}{3}\%$$

132. Let the population of town be P.

Net increase of population per annum = $4 - \frac{1}{2} = 3\frac{1}{2}\%$

Population after 3 years =
$$P\left(1 + \frac{3\frac{1}{2}}{100}\right)^3$$

$$P\left(1 + \frac{3\frac{1}{2}}{100}\right)^{3} - P$$

$$\therefore \qquad \text{Increase percentage} = \frac{P\left(1 + \frac{3\frac{1}{2}}{100}\right)^{3} - P}{P} \times 100$$

$$= \left[\left(1 + \frac{3\frac{1}{2}}{100} \right)^3 - 1^3 \right] \times 100$$

$$= \left[\frac{207}{200} \times \frac{207}{200} \times \frac{207}{200} - 1 \right] \times 100$$

$$= \left[\frac{207^3}{200^3} - 1 \right] \times 100$$

$$= [1.1087 - 1] \times 100 = 10.87\% = 10.8\%$$

133. Total population P = 5000

Let number of males be M.

 \therefore Number of females = 5000 - M

New number of males =
$$M \times \left(1 + \frac{10}{100}\right) = 1.1 M$$

Total population =
$$1.1M + (5000 - M) \cdot 1.15 = 5600$$

i.e.
$$5750 + (1.1 - 1.15) M = 5600$$

i.e.
$$(1.15 - 1.1) M = 5750 - 5600$$

$$0.05 M = 150$$

$$M = \frac{150}{0.05} = \frac{30}{0.01} = 3000$$

134. Let the value of machine be x.

Value after 3 years of depreciation is 20% per annum =
$$x \left(1 - \frac{R}{100}\right)^N$$

= $x \left(1 - \frac{20}{100}\right)^3$
= $x \times \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100}$
= $x \times \frac{512}{1000} = 0.512x$

i.e. value is less by 48.8%

135. Population 2 years ago = 62500

Decrease in population @ 4% per annum

:. Present population =
$$62500 \left(1 - \frac{4}{100}\right)^2$$

= $62500 \left(\frac{96}{100}\right)^2 = 62500 \left(\frac{24}{25}\right)^2$
= $100 \times (24)^2 = 57600$

136. Present population of district = 64000

Population after 3 years with its increase @ $2\frac{1}{2}\%$ per annum = $64000\left(1 + \frac{2\frac{1}{2}}{100}\right)^3$ = $64000 \times \left(1 + \frac{10}{400}\right)^3 = 64000 \times \left(\frac{410}{400}\right)^3$ = $\frac{64,000 \times (410)^3}{64,00,0000} = (41)^3 = 68921$

137. Value of a machine at present after depreciation @ 10% per annum for 3 years = $\mathbf{\xi}$ 8784 Let x be the purchase price

$$\therefore \qquad x \left(1 - \frac{10}{100}\right)^3 = 8748$$

=
$$8748 \times \frac{100^3}{90^3} = 8748 \times \frac{10^3}{9^3}$$

= $972 \times \frac{10^3}{9^2} = 12 \times 1000 = ₹ 12000$

138. If inflation increases @ 8% per annum, the cost of article worth ₹ 20, at present will be having a value after 2 years equal to $20 \times \left(1 + \frac{8}{100}\right)^2$.

$$20 \times \left(\frac{108}{100}\right)^2 = 20 \times \left(\frac{27}{25}\right)^2 = 27^2 \times \frac{4}{125} = \frac{729 \times 4}{125} = 23.328$$

Its value will be between ₹ 23 and ₹ 24.

139. Population 3 years ago = 160000

Present population =
$$160000 \times \left(1 + \frac{3}{100}\right) \left(1 + \frac{2.5}{100}\right) \left(1 + \frac{5}{100}\right)$$

= $160000 \times \left(\frac{103}{100}\right) \left(\frac{102.5}{100}\right) \left(\frac{105}{100}\right)$
= $16 \times 103 \times 102.5 \times \frac{21}{20} = 16 \times 103 \times \frac{205}{2} \times \frac{21}{20}$
= $4 \times 103 \times 205 \times \frac{21}{10} = 177366$

140. Let the population is 1998 be P.

$$P \left(1 + \frac{5}{100}\right)^{3} = 138915$$

$$P = 138915 \times \frac{100^{3}}{105^{3}}$$

$$= 138915 \times \frac{20^{3}}{21^{3}} = 6615 \times \frac{20^{3}}{21^{2}}$$

$$= 315 \times \frac{20^{3}}{21} = 15 \times 20^{3} = 120000$$

- **141.** Let production of a company last year be P.
 - .. For 4 years from 1998 to 2002, resultant effect in production

$$= P\left(1 + \frac{15}{100}\right)\left(1 + \frac{15}{100}\right)\left(1 - \frac{10}{100}\right)\left(1 + \frac{15}{100}\right)$$

$$= P \times \frac{115}{100} \times \frac{115}{100} \times \frac{90}{100} \times \frac{115}{100}$$

$$= P \times \frac{23}{20} \times \frac{23}{20} \times \frac{9}{10} \times \frac{23}{20}$$

$$= P \times \frac{12149 \times 9}{80000} = P \times \frac{109341}{80000} = P \times 1.3667625$$

:. Resultant effect = 36.68% Increase = 37% Increase

125

In February, number of ticketless travellers = $4000 \times \left(1 + \frac{5}{100}\right) = 4200$

In March, number of ticketless travellers = $4200 \times \left(1 - \frac{5}{100}\right) = 3990$

In April, number of ticketless travellers = $3990 \left(1 - \frac{10}{100}\right) = 3591$

143. Let the number of bushes at beginning be x.

$$\therefore x \left(1 + \frac{10}{100}\right) \left(1 + \frac{8}{100}\right) \left(1 - \frac{10}{100}\right) = 26730$$

 $x = \frac{26730}{110 \times 108 \times 90} \times 100^3$

$$= \frac{26730}{11 \times 108 \times 9} \times 100^2 = \frac{2430}{108 \times 9} \times 100^2$$

$$= \frac{270}{108} \times 100^2 = \frac{5}{2} \times 100^2 = 25000$$

144. Alcohol content in 9 ml shaving lotion of 50% concentration = 9(0.5) = 4.5 ml To reduce it to 30% concentration, water required =?

In 30% lotion, alcohol content = 4.5 ml

i.e. 30% quantity = 4.5 ml

:.
$$100\%$$
 quantity = $\frac{4.5}{30} \times 100 = 15 \text{ ml}$

 \therefore Quantity of water in the mixture = 15 - 4.5 = 10.5 ml

Quantity of water now added = 15 - 9 = 6 ml

145. $600 + 0.25 + 1200 \times 0.5 = 150 + 600 = ₹750$

Amount removed =
$$\frac{12}{100}$$
 (150) + $\frac{24}{100}$ (600) = 18 + 144 = ₹ 162

∴ Percentage of money removed = $\frac{162}{750} \times 100$

$$=\frac{5.4}{250} \times 100 = \frac{216}{1000} \times 100 = 21.6\%$$

Average

Average can be defined as the ratio of sum of given data to the number of data. For example, if $x_1, x_2, x_3, ..., x_n$ are n different values of a variable, then the average value of

that variable is given by $\frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$.



Note: If a group of n people is having an average height of h and if a new person joined and the average height of people in the group is increased by x, then the height of the new person is given by

New average + Number of old members \times Increase in average

i.e. $h = x + n \times x$

i.e. h = (n + 1)x

Age of new entrant = New average + Number of old members \times Change in average If average is increased, change is +ve, else it is -ve.

When a person of known height is replaced (from a group) by another person and the average height of the group of person is increased by x, then the height of new person is given by

Height of new person = Height of replaced person + Number of persons × Increase in average

EXAMPLE 1 The average age of 25 boys in a class is equal to 16. When a new boy was admitted to the class, the average age got decreased by one month. Find the age of new boy admitted.

 $\textbf{Solution} \quad \text{Total age of 25 boys} = 25 \times 16 = 400 \text{ years}$

When a new boy was admitted,

Total age of 26 boys = 26×15 years 11 months

= 413 years 10 months

Age of new entrant = 413 years 10 months – 400 years

= 13 years 10 months



Short cut: Age of new entrant = New average + Number of old members × Change in average

$$= 15 \text{ years } 11 \text{ months} + 25 (-1 \text{ month})$$

EXAMPLE 2 Average weight of 8 men is increased by 2 kg when one of them who weighs 130 kg is replaced by another man. What is the weight of the new man?

Solution Average weight is increased by 2 kg.

 \therefore Total weight is increased by $8 \times 2 = 16 \text{ kg}$

This is due to the extra weight of person included at present.

 \therefore Weight of new man = 130 + 16 = 146 kg



Short cut: The weight of new person = Weight of replaced person + Number of persons

$$= 130 + 8 \times 2 = 146 \text{ kg}$$

EXAMPLE 3 The average of 15 results is 60. If the average of first eight results is 58 and that of last eight is 61, find the eighth result.

Solution Sum of 15 results = $15 \times 60 = 900$

Sum of first 8 results = $8 \times 58 = 464$

Sum of last 8 results = $8 \times 61 = 488$

Eighth result =
$$464 + 488 - 900 = 952 - 900 = 52$$



Short cut: Eighth result = 60 + (61 - 60)8 + (58 - 60)8

$$= 60 + 1 \times 8 + (-2) \times 8$$

$$=60+8-16=52$$

EXAMPLE 4 The average age of three members of a family was 24 five years ago. At present the average age of the family including a new member is 22. Find the age of the new member.

Solution Total age of family members 5 years before = $24 \times 3 = 72$ years

Total age of family members at present = $(24 + 5) \times 3 = 87$ years

Total age of family members including a new member at present = $22 \times 4 = 88$ years

 \therefore Age of new member = 88 - 87 = 1 year



Short cut: Age of new member = New average + Old number of members

$$= 22 + 3 \times (-7) = 1$$
 year

EXAMPLE 5 The average of 15 results is 50, that of the first seven is 45 and that of last seven is 48. Find the value of the 8th number.

Solution 8th number = Sum of 15 results – (Sum of first 7 results + Sum of last 7 results) $= 15 \times 50 - (7 \times 45 + 7 \times 48)$ $= 750 - (7 \times 93) = 750 - 651 = 99$



Short cut: 8th number = 50 + (50 - 45)7 + (50 - 48)7 = 50 + 35 + 14 = 99

EXAMPLE 6 The average age of 16 boys in a class is decreased by 3 months, when a boy aged 17 years is replaced by a new boy. Find the age of new boy.

Solution Age of new boy = Age of replaced boy + Number of boys × Change in average

=
$$17 + 16 \times \left(\frac{-3}{12}\right) = 17 - 4 = 13 \text{ years}$$

RULES

Average Speed

Rule 1: If a man travels a certain distance at a speed of x km/h and the same distance at a speed of y km/h, then the average speed during the whole journey is given by $\frac{2xy}{x+y} \text{ km/h}$.

Proof: Let the distance travelled by the man be S km.

 \therefore Time taken for the first journey at a speed of $x \text{ km/h} = \frac{S}{x} \text{h}$

Time taken for the second journey at a speed of $y \text{ km/h} = \frac{S}{v} h$

 \therefore Total time take for the whole journey = $\frac{S}{x} + \frac{S}{y}h$

Total distance travelled = S + S = 2S

Average speed =
$$\frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{2S}{S/x + S/y} = \frac{2S \times xy}{S \times y + S \times x}$$
$$= \frac{S \times 2xy}{S \times (x + y)} = \frac{2xy}{x + y} \text{ k m/h}$$

Rule 2: If a man travels three equal distances at a speed of x km/h, y km/h and z km/h respectively, then the average speed of the man for the whole journey is given by $\frac{3xyz}{xy + yz + zx}$ km/h.

Proof: Let the three equal distances be S km.

Time taken for travelling the first distance = $\frac{S}{x}$ h

Time taken for travelling the second distance = $\frac{S}{y}$ h

 \therefore Total distance travelled = S + S + S = 3S km

Total time taken =
$$\frac{S}{x} + \frac{S}{y} + \frac{S}{z}$$
 h

$$\therefore \text{ Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{3S}{\left(\frac{S}{x} + \frac{S}{y} + \frac{S}{z}\right)}$$
$$= \frac{3xyz}{yz + xz + xy} \, \text{km/h}$$

EXAMPLE 7 A person travels from a place P to Q at the speed of 40 km/h and returns at a speed of 10 km/h. What is the average speed of the person for the whole journey?

Solution Average speed =
$$\frac{2 \times 40 \times 10}{40 + 10} = 16$$
 km/h

EXAMPLE 8 A train travels three equal distances at a speed of 80 km/h and 90 km/h and 120 km/h respectively. Find the average speed of the train for the whole journey.

Solution Average speed =
$$\frac{3 \times 80 \times 90 \times 120}{80 \times 90 + 90 \times 120 + 120 \times 80}$$
$$= \frac{3 \times 8 \times 9 \times 12 \times 1000}{100(72 + 108 + 96)} = 93\frac{21}{23} \text{ km/h}$$

Solution Let the required number of workers be x.

٠.

$$1900x + 5500 \times 16 = 2000(16 + x)$$
$$1900x - 2000 x = 2000 \times 16 - 5500 \times 16$$
$$-100x = -3500 \times 16$$
$$x = \frac{3500 \times 16}{100} = 560$$



Short cut:

Number of workers = Number of officers $\times \frac{\text{(Average salary of officers - Mean average)}}{\text{(Mean average - Average salary of workers)}}$ $= 16 \times \left(\frac{5500 - 2000}{2000 - 1900}\right)$ $= 16 \times \frac{3500}{100} = 16 \times 35 = 560$

EXAMPLE 10 The average of ten two-digit numbers is calculated. While calculating the average one number was mistakenly reversed and the average got reduced by 2.7. If the sum of the digits of the reversed number is 11, find the number.

Solution Let the number be xy.

- \therefore Value of number = $10 \times x + 1 \times y$
- \therefore Reversed number = yx
- \therefore Value of reversed number = $10 \times y + 1 \times x$
- : Difference of number and its reverse

$$= (10x + y) - (10y + x)$$

$$= 10(x - y) + (y - x)$$

$$= (10 - 1)(x - y) = 9(x - y)$$

i.e. difference of number and its reverse is a multiple of 9.

In this problem, the given difference of number and its reverse = $2.7 \times 10 = 27$

i.e.
$$9(x-y) = 27$$

 $\therefore x-y=3$ (1)

Sum of the digits is 11.

i.e.
$$x + y = 11(2)$$

Solving Eqs. (1) and (2), we get

$$(1) + (2)$$
 \Rightarrow $2x = 14$ or $x = 7$
 $(1) - (2)$ \Rightarrow $2y = 8$ or $y = 4$

.: The required number is 74.



Short cut: Difference of digits = $\frac{\text{Difference of number and its reverse}}{\alpha}$

The sum of digits will be given.

If $x_1, x_2, x_3, ..., x_n$ are in Harmonic Progression (H.P.), their reciprocals will be in Arithmetic Progression (A.P.).

i.e.
$$\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \dots, \frac{1}{x_n}$$
 are in A.P.

$$\therefore \text{ Arithmetic mean (A.M.)} = \frac{1}{n} \left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right]$$

Harmonic Mean (H.M.) =
$$\frac{1}{\frac{1}{n} \left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

EXAMPLE 11 A man goes to a place at the rate of 4 km/h. He comes back on a bicycle at 16 km/h. What is his average speed for the complete to and fro journey?

Solution Average speed =
$$\frac{2xy}{x+y} = \frac{2 \times 4 \times 16}{4+16} = \frac{32}{5} = 6.4 \text{ km/h}$$

Important Formulae

Average of consecutive natural numbers upto $n = \frac{(n+1)}{2}$

Average of squares of all natural numbers till $n = \frac{(n+1)(2n+1)}{6}$

Average of cubes of first *n* natural numbers = $\frac{n(n+1)^2}{4}$

Average of first n consecutive odd numbers = n

Average of squares of all consecutive odd numbers till $n = \frac{n(n+1)}{3}$

Average of squares of all consecutive even numbers till $n = \frac{(n+1)(n+2)}{3}$



Note: Average of any *n* consecutive odd numbers

or

Average of any n consecutive even numbers = First term + (n-1)



Short cut: When a new member joins a group without replacing any one,

Age of new person = Initial average of group + New number of members in the group \times Change in average

When a person leaves a group without any replacement being made, the age of the separated person = Initial average - New number of persons × Change in average

EXAMPLE 12 If a person covers X km at x km/h and Y km at y km/h and Z km at z km/h, find his average speed in covering the complete distance.

Solution Average speed =
$$\frac{\text{Total distance covered}}{\text{Total time}} = \frac{X + Y + Z}{\left(\frac{X}{x} + \frac{Y}{y} + \frac{Z}{z}\right)} \text{ km/h}$$

EXAMPLE 13 If a person covers Ath part of the distance at x km/h, Bth part of the distance at y km/h and the remaining Cth part at z km/h, find his average speed.

Solution Let the total distance covered be D km.

Time taken to cover Ath part (i.e. $A \times D \text{ km}$) = $\frac{AD}{x}$ h

Time taken to cover Bth part (i.e. $B \times D \text{ km}$) = $\frac{BD}{V}$ h

Time taken to cover Cth part (i.e. $C \times D \text{ km}$) = $\frac{CD}{r}$ h

Total time taken = $\left(\frac{AD}{x} + \frac{BD}{y} + \frac{CD}{z}\right)h$

$$= D\left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z}\right)h$$
Average speed
$$= \frac{D}{D\left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z}\right)}$$

$$= \frac{1}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}} k m/h$$



Short cut: If A, B, C are given as percentage, instead of part, expression change to

Average speed =
$$\frac{100}{\left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z}\right)} \text{ km/h}$$

EXAMPLE 14 A batsman makes a score of 82 runs in the 17th inning and thus increases his average by 2. Find his average after 17th inning.

Solution Let his average after 17th inning be x

$$\therefore$$
 Average after 16th inning = $(x-2)$

$$\therefore 16(x-2) + 82 = 17x$$

or
$$x = 82 - 32 = 50$$

EXAMPLE 15 Among the given three numbers, the second is twice the first and is also thrice the third. If the average of three numbers is 55, find the largest number.

Solution Let the first number be x

Second number =
$$2x$$

= $3 \times \text{third number}$
or third number = $\frac{2x}{3}$

$$\therefore \qquad x + 2x + \frac{2x}{3} = 55 \times 3 = 165$$
i.e.
$$\frac{(3x + 6x + 2x)}{3} = 165$$
or
$$11x = 165 \times 3$$
or
$$x = \frac{165 \times 3}{11} = 45$$

 \therefore Largest number = second number = $2x = 2 \times 45 = 90$

EXAMPLE 16 Average of 11 results is 65. If the average of first six results is 63 and the last six results is 67, find the sixth result.

Solution Sixth result =
$$(6 \times 63 + 6 \times 67 - 11 \times 65)$$

= $378 + 402 - 715 = 65$

EXAMPLE 17 The average weight of P, Q and R is 50 kg. If average weight of P and Q is 55 kg and that of Q and R is 43 kg, find the weight of Q.

Solution Let p, q and r be individual weight

$$\therefore \qquad p + q = 2 \times 55 = 110 \text{ kg} \tag{1}$$

$$q + r = 2 \times 43 = 86 \text{ kg}$$
 (2)

$$p + q + r = 3 \times 50 = 150 \text{ kg}$$
 (3)

$$(2) + (1) - (3) \Rightarrow (p+q) + (q+r) - (p+q+r) = 110 + 86 - 150 \text{ kg}$$

i.e. q = 46 kg

EXAMPLE 18 A class consists of 36 girls and 44 boys. If the average weight of girls is 40 kg and that of boys is 35 kg, find the average weight of all students in the class?

Solution Total weight of students = $36 \times 40 + 44 \times 35 = 2980 \text{ kg}$

Average weight of whole class =
$$\frac{2980}{36 + 44} = \frac{2980}{80} = 37.25 \text{ kg}$$

EXAMPLE 19 While entering marks scored by students of a class, mark of one student was mistakenly entered as 83 instead of 63. So the average marks for the class got increased by half. The number of students in the class is

Solution Let *n* be the number of students in the class

Change in average =
$$\frac{1}{2} = \frac{83 - 63}{N} = \frac{20}{N}$$

N = 40



EXERCISES

- 1. In an examination the average marks was found to be 60. The marks of 80 candidates had to be changed from 95 to 70 each and the average came down to 55 marks. The total number of candidates attended the examination were
 - (a) 250
- (b) 300
- (c) 400
- (d) 450
- (e) None of these
- 2. The average age of a group of 15 students is 10 years. When 5 more students joined the group, the average got increased by 1 year. The age of the new student is
 - (a) 18 years
- (b) 16 years
- (c) 14 years
- (d) 12 years
- (e) None of these
- 3. A person travels 1/3 of his journey by bus at 60 km/h, 1/3 by scooter at 30 km/h and the rest by walking at 10 km/h. Find his average speed for the whole journey.
 - (a) 30 km/h
- (b) $33\frac{1}{3}$ km/h (c) 20 km/h
- (d) 50 km/h
- (e) None of these
- 4. The average speed of a train in the onward journey is 25% less than that of the return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is
 - (a) 45 km/h
- (b) 47.06 km/h
- (c) 43.75 km/h
- (d) 56.25 km/h
- 5. The average of X_1 , X_2 and X_3 is 14. Twice the sum of X_2 and X_3 is 30. What is the value of X_1 ?
 - [IAS]

- (a) 20
- (b) 27
- (c) 16
- (d) 12

entire trip?

	(a) 45	(b) 2	$20\sqrt{5}$	(c) 400/9	
	(d) Impossible to	o find out unless th	e distance betweer	n the two place is k	known.
7.		of a cricket team re ingsters of equal ag			aged 32 and 30 are [IAS]
	(a) 20	(b) 22	(c) 24	(d) 26	
8.		ht of the students o join this class, the			_
	(a) 117.5 cm	(b) 115 cm	(c) 110 cm	(d) 112.5 cm	(e) None of these
9.	The average of fir	rst 46 natural num	ber is		
	(a) 23	(b) 23.5	(c) 24	(d) 25	(e) None of these
10.	The average of 13 is 49, find the 7th	B results is 50. If the h result.	e average of first 7	results is 52 and th	nat of last 7 results
	(a) 50	(b) 52	(c) 56	(d) 57	(e) None of these
11.		of a group of 18 per ne person of age 50 oup will be			
	(a) 26 years	(b) 27 years	(c) 28 years	(d) 29 years	(e) None of these
12.	-	rks of a student ob 90 marks in Math		•	ology is 70. If the
	(a) 80	(b) 78	(c) 76	(d) 75	(e) None of these
13.		tht of 3 person is 65 te average weight o		son having a weigh	t of 45 kg joins the
	(a) 65 kg	(b) 63 kg	(c) 61 kg	(d) 60 kg	(e) None of these
14.		B of his journey by h. Find the average			
	(a) 65 km/h	(b) 45 km/h	(c) 48 km/h	(d) $53\frac{1}{3}$ km/h	(e) None of these
15.		ght of 6 persons is is we man. The weight		hen one of them w	ho weighs 40 kg is
	(a) 76 kg	(b) 74 kg	(c) 72 kg	(d) 71 kg	(e) None of these
16.		of a graph of 9 stude falls by 2 years. The			
	(a) 20 years	(b) 18 years	(c) 19 years	(d) 16 years	(e) None of these
17.		dents made an aver What is the overall		irks and another ba	atch of 40 students
	(a) 49	(b) 48	(c) 47	(d) 46	(e) None of these
18.		a car to his office e average speed of			the same route at
	(a) 40 km/h	(b) 50 km/h	(c) 30 km/h	(d) 25 km/h	(e) None of these
19.		nean of the scores of a mean score of 80 a % is			
	(a) 45	(b) 50	(c) 51.4 approx	(d) 54.6 approx.	-

6. A person travelled from one place to another at an average speed of 40 km/h and back to the original place at an average speed of 50 km/h. What is his average speed in km/h during the

90	10 years are the average are of	o family of 1 mamb	one week 94 weeks Two	ahilduan harring				
40.	10 years ago, the average age of been born (with age difference of							
		The present age of the youngest child is [SSC						
	(a) 1 year (b) 2 years	(c) 3 years	(d) 5 years					
21.	The average weight of 3 men A,							
	average now becomes 80 kg. If a							
	replaces A, then the average we	ight of B, C, D and E	becomes 79 kg. The v	veight of A is [Bank PO]				
	(a) 70 kg (b) 72 kg	(c) 75 kg	(d) 80 kg	[Dank I O]				
22.	A cricketer has a certain average	` ,	` ,	scored 108 runs.				
	thereby increasing his average b			[AAO Exam]				
	(a) 48 runs (b) 52 runs	(c) 55 runs	(d) 60 runs					
23.	The average age of 8 men is incr and 23 years are replaced by two							
	(a) 22 years (b) 24 years		(d) 30 years					
24.	The average weight of 45 studen	* * * * * * * * * * * * * * * * * * * *		verage weight is				
	48 kg leave the class and other 5							
	is the new average weight (in kg	g) of the class		[RRB]				
	(a) $52\frac{1}{3}$ (b) $52\frac{1}{2}$	(c) $52\frac{2}{3}$	(d) None of these					
25.	The average age of 36 students	in a group is 14 years	s. When teacher's age	is included to it,				
	the average increases by one wh	at is the teacher's ag	ge in years?	[RBI]				
	(a) 31 (b)	o) 36	(c) 51					
	(d) Cannot be determined (e	e) None of these						
26	The average weight of three boy	vs A R and C is 54 1	ka while the everen	woight of throo				
40.	The average weight of three boy	3 n, D and O is 34 –	- kg, while the average	e weight of three				
	boys B, D and E is 53 kg. What i	is the average weigh	t of A, B, C, D and E?	[SBI PO]				
	(a) 52.4 kg (b)	o) 53.2 kg	(c) 53.8 kg					
	(d) Data inadequate (e	e) None of these						
27 .	The mean of 50 observations wa		er that an observation					
	taken as 23. The corrected new 1		(1) 20 1	[SSC]				
90	(a) 35.2 (b) 36.1 Of the three numbers, the first	(c) 36.5	(d) 39.1	o the third The				
40.		_		the third. The				
	average of the reciprocal of the r	· -	numbers are	[CBI]				
	,, , , , , , , , , , , , , , , , , , , ,		(d) 36, 18, 9					
29.	In the first 10 overs of a cricket g in the remaining 40 overs to rea			d be the run rate [MAT]				
	(a) 6.25 (b) 6.5	(c) 6.75	(d) 7					
30.	The average of 6 numbers is 3.95 other two is 3.85. What is the av			he average of the [Bank PO]				
	(a) 4.5 (b) 4.6	(c) 4.7	(d) 4.8					
31.	16 children are to be divided in	to two groups A and	l B of 10 and 6 childs	en. The average				
	per cent marks obtained by the cl							
	the 16 children is 76. What is the	e average per cent ma	arks of children of grou	ip B? [BSRB]				
	(a) $77\frac{1}{3}$ (b) $77\frac{2}{3}$	(c) $78\frac{1}{3}$	(d) $78\frac{2}{3}$					

32 .	The average of five excluded number is		. If on	e number is ex	cclud	ed, the average becon [Section	nes 25. The Officer's]
	(a) 25	(b) 27	(c)	30	(d)	35	
33.						The nineth one spen ney spent by all of the	
	(a) ₹ 260	(b) ₹ 290	(c)	₹ 292.50	(d)	₹ 400.50	
34.		years, that of	the pa	rents is 35 yea		l children. The averag nd that of the grand cl	
	(a) $28\frac{4}{7}$ years	•		•			
35.	The sum of three co What is the first of			ers is 38 more	e tha	n the average of these	e numbers. Bank PO]
	(a) 13	(b)	17			(c) 19	
	(d) Data inadequa	te (e)	None	of these			
36.	The average of 7 co	nsecutive num	bers is	s 20. The large	est of	these numbers is	[SSC]
	(a) 20	(b) 22	(c)	23	(d)	24	
37.	The average of the their position is	two-digit num			the sa	ame when the digits is	nterchange [CDS]
		(b) 44	(c)		(d)		
38.	If the mean of a , b ,					_	[ISTTM]
	, ,	(b) $3M^2$	` '	$6M^2$	(d)	$9M^2$	
39 .	The average of first	t nine prime n	umber	s is			[CBI]
	(a) 9	(b) 11	(c)	$11\frac{1}{9}$	(d)	$11\frac{2}{9}$	
40.	Find the average of	f all the numbe	ers bet	ween 6 and 34	whi	ch are divisible by 5	[CBI]
	(a) 18	(b) 20	(c)	24	(d)	30	
41.						nbers 3, 11, 7, 9, 15, 15 the number in place of	of x ?
	(a) 3	(b) 7	(c)	17	(d)		Officer's]
42.	` '	servation x, x	` '			is 11, then the mean	of the last
	(a) 11	(b) 13	(c)	15	(d)	17	
43.	The average of a no	on-zero numbe	r and i	ts square is 5	time	s the number. The nu	mber is [SSC]
	(a) 9	(b) 17	(c)	29	(d)	295	
44.	The average of the for the whole class		s is 16	years and that	t of g	irls is 15 years. The a	verage age [SSC]
	(a) 15 years						
	(b) 15.5 years						
	(c) 16 years						
	(d) Cannot be com	puted with the	given	information			
45.	A library has an arnumber of visitors					240 on other days. T g with a Sunday is	he average [MAT]
	(a) 250	(b) 275	(c)	280	(d)	285	

46.	If the average then the average	-				ıd 45 stud	ents respecti	vely is 50, 55 and 60, [CBI]
	(a) 53.33	-	54.68		55	(d)	None of the	
47.				` '				ccessive years. What
								each year? [MAT]
	(a) ₹ 7.98	(b)	₹8	(c)	₹ 8.50	(d)	₹9	
48.	The average of the remain				numbers	, 35 and 4	0 are discard	ed, then the average [RRB]
	(a) 28.32	(b)	28.78	(c)	29.27	(d)	29.68	
49.	The average What is the						average age	of 21 students is 14. [SBI PO]
	(a) 15 year	s (b)	17 years	(c)	18 years	s (d)	19 years	
50.								00. If the manager's e manager's monthly [RRB]
	(a) ₹ 2000	, ,	₹ 2400		₹ 3600	(d)		
51.								$\begin{array}{c} \text{ly income of } Q \text{ and } R \\ \text{onthly income of } P \text{ is} \\ \textbf{[RRB]} \end{array}$
	(a) ₹ 3500	(b)	₹ 4000	(c)	₹ 4050	(d)	₹ 5000	
52 .								age age of 5 students 15th student is [SSC]
	(a) 11 year	s (b)	14 years	(c)	15 years	s (d)	$15\frac{2}{7}$ years	
53.	The average of B and C i					average w	eight of A an	d B is 40 kg and that [SSC]
	(a) 17 kg	(b)	$20~\mathrm{kg}$	(c)	$26~\mathrm{kg}$	(d)	31 kg	
54.	The average	e of 8 num	bers is 20	. The av	erage of	first two ı	numbers is 1	$5\frac{1}{2}$ and that of next
	three is 21	$\frac{1}{3}$. If the s	sixth num	ber is le	ss than t	he sevent	h and eighth	numbers by 4 and 7
	respectively			ımber is				[SSC]
	(a) 18	` ′	22		25	` '	27	
55.		and the thi						than the average of and the third of the [SBI PO]
	(a) 15		(b) 45			(c) 60	
	(d) Data in	adequate	((e) None	of these			
56.	The average in his next							runs must he make [SSC]
	(a) 2	(b)	4	(c)	70	(d)	76	
57.	_	ıs is ₹ 120	00 and th			_		e average salary of The total number of [SSC]
	(a) 20		21	(c)	22	(d)	23	

1. (c)

11. (b)

21. (c)

31. (b)

41. (b)

51. (b)

61. (a)

3. (c)

13. (d)

23. (d)

33. (c)

43. (a)

53. (d)

63. (d)

2. (c)

12. (d)

22. (a)

42. (b)

52. (a)

62. (e)

32. (d)

4. (c)

14. (c)

24. (c)

34. (b)

44. (d)

54. (c)

64. (b)

5. (b)

15. (e)

25. (c)

35. (b)

45. (d)

55. (e)

65. (d)

6. (c)

16. (e)

26. (d)

36. (c)

46. (d)

56. (d)

7. (a)

17. (b)

27. (c)

37. (c)

47. (b)

57. (b)

8. (c)

18. (c)

28. (c)

38. (d)

48. (d)

58. (c)

9. (b)

19. (c)

29. (a)

39. (c)

49. (d)

59. (d)

58.					years. A baby having ent age of the baby is [SSC]
	(a) 1 years	(b) $1\frac{1}{2}$ yea	rs (c) 2 years	(d) 3 years	
59.					ickets for 26 runs and him till the last match [SSC]
	(a) 64	(b) 72	(c) 80	(d) 85	
60.	of them weighi		might be the weig	ht of the new person	= =
	(a) 76 kg		(b) 76.5 kg	(c) 85 kg	;
	(d) Data inade	quate	(e) None of these		
61.	-	-	of 24 students is 35 e weight of the teac		he teacher is included, [SSC]
	(a) 45 kg	(b) 50 kg	(c) 53 kg	(d) 55 kg	
62.			and 85 marks (out is his average mar		Mathematics, Physics, [Bank PO]
	(a) 65	(b) 69	(c) 72	(d) 76	(e) None of these
63.	workers is T . The	he number of ag		is 11 times that of ot	s S and that of other her workers. Then the [SSC]
	(a) $\frac{S+T}{2}$	(b) $\frac{S + 117}{2}$	(c) $\frac{1+T}{11S}$	(d) $\frac{11S + T}{12}$	
64.		that of the girls			of boys in the class is of boys to the number [SBI PO]
	(a) 1:2	(b) 2:3	(c) 3:4	(d) 3:5	
65.	` '	, ,	` '	` '	12%, then the average
	(a) will remain			may decrease	1270, then the average
	` '	· ·	` '	increased by 12%	
	(c) may increa	ise	(u)	increased by 12%	
R	HERS				

10. (d)

20. (c)

30. (b)

40. (b)

50. (c)

60. (c)

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 \Rightarrow

 \Rightarrow

 \Rightarrow

Solutions with Necessary Explanation

- 1. Let the number of candidates attended the examination be 'n'.
 - Total marks in the examination = 60n.

80 candidates marks are changed from 95 to 70 and average reduced to 55 marks.

$$\therefore \qquad \text{Old average} = 60 \quad \Rightarrow \quad \text{Total} = 60n \tag{1}$$

As per new average, total marks in the examination = 55n

$$= 60n - 80 \times (95 - 70)$$

$$=60n-2000$$

$$60n - 55n = 2000$$

$$5n = 2000$$
$$n = 400$$

2. Old average of 15 students = 10 years. 5 more students joined the group.

New average = 10 + 1 = 11 years.

Let Age of new student= x years

$$\therefore \frac{15 \times 10 + 5 \times x}{20} = 11$$

$$\Rightarrow$$
 $5x = 20 \times 11 - 15 \times 10 = 70$

:.
$$x = \frac{70}{5} = 14 \text{ years}$$

3. Let 'S' be the total distance travelled.

Time taken for 1st 1/3rd of journey = $\frac{S/3}{60} = \frac{S}{180}$ hour.

Time taken for 2nd 1/3rd of journey = $\frac{S/3}{30} = \frac{S}{90}$ hour.

Time taken for 3rd 1/3rd of journey = $\frac{S/3}{10} = \frac{S}{30}$ hour.

$$\therefore \qquad \text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{S}{\frac{S}{180} + \frac{S}{90} + \frac{S}{30}}$$

$$= \frac{S}{S\left(\frac{1}{180} + \frac{1}{90} + \frac{1}{30}\right)}$$

$$= \frac{180 \times 90 \times 30}{90 \times 30 + 180 \times 30 + 180 \times 90}$$

$$= \frac{180 \times 9 \times 3}{27 + 54 + 162}$$
$$= \frac{180}{1 + 2 + 6}$$
$$= 20 \text{ km/hr.}$$

- 4. Let the average speed of train for return journey be 'x' km/hr
 - \therefore Average speed in the onward journey = $\frac{3}{4}x$ km/hr
 - i.e. Average speed for the whole journey = $\frac{800}{16}$ = 50 km/hr

i.e.
$$50 = \frac{2(x)\left(\frac{3}{4}x\right)}{x + \frac{3}{4}x} = \frac{\frac{3}{2}x^2}{\frac{7}{4}x} = \frac{3}{2} \times \frac{4}{7}x = \frac{6}{7}x$$

$$\therefore x = \frac{7 \times 50}{6} \, \text{km/hr}.$$

- \therefore Speed of train in onward journey = $\frac{3}{4} \times \frac{7 \times 50}{6} = \frac{175}{4} = 43.75 \text{ km/hr}$
- **5.** Given average of X_1 , X_2 and $X_3 = 14$.

i.e.
$$X_1 + X_2 + X_3 = 14 \times 3 = 42$$
 (1)

$$2(X_2 + X_3) = 30$$

$$X_2 + X_3 = 15$$
(2)

$$\Rightarrow X_2 + X_3 = 15 \therefore (1) - (2) \Rightarrow X_1 = 42 - 15 = 27$$

6. Average speed of the person for his whole journey =
$$\frac{2\times40\times50}{(40+50)} = \frac{4000}{90} = \frac{400}{9}$$
 km/hr

7. A cricket team consists of 11 players.

Let its average age be x, and age of replaced player be y.

$$\therefore 11x - 32 - 30 + 2y = 11(x - 2)$$

i.e.
$$32 + 30 - 22 = 2y$$

or
$$y = \frac{32 + 30 - 22}{2} = \frac{40}{2} = 20 \text{ years}$$

8. Average height of 20 students in a class = 105 cm.

After joining 10 students of average height 120 cm, average height of students of the class

$$= \frac{20 \times 105 + 10 \times 120}{30} = 2 \times 35 + 10 \times 4 = 70 + 40 = 110 \text{ cm}$$

- **9.** Average of first 46 natural numbers $=\frac{1}{46} \left[\frac{46}{2} \times (1+46) \right] = \frac{47}{2} = 23.5$
- 10. Average of 13 results = 50

Average of first 7 results = 52

Average of last 7 results = 49

:. The 7th result = $7 \times 52 + 7 \times 49 - 13 \times 50 = 57$

11. Average age of 18 persons = 32 years 6 months

Age of persons left = 65 years \times 2 + 50 years \times 1

:. The sum of ages of persons remaining

=
$$18 \times 32$$
 years 6 months $-2 \times 65 - 1 \times 50$

$$=585-130-50=405$$

 \therefore Average age of remaining persons = $\frac{405}{15}$ = 27 years

12. Average marks in Physics, Chemistry and Biology = 70

Marks obtained in Maths = 90

 \therefore New average for the 4 subjects = $\frac{70 \times 3 + 90 \times 1}{4} = \frac{300}{4} = 75$ marks.

13. Average weight of 3 persons = 65 kg

Weight of 4th person = 45 kg

Average weight of new group = $\frac{65 \times 3 + 45 \times 1}{4} = \frac{195 + 45}{4} = \frac{240}{4} = 60 \text{ kg}$

14. Average speed of the man for the whole journey = $\frac{3 \times 80 \times 60 \times 30}{80 \times 60 + 60 \times 30 + 30 \times 80}$

$$= \frac{3 \times 80 \times 6 \times 3}{8 \times 6 + 6 \times 3 + 3 \times 8} = \frac{3 \times 80 \times 3}{8 + 3 + 4} = \frac{9 \times 80}{15} = 3 \times 16 = 48 \text{ km/h}$$



Short cut: Weight of new man = Weight of replaced man + Increase in average

$$=40+5\times 6$$

$$= 40 + 30 = 70 \text{ kg}$$

16. Average age of new student =

Old number of student × Old average of students ~ New average × New number of students

Number of new students

$$=\frac{9\times20\sim18\times15}{6}=\frac{180\sim270}{6}=\frac{90}{6}=15 \text{ years}$$

17. Overall average score = $\frac{60 \times 50 + 40 \times 45}{60 + 40}$

$$= \frac{60 \times 50 + 40 \times 45}{100}$$

$$= 30 + 18 = 48$$

18. Average speed = $\frac{20 \times 60 \times 20}{60 + 20} = \frac{2 \times 60 \times 20}{80} = 30 \text{ km/h}$

19. Let the number of students be 100.

Then total score =
$$52 \times 100 = 5200$$

first 20% scored = $20 \times 80 = 1600$
last 25% scored = $25 \times 31 = 775$

 \therefore Remaining 55% student's score = 5200 - 1600 - 775 = 2825

$$\therefore \qquad \text{Average score among } 55\% = \frac{2825}{55} = 51.4 \text{ approx}$$

- 20. Average age of 4 members (10 years ago) = 24 years
 - :. Average age of 4 members at present = 34 years

Let the age of children be x and x + 2 years

Present average age of family =
$$\frac{34 \times 4 + x + x + 2}{6} = 24$$

i.e.
$$\frac{136 + 2 + 2x}{6} = 24$$

or $138 + 2x = 144$

or $2x = 6$

or $x = 3 \text{ years}$
 $\therefore x + 2 = 5 \text{ years}$

- :. Age of youngest child = 3 years
- 21. Average weight of A, B and C = 84 kg

On joining D, average reduces to 80 kg.

Let weight of A, B, C, D and E be a, b, c, d and e respectively

$$\frac{a+b+c}{3} = 84 \tag{1}$$

$$\frac{a+b+c+d}{4} = 80 (2)$$

$$a+b+c=84\times3$$
 (a)

$$a+b+c+d=80\times 4\tag{b}$$

(b) - (a)
$$\Rightarrow$$
 $d = 80 \times 4 - 84 \times 3$
= $320 - 252 = 68$ years
 $e = d + 3 = 68 + 3 = 71$ years

$$\therefore \frac{b+c+d+71}{4} = 79 \tag{3}$$

or
$$b + c + d + 71 = 79 \times 4$$

$$= 245$$
 (c)

 $b + c + d = 79 \times 4 - 71$

(b) – (c)
$$\Rightarrow a = 320 - 245 = 75 \text{ years}$$

22. Let his average in the tenth innings be x.

or

His score in 11th innings = 108 runs

$$\therefore \text{ Average in eleventh innings} = \frac{x \times 10 + 108}{11} = x + 6$$

i.e.
$$10x + 108 = (x + 6)11$$

$$11x - 10x = 108 - 66$$
$$x = 42$$

:. New average = x + 6 = 42 + 6 = 48 runs

23. Let average age of 8 men be x.

Let the average age of two men joining be y.

On joining 2 persons by replacing two persons of ages 21 and 23 years

.. New average =
$$\frac{8 \times x - 21 - 23 + 2y}{8} = x + 2$$

i.e.
$$8x - 44 + 2y = 8x + 16$$

or
$$2y = 44 + 16 = 60$$

$$\therefore$$
 $y = 30 \text{ years}$

24. Average weight of 45 students = 52 kg

$$\therefore$$
 Total weight = $45 \times 52 \text{ kg}$

Weight of students left = $5 \times 48 \text{ kg}$

Weight of students joined = 5×54 kg

.. New average weight =
$$\frac{45 \times 52 - 5 \times 48 + 5 \times 54}{45}$$

= $52 + \frac{5(54 - 48)}{45}$
= $52 + \frac{30}{45} = 52 + \frac{2}{3} + \frac{2}{3} = 52 + \frac{2}$

25. Average age of 36 students = 14 years

Let teacher's age be x years.

$$\therefore \frac{14 \times 36 + x}{37} = 14 + 1 = 15$$

or
$$14 \times 36 + x = 15 \times 37$$

$$\therefore \qquad x = 15 \times 37 - 14 \times 36 = 51$$

26. Average weight of A, B and C = $\frac{a+b+c}{3} = 54 \frac{1}{3} \text{ kg}$

Average weight of B, D and E = $\frac{b+d+e}{3}$ = 53 kg

Data inadequate

27. Corrected new mean =
$$\frac{50 \times 36 - 23 + 48}{50} = 36 + \frac{25}{50} = 36.5$$

28. Let the third number be x

$$\therefore$$
 Second number = $2 \times x = 2x$

First number =
$$2 \times 2x = 4x$$

 \therefore Average of reciprocal of numbers = $\frac{1}{3} \left(\frac{1}{x} + \frac{1}{2x} + \frac{1}{4x} \right)$

$$= \frac{1}{3x} \left(\frac{4+2+1}{4} \right) = \frac{7}{12x} = \frac{7}{72}$$

$$\therefore$$
 $x = 6$

∴ Numbers are 6, 12, 24

29. Score in the first 10 overs = $3.2 \times 10 = 32$

To achieve a target of 282 runs,

Balance runs required = 282 - 32 = 250

Balance overs available = 40

$$\therefore \qquad \text{Required run rate} = \frac{250}{40} = 6.25$$

30. Required average =
$$\frac{3.95 \times 6 - 3.4 \times 2 - 3.85 \times 2}{2}$$
$$= \frac{23.70 - 6.8 - 7.7}{2}$$
$$= \frac{23.70 - 14.50}{2} = \frac{9.20}{2} = 4.60$$

31. Group A average = 75

Group (A + B) average = 76

Required average =
$$\frac{16 \times 76 - 10 \times 75}{6}$$

= $\frac{760 + 456 - 750}{6} = \frac{466}{6}$
= $77\frac{4}{6} = 77\frac{2}{3}\%$

32. Total of 5 numbers = 27×5

Excluded number = $27 \times 5 - 25 \times 4 = 135 - 100 = 35$

33. Let the total money spent be x

∴ Average expenditure =
$$\frac{x}{9}$$

$$\therefore 8 \times 30 + \left(\frac{x}{9} + 20\right) = x$$

i.e.
$$260 = x - \frac{x}{0}$$

$$260 = \frac{8x}{9}$$

34. Total age of a family =
$$2 \times 67 + 2 \times 35 + 3 \times 6$$

= $134 + 70 + 18 = 222$ years

$$\therefore$$
 Average age of family = $\frac{222}{7} = 31\frac{5}{7}$ years

35. Let (x - 2), x, (x + 2) be the numbers.

Sum of numbers =
$$x - 2 + x + x + 2 = 3x$$

Average of numbers =
$$\frac{3x}{3} = x$$

$$\therefore \qquad 3x = x + 38$$

$$\therefore \qquad 2x = 38$$

or
$$x = 19$$

$$\therefore \qquad \text{First number} = 19 - 2 = 17$$

36. Let number be x, (x + 1), (x + 2), ..., (x + 6).

:. Average =
$$\frac{1}{7} \times \frac{7}{2} (x + x + 6) = x + 3$$

i.e.
$$x + 3 = 20$$

$$\therefore$$
 $x = 20 - 3 = 17$

 \therefore Largest number = x + 6 = 17 + 6 = 23

37. Such two digit numbers are 11, 22, 33, 44, 55, 66, 77, 88 and 99.

$$\therefore$$
 Sum of these number = 11 (1 + 2 + 3 + \cdots + 9)

$$= 11 \times \frac{9}{2} (1+9) = 11 \times \frac{9}{2} \times 10 = 495$$

:. Average =
$$\frac{495}{9} = 55$$

38. Given
$$\frac{a+b+c}{3}=M$$

or
$$(a+b+c)=3M$$

$$(a+b+c)^2 = 9M^2$$

i.e.
$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 9M^2$$

$$\therefore \qquad ab + bc + ca = 0$$

$$a^2 + b^2 + c^2 = 9M^2$$

39. First nine prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and 23.

Average of above numbers =
$$\frac{2+3+5+7+11+13+17+19+23}{9} = \frac{100}{9} = 11\frac{1}{9}$$

40. Numbers between 6 and 34, which are divisible by 5 are 10, 15, 20, 25 and 30.

:. Average of above numbers =
$$\frac{10 + 15 + 20 + 25 + 30}{5} = \frac{100}{5} = 20$$

41. Given 12 numbers and mean to be 12.

$$\therefore$$
 Sum of 12 numbers = $12 \times 12 = 144$

$$\therefore$$
 $x = 144 - (3 + 11 + 7 + 9 + 15 + 13 + 8 + 19 + 17 + 21 + 14) = 144 - 137 = 7$

42. Mean of 5 observations given,

$$\frac{x+x+2+x+4+x+6+x+8}{5} = 11$$

i.e.
$$x + 4 = 11$$

or
$$x = 11 - 4 = 7$$

.. Mean of last 3 observations =
$$\frac{x+4+x+6+x+8}{3} = x+6=7+6=13$$

43. Let the non-zero number be x.

$$\frac{x+x^2}{2} = 5x$$

i.e.
$$x + x^2 = 10x$$
or
$$x^2 - 9x = 0$$
or
$$x - 9 = 0$$

$$x = 9$$

- 44. Cannot be computed as the number of boys and girls are not given.
- 45. In a month starting with Sunday, there will be 5 Sundays and 25 week days

$$\therefore \text{ Average visitor/day} = \frac{5 \times 510 + 25 \times 240}{30} = 5 \times 17 + 25 \times 8 = 85 + 200 = 285$$

46. Average marks of all students =
$$\frac{55 \times 50 + 60 \times 15 + 45 \times 60}{160}$$
$$= \frac{55 \times 5 + 6 \times 15 + 45 \times 6}{16} = \frac{275 + 90 + 270}{16} = \frac{635}{16} = 39.68$$

47. In the first year petrol consumed = $\frac{4000}{7.5}$ litres

In the second year petrol consumed = $\frac{4000}{8}$ litres

In the third year petrol consumed = $\frac{4000}{8.5}$ litres

Average price/litre of petrol =
$$\frac{4000 \times 3}{\frac{4000}{7.5} + \frac{4000}{8} + \frac{4000}{8.5}}$$

$$= \frac{3 \times 7.5 \times 8 \times 8.5}{8 \times 8.5 + 7.5 \times 8.5 + 7.5 \times 8}$$

$$= \frac{1530}{68 + 63.75 + 60} = \frac{1530}{191.75} = ₹8/litre$$



Short cut: Price/litre of petrol for 3 successive years are ₹ 7.5, ₹ 8 and ₹ 8.5/ litre

Average price =
$$\frac{7.5 + 8 + 8.5}{3}$$
 = ₹8/litre

48. The sum of 50 numbers = $50 \times 30 = 1500$

Discarding 35 and 40, we get

Average of remaining 48 numbers =
$$\frac{1500 - 35 - 40}{48}$$

= $\frac{1500 - 75}{48} = \frac{1425}{48} = 29.68$

49. Average age of 35 students = 16

Average age of 21 students = 14

.. Average age of remaining 14 students =
$$\frac{35 \times 16 - 21 \times 14}{14}$$
$$= 5 \times 8 - 21 = 40 - 21 = 19 \text{ years}$$

50. Average monthly salary of 20 employees = $\mathbf{\xi}$ 1.500

On adding manager's salary (let it be x)

New average =
$$\frac{1500 \times 20 + x}{21}$$
 = 1500 + 100 = 1600

 \therefore 1500 × 20 + $x = 21 \times 1600$

or
$$x = 21 \times 1600 - 20 \times 1500$$

= 33600 - 30000 = ₹ 3600

51. Monthly income of *P* and
$$Q = 2 \times 5050 = 10100$$
 (1)

Monthly income of
$$Q$$
 and $R = 2 \times 6250 = 12500$ (2)

Monthly income of
$$P$$
 and $R = 2 \times 5200 = 10400$ (3)

$$\frac{(1) + (2) + (3)}{2} - (2) \Rightarrow$$

Monthly income of
$$P = \frac{10100 + 12500 + 10400}{2} - 12500$$

= $5050 + 6250 + 5200 - 12500$

52. Total age of 15 students =
$$15 \times 15 = 225$$
 years (1)

Sum of ages of 5 students =
$$5 \times 14 = 70$$
 years (2)

Sum of ages of 9 students =
$$9 \times 16 = 144$$
 years (3)

 \therefore Age of 15th student = (1) - (2) - (3) = 225 - (70 + 144) = 11 years

53. Weight of
$$A + B + C = 45 \times 3 = 135 \text{ kg}$$
 (1)

Weight of A + B =
$$40 \times 2 = 80 \text{ kg}$$
 (2)

Weight of B + C =
$$43 \times 2 = 86 \text{ kg}$$
 (3)

Subtracting (2) from (1),

:. Weight of
$$C = 135 - 80 = 55 \text{ kg}$$
 (4)

Subtracting (3) from (1),

Weight of
$$A = 135 - 86 = 49 \text{ kg}$$
 (5)

Subtracting (5) from (2),

:. Weight of
$$B = 80 - 49 = 31 \text{ kg}$$
 (6)

54. Sum of 8 numbers =
$$8 \times 20 = 160$$
 (1)

Sum of first 2 numbers =
$$2 \times 15 \frac{1}{2} = 31$$
 (2)

Sum of next 3 numbers =
$$3 \times 21 \frac{1}{3} = 64$$
 (3)

Sixth number = Seventh number
$$-4$$
 (4)

Sixth number = Eighth number
$$-7$$
 (5)

$$(1) - (2) - (3) \Rightarrow 6th \text{ number} + 7th \text{ number} + 8th \text{ number}$$

$$= 160 - (31 + 64) = 160 - 95 = 65 \tag{6}$$

$$\therefore \qquad (6) \Rightarrow \quad (8\text{th number} - 7) + (8\text{th number} - 3) + (8\text{th number}) = 65$$

or
$$3 \times 8$$
th number = $65 + 7 + 3 = 75$

 \therefore 8th number = 25

55. Let x, y and z be the numbers.

$$\frac{x+y}{2} = \frac{y+z}{2} + 15$$

$$\therefore \frac{x-z}{2}=15$$

or
$$x - z = 2 \times 15 = 30$$

56. Total score in 10 innings = $32 \times 10 = 320$

To raise the average to 36,

Total score after 11 innings = $36 \times 11 = 396$

- \therefore Runs he has to take in the 11th innings = 396 320 = 76
- **57.** Let the total number of workers be x.
 - \therefore Number of workers except technician = x-7

$$\therefore 8000x = 7 \times 12000 + (x - 7) 6000$$

i.e.
$$(8000 - 6000)x = 7 \times 12000 - 7 \times 6000$$

$$2000x = 7 \times 6000$$

or
$$x = \frac{7 \times 6000}{2000} = 21$$

- 58. Average of age of family of 5 members (3 years ago) = 17 years
 - \therefore Present age of family of 6 members with a new born baby = 17 years Let the age of baby be x

$$\therefore \quad \frac{(17+3)\times 5+x}{6}=17$$

i.e.
$$\frac{100 + x}{6} = 17$$

or
$$100 + x = 102$$

or
$$x = 2$$
 years

59. Let the number of wickets taken be x.

$$\therefore \frac{12.4 \times x + 26}{x + 5} = 12$$

i.e.
$$12.4x + 26 = 12x + 60$$

or
$$0.4x = 60 - 26 = 34$$

or
$$x = \frac{34}{0.4} = 85$$

60. Let the weight of new person be $x \, \text{kg}$.

Weight of person replaced = 65 kg

i.e. Weight of new person = Weight of replaced person

+ old number of persons × change in average

$$x = 65 + 8 \times 2.5 = 85 \text{ kg}$$

61. Total weight of 24 students = $24 \times 35 = 840$

Let the weight of teacher be x kg.

$$\therefore \frac{840 + x}{25} = 35.4 \text{ kg}$$

or
$$840 + x = 25 \times 35.4 = 885$$

$$x = 885 - 840 = 45 \text{ kg}$$

62. Average marks =
$$\frac{76 + 65 + 82 + 67 + 85}{5} = \frac{375}{5} = 75$$

63. Average annual income of agricultural worker = S

Average annual income of other workers = T

Let other workers be n in number and agricultural works be 11n in number.

$$\therefore \frac{11n \times S + n \times T}{12n} = \frac{11S + T}{12}$$

64. Average age of boys = 16.4 years

Average age of girls = 15.4 years

Let m be number of boys and n be number of girls.

$$\therefore \frac{m \times 16.4 + n (15.4)}{m + n} = 15.8$$

$$16.4m + 15.4n = 15.8m + 15.8n$$

i.e.
$$0.6m = 0.4n$$

or
$$\frac{m}{n} = \frac{0.4}{0.6} = \frac{2}{3}$$

65. Since all numbers were increased by 12%, the average will also increase by 12%.

hapter

Problems Based on Ages

To solve the problems based on ages, the student should know how to solve linear equations within a short span of time. This chapter provides some typical questions based on ages.

EXAMPLE 1 The age of a person A was 7 times that of B 5 years ago. At present the age of A is 4 times that of B. What are the present ages of A and B?

Solution Let the present age of B be x years.

 \therefore The present age of A = $4 \times x = 4x$ years

Age of
$$A = 4x - 5$$
Age of $B = x - 5$

$$4x - 5 = 7(x - 5)$$
i.e. $7x - 4x = 35 - 5 = 30$
i.e. $3x = 30$

$$x = 10 \text{ years}$$
i.e. Present age of $B = 10 \text{ years}$
Present age of $A = 4 \times 10 = 40 \text{ years}$

EXAMPLE 2 At present the age of a father is five times that of his son. Three years hence, the father's age would be four times that of his son. Find the present ages of the father and the son.

Solution Let the present age of son be *x* years.

Then the present age of father = 5x years

Three years hence

or
$$4(x+3) = 5x + 3$$
or
$$4x + 12 = 5x + 3$$

$$x = 9 \text{ years}$$

$$Son's age = 9 \text{ years}$$
Father's age = $5 \times 9 = 45 \text{ years}$

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Solution Let the present age of son be x years.

Three years ago

Age of son =
$$(x - 3)$$
 years
Age of father = $7(x - 3)$ years

Three years hence

Age of son =
$$(x + 3)$$
 years
Age of father = $7(x - 3) + 6 = 7x - 15$
= $4(x + 3)$
 $7x - 15 = 4(x + 3)$
 $7x - 4x = 15 + 12 = 27$
 $3x = 27$
 $x = 9$ years
Age of son = 9 years
Age of father = $7(x - 3) + 3$
= $7(9 - 3) + 3$
= 9 years



i.e.

i.e.

i.e.

:.

Short cut

1. t_1 years earlier the father's age was x times that of his son. At present the father's age is y times that of his son. What is the present age of his son?

The son's age is given by
$$t_1 \frac{(x-1)}{(x-y)}$$

2. The present age of the father is y times the age of his son. t_2 years hence, the father's age becomes z times the age of his son. What are present ages of the father and his son?

Son's age is given by
$$\frac{t_2(z-1)}{(y-z)}$$

3. t_1 years earlier the age of the father was x times the age of his son. t_2 years hence, the age of the father becomes z times the age of his son. What is the present age of the son?

Son's age is given by
$$\frac{t_2(z-1) + t_1(x-1)}{(x-z)}$$

EXAMPLE 4 The age of a man is 4 times that of his son. Five years ago, the man was 7 times as old as his son at that time. What is the present age of the man?

Solution Son's age =
$$\frac{5(7-1)}{(7-4)} = 10$$
 years

 \therefore Father's age = $4 \times 10 = 40$ years

EXAMPLE 5 After 10 years, the age of a father will be twice that of his son. Five years ago, he was 5 times as old as his son. What are their present ages?

Solution Son's age =
$$\frac{5(5-1)+10(2-1)}{(5-2)} = \frac{30}{3} = 10$$
 years

:. Present age of father = 5(10-5) + 5 = 25 + 5 = 30 years

EXAMPLE 6 Ten year's ago, Anil's father was 4 times as old as his son. After 10 years, the father will be twice as old as his son. What is the present age of Anil?

Solution Age of Anil =
$$\frac{10(4-1)+10(2-1)}{(4-2)} = \frac{30+10}{2} = 20$$
 years

EXAMPLE 7 Two years ago the ratio of Ashok's age to Sanal's age was 4:3. Two years hence the ratio of their ages will be 6:5. What is the sum of their present ages in years?

Solution Two years ago, Ashok's age was $\frac{4}{3}$ of Sanal's age.

Two years hence, Ashok's age will be $\frac{6}{5}$ of Sanal's age.

$$\therefore \text{ Sanal's age} = \frac{2\left(\frac{4}{3} - 1\right) + 2\left(\frac{6}{5} - 1\right)}{\frac{4}{3} - \frac{6}{5}} = \frac{2/3 + 2/5}{2/15} = \frac{16/15}{2/15} = 8 \text{ years}$$

$$\therefore \qquad \frac{A-2}{S-2} = \frac{4}{3}$$

i.e.
$$\frac{A-2}{8-2} = \frac{4}{3}$$

$$A-2=\frac{4}{3}\times 6=8$$

$$A = 8 + 2 = 10 \text{ years}$$

i.e. Ashok's age = 10 years.

 \therefore The sum of their present ages = 10 + 8 = 18 years

EXAMPLE 8 The sum of the ages of a father and his son is 60 years. Five years ago, father's age was 4 times that of his son. What are the present ages of the father and son?

Solution Let the age of son be x years.

 \therefore The age of father = (60 - x) years

Five years ago

i.e.
$$[(60-x)-5] = 4(x-5)$$
i.e.
$$55-x = 4x-20$$
i.e.
$$5x = 75$$
or
$$x = 15 \text{ years}$$

 \therefore The present age of son = 15 years

The present age of father = 60 - 15 = 45 years

EXAMPLE 9 The sum of ages of a mother and her daughter is 52 years. After 4 years, the age of the mother will be three times that of the daughter. What is the age of the daughter?

Solution Let the age of daughter be x years.

 \therefore The age of mother = (52 - x) years

After 4 years

$$(52-x) + 4 = 3(x + 4)$$

$$56-x = 3x + 12$$

$$4x = 56-12 = 44$$

$$x = 11 \text{ years}$$

i.e. The age of daughter = 11 years



or

or

Short cut: Daughter's age =
$$\frac{\text{Total ages - Number of years after} \times (\text{Times - 1})}{52 - 4 \times (3 - 1)}$$
$$= \frac{52 - 4 \times (3 - 1)}{3 + 1}$$
$$= \frac{52 - 8}{4} = 11 \text{ years}$$

EXAMPLE 10 The ratio of father's age to son's age is 5:1. The product of their ages is 245. What will be the ratio of their ages after 5 years?

Solution Let the ages of son and father be x and 5x respectively.

Then $5x \times x = 245$ or $5x^2 = 245$ i.e. $x^2 = \frac{245}{5} = 49$ $\therefore \qquad x = 7$

 \therefore Son's age = 7 years

Father's age = 35 years

After 5 years

Father's age = 40 years Son's age = 12 years

 \therefore The ratio of their ages = 40 : 12 = 10 : 3

EXAMPLE 11 The ratio of Ram's age to his mother's age is 4:13. The difference of their ages is 27. What will be the ratio of their ages after 6 years?

Solution The present age ratio = 4:13

The difference of their ages = $4k \sim 13k = 9k$

$$\begin{array}{ccc} \therefore & 9k = 27 \\ \therefore & k = 3 \end{array}$$

After 6 years, age ratio = 4k + 6: 13k + 6

i.e.
$$4 \times 3 + 6 : 13 \times 3 + 6$$

i.e. 18:45=2:5

EXAMPLE 12 The ratio of the ages of a father and his son is 7:2 at present. After 5 years the ratio will become 8:3. What is the present age of the son?

Solution The present age ratio between father and son = 7:2 After 5 years, age ratio = 8:3

:. Son's age =
$$\frac{2 \times 5(8-3)}{7 \times 3 - 8 \times 2} = \frac{2 \times 5 \times 5}{21 - 16} = \frac{2 \times 5 \times 5}{5} = 10$$
 years

Father's age =
$$\frac{7 \times 5(8-3)}{7 \times 3 - 8 \times 2} = \frac{7 \times 5 \times 5}{21 - 16} = \frac{7 \times 5 \times 5}{5} = 35 \text{ years}$$

Short cut: If the present age ratio between father and son is x : y and after t years, age ratio becomes a : b, then

Son's age =
$$\frac{y \times t(a-b)}{\text{Difference of cross product}}$$

Father's age =
$$\frac{x \times t(a-b)}{\text{Difference of cross product}}$$

EXAMPLE 13 The ratio of the ages of a father and his son at present is 3:1, 6 years earlier the ratio was 4:1. What are the present ages of the son and the father?

Solution The present age ratio = 3:1 6 years before age ratio = 4:1

$$\therefore \qquad \text{Son's age} = \frac{1 \times 6(4-1)}{4 \times 1 - 3 \times 1} = 18 \text{ years}$$

Father's age =
$$\frac{3 \times 6(4-1)}{4 \times 1 - 3 \times 1} = 54$$
 years

EXAMPLE 14 A father said to his son, "I was as old as you are at present at the time of your birth". If the father's age is 38 years now, the son's age five years back was.

[Assistant Grade Exam]

Solution Let the son's age 5 years back be x.

$$\therefore$$
 Present age of son = $x + 5$

$$\therefore \qquad 38 - (x+5) = (x+5)$$

or
$$38 - 5 - 5 = 2x$$

or
$$x = \frac{28}{2} = 14 \text{ years}$$

EXAMPLE 15 The sum of the ages of 5 children born at the intervals of 3 years each is 50 years what is the age of the youngest child? [SSC]

Solution Let the age of youngest child be x.

$$\therefore \qquad x + (x+3) + (x+6) + (x+9) + (x+12) = 50$$

i.e.
$$5x + 30 = 50$$

or
$$5x = 50 - 30$$
or
$$5x = 20$$
or
$$x = \frac{20}{5} = 4 \text{ years}$$

Hence, age of youngest child = 4 years.

EXAMPLE 16 The sum of the present ages of a father and his son is 60 years. Six years ago, father's age was five times the age of his son. After six years, son's age will be.

[RRB]

Solution Let father's age be x and son's age be y

$$\therefore \qquad x + y = 60 \tag{1}$$

Six years before,

$$x - 6 = 5(y - 6) \tag{2}$$

Solving (1) and (2),

$$x + y = 60 \tag{1}$$

(2)
$$\Rightarrow x - 5y = 6 - 30 = -24$$
 (2)
(1) - (2) $\Rightarrow y + 5y = 60 - (-24) = 84$

i.e. 6y = 84

or
$$y = \frac{84}{6} = 14$$

$$\therefore (1) \Rightarrow x = 60 - y \\ = 60 - 14 = 46$$

After six years,

Age of father will be x + 6 = 46 + 6 = 52 years Age of son will be y + 6 = 14 + 6 = 20 years

EXAMPLE 17 A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?

[IGNOU]

Solution Let the present age of mother be x.

$$\therefore \qquad \text{Person's age at present} = \frac{2}{5} x$$

After 8 years,

Person's age will be
$$\frac{2}{5}x + 8 = \frac{1}{2}(x + 8)$$

i.e. $\frac{2}{5}x + 8 = \frac{x}{2} + 4$
or $\frac{x}{2} - \frac{2}{5}x = 8 - 4$
or $\frac{5x - 4x}{10} = 4$

$$\frac{x}{10} = 4$$

$$x = 40 \text{ years}$$

EXAMPLE 18 The ratio between the present ages of A and B is 5: 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1: 1. What is the ratio between A's age 4 years hence and B's age 4 years ago? [SIDBI]

Solution Let the present ages of A and B be x and y.

$$\frac{x}{y} = \frac{5}{3} \tag{1}$$

$$\frac{x-4}{y+4} = \frac{1}{1} \tag{2}$$

$$\frac{x+4}{y-4} = ?$$

$$x = \frac{5}{3}y$$

$$\frac{5}{3}y - 4 = y + 4$$

$$\left(\frac{5}{3} - 1\right)y = 4 + 4 = 8$$

$$\frac{2}{3}y = 8$$

$$y = \frac{8 \times 3}{2} = 12$$

From (1),

$$x = \frac{5}{3}y$$

$$=\frac{5}{3}\times12=20$$

$$\frac{x+4}{y-4} = \frac{20+4}{12-4} = \frac{24}{8} = 3 \text{ or } 3:1$$



:.

EXERCISES

- 1. The ratio of ages of Sunil and Suresh is 2:3. Four years earlier their age ratio was 5:8. Find the present ages of Sunil and Suresh.
 - (a) 12 years, 18 years
- (b) 24 years, 36 years
- (c) 20 years, 32 years

- (d) 30 years, 48 years
- (e) None of these
- 2. The ratio of ages of Mohan and Madhu is 3:4. After 8 years their age ratio becomes 4:5. What are the present ages of Mohan and Madhu?
 - (a) 21 years, 28 years
- (b) 27 years, 36 years
- (c) 24 years, 32 years

- (d) 30 years, 40 years
- (e) None of these

3.	In a family, a couple has a son and a daughter. The age of the father is three times that of his daughter and the age of son is half of his mother. The wife is 9 years younger to her husband and the brother is seven years older than his sister. What is the age of the mother?					
	· ·	[IAS]				
	(a) 40 years (b) 45 years (c) 50 years (d) 60 years					
4.	4. A father is now three times as old as his son. Five years back, he was four times a son. The age of the son is	s old as his [IAS]				
	(a) 12 (b) 15 (c) 18 (d) 20					
5.	5. The age of a father is twice that of his elder son. Ten years hence, the age of the be 3 times that of the younger son. If the difference of ages of two sons is 15 years the father is					
	(a) 60 years (b) 90 years (c) 80 years (d) 50 years					
6.	6. If the present age of X is half that of Y and after 20 years if the age of Y is $1\frac{1}{2}$	times more				
	than X , what is the age of X now?	[IAS]				
	(a) 20 (b) 15 (c) 10 (d) 25					
7.	7. Father is aged three times more than his son Ronit. After 8 years, he would be two times of Ronit's age. After further 8 years, how many times would he be of Ronit's a					
	(a) 2 times (b) $2\frac{1}{2}$ times (c) $2\frac{3}{4}$ times (d) 3 times					
8.	8. Q is as much younger than R as he is older than T. If the sum of the ages of 60 years, what is definitely the difference between R and Q's age?	R and T is Bank PO]				
	(a) 1 year (b) 2 years (c) 25 years					
	(d) Data inadequate (e) None of these					
9.	9. The age of the father 10 years ago was thrice the age of his son. Ten years hence, f will be twice that of his son. The ratio of their present age is	ather's age LIC AAO]				
	(a) 5:2 (b) 7:3 (c) 9:2 (d) 13:4					
10.	10. Eighteen years ago, a father was three times as old as his son. Now the father is as old as his son. Then the sum of the present ages of the son and the father is	only twice [SSC]				
	(a) 54 (b) 72 (c) 105 (d) 108					
l 1.	-					
	(a) 25 years (b) 30 years (c) 36 years					
	(d) Cannot be determined (e) None of these					
12.	12. The ratio of the present ages of two brothers is 1 : 2 and 5 years back, the ratio What will be the ratio of their ages after 5 years?	was 1 : 3. [SSC]				
	(a) 1:4 (b) 2:3 (c) 3:5 (d) 5:6					
13.	13. A man is 24 years older then his son. In two years, his age will be twice the age The present age of the son is	of his son. [RRB]				
	(a) 14 years (b) 18 years (c) 20 years (d) 22 years					
14.	14. Present ages of X and Y are in the ratio 5 : 6. Seven years hence the ratio will be What is present age of X in years?	come 6 : 7. Bank PO]				
	(a) 35 (b) 42 (c) 49					
	(d) Cannot be determined (e) None of these					

15 .	The ratio between the	present age	s of P and Q is	5:7. If the	difference betv	veen Q's	present
	age and P's age after 6	years is 2,	what is the sun	n of ages of	P and Q at pre	esent?	[RRB]
	(a) 18 years	(h)	59 vears		(c) 56 years		

(a) 48 years

(d) Cannot be determined

(e) None of these

16. The ratio between the ages of Arun and Deepak at present is 4:3. After 6 years, Arun's age will be 26 years. What is the age of Deepak at present? [RRB]

(a) 12 years

(b) 15 years

(c) $19\frac{1}{2}$ years (d) 21 years

17. Present ages of Sameer and Anand are in the ratio 5: 4. Three years hence, the ratio of their ages will become 11:9. What is Anand's present age in years? [RBI]

(a) 24

(b) 27

(c) 40

(d) Cannot be determined

(e) None of these

18. The ratio between the present ages of P and Q is 6:7. If Q is 4 years older than P, what will be the ratio of ages of P and Q after 4 years? **ISBI POI**

(a) 3:4

(b) 3:5

(c) 4:3

(d) Data inadequate

(e) None of these

19. Six years ago, the ratio of the ages of Kunal and Sagar was 6: 5. Four years hence, the ratio of their ages will be 11:10. What is Sagar's age at present? [Bank PO]

(a) 16 years

(b) 18 years

(c) 20 years

(d) Cannot be determined

(e) None of these

20. A person was asked to state his age in years. His reply was "Take my age three years hence, multiply it by 3 and then subtract three times my age three years ago and you will know how old I am" What was the age of the person? **ISSCI**

(a) 18 years

(b) 20 years

(c) 24 years

(d) 32 years



or

 \Rightarrow

or

2. (c) **3.** (d) **4.** (b)

5. (d)

6. (a) 7. (a)

8. (e)

9. (b)

10. (d)

11. (d) **14.** (a) **12.** (c) **13.** (d) **15.** (a) **16.** (b) 17. (a) **18.** (e) **19.** (a) **20.** (a)



Solutions with Necessary Explanation

1. Let the ages of Sunil and Suresh at present be 2x and 5x respectively Four years before, age ratio will be given by

$$\frac{2x-4}{3x-4} = \frac{5}{8}$$

$$8(2x-4) = 5(3x-4)$$

$$16x-32 = 15x-20$$

$$x = 32-20 = 12$$

 \therefore Present age of Sunil = $2x = 2 \times 12 = 24$ years

Present age of Suresh = $3x = 3 \times 12 = 36$ years

2. Let the ages of Mohan and Madhu be 3x and 4x respectively After 8 years, their age ratio will be given by

$$\frac{3x + 8}{4x + 8} = \frac{4}{5}$$

or
$$5(3x + 8) = 4(4x + 8)$$
 or
$$5(3x) - 4(4x) = 4 \times 8 - 5 \times 8$$

$$-x = (4 - 5)8 = -8$$
 or
$$x = 8$$

Age of Mohan = $3x = 3 \times 8 = 24$ years

Age of Madhu = $4x = 4 \times 8 = 32$ years

3. Let the age of mother be x years.

Age of son =
$$\frac{x}{2}$$

Age of daughter = $\frac{x}{2} - 7$

$$\therefore \qquad \text{Age of father} = 3\left(\frac{x}{2} - 7\right)$$

$$3\left(\frac{x}{2} - 7\right) - x = 9$$

$$\Rightarrow \qquad \frac{3x}{2} - 21 - x = 9$$
or
$$\frac{x}{2} = 9 + 21 = 30$$
or
$$x = 60 \text{ years}$$

4. Let the age of son be x.

 \therefore Age of father = 3x

Five years ago,

or

Age of father =
$$3x - 5$$

= $4(x - 5)$
i.e. $3x - 5 = 4x - 20$
or $x = 20 - 5 = 15$ years

5. Let the ages of elder and younger sons be x_1 and x_2 .

$$\therefore$$
 Age of father = $2x_1$

Ten years hence,

Age of father =
$$2x_1 + 10 = 3(x_2 + 10)$$
 (1)

Given

$$x_1 - x_2 = 15$$

From (1), $2x_1 + 10 - (3x_2 + 30) = 0$

$$2x_1 - 3x_2 - 20 = 0$$
$$(2x_1 - 2x_2) - x_2 - 20 = 0$$

$$2(15) - x_2 - 20 = 0$$

$$10 - x_2 = 0$$

or∴.

$$x_2 = 10$$

Age of elder son $x_1 = x_2 + 15 = 10 + 15 = 25$ years

$$\therefore$$
 Age of father = $2x_1 = 2 \times 25 = 50$ years

6. Let their present ages be x and y respectively

$$\therefore \qquad \qquad x = \frac{1}{2}y \tag{1}$$

After 20 years,

$$y + 20 = 1\frac{1}{2}(x + 20) \tag{2}$$

Multiplying (1) and (2),
$$y = 2x$$
 (3)

Subtracting (3) from (2),
$$20 = \frac{3}{2}(x+20) - 2x$$

or
$$20 = \frac{-x}{2} + 30$$

or
$$\frac{x}{2} = 30 - 20 = 10$$

or $x = 10 \times 2 = 20$

7. Let the age of Ronit be x.

Age of father
$$y = x + 3x = 4x$$

After 8 years,

Age of father =
$$y + 8 = 2\frac{1}{2}(x + 8)$$

i.e.
$$4x + 8 = 2\frac{1}{2}(x + 8)$$

i.e.
$$4x + 8 = \frac{5}{2}x + 20$$

i.e.
$$4x - \frac{5}{2}x = 20 - 8$$
$$\frac{3}{2}x = 12$$

$$x = \frac{12 \times 2}{3} = 8 \text{ years}$$

After next 8 years,

i.e.

Age of Ronit =
$$x + 16$$

= $8 + 16 = 24$ years
Age of father = $y + 16 = 4x + 16$
= $4(8) + 16 = 48$ years
Age of father = $2 \times$ age of Ronit

 $\therefore \qquad \text{Age of father} = 2 \times \text{age of Ronit}$

 \therefore Age of father = **Two times** Ronit's age.

8. Given T < Q < R.

Let their ages be t, q and r.

∴
$$r + t = 60$$
 (1)
 $r - q = q - t$
or $2q = r + t = 60$
∴ $q = 30$
∴ $r = 45$ and $t = 15$ years
i.e. $r - q = 45 - 30 = 15$ years

9. Let the age of son and father at present be x and y respectively

∴ 10 years ago,

$$y - 10 = 3(x - 10) \tag{1}$$

2(x+10)-10

10 years hence,

$$y + 10 = 2 (x + 10) (2)$$

Subtracting (1) from (2), 10 - 10 = (2x + 20) - (3x - 30)

$$20 = -x + 20 + 30$$

or

$$x = 30 \text{ years}$$

$$y =$$
= 2(x + 10 - 5) = 2(x + 5)
= 2(30 + 5) = 70 years

:.

$$y: x = 7:3$$

10. Let the ages of son and father at present be x and y respectively

Eighteen years ago,

$$y - 18 = 3(x - 18) \tag{1}$$

$$y = 2x \tag{2}$$

$$x + y = ?$$

Subtracting (1) from (2),

$$18 = 2x - 3x + 54$$

or

$$x = 54 - 18 = 36$$

From (2),

$$y = 2x = 2 \times 36 = 72$$

:.

$$x + y = 36 + 72 = 108$$

11. Let school ages of Neelam and Shaan be x and y respectively

x: y = 5: 6 = 5k: 6k

Given, $\frac{5k}{3}:\frac{6k}{2}=5:9$

i.e. $\frac{\frac{5k}{3}}{\frac{6k}{9}} = \frac{5}{9}$

or

$$\frac{5\times9}{3}k = \frac{6\times5}{2}k$$

$$15k = 15k$$

Shaan's school age cannot be determined.

12. Let the present ages of brothers be x and y respectively

$$x: y=1:2$$

5 years ago,

$$(x-5):(y-5)=1:3$$

Then

$$(x+5):(y+5)=?$$

$$\frac{x}{y} = \frac{1}{2} \tag{1}$$

$$\frac{x-5}{y-5} = \frac{1}{3} \tag{2}$$

From (1),
$$2x = y \tag{A}$$

From (2),
$$3(x-5) = y-5$$
 (B)

Subtracting (B) from (A)
$$2x - 3(x - 5) = y - (y - 5)$$

or 2x - 3x + 15 = y - y + 5

or
$$-x + 15 = 5$$

or $x = 15 - 5 = 10 \text{ years}$
From (A), $y = 2x = 20 \text{ years}$

$$\therefore \frac{(x+5)}{(y+5)} = \frac{10+5}{20+5} = \frac{15}{25} = \frac{3}{5}$$

13. Let the age of son at present be x.

$$\therefore \qquad \text{Age of man } y = x + 24 \tag{1}$$

After 2 years,

$$y + 2 = 2(x + 2) \tag{2}$$

Subtracting (1) from (2), (y+2)-y=2(x+2)-(x+24)

i.e. 2 = 2x + 4 - x - 24

i.e. 2 = x - 20

or x = 2 + 20 = 22 years

14. Given x : y = 5 : 6

$$\frac{x}{y} = \frac{5}{6} \tag{1}$$

After 7 years,

$$\frac{x+7}{y+7} = \frac{6}{7} \tag{2}$$

From (1),
$$6x = 5y$$
 (a)

From (2),
$$7(x+7) = 6(y+7)$$
 (b)

Multiplying (a) by 6
$$36x = 30y$$
 (i)

Multiplying (b) by 5
$$35(x+7) = 30(y+7)$$
 (ii)

Subtracting (i) from (ii), 35(x+7) - 36x = 30(y+7) - 30y

i.e.
$$35x - 36x + 35 \times 7 = 30y - 30y + 30 \times 7$$

or
$$-x + 35 \times 7 = 30 \times 7$$

or
$$x = (35 - 30) \times 7 = 35 \text{ years}$$

15. Let the ages of P and Q be 5k and 7k respectively.

or
$$k = \frac{2+6}{2} = 4$$

$$\therefore$$
 Required sum = $5k + 7k = 12k = 12 \times 4 = 48$ years

16. Let ages of Arun and Deepak be 4k and 3k respectively After 6 years,

Arun's age =
$$4k + 6 = 26$$

$$\therefore \qquad \qquad k = \frac{26-6}{4} = 5$$

 \therefore Present age of Deepak = $3k = 3 \times 5 = 15$ years

17. Let present ages of Sameer and Anand be 5k and 4k.

After 3 years, their age ratio will be given by $\frac{5k+3}{4k+3} = \frac{11}{9}$

i.e.
$$9(5k+3) = 11(4k+3)$$

 $45k+27 = 44k+33$
 $k = 33-27 = 6$

 \therefore Present age of Anand = $4k = 4 \times 6 = 24$ years

18. Let the present ages of P and Q be 6k and 7k respectively

Given
$$7k - 6k = 4$$
 or
$$k = 4$$

After 4 years, their age ratio will be given by,

$$\frac{6k+4}{7k+4} = \frac{6(4)+4}{7(4)+4} = \frac{28}{32} = \frac{7}{8}$$

19. Let ages of Kunal and Sagar at present be x and y. Six years ago,

$$\frac{x-6}{y-6} = \frac{6}{5} \tag{1}$$

Four years after,

$$\frac{x+4}{y+4} = \frac{11}{10} \tag{2}$$

From (1),
$$5(x-6) = 6(y-6)$$
 (a)

From (2),
$$10(x+4) = 11(y+4)$$
 b)

Multiplying (a) by 2
$$10(x-6) = 12(y-6)$$
 (c)

Subtracting (b) from (c)
$$10\{(x-6)-(x+4)\}=12(y-6)-11(y+4)$$

or
$$10\{-6-4\} = 12y - 72 - 11y - 44$$

$$-100 = y - 116$$

or

$$y = 116 - 100 = 16$$

.: Present age of Sagar = 16 years

20. Let the present age of person be x.

$$x = (x + 3) \times 3 - (x - 3) 3$$

= 3x + 9 - 3x + 9
= 18

.. Present age of person = 18 years

Profit and Loss

Profit and loss are calculated on the cost price. The price at which a trader buys an article is called the *cost price* (C.P.). The price at which a trader sells the article is called the *selling price* (S.P.)

Important Formulae

% profit =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$
 % loss = $\frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$
S.P. = $\left(\frac{100 + \% \, \text{profit}}{100}\right) \times \text{C.P.}$ in case of profit
S.P. = $\left(\frac{100 - \% \, \text{loss}}{100}\right) \times \text{C.P.}$ in case of loss
or
C.P. = $\frac{100}{(100 + \% \, \text{profit})} \times \text{S.P.}$ in case of profit
C.P. = $\frac{100}{(100 - \% \, \text{loss})} \times \text{S.P.}$ in case of loss

If a person sells two similar items one at a gain of x% and the other at a loss of x%, then the seller will be always at a loss of $\left(\frac{x}{10}\right)^2$

i.e. Loss % =
$$\left(\frac{\text{Common loss and profit \%}}{10}\right)^2 = \left(\frac{x}{10}\right)^2$$

EXAMPLE 1 A man buys x tables for $\forall y$ and sells w tables for $\forall z$. Find the net and % profit or loss that he makes.

Solution S.P. of w table = \mathbb{Z} z.

S.P. of
$$x$$
 table = $\mathbf{z} \frac{z}{w} \cdot x$

Net profit =
$$\left(\frac{z}{w}\right) \cdot x - y$$

% Profit =
$$\frac{\left(\frac{z}{w}\right)x - y}{y} \times 100\%$$

% Profit =
$$\left(\frac{zx}{wy} - 1\right) \times 100\%$$



Note: Successive profits and losses

S.P. = C.P. × Product of change factors for each stage

$$\frac{1}{\text{Change factor}} = \frac{1}{\left(\frac{100 + \% \text{ profit}}{100}\right)} \text{ or } \frac{1}{\left(\frac{100 - \% \text{ loss}}{100}\right)}$$

% gain =
$$\frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100$$
Error

$$= \frac{Error}{True \ value - Error} \times 100$$

EXAMPLE 2 A dishonest dealer professes to sell his goods at the cost price, but he uses a weight of 950 g for 1 kg. Find his gain per cent.

Solution Let the goods cost be ₹ 10 per kg.

He sells for ₹ 10 goods worth ₹ 9.50

∴ Gain = ₹
$$10 - ₹ 9.50 = ₹ 0.50$$

$$\therefore \qquad \text{\% Gain} = \left(\frac{0.5}{10 - 0.5}\right) \times 100 = 5\frac{5}{19}\%$$

or
$$\left(\frac{1000 - 950}{950}\right) \times 100 = \left(\frac{50}{950}\right) \times 100 = \frac{100}{19} = 5\frac{5}{19}\%$$

EXAMPLE 3 A dishonest dealer sells goods at 4% loss on the cost price but uses 16 g instead of 18 g. What is his percentage profit or loss?

Solution Let the cost price of goods be $\stackrel{?}{\stackrel{?}{\sim}} x/kg$.

Selling price =
$$\frac{24}{100} = \frac{24}{25} x/kg$$

Let the quantity of goods bought be y kg.

 $\therefore \text{ Total investment} = ₹ xy$

:.

% profit =
$$\frac{\frac{27}{25}xy - xy}{xy} \times 100 = \frac{2xy/25}{xy} \times 100 = \frac{200}{25} = 8\%$$



Short cut: % profit or loss =
$$(100 - 4) \left[\frac{18}{16} \right] - 100 = 108 - 100 = 8\%$$

To find the selling price:

EXAMPLE 4 A person bought an article for ₹ 240. For how much should he sell it so as to gain 20%?

Solution If the C.P. is ₹ 100, then S.P. will be ₹ 120.

Here C.P. is ₹ 240.

∴ S.P. =
$$240 \times \frac{120}{100} = ₹ 288$$

Rule of fraction

If x% is the loss in the trade, then the calculating figures are 100 and (100 - x).

$$\therefore S.P. = \frac{100 - x}{100} \times C.P.$$

If x% is the gain in the trade, then the calculating figures are 100 and (100 + x).

$$\therefore S.P. = \frac{100 + x}{100} \times C.P.$$

EXAMPLE 5 By selling goods for ₹ 356.89, a trader lost 11%. Find the cost price.

Solution Here the C.P. is more than S.P.

C.P. =
$$\frac{100}{100 - x}$$
 × S.P. = $\frac{100}{100 - 11}$ × 356.89 = $\frac{100}{89}$ × 356.89 = ₹ 401

RULES -

Rule 1: When an article is passed through three persons, and each time trade was at a profit of x% and y% respectively. If the cost price for the last person is C rupees, the cost price for the first person is given by

the cost price for the first person = cost price for the third person $\times \frac{100}{100 + x} \times \frac{100}{100 + y}$.

Rule 2: When there are two successive profits of x% and y%, then the resultant profit per cent is given by $\left(x+y+\frac{xy}{100}\right)$.

Rule 3: When there is a profit of x% and loss of y% in a transaction, then the resultant profit or loss per cent is given by $\left(x-y-\frac{xy}{100}\right)$.

Thus in Q. 6., the resultant profit or loss per cent = $\left(20 - 10 - \frac{20 \times 10}{100}\right) = 8\%$ profit

EXAMPLE 6 By selling an article for ₹ 570, a tradesman would lose 5%. At what price must he sell it to gain 10%?

Solution (100-5)% C.P. = 570

$$\therefore$$
 C.P. = 570 $\times \frac{100}{95}$

∴
$$(100 + 10)\%$$
 of C.P. = $\frac{570}{95} \times 110 = ₹ 660$

EXAMPLE 7 A person sold an article at a profit of 15%. If he had bought it at 10% less and sold it for $\stackrel{?}{\sim} 4.5$ more, he would have gained 25%. Find the cost price.

Solution Let the cost of article be ₹ 100

If C.P. is less by 10%, then new C.P. = 100 - 10 = 790

∴ Selling price at 25% gain =
$$90 \times \frac{125}{100} = ₹112.5$$

∴ For difference of ₹4.5, C.P. =
$$\frac{100}{2.5}$$
 × 4.5 = ₹180

Short cut: If a person sells an article at x% profit and if he bought it at y% less and sold it for $\angle z$ less he would have gained k%. Then the cost price is given by

C.P. =
$$\frac{z \times 100 \times 100}{(100 + x)100 - (100 - y)(100 + k)}$$
=
$$\frac{4.5 \times 100 \times 100}{(100 + 15)100 - (100 - 10)(100 + 25)}$$
=
$$\frac{4.5 \times 100 \times 100}{115 \times 100 - 90 \times 125} = \frac{45000}{11500 - 11250}$$
=
$$\frac{45000}{250} = ₹180$$

EXAMPLE 8 A buys a watch for ₹ 600 and sells it to B at 10% profit. B sells it to C for 5% profit. C uses it for two years and then sells it to D at a loss of 20%. For how much does C sell the watch to D?

Solution C.P. = ₹ 600

For the first stage, i.e. A to B; change factor =
$$\frac{100 + 10}{100} = \frac{110}{100}$$

For the second stage, i.e. B to C, change factor =
$$\frac{100 + 5}{100} = \frac{105}{100}$$

For the third stage, i.e. C to D; change factor =
$$\frac{100-20}{100} = \frac{80}{100}$$

Cost price to C =
$$600 \times \frac{110}{100} \times \frac{105}{100}$$

= $\frac{6 \times 11 \times 105}{10} = 3 \times 11 \times 21 = ₹ 693$
S.P. to D by C = $693 \times \frac{80}{100} = ₹ 554.4$

Dishonest seller/buyer using faulty measures of length/weight

EXAMPLE 9 A dishonest shopkeeper professes to sell cloth at cost price, but he uses faulty metre rod. His metre rod measures 95 cm only. Find his gain per cent.

Solution Profit = Price of 5 cm cloth.

C.P. of the cloth sold as one metre = price of 95 cm cloth.

:. $\% \text{ profit} = \frac{5}{95} \times 100\% = 5\frac{5}{19}\% \text{ or } 5.26\%$



Short cut: The general relationship can be written as

$$%$$
 gain = $\frac{\text{Difference between true measure and False measure}}{\text{Amount for which payment made}}$

EXAMPLE 10 A person gains x% when he sells an article for \mathcal{T} z. What would be the profit or loss per cent, if he sold it for \mathcal{T} w.

Solution C.P. =
$$z \times \left(\frac{100}{100 + x}\right)$$

$$\therefore \qquad \text{profit} = \frac{w - z \left(\frac{100}{100 + x}\right)}{z \left(\frac{100}{100 + x}\right)} \times 100\% = \left[\frac{w}{z} \left(\frac{100 + x}{100}\right) - 1\right] \times 100\%$$

or % profit =
$$\left[\frac{w}{z}(100 + x) - 100\right]$$
%

Short cut: A man sells an article at a profit of x%. If he sells it for \overline{z} more, he would make a profit of y%. Find the cost price of the article.

or C.P. =
$$\frac{z}{y\% - x\%}$$

 $\therefore \qquad \text{Cost price of article} = \frac{z}{y - x} \times 100$

EXAMPLE 11 A man sold an article at a loss of x%. If he sells it for $\not \in z$ more, he would gain y%. Find the cost price of the article.

Solution Initial S.P. =
$$\frac{100 - x}{100} \times \text{C.P.}$$

New S.P. = $\frac{100 + y}{100} \times \text{C.P.}$
So,
$$\left(\frac{100 + y}{100}\right) \text{C.P} - \left(\frac{100 - x}{100}\right) \text{C.P.} = z$$
or
$$\left[\frac{(100 + y) - (100 - x)}{100}\right] \times \text{C.P.} = z$$
or
$$\frac{y + x}{100} \times \text{C.P.} = z$$
or
$$\text{C.P.} = \left[\frac{z}{y + x} \times 100\right]$$

EXAMPLE 12 A man sold an article at a profit of x%. If he had bought it for w% less and sold it for ξz more, he would have made a profit of y%. Find the cost price of the article.

Solution Let the original cost price be $\not\in P$.

Initial S.P. =
$$\mathbf{\xi} \left(\frac{100 + x}{100} \right) P$$

New C.P. = $\mathbf{\xi} \left(\frac{100 - w}{100} \right) P$
New S.P. = $\mathbf{\xi} \left(\frac{100 + y}{100} \right) \left(\frac{100 - w}{100} \right) P$

i.e.
$$\left(\frac{100+y}{100}\right) \left(\frac{100-w}{100}\right) P - \left(\frac{100+x}{100}\right) P = z$$
 or
$$P = \frac{z}{\left(\frac{100+y}{100}\right) \left(\frac{100-w}{100}\right) - \left(\frac{100+x}{100}\right)}$$
 or
$$P = \frac{z \times 100 \times 100}{(100+y)(100-w) - 100(100+x)}$$

EXAMPLE 13 A man buys certain quantity of an article for \mathcal{Z} . He sells mth part of the stock at a loss of x%. At what per cent gain should he sell the remaining stock so as to make an overall profit of y%. on the total transaction?

Solution The C.P. of mth part of stock = \mathbb{Z} $m \cdot z$.

The S.P. of *m*th part of stock =
$$\left(\frac{100 - x}{100}\right)m \cdot z$$
.

For y% profit on total transaction, its S.P. = $\left(\frac{100 + y}{100}\right)z$

The S.P. of remaining (1-m)th part of stock

$$= \left(\frac{100 + y}{100}\right)z - \left(\frac{100 - x}{100}\right)m \cdot z = \left[\frac{(100 + y) - (100 - x)m}{100}\right]z$$

C.P. of the remaining (1-m)th part of stock = (1-m)z.

$$\therefore \qquad \% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

$$\% \text{ profit} = \frac{\left[\frac{(100 + y) - (100 - x)m}{100}\right]z - (1 - m)z}{(1 - m)z} \times 100\%$$

$$= \left[\frac{(100 + y) - (100 - x)m - 100(1 - m)}{(1 - m)z \times 100}\right]z \times 100\%$$
The required % profit =
$$\frac{(y + xm)}{(1 - m)}\%$$

EXAMPLE 14 A man invests \overline{z} in shares. He sells mth part at a profit of x%, and the remaining at a loss of y%. Find his overall per cent profit or loss.

Solution C.P. of mth part = mz

S.P. of mth part =
$$\left(\frac{100 + x}{100}\right) mz$$

C.P. of $(1 - m)$ th part = $(1 - m)z$
S.P. of $(1 - m)$ th part = $\frac{100 - y}{100} (1 - m)z$
Total S.P. = $\left(\frac{100 + x}{100}\right) mz + \left(\frac{100 - y}{100}\right) (1 - m)z$
= $\frac{[(100 + x)m + (100 - y)(1 - m)]z}{100}$
Total C.P. = $\overline{<}z$
% profit = $\frac{\text{S.P. - C.P.}}{\text{C.P.}} \times 100\%$
= $\left[\frac{\left(\frac{100 + x}{100}\right)m + \left(\frac{100 - y}{100}\right)(1 - m)}{z}\right]z - z}{z} \times 100\%$
= $\left[\frac{(100 + x)m + (100 - y)(1 - m)}{100} - 1\right] \times 100\%$

$$= [(100 + x)m + (100 - y)(1 - m) - 100]\%$$
$$= [xm - y + ym]\%$$

i.e.

% profit =
$$[mx - y(1 - m)]$$
%

For no profit, no loss,

$$mx - y(1 - m) = 0$$

$$\frac{x}{y} = \frac{1 - m}{m}$$

or

EXAMPLE 15 A man buys certain quantity of an article for \mathcal{E} z. He sells 1/4th part of the stock at a loss of 20%. At what per cent gain should he sell the remaining stock so as to make an overall profit of 10% on the total transaction.

Solution The required % profit = $\frac{y + xm}{1 - m}$ %

Here x = 20; y = 10; m = 1/4;

The required % profit =
$$\frac{10 + 20/4}{1 - 1/4} = \frac{10 + 5}{3/4} = 20\%$$

EXAMPLE 16 A reduction of x% in the price of an article enables a person to buy y kg more for \overline{z} . Find the reduced and the original price per kg of the article.

Solution Saving due to price reduction = $\frac{xz}{100}$

Cost of y kg at the reduced price =
$$\neq \frac{xz}{100}$$

∴ Cost of 1 kg at the reduced price =
$$₹ \frac{xz}{100y}$$

$$\therefore \qquad \text{Reduced price} = \frac{xz}{100 \, \text{y}} \text{ per kg}$$

$$\therefore \qquad \text{Original price} = \frac{100}{100 - x} \times \text{Reduced price}$$

$$= \frac{100}{100 - x} \times \frac{xz}{100y}$$

Original price =
$$\frac{xz}{(100 - x)y}$$
 per kg

If x% is hike, then

Original price =
$$\frac{xz}{(100 + x)y}$$
 per kg

EXAMPLE 17 A man purchased m oranges at x a rupee and n oranges at y a rupee. He mixed them together and sold them at z a rupee. What is his per cent loss or gain. What happens when m = n?

Solution C.P. of *m* oranges at *x* a rupee =
$$\neq \frac{m}{x}$$

C.P. of *n* oranges at *y* a rupee =
$$\mathbf{z} = \frac{n}{y}$$

Total C.P. of
$$(m + n)$$
 oranges = $\neq \left(\frac{m}{x} + \frac{n}{y}\right)$

Total S.P. of
$$(m + n)$$
 oranges at z a rupee = $\frac{\pi}{z}$

% profit =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

= $\left(\frac{\text{S.P}}{\text{C.P.}} - 1\right) \times 100\%$

i.e.

% profit =
$$\left(\frac{\frac{m+n}{z}}{\frac{m}{x} + \frac{n}{y}} - 1 \right) \times 100\%$$

$$= \left(\frac{\frac{m+n}{z}}{\frac{my+nx}{xy}} - 1\right) \times 100\%$$

$$= \left[\frac{(m+n)xy}{(my+nx)z} - 1 \right] \times 100\%$$

When m = n,

%profit =
$$\left[\frac{2mxy}{m(x+y)z} - 1\right] \times 100\%$$

= $\left[\frac{2xy}{(x+y)z} - 1\right] \times 100\%$

EXAMPLE 18 A man buys a certain number of mangoes at 6 a rupee and the same number at 9 a rupee too. He mixes and sells them at 8 a rupee. Find his per cent profit or loss.

Solution % profit =
$$\left[\frac{2xy}{(x+y)z} - 1\right] \times 100\%$$

Let the man purchase N number at each rate.

:. Total C.P. =
$$\frac{N}{6} + \frac{N}{9} = \frac{5}{18}N$$
;

$$x = 6$$
; $y = 9$; $z = 18$

$$\therefore Total S.P. = \frac{2N}{8} = ₹ \frac{N}{4}$$

$$\therefore \% profit = \left(\frac{N/4 - 5N/18}{5N/18}\right) \times 100\%$$

$$= \left[\frac{N/4}{5N/18} - 1\right] \times 100\%$$

$$= \left[\frac{18}{4 \times 5} - 1\right] \times 100\%$$

$$= -10\%$$

$$\therefore loss = 10\%$$

EXAMPLE 19 m kg of rice costs n kg of wheat; p kg of wheat costs q kg of sugar; r kg of sugar costs s kg of coffee; t kg of coffee costs u kg of milk; v kg of milk costs w kg of tea. If x kg of tea cost v y, find the cost of z kg of rice.

Solution Cost of m kg of rice = Cost of n kg of wheat;

Cost of p kg of wheat = Cost of q kg of sugar;

Cost of r kg of sugar = Cost of s kg of coffee;

Cost of t kg of coffee = Cost of u kg of milk;

Cost of v kg of milk = Cost of w kg of tea;

? = Cost of z kg of rice

$$\therefore \qquad m \times p \times r \times t \times v \times x ? = n \times q \times s \times u \times w \times y \times z$$

$$? = \frac{n \times q \times s \times u \times w \times y \times z}{m \times p \times r \times t \times v \times x}$$

EXAMPLE 20 Alfred buys an old scooter for $\stackrel{?}{\stackrel{\checkmark}}$ 4700 and spends $\stackrel{?}{\stackrel{\checkmark}}$ 800 on its repairs. If he sells the scooter for $\stackrel{?}{\stackrel{\checkmark}}$ 5800, what is his gain per cent?

$$\frac{\text{gain }\% = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{5800 - 5500}{5500} \times 100}{5500} \times 100}{\text{mod } 100} = \frac{300}{5500} \times 100 = 5\frac{25}{55} = 5\frac{5}{11}\%$$

EXAMPLE 21 If the selling price of an article is $\frac{5}{3}$ of its cost price, find the profit in the transaction.

Solution Profit =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{\frac{5}{3} \text{C.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{2}{3} \times 100 = 66 \frac{2}{3} \%$$

(b) 30%

(b) 30% loss

(b) 10% gain

(b) 180

? EXERCISES

(a) 20%

make?
(a) 30% gain

per cent?

(a) 10% loss

percentage gain.

	(a) 160	(b)	240	(c)	320	(d)	480	(e)	None of these
6.		n article to B at price did		0% a	and B sells it to	Са	t a loss of 10%	. If (C pays ₹ 540 for
	(a) 400	(b)	500	(c)	600	(d)	700	(e)	None of these
7.		ld a cow at a	loss of 12%. If	he l		o se	ll it at a gain o	f 6%	, it would have
	(a) 300	(b)	500	(c)	400	(d)	600	(e)	None of these
8.		e is sold for₹ been sold for	-	of 1	10%. What wou	ıld h	ave been the g	ain	or loss per cent
	(a) 10%	gain (b)	10% loss	(c)	5% gain	(d)	5% loss	(e)	None of these
9.		ne is sold at a 'ind the cost	_	. На	d it been sold	for ₹	540 more 249	% w	ould have been
	(a) 4800	(b)	5400	(c)	6000	(d)	6400	(e)	None of these
10.	-				ad he sold it forder to gain			wou	ld have gained
	(a) 645	(b)	954	(c)	654	(d)	945	(e)	None of these
11.		uld be 5% m	ore. Find the c	ost j	price.				are ₹ 150 less,
	(a) 750	(b)	650	(c)	950	(d)	850	(e)	None of these
12.	A trader	sells his table	e at a profit of	15%	and the chair	at a	loss of $7\frac{1}{2}\%$, k	out	on the whole he
	gains₹50	O. If he sells t	he table at a lo	ss o	$f^{7}\frac{1}{2}\%$, and ch	air a	at a profit of 15	5%, 1	then he neither
			the cost price o						
	(a) $\frac{5000}{9}$	(b)	$\frac{4000}{9}$	(c)	$\frac{4400}{9}$	(d)	$\frac{5300}{9}$	(e)	None of these
13.	he reduce		o 8.10/kg, but						at this rate. So, and the per cent
	(a) 5%	(h)	10%	(c)	15%	(b)	20%	(e)	None of these

1. A trader sells rice at a profit of 5% and uses a weight which is 25% less. Find the total

2. A person purchases 8 pens for ₹ 10 and sells 10 pens for ₹ 8. How much profit or loss does he

(c) 36% gain

3. A shopkeeper buys an article at 8 for ₹ 16 and sells them at 9 for ₹ 20. What is his gain or loss

(c) $11\frac{1}{9}\%$ gain

4. A person bought an article for ₹ 240. For how much should he sell it so as to lose 20%?

(c) 240

5. By selling an article for ₹384, a trader gains 20%. Find the cost price of article.

(d) 50%

(d) 36% loss

(d) $11\frac{1}{9}\%$ loss

(d) 192

(e) None of these

(e) None of these

(e) None of these

(e) None of these

(c) 40%

	(a) 25%	(b) 60%	(c) 50%	(d) 80%	(e) None of these
15.	If cost price of 20	tables is equ	al to selling price of	£ 25 tables, find the	loss%
	(a) 10%	(b) 20%	(c) 30%	(d) 40%	(e) None of these
16.	A man sells an a	rticle at a los	s of 5%. If he had bo	ought it at 20% mor	e and sold it for ₹ 115
	less, he would ha	ve made a lo	ss of 40%. Find the o	cost price of the art	icle.
	(a) 400	(b) 450	(c) 500	(d) 600	(e) None of these
17.	When Ram sold hit if he wanted to			of 15%. For how muc	ch he should have sold
	(a) 275	(b) 315	(c) 295	(d) 335	(e) None of these
18.					d of total shares at a 'ind his overall profit
	(a) 10% profit		(b) 10% loss	(c) No pr	rofit no loss
	(d) 5% loss		(e) None of these		
19.	A man buys 100 l	g of sugar for	r₹2400. He sold 1/4	th of the stock at a l	oss of 5%. At what per
	cent profit should whole transaction	_	remaining stock so a	s to make an overa	ll profit of 20% on the
	(a) 25%	(b) $27\frac{1}{3}\%$	(c) 28%	(d) $28\frac{1}{3}\%$	(e) None of these
20.			nas for ₹ 3,600. He se a gain of 10%. Find		ck at a loss of 10% and profit or loss.
	(a) $\frac{10}{3}$ % gain	(b) $\frac{10}{3}$ % 10	ess (c) $\frac{14}{3}$ % loss	(d) $\frac{14}{3}\%$ gain	(e) None of these
	3	3		-	
21.	A shopkeeper mi	xes 20 kg of			kg of rice which cost tal profit and per cent
21.	A shopkeeper mi him ₹ 7 per kg. T profit.	xes 20 kg of hen he sells	the mixture at ₹ 6.5	per kg. Find his to	
	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \frac{41}{69}\% A shopkeeper sol	exes 20 kg of hen he sells (b) $15, 4\frac{47}{69}$ d an article of	the mixture at $\stackrel{?}{=} 6.5$ (c) 12.5, 3 $\frac{43}{69}$ offering a discount of	per kg. Find his tot $\%$ (d) $10, 2\frac{37}{69}\%$ of 5% and earned a	tal profit and per cent (e) None of these profit of 23.5%. What
	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \frac{41}{69}\% A shopkeeper sol	exes 20 kg of hen he sells (b) $15, 4\frac{47}{69}$ d an article of	the mixture at $\stackrel{?}{=} 6.5$ (c) 12.5, 3 $\frac{43}{69}$	per kg. Find his tot $\%$ (d) $10, 2\frac{37}{69}\%$ of 5% and earned a	tal profit and per cent (e) None of these profit of 23.5%. What
	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \frac{41}{69}\% A shopkeeper sol would have been	(b) 15, 4 $\frac{47}{69}$ d an article of the percenta	the mixture at ₹ 6.5 (c) 12.5, 3 \(\frac{43}{69}\) offering a discount oge of profit earned if	per kg. Find his total $\%$ (d) $10, 2\frac{37}{69}\%$ of 5% and earned a f no discount was of	tal profit and per cent (e) None of these profit of 23.5%. What
22.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate The labelled price	(b) 15, 4 $\frac{47}{69}$ d an article of the percental uate	the mixture at $\stackrel{?}{\sim} 6.5$ (c) 12.5, $3\frac{43}{69}$ offering a discount of ge of profit earned if (b) 28.5 (e) None of these of is $\stackrel{?}{\sim} 6500$. The short	per kg. Find his total $\%$ (d) $10, 2\frac{37}{69}\%$ of 5% and earned a fino discount was of (c) 30 opkeeper sold it by	tal profit and per cent (e) None of these profit of 23.5%. What
22.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate The labelled price the labelled price.	(b) 15, 4 $\frac{47}{69}$ d an article of the percental uate	the mixture at $\stackrel{?}{\sim} 6.5$ (c) 12.5, $3\frac{43}{69}$ offering a discount of ge of profit earned if (b) 28.5 (e) None of these of is $\stackrel{?}{\sim} 6500$. The short	per kg. Find his total $\%$ (d) $10, 2\frac{37}{69}\%$ of 5% and earned a fino discount was of (c) 30 opkeeper sold it by	tal profit and per cent (e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the
22.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate The labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked	(b) 15, 4 $\frac{47}{69}$ d an article of the percental uate e of a cupboa e and earned (b) ₹ 5350 the selling p	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these rd is ₹ 6500. The shot a profit of 15%. When (c) ₹ 5600 orice of an article at	per kg. Find his too (d) 10, 2 \frac{37}{69} \% of 5\% and earned a f no discount was of (c) 30 opkeeper sold it by nat approximately i (d) ₹ 5800 10\% above the cos	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the [Bank PO] (e) ₹ 6000 t price. At the time of
22.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate The labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked	(b) 15,4 $\frac{47}{69}$ d an article of the percental uate (b) ₹ 5350 the selling percental disco	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these (c) ₹ 5600 Orice of an article at unt and suffers a los	per kg. Find his too (d) 10, 2 \frac{37}{69} \% (f 5\% and earned a f no discount was of (c) 30 (c) 30 (d) ₹ 5800 10\% above the cos s of 1\%. He allowed	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the [Bank PO] (e) ₹ 6000
22. 23.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate the labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked selling, he allows (a) 9% At what price shows	(b) 15, 4 47/69 d an article of the percental uate e of a cupboa e and earned (b) ₹ 5350 the selling pertain disco (b) 10% buld a shopke	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these ard is ₹ 6500. The shot a profit of 15%. When the control of t	yer kg. Find his total (d) 10, 2 \frac{37}{69} \% (e) 5% and earned a (f) no discount was of (c) 30 (d) ₹ 5800 10% above the coss (d) 11% (d) 11% (d) 11% (e) 1206	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the [Bank PO] (e) ₹ 6000 t price. At the time of a discount of [SSC]
22. 23.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69}\)\% A shopkeeper sol would have been (a) 24.5 (d) Data inadequate the labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked selling, he allows (a) 9% At what price shows	(b) 15, 4 47/69 d an article of the percental uate e of a cupboa e and earned (b) ₹ 5350 the selling pertain disco (b) 10% buld a shopke	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these ard is ₹ 6500. The shot a profit of 15%. When the control of t	yer kg. Find his total (d) 10, 2 \frac{37}{69} \% (e) 5% and earned a (f) no discount was of (c) 30 (d) ₹ 5800 10% above the coss (d) 11% (d) 11% (d) 11% (e) 1206	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the [Bank PO] (e) ₹ 6000 t price. At the time of a discount of [SSC]
22.23.24.25.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \frac{41}{69}\% A shopkeeper sol would have been (a) 24.5 (d) Data inadeque The labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked selling, he allows (a) 9% At what price sho offer a discount of (a) ₹ 1675 By selling an united the shop of	(b) 15, 4 $\frac{47}{69}$ d an article of the percental uate e of a cupboa e and earned (b) ₹ 5350 the selling pertain disco (b) 10% ould a shopked of 20% on the (b) ₹ 1875 abrella for ₹	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these rd is ₹ 6500. The short a profit of 15%. When the control of the second is ₹ 6500 or control of an article at the control of the second is ₹ 6500 or control of an article at the control of the second is ₹ 1900 300, a shopkeeper is ₹ 6.500.	per kg. Find his too (d) 10, 2 \frac{37}{69} \% of 5\% and earned a f no discount was of (c) 30 opkeeper sold it by nat approximately i (d) ₹ 5800 10\% above the cos s of 1\%. He allowed (d) 11\% nat costs him ₹ 1200 till make a profit of (d) ₹ 2025 gains 20\%. During	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on s the cost price of the [Bank PO] (e) ₹ 6000 t price. At the time of a discount of [SSC] 0 in order that he may 25%? [Bank PO]
22.23.24.25.	A shopkeeper mi him ₹ 7 per kg. T profit. (a) 17,5 \(\frac{41}{69} \) % A shopkeeper sol would have been (a) 24.5 (d) Data inadequate The labelled price the labelled price the labelled price cupboard? (a) ₹ 5000 A trader marked selling, he allows (a) 9% At what price shooffer a discount of (a) ₹ 1675 By selling an unshopkeeper allows	(b) 15, 4 $\frac{47}{69}$ d an article of the percental uate e of a cupboa e and earned (b) ₹ 5350 the selling pertain disco (b) 10% ould a shopked of 20% on the (b) ₹ 1875 abrella for ₹	the mixture at ₹ 6.5 (c) 12.5, 3 \frac{43}{69} offering a discount of ge of profit earned if (b) 28.5 (e) None of these rd is ₹ 6500. The short a profit of 15%. When the control of the second is ₹ 6500 or control of an article at the control of the second is ₹ 6500 or control of an article at the control of the second is ₹ 1900 300, a shopkeeper is ₹ 6.500.	per kg. Find his too (d) 10, 2 \frac{37}{69} \% of 5\% and earned a f no discount was of (c) 30 opkeeper sold it by nat approximately i (d) ₹ 5800 10\% above the cos s of 1\%. He allowed (d) 11\% nat costs him ₹ 1200 till make a profit of (d) ₹ 2025 gains 20\%. During	(e) None of these profit of 23.5%. What ffered? [Bank PO] giving 5% discount on the sthe cost price of the [Bank PO] (e) ₹ 6000 t price. At the time of a discount of [SSC] 0 in order that he may 25%? [Bank PO] (e) None of these a clearance sale, then per cent during the

14. The cost price of 6 pens is equal to the selling price of 4 pens. Find the profit per cent.

27 .	The price of an article is raised by 30% and then two successive discount of 10% each are allowed. Ultimately, the price of the article is [SSC]			
	(a) decreased by 5.3% (b) increased by 3%			
	(c) increased by 5.3% (d) increased by 10%			
98	The difference between a discount of 35% and two successive discount of 20% on a certain			
20.	bill was ₹ 22. Find the amount of the bill [Bank PO]			
	(a) ₹ 200 (b) ₹ 1100 (c) ₹ 2200			
	(d) Data inadequate (e) None of these			
29 .	Successive discounts of 10%, 12% and 15% amount to a single discount of [RRB]			
	(a) 32.68% (b) 35.28% (c) 36.68% (d) None of these			
30.	A fan is listed at ₹ 1500 and a discount of 20% is offered on the list price. What additional discount must be offered to the customer to bring the net price to ₹ 1104? [SSC]			
	(a) 8% (b) 10% (c) 12% (d) 15%			
31.	A pair of article, was bought for ₹37.40 at a discount of 15%. What must be the marked price of each of the articles? [AAO Exam]			
	(a) $\stackrel{?}{_{\sim}} 11$ (b) $\stackrel{?}{_{\sim}} 22$ (c) $\stackrel{?}{_{\sim}} 33$ (d) $\stackrel{?}{_{\sim}} 44$			
32.	Albert buys 4 horses and 9 cows for ₹ 13400. If he sells the horse at 10% profit and the cows at 20% profit, then he earns a total profit of ₹ 1880. The cost of a horse is [CDS]			
	(a) ₹ 1000 (b) ₹ 2000 (c) ₹ 2500 (d) ₹ 3000			
33.	The cash difference between the selling prices of an article at a profit of 4% and 6% is ₹ 3. The ratio of the two selling price is [CBI]			
	(a) 51:52 (b) 52:53 (c) 51:53 (d) 52:55			
34.	If a men reduces the selling price of a fan from ₹ 400 to ₹ 380, his loss increases by 2%. The cost price of the fan is [RRB]			
	(a) ₹ 480 (b) ₹ 500 (c) ₹ 600 (d) None of these			
35.	A sells a bicycle to B at a profit of 20%. B sells it to C at a profit of 25%. If C pays ₹ 225 for it, the cost price of the bicycle for A is [BSF]			
	(a) ₹ 110 (b) ₹ 120 (c) ₹ 125 (d) ₹ 150			
36.	A shopkeeper cheats to the extent of 10% while buying as well as selling, by using false weights. His total gain is [Bank PO]			
	() 100(
	(a) 10% (b) 11% (c) 20% (d) 21% (e) $22\frac{2}{9}$ %			
37.	A dishonest dealer use a scale of 90 cm instead of a metre scale and claims to sell at cost price. His profit is [NIFT]			
	(a) 9% (b) 10% (c) 12% (d) None of these			
38.	A dairyman pays ₹ 6.40 per litre of milk. He adds water and sells the mixture at ₹ 8 per litre, thereby making 37.5% profit. The proportion of water to milk received by the customers is [MAT]			
	(a) 1:10 (b) 1:12 (c) 1:15 (d) 1:20			
39.	Arun purchased 30 kg of wheat at the rate of $\stackrel{?}{\stackrel{\checkmark}}$ 11.50 per kg and 20 kg of wheat at the rate of $\stackrel{?}{\stackrel{\checkmark}}$ 14.25 per kg. He mixed the two and sold the mixture. Approximately at what price per kg should he sell the mixture to make 30% profit? [Bank PO]			
	(a) ₹ 14.80 (b) ₹ 15.40 (c) ₹ 15.60 (d) ₹ 16.30 (e) ₹ 18.20			
40.	A men bought apples at the rate of 8 for $\stackrel{?}{\stackrel{\checkmark}}$ 34 and sold them at the rate of 12 for $\stackrel{?}{\stackrel{\checkmark}}$ 57. How many apples should be sold to earn a net profit of $\stackrel{?}{\stackrel{\checkmark}}$ 45? [SSC]			
	(a) 90 (b) 100 (c) 135 (d) 150			

41.	If on selling 12 note books, a se is his per cent profit?	eller makes a profit equ	al to selling price of 4 r	otebooks. What [Bank PO]
	(a) $16\frac{2}{3}$	(b) 25	(c) 50	
	(d) Data inadequate	(e) None of these		
42.	The cost price of 20 articles is the value of x is	the same as selling pric	e of x articles. If the pr	ofit is 25%, then [MAT]
	(a) 15 (b) 16	(c) 18	(d) 25	
43.	The percentage profit earned incurred by selling the same make 25% profit?			
	(a) ₹ 2000	(b) ₹ 2200	(c) ₹ 2400	
	(d) Data inadequate	(e) None of these		
44.	In a certain store, the profit is price remains constant, appro			_
	(a) 30% (b) 70%	(c) 100%	(d) 250%	
45 .	If selling price is doubled, pro	fit triples. Find the pro	ofit per cent.	[MAT]
	(a) $66\frac{2}{3}$ (b) 100	(c) $105\frac{1}{3}$	(d) 120	
46.	The ratio between selling price the profit and the cost price of			ne ratio between [Bank PO]
	(a) 2:7	(b) 5:2	(c) 7:2	
	(d) Data inadequate	(e) None of these		
47.	When a plot was sold for ₹ 187 order to gain 15%?			e plot be sold in [AAO Exam]
	(a) $\stackrel{?}{\sim} 21000$ (b) $\stackrel{?}{\sim} 22500$		(d) ₹ 25800	
48.	A manufacturer undertakes to piece. According to his estimate a profit of 25%. However, as it loss to the manufacturer?	tes, even if 5% fail to pa turned out, 50% of the	ass the quality tests, th components were rejec	en he will make
40	(a) ₹ 12000 (b) ₹ 13000	* *	(d) ₹ 15000	4 h a 4i a a
49.	A fruitseller sells mangoes at kg he should have sold them t	to make a profit of 5%?	·	(RRB)
- 0	(a) ₹ 11.81 (b) ₹ 12	(c) ₹ 12.25	(d) ₹ 12.31	
50.	Peter purchased a machine for and sold it with 25% profit. At	t what price did he sell	the machine?	[Bank PO]
	(a) ₹ 105100 (b) ₹ 10628			
51.	Hundred oranges are bought a percentage of profit or loss is	at the rate of₹350 and	sold at the rate of ₹ 48	B per dozen. The [SSC]
	(a) $14\frac{2}{7}\%$ gain (b) 15% ga	in (c) $14\frac{2}{7}\%$ loss	(d) 15% loss	
52.	A shopkeeper purchased 70 k ₹ 6.5 per kg. What will be his		and sold the whole lo	ot at the rate of [SSC]
	(a) $4\frac{1}{6}\%$ (b) $6\frac{1}{4}\%$	(c) $8\frac{1}{3}\%$	(d) 20%	

(a) ₹50

53 .	The ratio of the cos	st price and	the sellin	ig price is	4:5. Th	e profit p		_
	()	a	, ,		4.30		[Hotel M	anagement]
	` '	(b) 20%		25%	` '	30%		
54.	The profit earned by article is sold for ₹							
	` '	(b) ₹ 960		₹ 1060		₹ 1200	` '	None of these
55.	If the selling price per cent is	of 50 article	es is equal	to the cos	st price o	f 40 articl		e loss or gain anagement]
	(a) 20% loss	(b) 20% ga	in (c)	25% loss	(d)	25% gair	n	
56.	On an order of 5 do This is equivalent				ıct, a reta	iler recei	ves an ext	ra dozen free. [CBI]
	(a) 15%	(b) $16\frac{1}{6}\%$	(c)	$16\frac{2}{3}\%$	(d)	20%		
57 .	On selling 17 balls a ball is	at ₹ 720, th	ere is a lo	ss equal t	o the cos	t price of	5 balls. Th	e cost price of [SSC]
	(a) ₹ 45	(b) ₹ 50	(c)	₹ 55	(d)	₹ 60		
58.	Some articles were	bought at	6 for ₹ 5 a	nd sold at	5 for ₹ 6	. Gain pe	er cent is	[SSC]
	(a) 30%	(b) $33\frac{1}{3}\%$	(c)	35%	(d)	44%		
59 .	A man bought som What is the profit		the rate o	f 16 for ₹	24 and so	old them	at the rate	of 8 for ₹ 18. [Bank PO]
	(a) 25%	(b) 40%	(c)	50%	(d)	60%	(e) N	None of these
60.	A trader mixes thr 2:4:3 in terms o does he make?						hat percen	
	(a) 8%	(b) 9%	(c)	10%	(d)	None of	these	
61.	A trader mixes 26 l and sells the mixtu					e of anoth	er variety	at₹36 per kg [Bank PO]
	(a) No profit, no lo		(b) 5%			(c) 8%		
	(d) 10%		(e) None	e of these				
62 .	A vendor bought to	offees at 6 fe	or a rupee	. How ma	ny for a i	rupee mu	st he sell t	o gain 20%? [CBI]
	(a) 3	(b) 4	(c)	5	(d)	6		
63.	A shopkeeper profe kilogram weight. T		-	_	rice but	uses a we	eight of 80	0 g instead of [CBI]
	(a) 20%	(b) $16\frac{2}{3}\%$	(c)	25%	(d)	None of	these	
64.	A fair price shopke loss per cent is	eeper makes	s 10% prof	fit on his g	goods. He	lost 20%	goods dur	ing theft. His [SSC]
	` '	(b) 10		11		12		
65.	A man buys an art His gain or loss pe		less thai	n its value	e and sell	ls it for 1	0% more tl	nan its value. [SSC]
	(a) No profit, no lo	oss		(b)	20% pro	${f fit}$		
	(c) Less than 20%	profit		(d)	More tha	an 20% p	rofit	
66.	If 5% more is gaine article is	ed by selling	an article	e for ₹ 350	than by	selling it	for ₹ 340, t	the cost of the [CBI]

(c) ₹ 200

(d) ₹ 225

(b) ₹ 160

[BSF]

68.	A cloth merchant the rest was sold							oss and [SSC]
	(a) Neither loss	nor gain		(b) 5	% loss			
	(c) 5% gain			(d) 1	0% gair	1		
69.	If a company sel ₹ 200000 and 2.59 company for the	% on the rei						
	(a) ₹ 250000	(b) ₹ 255	000 (c)	₹ 260100	(d)	₹ 262200		
70.	A shopkeeper gived discount of 20% of then what is its 1	n the label	led price of					
	(a) ₹ 844.80	(b) ₹ 929	` '	₹ 1000	` '	₹ 1044.80		
71.	A discount of 159 costs of the two a			same as a	discoun	t of 20% on	another artic	cle. The [SSC]
	(a) ₹ 40, ₹ 20	(b) ₹ 60,		₹ 80, ₹ 60				
72.	The list price of a of 20% and 10%.				nd it is l	oeing sold at	successive di	scounts [SSC]
	(a) ₹ 1400	(b) ₹ 144	` '	₹ 1520	` '	₹ 1700		
73.	The marked price successive discou			_			_	ing two [SSC]
	(a) 12%	(b) 14%	(c)	15%	(d)	18%		
74.	By selling an arti	cle at $\frac{2}{5}$ of	the marked	price, there	is a los	s of 25% . Th	e ratio of the	marked
	price and the cos	_	e article is					[SSC]
	(a) 2:5	(b) 5:2	` '	8:15	` '	15:8		
75.	The cost price of allowing a discou			_	orice. Ca	alculated th	e gain per ce	nt after [CBI]
	(a) 37.5%	(b) 48%	` '	50.5%	` '	52%		
76.	A trader marked discount of 16% of	on the mark	ed price. H	is actual pr	ofit was	3	t of 25%. He	allowed [SSC]
	(a) 5%	(b) 9%	` '	16%	` '	25%		TT 11
77.	A shopkeeper sol not given the disc cost price of each	count, he wo					t price. What	
	(a) ₹ 240		(b) ₹ 260)		(c) ₹ 280		
	(d) Data inadequ	ıate	(e) None	e of these				
78.	A shopkeeper sel profit. If the disc		_		-	ing 10% dis	count and ear	ns 50%
	(a) 60%	(b) $60\frac{2}{3}$	% (c)	66%	(d)	$66\frac{2}{3}\%$	(e) None o	of these

67. A shopkeeper sells two watches for ₹ 308 each. On one he gets 12% profit and on the other

(b) $1\frac{11}{25}\%$ loss

(d) $3\frac{2}{25}\% loss$

12% loss. His profit or loss in the entire transaction was

(a) Neither profit, nor loss

(c) $1\frac{11}{25}\%$ profit

79.	A shopkeeper sold a TV set for ₹ 17940 with a discount of 8% and earned a profit of 19.6% What would have been the percentage of profit earned if no discount was offered? [Bank PO]				
	(a) 24.8%	(b) 25%	(4	[Bank PO]	
	(d) Cannot be determined	` '		20.470	
80	` '	, .		a profit of ₹70. What was its	
00.	cost price?	1070 Tebate on th	e listed price gave	[Bank PO]	
	(a) ₹ 200	(b) ₹ 350	(0	e) ₹700	
	(d) Cannot be determined	l (e) None of	these	,	
81.	A shopkeeper fixes the madiscount allowed to gain 8		tem 35% above its	cost price. The percentage of [Assistant Grade]	
	(a) 20% (b) 27%		% (d) 43		
82.	Kunal bought a suitcase v	vith 15% discoun	t on the labelled p	orice. He sold the suitcase for	
	₹ 2880 with 20% profit on				
	(a) ₹ 2040 (b) ₹ 2	400 (c) ₹ 2	604 (d) ₹ 2	(e) None of these	
83.	A trader marked his goods	at 20% above the	cost price. He sol	d half the stock at the marked	
	price, one quarter at a disc on the marked price. His t		e marked price an	d the rest at a discount of 40% [SSC]	
	(a) 2% (b) 4.5	% (c) 13	5% (d) 15	%	
84.		g₹1.50 free with		₹30, at a discount of 15% and then he makes a profit of 20%. [SSC]	
	(a) ₹ 19.75 (b) ₹ 26	(c) ₹ 2	1 (d) ₹ 2		
85.	A shopkeeper expects a ga	in of $22\frac{1}{2}\%$ on h	is cost price. If in	a week, his sale was of ₹ 392,	
	what was his profit?	2		[Bank PO]	
	(a) ₹ 18.20 (b) ₹ 7) (c) ₹ 7	2 (d) ₹ 8	38.25	
86.	A man buys a cycle for ₹ : cycle?	1400 and sells it a	at a loss of 15%. W	That is the selling price of the [SSC]	
	(a) ₹ 1090 (b) ₹ 1	160 (c) ₹ 1	190 (d) ₹ 1	1202	
87.	A men gains 20% by selling what will be the percentage.		certain price. If l	ne sells it at double the price, [SSC]	
	(a) 10% (b) 30%	(c) 15°	% (d) 14	0%	
88.	If the cost price of 12 pen	s is equal to selli	ng price of 8 pens	find the gain per cent. [SSC]	
	(a) 25% (b) 33%	(c) 50°	% (d) 66	$-\frac{2}{3}\%$	
	wers				
1.	(c) 2. (d) 3. (c)	4. (d) 5. (c)	6. (b) 7. (c)	8. (d) 9. (c) 10. (d)	
11.	(a) 12. (b) 13. (b)	4. (c) 15. (b)	16. (c) 17. (b)	18. (c) 19. (d) 20. (b)	
21.	(c) 22. (c) 23. (b) 2	24. (b) 25. (b)	26. (c) 27. (c)	28. (c) 29. (a) 30. (a)	
31.	(b) 32. (b) 33. (b) 3	34. (d) 35. (d)	36. (d) 37. (d)	38. (a) 39. (d) 40. (a)	
41.	(c) 42. (b) 43. (a)	44. (b) 45. (b)	46. (e) 47. (c)	48. (b) 49. (a) 50. (c)	
51.	(a) 52. (c) 53. (c) 5	54. (b) 55. (a)	56. (c) 57. (d)	58. (d) 59. (c) 60. (c)	

- **61.** (b)
- **62.** (c)

82. (a)

63. (c)

83. (a)

- **64.** (d)
- 65. (d) **66.** (c)
- **67.** (b)

87. (d)

- **68.** (c)
 - **69.** (d)
- **70.** (c)

- **71.** (c)
- **72.** (b)

- **76.** (a)
- 77. (c)
- 78. (d) 88. (c)

- 81. (a)
- **73.** (c)
- **74.** (d) 84. (b)
- **75.** (a) 85. (c)
- 86. (c)
- **79.** (e)
- 80. (d)



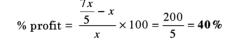
Solutions with Necessary Explanation

1. Let the cost price of rice be $\not\in x/kg$.

∴ Selling price of rice =
$$₹$$
 $\left(\frac{105x}{100}\right)$ per $\frac{75}{100}$ kg

S.P. =
$$\Re \left(\frac{105x}{100} \times \frac{100}{75} \right) / kg$$

$$= \frac{7x}{5} / kg$$





Short cut: Total percentage profit = $100 \times \frac{\% \text{ profit} + \% \text{ less in weight}}{100 - \% \text{ less in weight}}$

=
$$100 \times \left(\frac{5+25}{100-25}\right) = \left(\frac{100 \times 30}{75}\right) = 40\%$$

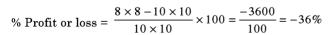
If a shopkeeper sells his goods at x% loss on the cost price but uses y g instead of z g then his % profit or loss is

$$(100 - x)\frac{z}{y} - 100$$
 as the sign is +ve or -ve.

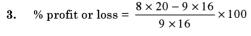
2. Price at which he bought the pencil = $\frac{10}{9}$ = $\frac{10}{9}$ = $\frac{10}{9}$ = $\frac{10}{9}$ = $\frac{10}{9}$

Price at which he sold the pencil = $\frac{8}{10} = \frac{10}{10} = \frac{10}$

% Profit or loss =
$$\frac{0.8 - 1.25}{1.25} = \frac{-0.45}{1.25} = -36\%$$



$$\therefore$$
 % loss = 36%



$$=\frac{1600}{9\times16}=\frac{100}{9}=11\frac{1}{9}\%$$

$$\therefore \text{ Profit} = 11\frac{1}{9}\%$$



4. If C.P. is ₹ 100, S.P. will be ₹ 80.

∴ S.P. =
$$\frac{80}{100}$$
 × 240 = ₹ 192

5. If the C.P. is ₹ 100, the S.P. will be ₹ 120.

∴ For S.P. = ₹ 384, C.P. will be
$$\frac{100}{120} \times 384 = 5 \times 64 = ₹ 320$$



Short cut: By rule of fraction

C.P. = S.P. ×
$$\frac{100}{100 + x}$$
 = 384 × $\frac{100}{100 + 20}$ = 384 × $\frac{100}{120}$ = ₹ 320

6. In the transaction there is a profit of 20% and a loss of 10%.

B's cost price =
$$540 \times \frac{100}{90} = ₹600$$

A's cost price =
$$600 \times \frac{100}{120} = ₹ 500$$

7. (106)% of cost price – (100 - 12)% of cost price = ₹ 72

∴ Cost price =
$$\frac{72}{\frac{106 - 88}{100}} \times \frac{72 \times 100}{18} = ₹ 400$$

8. C.P. = S.P. ×
$$\frac{100}{100+10}$$
 = 4950 × $\frac{100}{110}$ = ₹ 4500

If S.P. = 4275, loss % =
$$\frac{4500 - 4275}{4500} \times 100 = \frac{225}{4500} \times 100 = 5\%$$

- 9. 124% of cost -115% of cost = ₹540
 - ∴ 9% of cost = ₹ 540

∴ Cost =
$$\frac{540 \times 100}{9}$$
 = ₹ 6000



Short cut: Cost price = $\frac{\text{More gain} \times 100}{\text{Discourse}}$

Difference in percentage profit

Cost price =
$$\frac{540 \times 100}{24 - 15} = \frac{540 \times 100}{9} = ₹6000$$

10. Let the cost price of radio be \overline{x} .

S.P. at 5% loss =
$$\frac{95}{100}x$$

S.P. at 25% gain =
$$\frac{125}{100}x$$

$$\therefore \frac{125x}{100} - \frac{95x}{100} = \text{ ? 210}$$

$$30x = 210 \times 100$$

$$\therefore \qquad \qquad x = \mathbf{₹700}$$

∴ To gain 35% he would sell at ₹700 ×
$$\frac{135}{100}$$
 = 7 × 135= ₹ 945



Short cut: Selling price = More rupees $\frac{(100 + \% \text{ final gain})}{(\% \text{ gain} + \% \text{ loss})}$

$$= 210 \times \frac{(100 + 35)}{(25 + 5)} = \frac{210 \times 135}{30} =$$
₹ 945

11. The cost price of the article is ξx .

$$\therefore$$
 The selling price = $\frac{120}{100} \times x$

New selling price =
$$\overline{ } \left(\frac{120x}{100} - 150 \right)$$

New profit % =
$$\frac{\left(\frac{120x}{100} - 150\right) - (x - 150)}{(x - 150)} \times 100$$

$$= \frac{\left(\frac{120x}{100} - x\right) \times 100}{(x - 150)} = \frac{\frac{20x}{100} \times 100}{x - 150} = \frac{20x}{x - 150} \%$$

New % profit =
$$20 + 5 = 25\%$$

$$\frac{20x}{x-150} = 25 \text{ or } (x-150)25 = 20x$$

$$25x - 20x = 150 \times 25$$

$$\therefore 5x = 150 \times 25$$

or
$$x = 150 \times 5 = 750$$



When cost price and selling price are reduced by the same amounts (say D),

then the cost price = $\frac{[Initial\ profit\ \% + Increase\ in\ profit\ \%] \times D}{Increase\ in\ profit\ \%}$

Increase in profit %

∴ Cost price =
$$\frac{[20+5] \times 150}{5} = ₹750$$

12. Let the cost price of table be $\overline{\xi}$ t and that of chair $\overline{\xi}$ c.

Then 15% of
$$t - 7\frac{1}{2}$$
% of $c = 50$

$$15t - 7\frac{1}{2}c = 5000\tag{1}$$

$$-7\frac{1}{2}t + 15c = 0 (2)$$

Solving Eqs. (1) and (2), we get

$$c = \frac{2000}{9}$$
 and $t = \frac{4000}{9}$

13. After reduction, 900 g costs ₹ 8.10.

∴ 1000 g will cost
$$\frac{1000}{900} \times 8.10 = ₹ 9$$

% Change in actual price or loss =
$$\frac{10-9}{10} \times 100\% = 10\%$$

14. S.P. of 4 pens = C.P. of 6 pens.

i.e. S.P. of 2 pens =
$$C.P.$$
 of 3 pens

Profit =
$$C.P.$$
 of 3 pens

Investment = C.P. of 6 pens

% profit =
$$\frac{6-4}{4} \times 100 = 50\%$$

15. The C.P. of 20 tables = S.P. of 25 tables

$$\% \text{ loss} = \frac{25 - 20}{25} \times 100\% = 20\%$$



Note: In general, if the C.P. of x articles = S.P. of y articles, then profit % = $\left(\frac{x}{y} - 1\right) \times 100\%$ If result is negative, (then it represents loss)

16. Let the cost price of the article be 'P'.

Initial S.P. =
$$\frac{95}{100}P$$

Final S.P. =
$$\left(\frac{120}{100} \times \frac{60}{100}\right) P$$

$$\therefore \frac{95}{100}P - \left(\frac{120}{100} \times \frac{60}{100}\right)P = 115$$

$$\therefore P = \frac{115 \times 100 \times 100}{95 \times 100 - 120 \times 60} = \frac{115 \times 100 \times 100}{2300} = \text{ ₹ 500}$$

17. S.P. =
$$\left(\frac{255}{85} \times 100\right) \times \frac{105}{100} = \frac{255}{85} \times 105 = ₹315$$

18. C.P. of one-third shares =
$$\frac{₹2100}{3}$$
 = ₹7000

S.P. of one-third shares =
$$\frac{110}{100} \times 7000 = ₹7700$$

C.P. of remaining $\frac{2}{3}$ rd shares = ₹ (21000 – 7000) = ₹ 14000

S.P. of remaining $\frac{2}{3}$ rd shares = $\frac{95}{100} \times 14000 = ₹13300$

Total S.P. = ₹ 7700 + 13300 = ₹ 21000

Total C.P. = ₹ 21000

$$S.P. = C.P.$$

or there is no profit no loss.

19. Given x = -5, y = +20, m = 1/4

The required profit percentage on the remaining stock

$$\frac{y - xm}{1 - m}\% = \frac{20 + 5/4}{1 - 1/4} = \frac{85/4}{3/4} = \frac{85}{3} = 28\frac{1}{3}\%$$

Alternative method:

Let the required % profit be z

$$\frac{1}{4}(100 - 5) + \frac{3}{4}(100 + z) = 100 + 20$$
or
$$\frac{95}{4} + 75 + \frac{3}{4}z = 120$$
or
$$\frac{3}{4}z = 120 - 75 - \frac{95}{4}$$

$$= \frac{180 - 95}{4} = \frac{85}{4}$$

$$\therefore z = \frac{85}{4} \times \frac{4}{3} = \frac{85}{3} = 28\frac{1}{3}\%$$

20. % profit =
$$[mx - (1 - m)y]$$
%

$$x = 10$$
; $y = 10$; $m = \frac{1}{3}$; $1 - m = \frac{2}{3}$

:. % profit =
$$\frac{1}{3} \times 10 - \frac{2}{3} \times 10 = \frac{-10}{3}$$
 %

$$\therefore \log = \frac{10}{3}\%$$

21. Total C.P. =
$$20 \times 5 + 35 \times 7 = ₹345$$

Total S.P. =
$$(20 + 35) \times 6.5 = ₹ 357.5$$

Total profit = ₹ 357.5
$$-$$
 ₹ 345 $=$ ₹ **12.5**

Per cent profit =
$$\frac{12.5}{345} \times 100\% = 3\frac{43}{69}\%$$

22. Given discount =
$$5\%$$
 and profit = 23.5%

Let the marked price of article be x

$$\therefore \frac{95}{100} x = 123.50$$
or
$$x = \frac{123.50 \times 100}{95}$$

$$= \frac{12350}{95} = \frac{2470}{19} = ₹130$$

∴ Without discount, S.P. = ₹ 130, C.P. = ₹ 100

$$\therefore \qquad \text{Profit \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$
$$= \frac{130 - 100}{100} \times 100 = 30\%$$

23. Labelled price of cupboard = ₹ 6500

Selling price (S.P.) =
$$\frac{95}{100}$$
 × 6500 = ₹ 6175

Profit % = 15;

Let x be the cost price

$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \% \text{ profit}$$
i.e.
$$\frac{6175 - x}{x} \times 100 = 15$$
i.e.
$$\frac{6175 - x}{x} = \frac{15}{100}$$
i.e.
$$\frac{6175}{x} = 1 + \frac{15}{100} = \frac{115}{100}$$

$$x = \frac{6175 \times 100}{115} = 5369.56 \simeq ₹ 5350$$

24. Let the cost price of article be ₹ 100.

∴ Marked price = ₹ 110

Let x% be the discount given

$$S.P. = 110 \left(1 - \frac{x}{100} \right)$$

$$\% \text{ loss} = \frac{C.P. - S.P.}{C.P.} \times 100$$
or
$$100 \left(1 - \frac{S.P.}{C.P.} \right) = 1$$
or
$$1 - \frac{S.P.}{C.P.} = \frac{1}{100}$$
or
$$\frac{S.P.}{C.P.} = 1 - \frac{1}{100} = \frac{99}{100}$$

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25. Let the marked price of radio be $\stackrel{?}{\underset{\sim}{}} x$.

C.P. =
$$₹ 1200$$
; S.P. = $0.8 \times x$; % profit = 25%

$$\therefore \qquad \text{% profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

i.e.
$$\frac{0.8x - 1200}{1200} \times 100 = 25$$

i.e.
$$\frac{0.8x}{1200} = \frac{25}{100} + 1$$

i.e.
$$\frac{0.8x}{1200} = \frac{125}{100}$$

or
$$x = \frac{125 \times 1200}{100 \times 0.8} = 125 \times 15 = ₹ 1875$$

26. S.P. = C.P.
$$\left(1 + \frac{x}{100}\right)$$
 where x is profit %

$$\therefore 300 = \text{C.P.} \left(1 + \frac{20}{100} \right)$$

or C.P. =
$$\frac{300}{1.2}$$
 = ₹ 250

After discount, S.P. =
$$300 \times \left(1 - \frac{10}{100}\right) = ₹ 270$$

$$\therefore \qquad \text{Gain } \% = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$
$$= \frac{270 - 250}{250} \times 100 = 8\%$$

27. Let the initial price of article be ₹ 100.

After 1st rise of 30%, price = $1.3 \times 100 = ₹130$

After 1st discount of 10%, price =
$$130\left(1 - \frac{10}{100}\right) = ₹117$$

After 2nd discount of 10%, price =
$$117\left(1 - \frac{10}{100}\right) = 117 \times 0.9 = ₹105.3$$

.. Price is increased by 5.3%

Let the initial price of article be ₹ 100

∴ Marked price =
$$1.3 \times 100 = ₹ 130$$

∴ Price after 2 discounts of 10% each =
$$130 \left\{ 1 + \left(-10 - 10 + \frac{10^2}{100} \right) \% \right\}$$

= $130 \left(\frac{100 - 19}{100} \right) = 130 \times \frac{81}{100} = ₹105.3$

$$\text{::} \qquad \text{% profit} = \frac{105.3 - 100}{100} \times 100 = 5.3\%$$

i.e. price is increased by 5.3%

28. Let the amount of bill be ξx .

1st discount = 35%

2nd discount = 20%, each two times in succession.

:. Equivalent discount for 2nd one =
$$\left(-20 - 20 + \frac{400}{100}\right)\% = -36\%$$
 36% decrease

$$\therefore$$
 Difference of discounts = $x(36 - 35)\% = 22$

$$x \times \frac{1}{100} = 22$$

$$x = \mathbf{₹2200}$$

29. Let marked price be ₹ 100.

∴ S.P. =
$$\frac{85}{100} \times \frac{88}{100} \times \frac{90}{100} \times 100 = ₹ 67.32$$

$$\therefore$$
 Single discount = $100 - 67.32 = 32.68\%$

30. Listed price = ₹ 1500

First discount = 20%

Let additional discount be x%

$$\therefore 1500 \times \frac{80}{100} \times \left(\frac{100 - x}{100}\right) = 1104$$

$$\therefore 1 - \frac{x}{100} = \frac{1104 \times 100}{1500 \times 80}$$

$$\frac{x}{100} = 1 - \frac{1104 \times 100}{1500 \times 80}$$

or
$$x = \left(1 - \frac{1104 \times 100}{1500 \times 80}\right) \times 100 = \left(1 - \frac{368}{400}\right) \times 100 = 8\%$$

31. Let marked price be $\not\in x$.

$$\therefore \qquad \left(x \times \frac{85}{100}\right) \times 2 = ₹ 37.40$$

$$x = \frac{37.40 \times 100}{170} = \text{ ? 22}$$

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$$\therefore \qquad x + y = 13400 \tag{1}$$

:. Profit earned =
$$\frac{10}{100} \times 4x + \frac{20}{100} \times 9y = 1880$$

i.e.
$$4x + 18y = 18800$$
 (2)

Solving (1) and (2), we get x = 7 2000

- ∴ Cost of horse = ₹ 2000
- **33.** Let \overline{x} be the cost price.

$$\therefore \quad x \times \left\{ \left(1 + \frac{6}{100}\right) - \left(1 + \frac{4}{100}\right) \right\} = ₹ 3$$

i.e.
$$x\left\{\frac{6-4}{100}\right\} = ₹ 3$$

$$\therefore \qquad x = \frac{3 \times 100}{2} = 7.50$$

$$\therefore \qquad \text{Ratio of selling price} = \frac{150 \times 1.04}{150 \times 1.06} = \frac{52}{53}$$

$$\therefore$$
 Required ratio = 52:53

34. Let \overline{x} x be the cost price of the fan.

$$\therefore$$
 400 - 380 = 20 = 2% of $x = \frac{2}{100}x$

35. Let the cost price of bicycle to A be $\overline{\xi}$ x.

$$\therefore \qquad x \times \frac{120}{100} \times \frac{125}{100} = \text{ ? 225}$$

$$x = \frac{225 \times 100 \times 100}{120 \times 125}$$
$$= \frac{9 \times 100 \times 100}{120 \times 5} = \frac{300}{2} = ₹ 150$$

36. The shopkeeper pays for 900 g instead of 1000 g while buying and sells the item weighing 900 gm for 1000 g or 1 kg.

Gain % =
$$\frac{(100 + \text{common gain \%})^2}{100} - 100$$

= $\left[\frac{(100 + 10)^2}{100} - 100\right]$ %
= $\left[\frac{12100}{100} - 100\right]$ % = **21**%

37. Gain % =
$$\left(\frac{10}{90} \times 100\right)$$
% = **11** $\frac{1}{9}$ %

38. Profit = 37.5%

$$C.P. = 6.4;$$
 $S.P. = 8$

$$\therefore \text{ Mean cost price} = \frac{100}{137.5} \times 8 = \text{?} \frac{64}{11}$$

:. Required ratio =
$$\frac{64}{110}$$
: $\frac{64}{11}$ = 1:10

39. C.P. =
$$30 \times 11.50 + 20 \times 14.25 = 345 + 285 = ₹630/50 \text{ kg} = ₹12.6 \text{ kg}$$

S.P. =
$$1.3 \times 712.6 / \text{kg} = 716.38 / \text{kg} \approx 716.30 / \text{kg}$$

40. C.P. =
$$\frac{34}{8}$$
; S.P. = $\frac{57}{12}$

Let x be the number of apples sold

$$\therefore$$
 45 = (S.P. – C.P.) x

or
$$x = \frac{45}{\text{S.P.} - \text{C.P.}} = \frac{45}{\frac{57}{12} - \frac{34}{8}} = \frac{45}{\frac{114 - 102}{24}} = \frac{45 \times 24}{12} = 790$$

41. The cost price of 12 note books = Selling price of 8 note books

Water

₹ 0

 $\overline{110}$

Milk

₹ 6.40

64

11

$$\therefore \text{ % profit} = \frac{\text{Cost price of 6 note books}}{\text{Cost price of 12 note books}} \times 100 = 50\%$$

42. C.P. of 20 articles = S.P. of x articles

$$\therefore$$
 Profit (25%) = S.P. of (20 – x) articles

25% profit = C.P. of 5 articles = S.P. of
$$\frac{x}{4}$$
 articles

$$\therefore \frac{x}{4} = 20 - x$$

or
$$\frac{5}{4}x = 20$$

or
$$x = \frac{20 \times 4}{5} = 16$$

43. Let ₹ x be the C.P. of article.

Then
$$\frac{1920 - x}{x} = \frac{x - 1280}{x}$$

$$\frac{1920}{x} + \frac{1280}{x} = 1 + 1 = 2$$

or
$$x = \frac{1920 + 1280}{2} = 1600$$

For 25% profit S.P. = $1600 \times 1.25 = ₹2000$

44. Let x be the initial cost price.

$$\therefore$$
 S.P. = 420% of $x = 4.2 x$, % profit = 3.2 x

If C.P. is increased by 25%

New C.P. = 1.25x, New S.P. = 4.2x

$$\therefore$$
 Profit = S.P. – C.P. = $(4.2 - 1.25)x = 2.95x$

$$\therefore \quad \text{Required percentage} = \left(\frac{2.95x}{4.2x} \times 100\right)\% = \frac{1475}{21} = 70\%$$

45. Profit = S.P. - C.P.

If S.P. is doubled, new profit = 2S.P. – C.P. = 3 (S.P. – C.P.)

i.e.
$$3C.P. - C.P. = 3S.P. - 2S.P.$$

or
$$2C.P. = S.P.$$

.. Profit % =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{2 \text{ C.P.} - \text{C. P.}}{\text{C.P.}} \times 100 = 100 \%$$

46.
$$\frac{\text{S.P.}}{\text{C.P.}} = \frac{7}{5}$$

$$\therefore \frac{\text{Profit}}{\text{C.P.}} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} = \frac{7 - 5}{5} = \frac{2}{5}$$

$$\therefore$$
 Profit : C.P. = 2 : 5

$$\therefore \frac{85}{100}$$
 C.P. = ₹ 18700

To gain 15% it should be sold @ $\stackrel{?}{\underset{?}{\sim}} \frac{115}{100}$ C.P.

∴ Required S.P. =
$$\frac{18700}{85} \times 115 = ₹25300$$

$$\therefore$$
 Total S.P. = 2000 × 25; Profit % = 25

5% failed in quality test

:. Net S.P. =
$$\left(2000 \times \frac{95}{100}\right) \times 25 = 1900 \times 25$$

∴ Cost price =
$$\frac{100}{125} \times 1900 \times 25 = ₹38000$$

If 50% of components rejected,

So, loss incurred to manufacturer = $(38000 - 25 \times 1000) = 713000$

49. Required selling price =
$$\frac{9}{80}$$
 × 105 = ₹ 11.81/kg

50. Total cost price =
$$80000 + 5000 + 1000 = ₹86000$$

Required selling price @ 25% profit = $86000 \times \frac{125}{100} = ₹ 107500$

$$\therefore \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{4 - 3.5}{3.5} \times 100 = \frac{100}{7} = 14\frac{2}{7} \% \text{ profit}$$

52. Cost price of potatoes = ₹ 420/70 kg = ₹ 6/kg Selling price of potatoes = ₹ 6.5/kg

:. Gain per cent =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{6.5 - 6}{6} \times 100 = \frac{1}{12} \times 100 = 8 \frac{1}{3} \%$$

53.
$$\frac{\text{Cost price}}{\text{Selling price}} = \frac{4}{5}$$

:. Profit per cent =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{5-4}{4} \times 100 = 25\%$$

54. Let x be the C.P. of article.

$$\therefore 832 - x = x - 448$$

or
$$x = \frac{832 + 448}{2} = \text{ } 640$$

S.P. with 50% profit = C.P. × 1.5 =
$$640 \times 1.5 = ₹ 960$$

55. S.P. of **50** articles = C.P. of **40** articles

% loss =
$$\frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$$

= $\frac{\text{S.P. of 1.25 article} - \text{S.P. of 1 article}}{\text{S.P. of 1.25 article}} \times 100$
= $\frac{0.25}{1.25} \times 100 = 20\%$

56. Discount =
$$\frac{1}{5+1} \times 100 = \frac{1}{6} \times 100 = 16 \frac{2}{3} \%$$

$$(C.P. - S.P.)$$
 of 17 balls = $C.P.$ of 5 balls

$$\therefore S.P. of 17 balls = C.P. of (17 - 5) balls$$
$$= C.P. of 12 balls = 720$$

∴ C.P. of ball =
$$\frac{720}{12}$$
 = ₹ 60

58. C.P. = ₹
$$5/6$$
; S.P. = ₹ $6/5$

Gain % =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

= $\frac{\frac{6}{5} - \frac{5}{6}}{\frac{5}{6}} \times 100 = \frac{\frac{36 - 25}{30}}{\frac{5}{6}} \times 100$
= $\frac{11}{30} \times \frac{6}{5} \times 100 = \frac{11}{25} \times 100 = 44\%$

59. C.P. of fruits = ₹
$$24/16 = ₹ 3/2$$
;

S.P. of fruits = ₹
$$18/8 = ₹ 9/4$$
;

Profit % =
$$\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

= $\frac{\frac{9}{4} - \frac{3}{2}}{\frac{3}{2}} \times 100 = \frac{\frac{(9-6)}{4}}{\frac{3}{2}} \times 100$
= $\frac{\frac{3}{4}}{\frac{3}{2}} \times 100 = \frac{3}{4} \times \frac{2}{3} \times 100 = \mathbf{50}\%$

60. Cost price =
$$50 \times 2 + 20 \times 4 + 30 \times 3 = ₹ 270$$

Selling price = $33 \times 9 = ₹297$

$$\therefore \text{ Profit \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{297 - 270}{270} \times 100 = 10\%$$

61. C.P. of
$$(26 + 30 = 56 \text{ kg})$$
 of rice $= 26 \times 20 + 30 \times 36$

S.P. of 56 kg of rice @
$$₹ 30/\text{kg} = 56 \times 30 = ₹ 1680$$

$$\therefore \qquad \text{Profit } \% = \frac{1680 - 1600}{1600} \times 100 = \frac{80}{1600} \times 100 = 5\%$$

62. C.P. of toffees = ₹
$$1/6$$

S.P. of toffees with 20% gain =
$$1.2 \times \frac{1}{6} = ₹ 0.2$$
/each = ₹ 1/5

63. Profit % =
$$\frac{\text{True weight} \sim \text{False weight}}{\text{True weight}} \times 100\% = \frac{1000 \text{ g} - 800 \text{ g}}{800 \text{ g}} \times 100\% = 25\%$$

64. Let C.P. of goods be
$$\not\equiv x/\text{each}$$
.

S.P. of goods =
$$1.1 \times x$$

Let number of items be y

$$\therefore \quad \text{Total C.P.} = \overline{\xi} \ x \times y$$

S.P for
$$\frac{4y}{5}$$
 items = $1.1x \times \frac{4y}{5}$

$$S.P. = \frac{4.4}{5} xy$$

$$\therefore \qquad loss \% = \frac{C.P. - S.P.}{C.P.} \times 100$$

$$=\frac{xy-\frac{4.4}{5}xy}{xy}\times100$$

$$= \left(1 - \frac{4.4}{5}\right) \times 100 = \frac{0.6}{5} \times 100 = 12\%$$

65. Gain or loss % =
$$\frac{(1.1 - 0.9)}{0.9} \times 100 = \frac{1100 - 900}{9} = \frac{200}{9} = 22\frac{2}{9}\%$$

more than 20% profit

66. C.P.
$$\times$$
 0.05 = 350 - 340 = 10

∴ C.P. =
$$\frac{10}{0.05}$$
 = ₹ 200

67. Profit or loss % in the transaction =
$$\left(\frac{\text{Common loss and gain}\%}{10}\right)^2 = \left(\frac{12}{10}\right)^2$$

= 1.44% = 1 $\frac{11}{25}$ % loss

68. S.P. =
$$50\% \times 120\% \times \text{C.P.} + 25\%$$
 (80%) C.P. + $25\% \times 100\%$ C.P.

S.P. =
$$\left(\frac{6}{10} + \frac{2}{10} + \frac{1}{4}\right)$$
 C.P. = $\left(\frac{4}{5} + \frac{1}{4}\right)$ C.P. = $\left(\frac{16+5}{20}\right)$ C.P. = $\left(\frac{21}{20}\right)$ C.P. profit % = $\left(\frac{21}{20} - 1\right) \times 100 = 5$ %

69. Discount =
$$\frac{4}{100}$$
 × 200000 + $\frac{2.5}{100}$ × 72000 = 8000 + 1800 = ₹ 9800

∴ Price changed by company for the car = 272000 - 9800 = ₹ 262200

70. Net discount =
$$\left(-12 - 20 + \frac{240}{100}\right)\% = -29.6\%$$

:.
$$(100 - 29.6)\%$$
 of labelled price = 704

$$∴ Labelled price = \frac{704}{70.4} \times 100 = ₹1000$$

71.
$$\frac{15}{100} \times x = \frac{20}{100} \times y$$

or $\frac{x}{y} = \frac{20}{100} \times \frac{100}{15}$
or $\frac{x}{y} = \frac{4}{3}$
i.e. $x = 80, y = 60$

72. Net discount =
$$\left(-20 - 10 + \frac{200}{100}\right)\% = -28\%$$

∴ Selling price =
$$(100 - 28)\%$$
 of list price
= 72% of 2000
= $72 \times 20 = ₹ 1440$

73. Total discount =
$$\frac{720 - 550.80}{720} \times 100$$

= $\frac{169.20}{720} \times 100 = \frac{16920}{720} = 23.5\%$

$$\therefore \left(-10 - x + \frac{10x}{100}\right)\% = 23.5\%$$
i.e.
$$x - \frac{x}{10} = 23.5 - 10 = 13.5$$

or
$$\frac{9}{10}x = 13.5$$

or
$$x = 13.5 \times \frac{10}{9} = 15\%$$

74. $\frac{2}{5}$ of marked price = 75% of C.P.

$$\therefore \frac{\text{Marked price}}{\text{Cost price (or C.P.)}} = \frac{75\%}{2/5} = \frac{3/4}{2/5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

75. C.P. of article =
$$\frac{64}{100}$$
 of marked price

S.P. of article = (100 - 12)% of marked price

$$\therefore \quad \text{gain } \% = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{\frac{88}{100} - \frac{64}{100}}{\frac{64}{100}} \times 100 = \frac{24}{64} \times 100 = \frac{3}{8} \times 100 = 37.5\%$$

76. Marked price = $1.25 \times \text{cost price}$

S.P. =
$$(100 - 16)\%$$
 of marked price
= 84% of marked price
= $\frac{84}{100} \times 1.25 \times \text{cost price}$
= $\frac{21}{25} \times 1.25 \times \text{cost price}$ = $21 \times 0.05 \times \text{cost price}$
= $1.05 \times \text{cost price}$

:. profit % =
$$\frac{1.05 - 1}{1} \times 100 = 5\%$$

77. Let the cost price of saree be $\not\in x$.

 \mathbf{or}

 \mathbf{or}

Labelled price =
$$266 \times \frac{100}{95} = ₹280$$

profit % = $12 = \frac{280 - x}{x} \times 100$
 $x\left(\frac{12}{100} + 1\right) = 280$
 $x = \frac{100}{112} \times 280 = ₹250$

78. S.P. = ₹ 45/each; profit = 50%

S.P. without discount =
$$45 \times \frac{100}{90} = ₹50$$

$$\therefore \qquad \text{C.P.} = \frac{\text{S.P.}}{1.5} = \frac{45}{1.5} = ₹30$$

:. Without discount profit
$$\% = \frac{S.P. - C.P.}{C.P.} \times 100$$

$$= \frac{50 - 30}{30} \times 100 = \frac{2}{3} \times 100 = 66 \frac{2}{3} \%$$

79. S.P. = ₹ 17940; profit % = 19.6

$$\therefore \qquad \text{C.P.} = \frac{17940}{1.196} = ₹15000$$

Marked price =
$$17940 \times \frac{100}{92} = 195 \times 100 = ₹19500$$

:. Profit % without giving discount =
$$\frac{19500 - 15000}{15000} \times 100 = \frac{4500}{15000} \times 100 = 30\%$$

- 80. Data given is inadequate and therefore cannot be determined.
- 81. Marked price = $1.35 \times \text{cost price}$

For a gain of 8%,

$$S.P. = 1.08 \times cost price$$

.. % discount allowed on marke d price =
$$\frac{1.35 - 1.08}{1.35} \times 100 = \frac{0.27}{1.35} \times 100 = 20\%$$

82. C.P. =
$$\frac{85}{100} \times \text{labelled price}$$

S.P. =
$$2880 = \frac{120}{100}$$
 labelled price

∴ C.P. =
$$\frac{2880 \times 100}{120} \times \frac{85}{100} = 24 \times 85 = ₹ 2040$$

83. Marked price = $1.2 \times C.P.$

Total S.P. =
$$\frac{1}{2} \times (1.2 \times \text{C.P.}) + \frac{1}{4} \times 0.8 \times 1.2 \times \text{C.P.} + \frac{1}{4} \times 0.6 \times 1.2 \times \text{C.P.}$$

= $(0.6 + 0.24 + 0.18) \text{ C.P.} = 1.02 \text{ C.P.}$

 \therefore Total gain = 2%

84. Marked price = ₹ 30

S.P. after discount of
$$15\% = 0.85 \times 30$$
 = ₹ 25.5

Less cost of shuttle cock (free) worth $\stackrel{?}{\sim} 1.50 = -1.5$

$$\therefore \qquad 24 = \text{C.P.} \times 1.2$$

or C.P. =
$$\frac{24}{1.2}$$
 = ₹ 20

85. Let his profit be \mathbb{Z} x.

Cost price ×
$$\left(1 + \frac{22.5}{100}\right)$$
 = Selling price = ₹ 392

$$\therefore \qquad \text{Cost price} = \frac{392 \times 100}{122.5} = \frac{392 \times 1000}{1225} = 320$$

∴ Profit = S.P. – C.P. =
$$392 - 320 = ₹ 72$$

86. C.P. of cycle = ₹ 1400

S.P. cycle =
$$\frac{85}{100} \times 1400 = ₹1190$$

87. S.P.₁ =
$$\frac{120}{100} \times \text{C.P.}$$

$$S.P._2 = 2 S.P._1 = \frac{240}{100} \times C.P.$$

:. % profit =
$$\frac{\text{S.P.}_2 - \text{C.P.}}{\text{C.P.}} \times 100 = 140\%$$

88. Let C.P. of 1 pen be ξx .

٠.

C.P. of 12 pens = S.P. of 8 pens =
$$₹$$
 12 x
Profit = S.P. of 4 pens
= C.P. of 6 pens = $₹$ 6 x
gain % = $\frac{\text{C.P. of 6 pens}}{\text{C.P. of 12 pens}} \times 100$

 $=\frac{6x}{12x}\times100=50\%$

81

Squares and Square Roots

DUPLEX: For a single central digit, duplex is its square. For example, duplex of 7 is $7^2 = 49$.

The duplex of a number with even number of digits is double the cross-product of digits equidistant from the two ends.

For example,

Duplex of
$$79 = 2(7 \times 9) = 126$$

Duplex of $34 = 2(3 \times 4) = 24$
Duplex of $109 = 2(1 \times 9) + 0^2 = 18$
Duplex of $1357 = 2(1 \times 7) + 2(3 \times 5) = 44$

EXAMPLE 1 Find the value of $(209)^2$.

Solution
$$(209)^2 = D \text{ for } 2 \mid D \text{ for } 20 \mid D \text{ for } 209 \mid D \text{ for } 09 \mid D \text{ for } 9$$

$$= 2^2 \mid 2(2 \times 0) \mid 2 \mid (2 \times 9) + 0^2 \mid | \mid 2(0 \times 9) \mid 9^2$$

$$= 4 \mid 0 \mid 36 \mid 0 \mid 81$$

$$= 43681$$

$$= 43681$$

EXAMPLE 2 Find the value of $(873)^2$.

Solution
$$(873)^2 = D$$
 for $8 | D$ for $87 | D$ for $873 | D$ for $73 | D$ for 3

$$= 64 | 2(8 \times 7) | 2(8 \times 3) + 7^2 | 2(7 \times 3) | 3^2$$

$$= 64 | 112 | 97 | 42 | 9$$

$$= 762129$$

$$97$$

$$112$$

$$64$$

$$762129$$

Square of a two-digit number beginning with 5

Take the number as 5A.

$$(5A)^2 = 5^2 \mid 2 \times 5 \times A \mid A^2$$

= 25 \ \ \ \ 10A \ \ A^2

=
$$25 |A0|A^2$$

= $(25 + A)|A^2$, A^2 is written as a two-digit number.

EXAMPLE 3 Find the value of the following:

(a)
$$(54)^2$$
 (b) $(59)^2$ (c) $(52)^2$

Solution

(a)
$$(54)^2 = 25 + A \mid A^2 = 25 + 4 \mid 4^2 = 29 \mid 16 = 2916$$

(b)
$$(59)^2 = 25 + A \mid A^2 = 25 + 9 \mid 9^2 = 34 \mid 81 = 3481$$

(c)
$$(52)^2 = 25 + A | A^2 = 25 + 2 | 2^2 = 27 | 04 = 2704$$

Square of a number with unit digit as 5

Let the number be represented as A5.

$$(A5)^{2} = A^{2} | 2 \times A \times 5 | 5^{2}$$

$$= A^{2} | 10A | 25$$

$$= A^{2} | A0 | 25$$

$$= A^{2} + A | 25$$

$$= A(A + 1) | 25$$

EXAMPLE 4 (a)
$$(15)^2 = ?$$
 (b) $(25)^2 = ?$ (c) $(115)^2 = ?$ (d) $(225)^2 = ?$

Solution

(a)
$$(15)^2 = 1(1+1) \mid 25 = 2 \mid 25 = 225$$

(b)
$$(25)^2 = 2(2+1) \mid 25 = 6 \mid 25 = 625$$

(c)
$$(115)^2 = 11(11+1) \mid 25 = 132 \mid 25 = 13225$$

(d)
$$(225)^2 = 22(22+1) \mid 25 = 506 \mid 25 = 50625$$

Square of a number which is nearer to 10^x

Use the algebraic formula

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

EXAMPLE 5 Find the value of the following

(a)
$$(98)^2$$
 (b) $(103)^2$ (c) $(993)^2$ (d) $(1007)^2$

Solution

(a)
$$(98)^2 = (98)^2 - 2^2 + 2^2 = (98 + 2)(98 - 2) + 4 = 9600 + 4 = 9604$$

(b)
$$(103)^2 = (103)^2 - 3^2 + 3^2 = 100 \times 106 + 9 = 10609$$

(c)
$$(993)^2 = (993)^2 = (993^2 - 7^2) + 7^2 = 1000 \times 986 + 49 = 986049$$

(d)
$$(1007)^2 = (1007^2 - 7^2) + 7^2 = 1000 \times 1014 + 49 = 1014049$$

SQUARE ROOTS: To find the square root easily, the following points are to be kept in mind.

- 1. Make thorough squares of all natural numbers up to 32.
- 2. Perfect squares will never end with digits 2, 3, 7, 8.

The square root of a perfect square can be identified at a glance using the end digits of those numbers as shown in the following table.

If square ends with the digit	Square root will end with digit
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

EXAMPLE 6 Find the value of $\sqrt{4096}$.

Solution $\sqrt{4096} = 10 \times \sqrt{40.96}$

 $\sqrt{40.96}$ lies between 6 and 7. The root can end with either 4 or 6 as square ends with 6.

But 40.96 is nearer to $6^2 = 36$. \therefore The root is $6.4 \times 10 = 64$

EXAMPLE 7 Find the value of $\sqrt{35344}$.

Solution $\sqrt{35344} = 10 \times \sqrt{353.44}$

 $\sqrt{353.44}$ lies between 18 and 19.

Now the root can end with 2 or 8 as square ends with 4

 \therefore 353.44 is nearer to $19^2 = 361$

The root will be ending with 8

 $\therefore \text{ root} = 18.8 \times 10 = 188$



EXERCISES

Q (1-5) Find the squares of the following numbers:

- 1. 847
- **2.** 8347
- 3. 54613
- 4. 480028
- **5.** 63040586

- **Q** (6–15) Find values of the following:
 - 6. 108^2
- 7.96^2
- 8. 997²
- 9. 1004^2
- 10. 1009^2

- 11. 999²
- 12. 991^2
- 13. 94^2
- 14. 106^2
- 15. 109^2

- Q (16-30) Find the square roots of the following:
 - 16. $\sqrt{4624}$
- 17. $\sqrt{20044}$
- 18. $\sqrt{12321}$
- 19. $\sqrt{3481}$
- **20.** $\sqrt{7569}$

- **21.** $\sqrt{61009}$ **26.** $\sqrt{767376}$
- **27.** $\sqrt{357604}$
- **22.** $\sqrt{143641}$ **23.** $\sqrt{481636}$ **28.** $\sqrt{474721}$
- **24.** $\sqrt{906304}$ **29.** $\sqrt{802816}$
- **25.** $\sqrt{619369}$ **30.** $\sqrt{636804}$

Q (31-82) Find the most correct answer from the given choices:

31. If $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ and $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$, then the value of $(x^2 + y^2)$ is [SSC]

- (a) 10
- (b) 13
- (c) 14
- (d) 15

[SSC]

[CBI]

(d) $14 + 8\sqrt{3}$

33. If
$$\sqrt{2} = 1.414$$
, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to [CBI]

(a) 0.172 (b) 0.414 (c) 0.586 (d) 1.414

34. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, then [RRB]

(a) $a = -11$, $b = -6$ (b) $a = -11$, $b = 6$ (c) $a = 11$, $b = -6$ (d) $a = 6$, $b = 11$

35. $\left[\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} - \frac{6}{\sqrt{8}-\sqrt{12}} \right] = ?$ [RRB]

(a) $\sqrt{3} - \sqrt{2}$ (b) $\sqrt{3} + \sqrt{2}$ (c) $5\sqrt{3}$ (d) 1

36. $\left(2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} \right) =$ [SSC]

(a) $2 - \sqrt{2}$ (b) 2 (c) $2 + \sqrt{2}$ (d) $2\sqrt{2}$

37. Given $\sqrt{5} = 2.2361$, $\sqrt{5} = 1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}} =$ [SSC]

(a) 1.98 (b) 1.984 (c) 1.9841 (d) 2

38. What is the least number which should be subtracted from 0.000326 to make it a perfect square?

(a) 0.000002 (b) 0.000004 (c) 0.02 (d) 0.04

39. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square.

(a) 2 (b) 3 (c) 7 (d) 11

40. If $\sqrt{5} = 2.236$, then the value of $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125} =$ [MBA]

(a) 5.59 (b) 7.826 (c) 8.944 (d) 10.062

41. The value of $\frac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}}$ is nearly equal to [CBI]

(a) 0.6 (b) 0.7 (c) $0.\overline{8}$ (d) $0.\overline{9}$

42. Square root of $0.\overline{4}$ is $(2.0.05)$ (e) 1.1039 (d) 2.1039

32. If $x = (7 - 4\sqrt{3})$, then the value of $\left(x + \frac{1}{x}\right)$ is

(b) $8\sqrt{3}$

(c) 14

(a) $3\sqrt{3}$

44.
$$\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2$$
 simplifies to [RRB]

(a) $\frac{3}{4}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) None of these

45. Square root of $(7 + 3\sqrt{5})$ (7 - 3 $\sqrt{5}$) is [SSC]
(a) $\sqrt{5}$ (b) 2 (c) 4 (d) $3\sqrt{5}$

46. The value of $\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}}$ is [SSC]
(a) 0.1 (b) 10 (c) 10^2 (d) 10^3

47. If $3\sqrt{5} + \sqrt{125} = 17.88$, then what will be the value of $\sqrt{80} + 6\sqrt{5}$? [Bank PO]
(a) 13.41 (b) 20.46 (c) 21.66 (d) 22.35

48. If $\sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27}$, then the value of x is [CDS]
(a) 1 (b) 3 (c) 5 (d) 7

49. $\sqrt{\frac{48.4}{0.289}}$ is equal to [SSC]
(a) $1\frac{5}{17}$ (b) $12\frac{1}{17}$ (c) $12\frac{16}{17}$ (d) $129\frac{7}{17}$

50. If $\sqrt{x} + \sqrt{441} = 0.02$, then the value of x is [SSC]
(a) 0.1764 (b) 1.764 (c) 1.64 (d) 2.64

51. $28\sqrt{7} + 1426 = \frac{3}{4}$ of 2872 . The value in place of (?) is [BSRB]
(a) 576 (b) 676 (c) 1296 (d) 1444

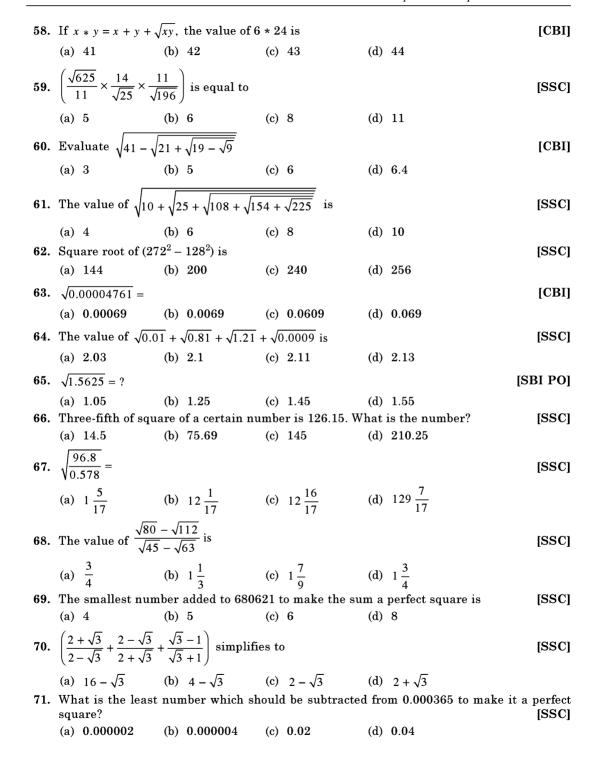
52. What number should be divided by $\sqrt{0.25}$ to give the result as 257 [CBI]
(a) 12.5 (b) 25 (c) 50 (d) 125

53. If $\frac{52}{x} = \sqrt{\frac{169}{289}}$, then the value of x is [CBI]
(a) 576 (b) 58 (c) 62 (d) 68

54. $\sqrt{1.5625} = ?$
(a) 1.05 (b) 1.25 (c) 1.45 (d) 1.55

55. $1.5^2 \times \sqrt{0.0225} = ?$
(a) 0.0375 (b) 0.3375 (c) 3.275 (d) 3.275

56. The value of $\sqrt{0.00041}$ is [SSC]
(a) 2 (b) 4 (c) 6 (d) 8



72. If $\sqrt{5} = 2.236$, then value of $\frac{1}{\sqrt{5}}$ is (a) 0.447 (b) 0.367 (c) 0.745(d) None of these 73. The value of $\frac{1+\sqrt{0.04}}{1-\sqrt{0.1}}$ is nearly equal to [CBI] (a) 0.6 (b) 1.1 (c) 1.6 (d) 1.7 74. Square root of $\frac{(0.75)^3}{1-0.75} + [0.75 + 0.75^2 + 1]$ is [SSC] (b) 2 (a) 1 (d) 4 75. Given $\sqrt{2} = 1.414$. The value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is [SSC] (b) 8.484 (c) 8.526 76. The least perfect square, which is divisible by 21, 36 and 66 completely is [CBI] (a) 213444 (b) 214344 (c) 214434 (d) 231444 77. The value of $\sqrt{\frac{0.16}{0.4}}$ is [IGNOU] (a) 0.02(c) 0.63(d) None of these 78. $\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}}$ is equal to [NIFT] (a) 0.9 (b) 0.99 (c) 9 (d) 99 **79.** If $0.13 + p^2 = 13$, then p equals [SSC] (a) 0.01(b) 0.1 (c) 10 (d) 100 80. What should come in place of both the question marks in the equation $\frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$ [Bank PO] (c) 144 (a) 12 (b) 14 (d) 196 81. For what value of? the statement $\left(\frac{?}{15}\right)\left(\frac{?}{135}\right)=1$ is true? [SSC] (a) 15 (b) 25 (c) 35 (d) 45 82. Given that $\sqrt{13} = 3.605$ and $\sqrt{130} = 11.40$, find the value of $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$ [SSC] (a) 36.164 (b) 36.304 (c) 37.164 (d) 37.304

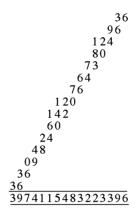


31. (c) **32.** (c) **34.** (c) **35.** (c) **36.** (b) **37.** (c) **40.** (b) **33.** (a) **38.** (a) **39.** (b) **41.** (c) **42**. (a) **43.** (c) **44.** (c) **45**. (b) **46.** (b) **47**. (d) **48.** (a) **49.** (c) **50**. (a) **51.** (b) **52.** (a) **53.** (d) **54.** (b) **55.** (c) **56.** (c) **57.** (c) **58.** (b) **59.** (a) **60.** (c) **61.** (a) **62.** (c) **63.** (b) **64.** (d) **65**. (b) **66.** (a) **67.** (c) **68.** (b) **69.** (a) **70.** (a) **71.** (b) **72.** (a) **73.** (d) **74.** (d) **75.** (b) **76.** (a) **77.** (c) **78.** (b) **79.** (b) **80.** (a) **81.** (d) **82.** (d)

Solutions with Necessary Explanation

= 3974115483223396

1. $(847)^2 = ?$ $(847)^2 = D$ for $8 \mid D$ for $84 \mid D$ for $847 \mid D$ for $47 \mid D$ for 7 $= 8^{2} | 2(8 \times 4) | 2(8 \times 7) + 4^{2} | 2(4 \times 7) | 7^{2}$ 64 = 64 | 64 | 128 | 56 | 49 = 717409717409 2. $(8347)^2 = ?$ 49 $(8347)^2 = D$ for $8 \mid D$ for $83 \mid D$ for $834 \mid D$ for $8347 \mid D$ for $347 \mid D$ for $47 \mid D$ for 756 $= 8^{2} | 2(8 \times 3) | 2(8 \times 4) + 3^{2} | 2(8 \times 7) + 2(3 \times 4) | 2(3 \times 7) + 4^{2}$ 136 $|2(4 \times 7)|7^2$ 73 = 64 | 48 | 73 | 136 | 58 | 56 | 49 | 48 64 = 6967240969672409 3. $(54613)^2 = ?$ $(54613)^2$ = D for $5 \mid D$ for $54 \mid D$ for $546 \mid D$ for $5461 \mid D$ for $54613 \mid D$ for 461309 06 | D for 613 | D for 13 | D for 3. 37 $= 5^{2} | 2(5 \times 4) | 2(5 \times 6) + 4^{2} | 2(5 \times 1) + 2(4 \times 6) | 2(5 \times 3) + 2(4 \times 1)$ $+6^{2} | 2(4 \times 3) + 2 (6 \times 1) | 2(6 \times 3) + 1^{2} | 2(1 \times 3) | 3^{2}$ = 25 | 40 | 76 | 58 | 74 | 36 | 37 | 6 | 9 |= 2982579769 4. $(480028)^2 = ?$ $(480028)^2$ = D for 4 | D for 48 | D for 480 | D for 4800 | D for 48002 | D for 480028 $|\,D\,\, for\,\, 80028\,|\,D\,\, for\,\, 0028\,|\,D\,\, for\,\, 028\,|\,D\,\, for\,\, 28\,|\,D\,\, for\,\, 8$ $= 4^{2} | 2(4 \times 8) | 2(4 \times 0) + 8^{2} | 2(4 \times 0) + 2(8 \times 0) | 2(4 \times 2)$ $+2(8\times0)+0^{2}|2(4\times8)+2(8\times2)+2(0\times0)|2(8\times8)$ $+2(0\times 2)+0^2|2(0\times 8)+2(0\times 2)|2(0\times 8)$ 16 $+2^{2} | 2(2 \times 8) | 8^{2}$ 00 64 = 16 | 64 | 64 | 0 | 16 | 96 | 128 | 0 | 4 | 32 | 64 = 230426880784 230426880784 5. $(63040586)^2 = ?$ $(63040586)^2$ = D for $6 \mid D$ for $63 \mid D$ for $630 \mid D$ for $6304 \mid D$ for $63040 \mid D$ for 630405| D for 6304058 | D for 63040586 | D for 3040586 | D for 040586 | D for 40586 D for 0586 | D for 586 | D for 86 | D for 6 $=6^{2} | 2(6 \times 3) | 2(6 \times 0) + 3^{2} | 2(6 \times 4) + 2(3 \times 0) | 2(6 \times 0) + 2(3 \times 4) + 0^{2}$ $|2(6\times5)+2(3\times0)+2(0\times4)|2(6\times8)+2(3\times5)+2(0\times0)+4^{2}$ $|2(6\times6)+2(3\times8)+2(0\times5)+2(4\times0)|2(3\times6)+2(0\times8)+2(4\times5)+0^{2}$ $|2(0\times6)+2(4\times8)+2(0\times5)|2(4\times6)+2(0\times8)+5^{2}|2(0\times6)+2(5\times8)$ $|2(5\times6)+8^2|2(8\times6)|6^2$ = 36 | 36 | 9 | 48 | 24 | 60 | 142 | 120 | 76 | 64 | 73 | 80 | 124 | 96 | 36



6.
$$108^2 = (100 + 8)^2 = 10000 + 1600 + 64 = 11664$$

7.
$$96^2 = (100 - 4)^2 = 10000 - 800 + 16 = 9216$$

8.
$$997^2 = (1000 - 3)^2 = 1000000 - 6000 + 9 = 944009$$

9.
$$1004^2 = (1000 + 4)^2 = 1000000 + 8000 + 16 = 1008016$$

10.
$$1009^2 = (1000 + 9)^2 = 10000000 + 18000 + 81 = 1018081$$

11.
$$999^2 = (1000 - 1)^2 = 1000000 - 2000 + 1 = 998001$$

12.
$$991^2 = (1000 - 9)^2 = 1000000 - 18000 + 81 = 982081$$

13.
$$94^2 = (100 - 6)^2 = 10000 - 1200 + 36 = 8836$$

14.
$$106^2 = (100 + 6)^2 = 10000 + 1200 + 36 = 11236$$

15.
$$109^2 = (100 + 9)^2 = 10000 + 1800 + 81 = 11881$$

16.
$$\sqrt{4624} = 10 \times \sqrt{46.24}$$

 $\sqrt{46.24}$ lies between 6 and 7. The root will end with either 2 or 8 as square ends with 4. Here 46.24 is nearer to $7^2 = 49$.

$$\therefore$$
 Root is $6.8 \times 10 = 68$

17. $\sqrt{20044} = 10 \times \sqrt{200.44}$

 $\sqrt{200.44}$ lies between 14 and 15. The root will end with either 2 or 8 as square ends with 4. Here 200.44 is nearer to $14^2 = 196$

Hence root is $10 \times 14.2 = 142$

18. $\sqrt{12321} = 10 \times \sqrt{123.21}$

 $\sqrt{123.21}$ lies between 11 and 12. The root will end with 1 or 9 as square ends with 1.

Here 123.21 is nearer to $11^2 = 121$.

Hence root is $10 \times 11.1 = 111$

21. $\sqrt{61009} = 10 \times \sqrt{610.09}$

 $\sqrt{610.09}$ lies between 24 and 25. Root will end with 3 or 7 as square ends with 9.

Here 610.09 is nearer to $25^2 = 625$

Hence root is $10 \times 24.7 = 247$

23. $\sqrt{481636} = 10 \times \sqrt{4816.36}$

 $\sqrt{4816.36}$ lies between 69 and 70. Root will end with either 4 or 6 as square ends with 6.

Here 4816.36 is nearer to $69^2 = 4761$

Hence root is $10 \times 69.4 = 694$

24.
$$\sqrt{906304} = 10 \times \sqrt{9063.04}$$

 $\sqrt{9063.04}$ lies between 95 and 96. Root will end with 2 or 8 as square ends with 4.

Here 9063.04 is nearer to $95^2 = 9025$

Hence root = $95.2 \times 10 = 952$

25.
$$\sqrt{619369} = 10 \times \sqrt{6193.69}$$

 $\sqrt{6193.69}$ lies between 78 and 79. Root will end with 3 or 7 as square ends with 9.

Here 6193.69 is nearer to $79^2 = 6241$

Hence root = $78.7 \times 10 = 787$

29.
$$\sqrt{802816} = 10 \times \sqrt{8028.16}$$

 $\sqrt{8028.16}$ lies between 89 and 90. Root will end with 4 or 6 as square ends with 6.

Here 8028.16 is nearer to $90^2 = 8100$.

Hence root = $89.6 \times 10 = 896$

30.
$$\sqrt{636804} = 10 \times \sqrt{6368.04}$$

 $\sqrt{6368.04}$ lies between 79 and 80. Root will end with 2 or 8 as square ends with 4.

Here 6368.04 is nearer to $80^2 = 6400$.

Hence root = $10 \times 79.8 = 798$

31.
$$x = \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1} = \frac{4 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

$$y = \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$\therefore x^2 + y^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 2 \times [2^2 + (\sqrt{3})^2] = 14$$

32.
$$x = 7 - 4\sqrt{3}$$

$$\therefore \frac{1}{x} = \frac{1}{7 - 4\sqrt{3}} = \frac{(7 + 4\sqrt{3}) \times 1}{(7 + 4\sqrt{3}) \times (7 - 4\sqrt{3})} = \frac{7 + 4\sqrt{3}}{7^2 - (4\sqrt{3})^2} = \frac{7 + 4\sqrt{3}}{49 - 48} = 7 + 4\sqrt{3}$$

$$\therefore x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = 14$$

33.
$$\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1^2} = \frac{2+1-\sqrt{2}}{(2-1)} = 3-2\sqrt{2}$$

Given $\sqrt{2} = 1.414$

$$\therefore$$
 3 - 2 $\sqrt{2}$ = 3 - 2 × 1.414 = 3 - 2.828 = **0.172**

34.
$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{(5+2\sqrt{3})}{(7+4\sqrt{3})} \times \frac{(7-4\sqrt{3})}{(7-4\sqrt{3})}$$
$$= \frac{5\times 7+7\times 2\sqrt{3}-5\times 4\sqrt{3}-2\times 4\times 3}{7^2-(4\sqrt{3})^2}$$

$$= \frac{35 - 24 + \sqrt{3} (14 - 20)}{49 - 48} = 11 - 6\sqrt{3}$$

$$\therefore \qquad a + b\sqrt{3} = 11 - 6\sqrt{3}$$

$$\Rightarrow \qquad a = 11; \qquad b = -6$$
35.
$$\frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} - \frac{6}{\sqrt{8} - \sqrt{12}}$$

$$= \frac{3\sqrt{2} \times (\sqrt{6} + \sqrt{3})}{(\sqrt{6} - \sqrt{3}) (\sqrt{6} + \sqrt{3})} - \frac{4\sqrt{3}}{(\sqrt{6} - \sqrt{2})} \times \frac{(\sqrt{6} + \sqrt{2})}{(\sqrt{6} + \sqrt{2})} - \frac{6}{(\sqrt{8} - \sqrt{12})} \frac{(\sqrt{8} + \sqrt{12})}{(\sqrt{8} + \sqrt{12})}$$

$$= \frac{3\sqrt{2}}{6 - 3} - \frac{4\sqrt{3}}{6 - 3} - \frac{4\sqrt{3}}{6 - 2} - \frac{6(\sqrt{8} + \sqrt{12})}{8 - 12}$$

$$= \frac{3\sqrt{12} + 3\sqrt{6}}{3} - \frac{(4\sqrt{18} + 4\sqrt{6})}{4} - \frac{(6\sqrt{8} + 6\sqrt{12})}{-4}$$

$$= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + \frac{3\sqrt{8}}{2} + \frac{3}{2} \sqrt{12}$$

$$= 2\sqrt{3} + \sqrt{6} - 3\sqrt{2} - \sqrt{6} + 3\sqrt{2} + 3\sqrt{3}$$

$$= 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$
36.
$$2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2} = 2 + \sqrt{2} + \frac{1}{(2 + \sqrt{2})} \times \frac{(2 - \sqrt{2})}{(2 - \sqrt{2})} + \frac{1}{(\sqrt{2} - 2)} \times \frac{(2 + \sqrt{2})}{(2 + \sqrt{2})}$$

$$= (2 + \sqrt{2}) + \frac{2 - \sqrt{2}}{2^2 - (\sqrt{2})^2} + \frac{1}{2^2 - (\sqrt{2})^2}$$

$$= 2 + \sqrt{2} + \frac{2 - \sqrt{2}}{4 - 2} - \frac{1(2 + \sqrt{2})}{4 - 2}$$

$$= 2 + \sqrt{2} + 1 - \frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} = 2$$
37.
$$\frac{1}{\sqrt{5} - \sqrt{3}} = \frac{1}{(\sqrt{5} - \sqrt{3})^2} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})}$$

$$= \frac{\sqrt{5} + \sqrt{3}}{5 - 3} = \frac{\sqrt{5} + \sqrt{3}}{5 - 3}$$
Given
$$\sqrt{5} = 2.2361; \qquad \sqrt{3} = 1.7321$$

$$\therefore \frac{\sqrt{5} + \sqrt{3}}{2} = \frac{2.2361 + 1.7321}{2} = \frac{3.9682}{2} = 1.9841$$

38. $0.000326 = 326 \times 10^{-6}$

 324×10^{-6} is a perfect square

 \therefore Required number to be subtracted = (326 – 324) \times 10^{-6} = 2 \times 10^{-6}

39. $5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11 = 2^4 \times 3 \times 11^2$

To make the above number a perfect square, it should be multiplied by 3.

40.
$$\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125} = \frac{\sqrt{5}}{2} - 2\sqrt{5} + 5\sqrt{5} = \left(5 - 2 + \frac{1}{2}\right)\sqrt{5} = \frac{7}{2}\sqrt{5}$$

Given $\sqrt{5} = 2.236$

$$\therefore \frac{7\sqrt{5}}{2} = 7 \times \frac{2.236}{2} = 7 \times 1.118 = 7.826$$

41.
$$\frac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}} = \frac{1 + 0.1}{1 - \sqrt{0.1}}$$

$$\frac{(1+0.1)(1+\sqrt{0.1})}{(1-\sqrt{0.1})(1+\sqrt{0.1})} = \frac{1+\sqrt{0.1}+0.1+0.1\sqrt{0.1}}{1^2-0.1}$$

$$= \frac{1.1+1.1\sqrt{0.1}}{0.9} = \frac{1.1\times1.361}{0.9}$$

$$= \frac{1.4971}{0.9} = \frac{1.4971}{9} = 1.663 \approx 1.6$$
(:: $\sqrt{0.1} = 0.361$ approx.)

42.
$$0.\overline{4} = 0.444 \dots = \frac{4}{9}$$

$$\therefore \frac{\sqrt{4}}{9} = \frac{2}{3} = 0.666 \dots = \mathbf{0.\overline{6}}$$

43.
$$\sqrt{4a^2 - 4a + 1} + 3a = \sqrt{(2a - 1)^2 + 3a} = \pm (2a - 1) + 3a$$

= $5a - 1$ or $3a - (2a - 1) = (a + 1)$

Given a = 0.1039

$$\therefore 5a - 1 = 5 \times 0.1039 - 1 = 0.5195 - 1 = 0.4705$$
$$a + 1 = 0.1039 + 1 = 1.1039$$

44.
$$\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 = 3 + \frac{1}{3} - 2 \times \sqrt{3} \times \frac{1}{\sqrt{3}} = 3 + \frac{1}{3} - 2 = 1 \cdot \frac{1}{3} = \frac{4}{3}$$

45.
$$\sqrt{(7+3\sqrt{5})(7-3\sqrt{5})} = \sqrt{[(7^2-(3\sqrt{5})^2]} = \sqrt{49-45} = \sqrt{4} = 2$$

46.
$$\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}} = \sqrt{\frac{10^2 \left[(0.003)^2 + (0.021)^2 + (0.0065)^2 \right]}{(0.003)^2 + (0.021)^2 + (0.0065)^2}} = \mathbf{10}$$

47.
$$3\sqrt{5} + \sqrt{125} = 3\sqrt{5} + 5\sqrt{5} = 8\sqrt{5}$$

Given that $8\sqrt{5} = 17.88$

$$\therefore \sqrt{80} + 6\sqrt{5} = 4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5} = \frac{17.88}{8\sqrt{5}} \times 10\sqrt{5} = 22.35$$

48.
$$\sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27}$$

$$\Rightarrow 1 + \frac{55}{729} = \left(1 + \frac{x}{27}\right)^2 = 1 + \frac{x^2}{27^2} + \frac{2x}{27}$$

$$\Rightarrow \frac{x^2}{27^2} + \frac{2x}{27} = \frac{55}{729}$$

$$\Rightarrow \frac{x^2 + 27 \times 2x}{27^2} = \frac{55}{729}$$

$$\Rightarrow x^2 + 54x - 55 = 0$$

$$\therefore (x + 55) (x - 1) = 0 \Rightarrow x = 1 \text{ or } x = -55$$

$$\therefore x = 1$$

49.
$$\sqrt{\frac{48.4}{0.289}} = \sqrt{\frac{48400}{289}} = \frac{220}{17} = 12 \frac{16}{17}$$

50.
$$\sqrt{x} \div \sqrt{441} = 0.02$$

$$\therefore x = (0.02 \times \sqrt{441})^2 = (0.02 \times \sqrt{441})^2 = (0.42)^2 = \mathbf{0.1764}$$

51.
$$28\sqrt{?} = \frac{3}{4} \times 2872 - 1426 = 3 \times 718 - 1426 = 728$$

:. Value in place of ? =
$$\left(\frac{728}{28}\right)^2 = (26)^2 = 676$$

52. The required number =
$$25 \times \sqrt{0.25} = 25 \times 0.5 = 12.5$$

$$53. \ \frac{52}{x} = \sqrt{\frac{169}{289}} = \frac{13}{17}$$

$$\therefore x = \frac{52 \times 17}{13} = 68$$

54.
$$\sqrt{1.5625} = 1.25$$

55.
$$1.5^2 \times \sqrt{0.0225} = 1.5^2 \times 10^{-2} \times 15 = 3.275$$

56.
$$\sqrt{0.000441} = \sqrt{10^{-6} \times 441} = 10^{-3} \times 21 =$$
0.021

57.
$$15876 = 158.76 \times 10^2$$

158.76 in between squares 144 and 169 and is nearer to 169. Therefore square root should end with 6.

58.
$$x * y = x + y + \sqrt{xy}$$

$$\therefore 6 * 24 = 6 + 24 + \sqrt{6 \times 24}$$

$$= 30 + \sqrt{144} = 30 + 12 = 42$$

59.
$$\left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}}\right) = \frac{25}{11} \times \frac{14}{5} \times \frac{11}{14} = \frac{25}{5} = 5$$

60.
$$\sqrt{41 - \sqrt{21 + \sqrt{19 - \sqrt{9}}}} = \sqrt{41 - \sqrt{21 + \sqrt{(19 - 3)}}}$$

= $\sqrt{41 - \sqrt{21 + 4}} = \sqrt{41 - 5} = \sqrt{36} = 6$

61.
$$\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}} = \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + 15}}}}$$

$$= \sqrt{10 + \sqrt{25 + \sqrt{108 + 13}}}$$

$$= \sqrt{10 + \sqrt{25 + 11}} = \sqrt{10 + 6} = \sqrt{16} = 4$$

62.
$$\sqrt{272^2 - 128^2} = \sqrt{(272 + 128)(272 - 128)} = \sqrt{400 \times 144} = 20 \times 12 = 240$$

63.
$$\sqrt{0.00004761} = \sqrt{10^{-6} \times 47.61} = 10^{-3} \times 6.9 = 0.0069$$

64.
$$\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009} = 0.1 + 0.9 + 1.1 + 0.03 = 2.13$$

65.
$$\sqrt{1.5625} = 1.25$$

66.
$$\frac{3}{5} \times x^2 = 126.15$$

$$\therefore \qquad x^2 = \frac{126.15 \times 5}{3} = 42.05 \times 5$$

$$x^2 = 210.25$$
or
$$x = \sqrt{210.25} = 14.5$$

67.
$$\sqrt{\frac{96.8}{0.578}} = \sqrt{\frac{96800}{578}} = \sqrt{\frac{48400}{289}} = 12\frac{16}{17}$$

68.
$$\frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}} = \frac{4\sqrt{5} - 4\sqrt{7}}{3\sqrt{5} - 3\sqrt{7}} = \frac{4(\sqrt{5} - \sqrt{7})}{3(\sqrt{5} - \sqrt{7})} = \frac{4}{3} = 1\frac{1}{3}$$

69. 4 is to added to 680621 to become a perfect square

$$\begin{array}{r} 825 \\ 8 \hline 680621 \\ 64 \\ 162 \hline 406 \\ 324 \\ 1645 \hline 8221 \\ 8225 \\ \hline -4 \\ \end{array}$$

70.
$$\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} + \frac{(2-\sqrt{3})}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} + \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} + \frac{(2-\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} + \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{4+3+4\sqrt{3}}{4-3} + \frac{4+3-4\sqrt{3}}{4-3} + \frac{3+1-2\sqrt{3}}{3-1}$$

$$= 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + \frac{4 - 2\sqrt{3}}{2}$$
$$= 7 + 7 + 2 + 4\sqrt{3} - 4\sqrt{3} - \sqrt{3}$$
$$= 16 - \sqrt{3}$$

71. $0.000365 = 365 \times 10^{-6}$ 4×10^{-6} should be subtracted to get a perfect square.

72.
$$\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.236}{5} = 0.4472 \approx 0.4472$$

73.
$$\frac{1+\sqrt{0.04}}{1-\sqrt{0.1}} = \frac{1+0.2}{1-0.316} = \frac{1.2}{0.684} = 1.7 \text{ approx.}$$

74.
$$\frac{(0.75)^3}{1 - 0.75} + [0.75 + 0.75^2 + 1] = (0.75)^2 \times 3 + 0.75 + 0.75^2 + 1$$
$$= 4 \times (0.75)^2 + 1.75$$
$$= 4 \times \frac{9}{16} + 1.75 = 2.25 + 1.75 = 4$$

75.
$$\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$$

= $2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$
= $6\sqrt{2} = 6 \times 1.414 = 8.484$

76. LCM of 21, 36 and $66 = 2^2 \times 3^2 \times 7 \times 11$

 \therefore Least perfect square divisible by 21, 36 and $66 = 2^2 \times 3^2 \times 7^2 \times 11^2 = 213444$

77.
$$\sqrt{\frac{0.16}{0.4}} = \sqrt{0.4} = 0.63$$

78.
$$\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}} = \sqrt{\frac{0.81 \times 0.0484}{0.0064 \times 6.25}} = \frac{0.9 \times 0.22}{0.08 \times 2.5}$$
$$= \frac{0.9 \times 0.22}{0.2} = 0.9 \times 1.1 = \mathbf{0.99}$$

79.
$$0.13 \div p^2 = 13$$

or $\frac{0.13}{p^2} = 13$
or $p^2 = \frac{0.13}{13} = 0.01$
or $p = 0.1$

80.
$$\frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$$

$$\therefore (?)^2 = \sqrt{128 \times 162} = \sqrt{64 \times 324} = 8 \times 18$$

$$\therefore ? = \sqrt{18 \times 8}$$

$$= \sqrt{36 \times 4} = 12$$

81.
$$\left(\frac{?}{15}\right)\left(\frac{?}{135}\right) = 1$$

or $(?)^2 = 15 \times 135$
 $= 15^2 \times 9$
or $? = \sqrt{15^2 \times 9}$
 $= 15 \times 3 = 45$

82.
$$\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013} = \sqrt{10^{-2} \times 130} + 10\sqrt{13} + \sqrt{10^{-4} \times 130}$$

$$= 0.1\sqrt{130} + 10\sqrt{13} + 0.01\sqrt{130}$$

$$= 10\sqrt{13} + 0.11\sqrt{130}$$
Given $\sqrt{13} = 3.605$; $\sqrt{130} = 11.4$

$$\therefore 10\sqrt{13} + 0.11\sqrt{130} = 10 \times 3.605 + 0.11 \times 11.4$$
$$= 36.05 + 1.254 = 37.304$$

Cubes and Cube Roots

To find out the cube root of a number, the reader is advised to recall the cubes of first ten natural numbers.

The cube root of a number can be identified using the end digit of those perfect cubes. The following table illustrates how to find easily the cube roots of the given numbers (perfect cubes).

If the number ends with digit	Its cube root will end with digit
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

TO FIND CUBES OF TWO-DIGIT NUMBERS

Step 1 Write down the cubes of digit in tenth place in a row of 4 figures. The other three numbers should be written in a geometrical ratio which exists between the unit's place and tenth place of given number.

Step II Write down under the second and third numbers, just two times of second and third number. Then add up the two rows.

EXAMPLE 1 Find the value of $(12)^3$.

Solution Step I Tens' digit is 1. So write the cube of 1. The ratio between the unit's digit and ten's digit is 2:1, so, the next digit will be double the previous one.

So the first row is

Step II In the above row, the 2nd and 3rd elements are 2 and 4. Double these numbers and write below them. Then add up the two rows.

1	2	4	8
	4	8	
1	6 ~ _	12	8
1	7	2	8

.. The number is 1728

EXAMPLE 2 Find the value of $(11)^3$.

Solution	Step I Step II	1	${1 \atop 2}$	$\frac{1}{2}$	1
		1	3	3	1

 $\therefore (11)^3 = 1331$

EXAMPLE 3 Find the value of $(16)^3$.

Solution	Step I	1^3	$1^2.6^1$	$1^{1}.6^{2}$	$1^{0}.6^{3}$
	i.e.	1	6	36	216
	Step II		12	72	
		1	18	108	216
Plac left	Place carry over to left in each place.			129	216
Rea	d all end digits i.e. 4096	3			
∴ (16)	$^{3} = 4096$				

EXAMPLE 4 Find the value of $(18)^3$.

		, .			
Solution	n Step I	1^3	$1^2.8^1$	$1^{1}.8^{2}$	$1^{0}.8^{3}$
	i.e.	1	8	64	512
	Step II		16	128	
	Add up the rows	1	24	192	512
]]	Place carry over to eft in each place.	5	48	243	512
]	Read all end digits i.e. 583 :	2			
∴ ($(18)^3 = 5832$				

EXAMPLE 5 Find the value of $(17)^3$.

	o i ilia ulle value of (- • , •			
Solution	Step I	1^3	$1^2.7^1$	$1^{1}.7^{2}$	$1^{0}.7^{3}$
	i.e.	1	7	49	343
	Step II		7×2	49×2	
	i.e.		14	98	
	Add up to two rows				
		1	7	49	343
			14	98	
		1	21	147	343
		4	39	181	343

- (i) In the unit's place, write down 3 and carry 34.
- (ii) In the tens' place 49 + 98 + 34 = 181. Write down 1 and carry 18.
- (iii) In the hundredth place, 7 + 14 + 18 = 39. Write down 9 and carry 3.
- (iv) In the thousandth place, 1 + 3 = 4.

$$\therefore (17)^3 = 4913$$

EXAMPLE 6 Find the value of $(19)^3$.

Add upto two rows,

- (i) In the unit's place write 9 and carry 72.
- (ii) In the tens' place, 81 + 162 + 72 = 315. Write down 5 and carry 31.
- (iii) In the hundredth place, 9 + 18 + 31 = 58. Write down 8 and carry 5.
- (iv) 1 + 5 = 6. Write it down in the thousandth place.

$$\therefore (19)^3 = 6859$$

EXAMPLE 7 Find $\sqrt[3]{1331}$.

Solution
$$\sqrt[3]{1331} = 10 \times \sqrt[3]{1.331}$$

 $\sqrt[3]{1.331}$ lies between 1 and 2. Since the end digit of the cube is 1, the cube root will also end with 1.

$$\therefore$$
 root is $10 \times 1.1 = 11$

$$\therefore$$
 $\sqrt[3]{1331} = 11$

EXAMPLE 8 Find $\sqrt[3]{103823}$.

Solution
$$\sqrt[3]{103823} = 10 \times \sqrt[3]{103.823}$$

 $\sqrt[3]{103.823}$ lies between 4 and 5 since the end digit of perfect cube is 3, the cube root will end with 7.

 \therefore The root is $10 \times 4.7 = 47$

EXAMPLE 9 Find $\sqrt[3]{1092727}$.

Solution
$$\sqrt[3]{1092727} = 10 \times \sqrt[3]{1092.727}$$

 $\sqrt[3]{1092.727}$ lies between 10 and 11. Since the end digit of perfect cube is 7, the cube root will end with 3.

 \therefore The root is $10 \times 10.3 = 103$



EXERCISES

- 1. Find the cubes of the following numbers:
 - (a) 26
- (b) 27
- (c) 32
- (d) 49
- (e) 43

- (f) 51 (k) 73
- (g) 53 (l) 92
- (h) 57 (m) 95
- (i) 64
- (j) 69

- 2. Find the cube roots of the following:
 - (a) 941192
- (b) 681472(g) 753571
- (c) 884736(h) 614125
- (d) 405224(i) 300763
- (e) 250047(j) 389017

- (f) 551368 (k) 704969
- 3. Find the cube root of 0.000216.
 - (a) 0.6
- (b) 0.06
- (c) 0.006
- (d) None of these

- **4.** Find the cube root of 0.000729.
 - (a) 0.9
- (b) 0.09
- (c) 0.009
- (d) None of these
- 5. Smallest five-digit number which is a perfect cube is
 - (a) 10961
- (b) 10648
- (c) 10968
- (d) 10689
- 6. What is the smallest number by which 5600 must be divided to make it a perfect cube?
 - (a) 7
- (b) 70
- (c) 14
- (d) 700
- 7. The least perfect cube which is completely divisible by 21, 24 and 27 is
 - (a) 70488
- (b) 78048
- (c) 74088
- (d) 78084
- 8. Find the least number by which 4800 should be multiplied to make it a perfect cube.
 - (a) 75
- (b) 50
- (c) 45
- (d) 30



- **3.** (b)
- **4.** (b)
- **5.** (b)
- **6.** (d)
- 7. (c)
- 8. (c)



Solutions with Necessary Explanation

[Hint: Students are advised to workout yourself remaining exercises for which only answers are given in the following manner.]

Write '6' in the unit place and carry 21 to 10th place. Then 216 + 21 = 237.

Write '7' in the tenth place and carry 23 to 100th place.

i.e.
$$72 + 23 = 95$$

Write '5' in the hundredth place and carry 9 to thousandth place i.e. 8 + 9 = 17. Hence $(26)^3 = 17576$

Hence $(32)^3 = 32768$

Hence $(49)^3 = 117649$

Hence $(57)^3 = 185193$

(j)
$$(69)^3$$
 6^3 $6^2 \times 9$ 6×9^2 9^3 Row I 216 324 486 729 II 648 972 I + II $216 + 972 + 1458 + 729$ Carry 112 153 72 328 1125 1530 5 0 9

Hence $(69)^3 = 328509$

(m)
$$(95)^3$$
 9^3 $9^2 \times 5$ 9×5^2 5^3 Row I 729 405 225 125 Row II 810 450

I + II 729 1215 + 675 + 125

Carry 128 68 12

857 1283 687

3 7 5

Hence $(95)^3 = 857375$

2. (a) $\sqrt[3]{941192} = 10 \times \sqrt[3]{941.192}$

 $\sqrt[3]{941.192}$ lies between 9 and 10. Cube ends with 2 and so cube root will end with 8.

$$\therefore \sqrt[3]{941.192} = 9.8$$
Hence $\sqrt[3]{941192} = 10 \times 9.8 = 98$

(c) $\sqrt[3]{884736} = 10 \times \sqrt[3]{884.736}$

³√884.736 lies between 9 and 10. Cube ends with 6 and so cube root will end with '6'.

$$\therefore \sqrt[3]{884.736} = 9.6$$

Hence
$$\sqrt[3]{884736} = 10 \times 9.6 = 96$$

(e) $\sqrt[3]{250047} = 10 \times \sqrt[3]{250.047}$

 $\sqrt[3]{250.047}$ lies between 6 and 7. Cube root will end with '3' as cube end with '7'.

$$\therefore \sqrt[3]{250.047} = 6.3$$

$$\therefore$$
 $\sqrt[3]{250047} = 10 \times 6.3 = 63$

(g) $\sqrt[3]{753571} = 10 \times \sqrt[3]{753.571}$

 $\sqrt[3]{753.571}$ lies between 9 and 10. Cube root will end with '1' as cube end with '1'.

$$\therefore \sqrt[3]{753.571} = 9.1$$

$$\therefore$$
 $\sqrt[3]{753571} = 10 \times 9.1 = 91$

(h) $\sqrt[3]{614125} = 10 \times \sqrt[3]{614.125}$

 $\sqrt[3]{614.125}$ lies between 8 and 9. Cube root will end with '5' as cube end with '5'.

$$\therefore \sqrt[3]{614.125} = 8.5$$

$$\therefore \sqrt[3]{614125} = 10 \times 8.5 = 85$$

(j) $\sqrt[3]{389017} = 10 \times \sqrt[3]{389.017}$

 $\sqrt[3]{389.017}$ lies between 7 and 8. Cube root will end with '3' as cube end with '7'.

$$\therefore \quad \sqrt[3]{389.017} = 7.3$$

$$\therefore$$
 $\sqrt[3]{389017} = 10 \times 7.3 = 73$

(k) $\sqrt[3]{704969} = 10 \times \sqrt[3]{704.969}$

 $\sqrt[3]{704.969}$ lies between 8 and 9. Cube root will end with '9' as cube end with '9'.

$$\therefore \sqrt[3]{704.969} = 8.9$$

Hence
$$\sqrt[3]{704969} = 10 \times 8.9 = 89$$

3.
$$\sqrt[3]{0.000216} = \sqrt[3]{216 \times 10^{-6}} = 6 \times 10^{-2} = 0.06$$

4.
$$\sqrt[3]{0.000729} = \sqrt[3]{729 \times 10^{-6}} = 9 \times 10^{-2} = \mathbf{0.09}$$

5. $(22)^3 = 484 \times 22 = 10648$

Hence smallest five-digit number which is a perfect cube is 10648.

6.
$$5600 = 2^3 \times 7 \times 10^2 = 2^5 \times 7 \times 5^2$$

:. The least number by which 5600 is to be divided to get a perfect cube

$$= 2^{2} \times 7^{1} \times 5^{2}$$
$$= 7 \times 10^{2} = 700$$

7. The least number completely divisible by 21, 24 and 27 is

3	21,	24,	27
3	7,	8,	9
2	7,	8,	3
2	7	4	3
	7	2	3

LCM of 21, 24 and $27 = 2^3 \times 3^3 \times 7$

:. Least perfect cube completely divisible by 21, 24 and 27 is

$$2^3 \times 3^3 \times 7^3 = LCM \times 7^2$$

= $LCM \times 49 = 74088$

8.
$$4800 = 2^3 \times 2 \times 3 \times 2^2 \times 5^2 = 2^6 \times 3^1 \times 5^2$$

:. Least number which should be multiplied with given number make it a perfect cube is $3^2 \times 5 = \textbf{45}$

Series

A series is a sequence or ordered collection of numbers.

PRIME NUMBERS: A prime number is a counting number greater than 1, which is divisible only by 1 and by itself. For example,

PERFECT SQUARE: A whole number whose square root is also a whole number is called a *perfect square*. For example,

PERFECT CUBE: A whole number whose cube root is also a whole number is called a *perfect cube*. For example,

ARITHMETIC PROGRESSION (AP)/ARITHMETIC SERIES: An arithmetic progression is the sequence of numbers in which a number in the series is obtained by adding a constant number called *common difference* to the previous number in the series.

The sum of first n terms of the series $S = \frac{n}{2} \{2a + (n-1)d\}$

where a is the first term and d is the common difference.

The sum of the series (of all n terms) = $\frac{n}{2} \{a + l\} = \frac{n}{2} \{2a + (n-1)d\}$

where 'a' is the first term and 'l' is the last term For example,

- (i) $2, 5, 8, 11 \dots$ common difference = 3
- (ii) $9, 15, 21, 27 \dots$ common difference = 6 are in A.P.

GEOMETRIC PROGRESSION (GP)/GEOMETRIC SERIES: A sequence of numbers is said to be in *geometric progression* (GP), if the ratio of two consecutive terms in the sequence is always the same and is called the *common ratio*.

The *n*th term of a series is given by

$$a_n = a \cdot r^{n-1};$$

 $l = a \cdot r^{N-1};$
 $S_n = a(r^{n-1})/(r-1);$
 $S_N = a(r^{N-1})/(r-1), r \neq 1$

where a =first term

r = common ratio

 $S_n = \text{Sum of first } n \text{ terms}$

l = last term

 $S_N = \text{Sum of all } N \text{ terms of the series}$

For example,

(i) $3, 15, 75, 375, \dots$ common ratio = 5 (ii) $2, 6, 18, 54 \dots$ common ratio = 3

COMBINATION SERIES: This can be a series combining different rules in a logical way.

- (a) Series of perfect squares: For example, 9, 25, 49, 81, 121 ... The series can be represented by the mathematical expression $f(x) = (1 + 2x)^2$ where x = 1, 2, 3 ...
- (b) Series of perfect cubes: For example,
 - (i) 1, 27, 729, 19683 ... can be represented by $f(x) = (27)^{x-1}$ where x = 1, 2, 3, 4
 - (ii) 1, 64, 373, 1000, ... can be represented by $f(x) = [1 + 3(x 1)]^3$ where x = 1, 2, 3, 4 ...

Second Degree Arithmetic Series: A series in which the differences between the successive terms themselves are in AP and forms an arithmetic series is called the *second degree arithmetic series*. For example, 2, 3, 6, 11, 18 ...

The difference of successive terms of the above series 1, 3, 5, 7 ... which forms an arithmetic series with common difference 2.

The given series can be represented by a quadratic mathematical function:

$$f(x) = x^2 + 2$$

where $x = 0, 1, 2, 3, 4 \dots$

Third Degree Arithmetic Series: A series in which the difference of the successive terms forms a second degree arithmetic series is termed third degree arithmetic series. Mathematically, such a series can be represented by a general cubic function and hence the name third degree. For example,

- (a) 20, 30, 42, 59, 84, 120, 170, ...

 The difference of successive terms of the above series is 10, 12, 17, 25, 36, 50... and 2, 5, 8, 11, 14 ... and this forms an arithmetic series with common difference 3.
- (b) 1, 15, 53, 127, 249, 431, 685, ...

 The difference of successive terms of the above series is 14, 38, 74, 122, 182, 254.

 The difference of successive terms of the above series is 24, 36, 48, 60, 72 and this forms an arithmetic series with common difference 12.

Mathematically, the given series can be represented by a cubic function

$$f(x) = 2x^3 - 1$$
 where $x = 1, 2, 3, 4 ...$

ARITHMETICO-GEOMETRIC (A-G) SERIES: The series is formed by combination of arithmetic and geometric progressions. Each succeeding term is obtained by first adding (or subtracting) the common difference of its AP to the previous term and then multiplying (or dividing) the sum by common ratio of its GP. An important property of such an A-G series is that the difference of successive terms are in GP.

For example,

Series is obtained by adding 3 to previous term and then multiplying the sum by 2. i.e. $a_2 = (a_1 + 3) \times 2$

GEOMETRICO-ARITHMETIC (G-A) SERIES: Each succeeding term in G-A series is found by first multiplying (or dividing) the previous term by the common ratio of its GP and then adding (or subtracting) to it the common difference of its AP. The important property of G-A series, like A-G series, is that the difference of its successive terms are in geometric progression. For example, 1, 2, 6, 22, 86, 342, ...

The terms in the above series are obtained by first multiplying the previous term by 4 and then subtracting 2 from the resultant product.

TWIN SERIES: As the name suggests, it consists of two series combined into a single series. The alternating terms of this series form an independent series. For example,

Odd terms of the series 2, 7, 12, 17, 22, ... is an arithmetic series with common difference 5. Even terms of the series 3, 9, 27, 81, 243 ... is a geometric series with common ratio 3.

ALTERNATING SERIES: In an alternating series, the successive terms increase and decrease alternately. It differs from irregular increase (or decrease) series wherein the successive terms increase (or decrease) continuously but the ratio of increase does not follow a particular trend.

While solving an alternate series we should focus on the following possibility of its formation:

- 1. Whether it is a twin series which has been formed by combining two different series.
- 2. It has been formed by performing two different kinds of operations alternately on successive terms. For example,

In the above series, there are two series 6, 12, 24, 48, 96 ... and 13, 40, 104, 229 ... i.e. the first one is a geometric series with first term 6 and common ratio 2. In the second series, terms are having differences 27, 64, 125, ... i.e., 3^3 , 4^5 , 5^3 , ...

ODD MAN OUT (WRONG NUMBER) IN A SERIES

For example,

(i) 3, 5, 7, 9, 11 ...

9 is the odd man out because all others are prime numbers.

(ii) 2, 3, 7, 10, 11 ...

Except 10, all are prime numbers.

(iii) 3, 10, 29, 66, 127, 220, 345 ... Except 220, all other terms can be expressed as $x^3 + 2$, where x = 1, 2, 3...

SPECIAL TYPE OF SERIES

(a) Numbers followed by their LCM or HCF.

For example,

In the above series, every four numbers forms a block in which the fourth number is the LCM of first three numbers. Therefore, the number next to 7 is 210. i.e. LCM of 5, 6, 7 = 210

(b) Numbers followed by their product.

For example,

$$1, 2, 2, 4, 8, 32, \dots$$

 $1 \times 2 = 2; 2 \times 2 = 4; 2 \times 4 = 8; 4 \times 8 = 32; 8 \times 32 = 256,$

which is the next number in the series.

(c) By use of digit sum.

For example,

Next number = Previous number + Digit-sum of previous number.

i.e.,
$$15 = 12 + (1+2)$$
$$21 = 15 + (1+5)$$
$$24 = 21 + (2+1)$$
$$30 = 24 + (2+4)$$

 \therefore Next number = 30 + (3 + 0) = 33

EXERCISES

In the following number series, one of the number does not fit into the series. Mark the wrong number in the series.

- 1. 1788 892 444 220 112 52 24
 - (a) 52
- (b) 112
- (c) 220
- (d) 444
- (e) 892

- **2.** 225 289 338 374 397 415 424
 - (a) 415
- (b) 289
- (c) 338
- (d) 374
- (e) 397

- **3.** 5 7.5 11.25 17.5 29.75 50 91.25

- - (a) 7.5
- (b) 17.5
- (c) 29.75
- (d) 91.25
- (e) None of these

- 4. 35 118 280 600 1238 2504 5036
 - (a) 118
- (b) 280
- (c) 600
- (d) 1238
- (e) 2504

- **5.** 10 12 28 90 368 1840 1112
 - (a) 1840
- (b) 368
- (c) 90
- (d) 28
- (e) 12

In each of the following questions, a number series is given. After the series, a number is given followed by (A), (B), (C), (D) and (E). You have to complete the series starting

				en foll	owing	the se	quence	of the g	iven se	ries. Ther	n answer the
_	_	estions	S.								
6.	80	50	130	100	180	150	230				
	900	(A)	(B)	(C)	(D)	(E)					
			e follo				ie in plac	ce of (E)?			
	(a)	1050		(b) 97	0	(c)	1000	(d)	950	(e)	None of these
7.	60	121	131	264	284	571	601				
	120	(A)	(B)	(C)	(D)	(E)					
	Whi	ch of th	e follo	_			ie in plac	e of (D)?			
	(a)	524		(b) 10	11	(c)	1010	(d)	514	(e)	None of these
8.	2	4	9	20	43	90					
	3	(A)	(B)	(C)	(D)	(E)					
	Whi	ch of th	e follo	wing nu	mbers	will con	ie in plac	e of (D)?			
	(a)	58		(b) 99		(c)	48	(d)	59	(e)	None of these
9.	200	184	193	157	182	118	167				
	150	(A)	(B)	(C)	(D)	(E)					
	Whi	ch of th	e follo	wing nu	mbers	will con	ie in plac	e of (E)?			
	(a)	132		(b) 11	5	(c)	68	(d)	117	(e)	None of these
10.	4	14	42	147	588	2058	3 10290)			
	8	(A)	(B)	(C)	(D)	(E)					
	Whi	ch of th	e follo	wing nu	mbers	will con	ie in plac	e of (C)?			
	(a)	28		(b) 11	76	(c)	343	(d)	216	(e)	None of these
											ven after the
				(A), (B)	, (C), (l	D) and	(E) are g	given. Fi	ind the	number ii	n place of the
		letter									
11.		14	5	18	0.5	-					
	19	(A)	(B)	(C)	(D)	(E)					
			ome ir	place o		, ,		. . .			
		12.75		(b) 25.		(c)	18.75	(d)	15.25	(e)	None of these
12 .		21.5	30.5	44	62						
	21	(A)	(B)	(C)	(D)	(E)					
			ome in	place o							
	` '	84.5		(b) 88.		(c)	90	(d)	95	(e)	None of these
13.		26	11	36	9						
	7	(A)	(B)	(C)	(D)	(E)					
			ome ir	place o	of (C)?						
	(a)			(b) 21		(c)			30		None of these
14.	Wha	it shoul	d com		place o	f the qu	estion m	ark(?) in	the follo	owing numl	ber series?
	11	28 55	92	139 ?							
	(a)	196		(b) 10	0	(c)	210	(d)	216	(e)	None of these
15.	In tl	he follo	wing n	umber s	series o	nly one	number	is wrong.	Find o	ut that wro	ng number?
	2 6	30	210 2								
	_ (210 2	2520 2	0790						
	(a)	30	210 2	2520 2 (b) 6	0790	(c)	210	(d)	2520	(e)	None of these
16.	(a)			(b) 6			210 , 15, 24, 3	, ,	2520	(e)	None of these

Fill i	n the blanks with	appropriate ch	oice	
17.	1 6 27 108			
	(a) 405		(c) 256	(d) 162
18.	4 10 33 136 _			
	(a) 385	(b) 685	(c) 786	(d) 812
19.	2 12 36 80			
	(a) 120		(c) 150	(d) 170
20.	2 18 66 210 6			
	(a) 1842 ((c) 1798	(d) 1938
21.	8 64 216 512		()	(3)
	(a) 1000 ((c) 1090	(d) 1024
22.	3 13 27 50 87		()	(3) (10
	(a) 148 ((c) 316	(d) 412
23.	4 30 160 810 40		() 20210	(1) 22122
	(a) 10160 ((d) 22168
	n the blanks with		ioice	
24.	243,, 108,		()	(1) 100
2=	(a) 81 ((c) 324	(d) 120
25.	, 24, 96,		() 0	(1) O
0.0	(a) 12 ((c) 6	(d) 8
26.	336, 210, 120, (a) 40 ((a) 49	(4) 20
97			(c) 48	(d) 30
41.	5, 14, 32,(a) 36	_, 140 (b) 40	(c) 68	(d) 84
28	2, 20,, 380		(c) 0 8	(u) 04
20.	(a) 92		(c) 112	(d) 124
Find	the odd man out:		(0) 112	(u) 124
	5, 10, 13, 20,			
20.	(a) 13 ((c) 10	(d) 29
30.	4096, 3072, 2048		(6) 10	(u) 2 0
	(a) 3072		(c) 1728	(d) 1296
31.	2, 10, 30, 70,			
	(a) 10 ((c) 70	(d) 130
32.	10, 25, 49, 81,		•	` ,
	(a) 81 ((c) 25	(d) 10
	190, 166, 145,			·
	(a) 166	(b) 145	(c) 128	(d) 112
34.	1, 3, 10, 21, 6	34, 129, 356,	777	
				(d) 777
35.	(a) 64 (445, 221, 109,	46, 25, 11, 4		
		(b) 109		(d) 25

In each of the following questions, a number series is given. After the series, a number is given below which is followed by letters A, B, C, D and E. You have to complete the

series starting with the number given and following the same property as in the given number series. **36.** 4 6 15 49 201 1011 B C D \mathbf{E} 15 Α What should come in place of D? (a) 115 (b) 465 (c) 455 (d) 475 (e) None of these **37.** 0 1 9 32 825 0 C 4 A В D Ε What should come in the place of C? (e) None of these (a) 279 (b) 81 (c) 269 (d) 272 **38.** 2 -128 -9 86 -25122 A В \mathbf{C} D \mathbf{E} What should come in place of B? (b) 328 (d) 228 (e) None of these (a) 124 (c) 9 **39.** 101 323 545 767 989 111011 34 В \mathbf{C} D \mathbf{E} Α What should come in place of A? (a) 56 (b) 78 (c) 43 (d) 100 (e) None of these **40.** -10 10 65 345 1750 -2C Ε Α В D What should come in place of E? (a) -1750(b) 1375 (c) -1325(d) -1475(e) None of these **41.** 4 29 74 231 928 4641 3 Α В \mathbf{C} D \mathbf{E} What will come in place of D? (a) 696 (b) 904 (c) 902 (d) 4520 (e) None of these **42.** 600 360 360 450 600 800 В C 1200 Α D \mathbf{E} What will come in place of B? (a) 600 (b) 660 (c) 560 (d) 720 (e) None of these **43.** 43 49 2197 130321 100000 1 E 25 Α В C D What will come in place of E? (a) 1 (b) 0.8 (c) 0.75(d) 0.63 (e) None of these 44. 703 803 1068 924 1237 1433 307 Α В \mathbf{C} D \mathbf{E} What will come in place of E? (d) 672 (e) None of these (a) 641 (b) 1037 (c) 841 **45**. 14 71 428.5 3004 24039 216361 \mathbf{E} 4 Α В \mathbf{C} D What will come in place of B? (a) 125 (b) 128 (c) 125.5 (d) 128.5 (e) None of these What will come in place of the question mark (?) in the following number series?

46. 283 505 ? 1282 1837 2503

(a) 828 (b) 848 (c) 882

(d) 884

(e) None of these

```
47. 7 36 125 256 ? 64 1
    (a) 81
                     (b) 162
                                     (c) 128
                                                      (d) 243
                                                                      (e) None of these
48. 5
      6 ?
                412 2185
            87
                                                                      (e) 21
    (a) 16
                     (b) 13
                                     (c) 20
                                                      (d) 18
49. 60
       20
            ?
               15 60 12
    (a) 60
                     (b) 40
                                     (c) 75
                                                      (d) 50
                                                                      (e) 48
50. 2 5 17.5 43.75
                      ?
                                     (c) 151.175
                                                      (d) 153.125
                                                                      (e) 153.175
    (a) 151.125
                     (b) 151.135
51. 3 20 87
               392
                    ?
    (a) 1612
                     (b) 1963
                                     (c) 847
                                                      (d) 2025
                                                                      (e) None of these
52. 12 28 75
                316 1555 ?
    (a) 9366
                                                      (d) 9336
                                                                      (e) None of these
                     (b) 9294
                                     (c) 7811
53. 112
         111
              119
                   92 156 31
    (a) 1375
                     (b) 287
                                     (c) 387
                                                      (d) 247
                                                                      (e) None of these
54. 1 15
          16
               31 47
                       78 125
    (a) 172
                     (b) 203
                                     (c) 139
                                                      (d) 167
                                                                      (e) None of these
55. 55 60 67
               78 91 108 ?
    (a) 125
                     (b) 121
                                     (c) 127
                                                      (d) 89
                                                                      (e) None of these
```



1. (b)	2. (e)	3. (c)	4. (d)	5. (a)	6. (b)	7. (a)	8. (d)	9. (c)	10. (e)
11. (c)	12. (b)	13. (b)	14. (a)	15. (d)	16. (c)	17. (a)	18. (b)	19. (c)	20. (d)
21. (a)	22. (b)	23. (c)	24. (b)	25. (c)	26. (b)	27. (c)	28. (a)	29. (c)	30. (b)
31. (c)	32. (d)	33. (c)	34. (c)	35. (c)	36. (b)	37. (b)	38. (d)	39. (a)	40. (e)
41. (b)	42. (c)	43. (a)	44. (b)	45. (d)	46. (e)	47. (d)	48. (c)	49. (a)	50. (d)
51. (e)	52. (a)	53. (d)	54 . (b)	55. (c)					



Oolutions with Necessary Explanation

- 1. The series is $(a_1 \div 2) 2 = a_2$
 - Hence the number 112 is wrong.
- 2. The series is formed by addition of 8^2 , 7^2 , 6^2 ... to the previous term. Hence the wrong number is 397.
- 3. The series is formed by first adding 2.5,

Then
$$(2.5)2 - 1.25 = 3.75$$
;

$$(3.75)2 - 1.25 = 6.25;$$

$$(6.25)2 - 1.25 = 11.25$$
 and so on.

Hence the number 29.75 is wrong.

4. The series is formed by first adding 83.

Then
$$(83)2 - 4 = 162$$
;

$$162 \times 2 - 4 = 320$$
;

$$320 \times 2 - 4 = 636$$
;

```
636 \times 2 - 4 = 1268;

1268 \times 2 - 4 = 2532
```

Hence the wrong number is 1238.

- 5. The series is formed by applying \times 1 + 2, \times 2 + 4, \times 3 + 6, \times 4 + 8, \times 5 + 10, \times 6 + 12. Hence the wrong number is 1840.
- 6. The series is an alternate series and is formed by successive addition of -30 and +80 alternatively to the previous term.

Therefore 900, 870, 950 ... is the series in which (E) will be 970.

7. Series is obtained as follows:

```
1st term = 60;

2nd term = 60 \times 2 + 1 = 121;

3rd term = 121 + 10 = 131;

4th term = 131 \times 2 + 2 = 264;

5th term = 264 + 20 = 284;

6th term = 284 \times 2 + 3 = 571;

7th term = 571 + 30 = 601.

\therefore Here, 1st term = 120;

2nd term = 120 \times 2 + 1 = 241;

3rd term = 241 + 10 = 251;

4th term = 251 \times 2 + 2 = 504;

5th term = 504 + 20 = 524.
```

8. The series is obtained as follows:

```
1st term = 2;

2nd term = 2 \times 2 = 4;

3rd term = 4 \times 2 + 1 = 9;

4th term = 9 \times 2 + 2 = 20;

5th term = 20 \times 2 + 3 = 43;

6th term = 43 \times 2 + 4 = 90.
```

Similarly, from the given numbers, series is obtained as follows:

```
1st term = 3;

2nd term = 3 \times 2 = 6;

3rd term = 6 \times 2 + 1 = 13;

4th term = 13 \times 2 + 2 = 28;

5th term = 28 \times 2 + 3 = 59;

6th term = 59 \times 2 + 4 = 122.
```

9. The given series is obtained as follows:

```
1st term = 200;

2nd term = 200 - 16 = 184;

3rd term = 184 + 9 = 193;

4th term = 193 - 36 = 157;

5th term = 157 + 25 = 182;

6th term = 182 - 64 = 118;

7th term = 118 + 49 = 167.
```

Similarly when series is started with a number 150, the number to be obtained as 6th term is given by 150 - 16 + 9 - 36 + 25 - 64 = 68

10. Series is obtained as follows:

1st term = 4;

2nd term = $4 \times 3.5 = 14$;

 $3rd term = 14 \times 3 = 42$:

4th term = $42 \times 3.5 = 147$;

 $5th term = 147 \times 4 = 588;$

6th term = $588 \times 3.5 = 2058$;

7th term = $2058 \times 5 = 10290$.

Similarly given series starts with 8.

i.e. 1st term = 8;

2nd term = $8 \times 3.5 = 28$;

 $3rd term = 28 \times 3 = 84;$

4th term = $84 \times 3.5 = 293$;

i.e. number in place of (C) = 293.

Here correct answer is not given among choice.

- :. Answer is (e).
- 11. Series is $\times 1 + 1$, $\div 2 2$, $\times 3 + 3$, $\div 4 4$, ...

$$A = 19 \times 1 + 1 = 20;$$

$$B = 20 \div 2 - 2 = 8;$$

$$C = 8 \times 3 + 3 = 27$$
;

$$D = 27 \div 4 - 4 = 2.75$$
;

$$E = 2.75 \times 5 + 5 = 18.75$$

- \therefore Number in place of (E) = 18.75.
- **12.** Series is +4.5, +9.0, +13.5, +18.0, ...

$$\therefore$$
 A = 21 + 4.5 = 25.5;

$$B = 25.5 + 9 = 34.5$$
:

$$C = 34.5 + 13.5 = 48.0;$$

$$D = 48.0 + 18 = 66.0;$$

$$E = 66.0 + 22.5 = 88.5$$
.

- \therefore Number in place of (E) = 88.5
- 13. The series is $\times 2 + 2$, $\div 2 2$, $\times 3 + 3$, $\div 3 3$...

$$A = 7 \times 2 + 2 = 16$$
;

$$B = 16 \div 2 - 2 = 6$$
;

$$C = 6 \times 3 + 3 = 21$$
;

$$D = 21 \div 3 - 3 = 4;$$

$$E = 4 \times 4 + 4 = 20.$$

- \therefore Number in place of (C) = 21
- 14. The series is 11, 28, 55, 92, 139?

Difference between consecutive term of the series are in AP is

- :. Number in place of (?) = 139 + 57 = 196
- 15. The series is 2, 6, 30, 210, 2520, 20790

$$1st term = 2;$$

```
2nd term = 2 \times 3 = 6;

3rd term = 6 \times 5 = 30;

4th term = 30 \times 7 = 210;

5th term = 210 \times 9 = 1890;

6th term = 1890 \times 11 = 20790.
```

- .. The number 2520 is wrong and it should be replaced by 1890.
- **16.** The series is 0, 3, 8, 15, 24, 35 ...

```
1st term = 0;

2nd term = 0 + 3 = 3;

3rd term = 3 + 5 = 8;

4th term = 8 + 7 = 15;

5th term = 15 + 9 = 24;

6th term = 24 + 11 = 35;

7th term = 35 + 13 = 48.
```

- .. The required number (next number) in the sequence is 48.
- 17. The series is 1, 2×3 , 3×3^2 , 4×3^3 , 5×3^4
 - \therefore Next term in the series = $5 \times 3^4 = 405$
- 18. The series is $4 \times 2 + 2$, $10 \times 3 + 3$, $33 \times 4 + 4$, $136 \times 5 + 5$.
 - \therefore Next term in the series = $136 \times 5 + 5 = 685$
- **19.** The series is $1^3 + 1^2$, $2^3 + 2^2$, $3^3 + 3^2$, $4^3 + 4^2$, $5^3 + 5^2$
 - \therefore Next term in the series = $5^3 + 5^2 = 150$
- 20. This is an A-G series (arithmetico-geometric series). Each term is first added by 4 and then sum is multiplied by 3.
 - \therefore Next term in the series = $(642 + 4) \times 3 = 1938$
- 21. The series is cubes of even numbers
 - \therefore Next term in the series is $10^3 = 1000$
- 22. The given series is a third degree arithmetic series
 - i.e. given series is 3, 13, 27, 50, 87, 143

Their differences are 10, 14, 23, 37, 56

Difference of the above series is 4, 9, 14, 19 (i.e. this series is in AP)

- \therefore Next term in the given series = 24 + 56 + 143 = 223
- 23. The given series is an A-G series (arithmetico-geometric series). Each term is first added by 2 and then sum is multiplied by 5.
 - \therefore Next term in the series = $(4060 + 2) \times 5 = 20310$
- **24.** 162
- 25. 6

- 26. 60 (Third degree AP)
- **27.** 68 (G–A series \times 2 + 4) **28.** 92 (A–G series + 3 \times 4)
- **29.** The series is $1^2 + 4$, $2^2 + 4$, $3^2 + 4$, $4^2 + 4$, $5^2 + 4$.
 - \therefore Odd man out = 10
- **30.** The series is a GP with common ratio = 3/4
 - .. Odd man out = 2048
- **31.** The series is $1^3 + 1$, $2^3 + 2$, $3^3 + 3$, $4^3 + 4$, $5^3 + 5$.
 - \therefore Odd man out = 70
- 32. The sequence forms square of consecutive odd numbers
 - \therefore Odd man out = 10

33. This is a second degree arithmetic series with their differences in AP. In this 128 is wrong.

∴ Odd man out = 128

- **34.** This is a Geometrico-Arithmetic series (\times 2 + 1, \times 3 + 1, \times 2 + 1, ...) Here **356** is odd.
- **35.** This is a A–G series $(-3 \div 2)$

Here the term 46 is odd.

36. The series is $\times 1 + 2$, $\times 2 + 3$, $\times 3 + 4$, $\times 4 + 5$...

$$A = 15 \times 1 + 2 = 17$$

$$B = 17 \times 2 + 3 = 37$$

$$C = 37 \times 3 + 4 = 115$$

$$D = 115 \times 4 + 5 = 465$$

Hence value of D = 465

37. The series is $\times 1^2 + 1^2$, $\times 2 - 2$, $\times 3^2 + 3^2$, $\times 4 - 4$...

$$A = 4 \times 1^{2} + 1^{2} = 5$$

$$B = 5 \times 2 - 2 = 8$$

$$C = 8 \times 3^{2} + 3^{2} = 81$$

Hence value of C = 81

38. The series is $1^2 + 1$, $3 - 2^2$, $5^2 + 3$, $7 - 4^2$, $9^2 + 5$, ...

Here
$$2 = 1^2 + 1$$
; $122 = 1171$
 $-1 = 3 - 2^2$; $A = 13 - 2^2 = 9$
 $28 = 5^2 + 3$; $B = 15^2 + 3 = 228$
 $-9 = 7 - 4^2$; $C = 17 - 4^2 = 1$.

: Answer is 228

- 39. In this series, every digit of the number increases by 2 in each step. i.e. 3 4 5 6 Hence A = 56
- **40.** The series is $(+1) \times 5$, $(+2) \times 5$, $(+3) \times 5$, $(+4) \times 5$, $(+5) \times 5$

 $\therefore \text{ Ans.} = -1375$

Here correct answer is not given. Hence answer is (e).

41. Here
$$4 \times 1 + 5^2 = 29$$
; $29 \times 2 + 4^2 = 74$; $74 \times 3 + 3^2 = 231$; $231 \times 4 + 2^2 = 928$; $928 \times 5 + 1^2 = 4641$.

Similarly
$$3 \times 1 + 5^2 = 28$$
; $28 \times 2 + 4^2 = 72$; $72 \times 3 + 3^2 = 225$; $225 \times 4 + 2^2 = 904$.

:. Answer is 904

42. Here
$$600 \times \frac{1}{2} + 60 = 360$$
; $360 \times \frac{2}{3} + 120 = 360$; $360 \times \frac{3}{4} + 180 = 450$; $450 \times \frac{4}{5} + 240 = 600$; $600 \times \frac{5}{6} + 300 = 800$.

Similarly,
$$1200 \times \frac{1}{2} + 60 = 660$$
; $660 \times \frac{2}{3} + 120 = 560$;

: Answer is 560.

43. Here
$$(4+3)^2 = 49$$
; $(4+9)^3 = 2197$; $(2+1+9+7)^4 = 130321$; $(1+3+0+3+2+1)^5 = 100000$; $(1+0+0+0+0+0)^6 = 1$.

Since digit sum of 43 and 25 are the same, remaining terms of the series will be in the same pattern.

∴ Answer is 1

44. Here
$$703 + 10^2 = 803$$
; $803 + 11^2 = 924$; $924 + 12^2 = 1068$; $1068 + 13^2 = 1237$; $1237 + 14^2 = 1433$.

 \therefore Required answer = 307 + (1433 - 703) = 1037

45. Here,
$$14 \times 5 + 100 \times 1 = 71; \qquad 71 \times 6 + 1.25 \times 2 = 428.5; \\ 428.5 \times 7 + 1.50 \times 3 = 3004; \qquad 3004 \times 8 + 1.75 \times 4 = 24039; \\ 24039 \times 9 + 2.00 \times 5 = 216361$$

Similarly, $4 \times 5 + 1.00 \times 1 = 21$; $21 \times 6 + 1.25 \times 2 = 128.5$

- **46.** The series is obtained by adding + 222, + 333, + 444 ... in succession to the previous term. Hence question mark (?) will be replaced by **838**.
- 47. The series is 7^1 , 6^2 , 5^3 , 4^4 , 3^5 , 2^6 and 1^7 ,
 - \therefore Answer is $3^5 = 243$
- **48.** The series is $\times 1 + 1^3$, $\times 2 + 2^3$, $\times 3 + 3^3$, ...
 - ∴ Answer is 20
- **49.** The series is $\times \frac{1}{3}$, $\times 3$, $\times \frac{1}{4}$, $\times 4$, $\frac{1}{5}$, $\times 5$...
 - \therefore Answer is $20 \times 3 = 60$
- **50.** This is an alternate series $\times 2.5, \times 3.5, \times 2.5, \times 3.5, \dots$

.: Answer is 153.125

51.
$$3 + 1^2 + 16 = 20$$
; $20 + 9^2 - 14 = 87$; $87 + 17^2 + 16 = 392$; $392 + 25^2 - 14 = 1003$

The answer is 1003.

Here correct answer is not given. Hence answer is (e).

52.
$$12 \times 2 + 2^2 = 28$$
; $28 \times 3 - 3^2 = 75$; $75 \times 4 + 4^2 = 316$; $316 \times 5 - 5^2 = 1555$; $1555 \times 6 + 6^2 = 9366$

53.
$$112 - 1^3 = 111$$
: $111 + 2^3 = 119$: $119 - 3^3 = 92$: $92 + 4^3 = 156$: $156 - 5^3 = 31$: $31 + 6^3 = 247$.

54. Here
$$1 + 15 = 16$$
; $15 + 16 = 31$; $16 + 31 = 47$; $31 + 47 = 78$; $47 + 78 = 125$; $78 + 125 = 203$. The series is similar to Fibonacci series i.e. each term of the series is obtained by taking sum of previous two terms.

55.
$$55 + 5 = 60$$
; $60 + 7 = 67$; $67 + 11 = 78$; $78 + 13 = 91$; $91 + 17 = 108$; $108 + 19 = 127$. Here the numbers added are consecutive prime numbers.

Progression and Sequence

ARITHMETIC PROGRESSION (AP): A sequence of numbers is said to be in Arithmetic Progression if the difference between any two consecutive numbers is always the same. The difference between two consecutive terms of the series is termed the *common difference* (d), for example, 2, 6, 10, 14 ... is an AP with first term a = 2 and common difference d = 6 - 2 = 10 - 6 = 4.

If in an AP, there are n terms, and if the first term is a and common difference is d, last term l = a + (n-1)d.

rth term from beginning = a + (r - 1)d

Sum of the series =
$$\frac{1}{2}n(a+l)$$

= $\frac{1}{2}n\{2a+(n-1)d\}$

GEOMETRIC PROGRESSION (GP): A sequence of numbers (or terms) is said to be in geometric progression if the ratio of two consecutive numbers (or terms) is constant. This constant ratio is called the *common ratio*.

If in a GP, there are n terms, and if the first term is a and common ratio is r, then nth term is given by a. r^{n-1} .

Sum of
$$n$$
 terms $S_n = \frac{a(r^n-1)}{(r-1)}$ if $r>1$ or $r<-1$
$$S_n = \frac{a(1-r^n)}{(1-r)}$$
 if $-1< r<1$

EXAMPLE 1 The sum of 3 numbers in a GP is 26 and their product is 216. Find the numbers.

Solution Let the numbers be $\frac{a}{r}$, a, ar.

The sum of 3 numbers is given by
$$\frac{a}{r} + a + ar = 26$$
 (1)

The product of 3 numbers is given by
$$\frac{a}{r} \times a \times ar = 216$$
 (2)

$$(2) \Rightarrow a^3 = 216 \Rightarrow a = \sqrt[3]{216} = 6$$

$$\therefore \qquad (1) \Rightarrow 6\left(\frac{1}{r} + r\right) = 26 - 6 = 20$$

$$\Rightarrow \qquad \frac{1}{r} + r = \frac{20}{6} = \frac{10}{3}$$

$$\therefore$$
 $r = 3$

 \therefore The numbers are 2, 6, 18.

EXAMPLE 2 The product of three numbers in a GP is 1728 and the sum of the products of them taken in pairs is 756. Find the numbers.

Solution Let the numbers be $\frac{a}{r}$, a, ar.

$$\therefore \qquad \frac{a}{r} \cdot a \cdot ar = 1728 \tag{1}$$

$$\Rightarrow$$
 $a^3 = 1728$

$$\Rightarrow$$
 $a = 12$

$$\frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 756$$

$$\frac{a^2}{r} + a^2 \cdot r + a^2 = 756$$
(2)

i.e.
$$a^2 \left(r + \frac{1}{r} \right) = 756 - a^2$$

= $756 - 12^2 = 756 - 144 = 612$

$$\therefore r + \frac{1}{r} = \frac{612}{144} = 4\frac{1}{4}$$

$$r=4$$

.. The numbers are $\frac{12}{4}$, 12 and 12 × 4, i.e. 3, 12 and 48

HARMONIC PROGRESSION (HP): A sequence of numbers is said to be in HP, if the reciprocals of its terms form an AP. Conversely, if the terms of a sequence are in AP, their reciprocals form an HP.

If
$$x_1, x_2, x_3, ...$$
 are in HP, then $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}$... are in AP.

For example, $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}$ are in HP.



Note: Three quantities a, b and c are in HP, then a: c = (a - b): (b - c).

The *n*th term of an HP = $\frac{1}{a + (n-1)d}$.

HARMONIC MEAN (HM): The Harmonic Mean (HM) of two given quantities a and b is given by HM = $\frac{2ab}{a+b}$.

ARITHMETIC MEAN (AM): If x_1 , x_2 , and x_3 are in AP, then the Arithmetic Mean between x_1 and $x_2 = \overline{x} = \frac{x_1 + x_2}{2}$, such that x_1, \overline{x} , and x_2 are in AP.

For example, Arithmetic Mean of 4 and $8 = \frac{4+8}{2} = 6$

:. 4, 6, 8 are in AP with common difference 2.

GEOMETRIC MEAN (GM): The geometric mean between the two numbers a and $ar^2 = \sqrt{a \times ar^2} = ar$. Then a, ar, ar^2 are in GP.

For example, the GM between 3 and $27 = \sqrt{3 \times 27} = 9$

∴ 3, 9, and 27 are in GP.

If x and y are two unequal positive distinct quantities, A, G, and H are their arithmetic mean, geometric mean and harmonic mean respectively, then

(i)
$$AH = G^2$$

(ii)
$$A > G > H$$
 or $A \ge G \ge H$.

Important Results

Finite Series

- 1. The sum of first *n* natural numbers, $S_n = \frac{n(n+1)}{2}$.
- 2. The sum of squares of first n natural numbers, $S_n = \frac{n(n+1)(2n+1)}{6}$
- 3. The sum of cubes of first *n* natural numbers, $S_n = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2$.

EXAMPLE 3 Find the sum of first *n* terms of the following series $\frac{1}{3.4}$, $\frac{1}{4.5}$, $\frac{1}{5.6}$, $\frac{1}{6.7}$,

$$\frac{1}{7.8}, \cdots$$

Solution $x_m = \frac{1}{(m+2)\cdot(m+3)} = \frac{1}{m+2} - \frac{1}{m+3}$

where m = 1, 2, 3, ...

$$x_1 = \frac{1}{3} - \frac{1}{4}; \quad x_2 = \frac{1}{4} - \frac{1}{5}; \quad x_3 = \frac{1}{5} - \frac{1}{6}, \text{ etc.}$$

$$x_n = \frac{1}{n+2} - \frac{1}{n+3}$$

$$S_n = \frac{1}{3} + \left(\frac{-1}{4} + \frac{1}{4}\right) + \left(\frac{-1}{5} + \frac{1}{5}\right) + \frac{-1}{n+3}$$

$$= \frac{1}{3} - \frac{1}{n+3} = \frac{n+3-3}{3(n+3)} = \frac{n}{3(n+3)}$$

EXAMPLE 4 Find the sum of squares of first *n* natural numbers.

Solution
$$S_n = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$$
$$= \frac{n(n+1)(2n+1)}{6}$$

EXAMPLE 5 Find the sum of the series $5 + 55 + 555 + 5555 + \cdots$ to n terms.

Solution
$$= 5(1 + 11 + 111 + \cdots \text{ to } n \text{ terms})$$

$$= \frac{5}{9}(9 + 99 + 999 + \cdots \text{ to } n \text{ terms})$$

$$= \frac{5}{9}\{(10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots \text{ to } n \text{ terms}\}$$

$$= \frac{5}{9}\{10 + 10^2 + 10^3 + \cdots + 10^n - n \times 1\}$$

$$= \frac{5}{9}\left\{10 \times \frac{10^n - 1}{10 - 1} - n\right\}$$

$$= \frac{5}{9}\left\{\frac{10}{9}(10^n - 1) - n\right\}$$

EXAMPLE 6 Find the sum of squares of first *n* odd natural numbers.

Solution The series of odd natural numbers is 1^2 , 3^2 , 5^2 , 7^2 , ... to n terms. The terms of the series is given by $(2m-1)^2$, where m=1, 2, 3, ...,

$$(2m-1)^2 = 4m^2 - 4m + 1$$

$$S_n = \sum_{m=1}^n (2m-1)^2 = 4\sum_{m=1}^n m^2 - 4\sum_{m=1}^n m + \sum_{m=1}^n 1$$

$$= 4 \times \frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$$

$$= \frac{2n}{2}(n+1)(2n+1) - 2n(n+1) + n$$

$$= 2n(n+1) \left\{ \frac{2n+1}{3} - 1 \right\} + n$$

$$= 2n(n+1) \frac{(2n+1-3)}{3} + n$$

$$= n \left\{ 2(n+1) \frac{(2n-2)}{3} + 1 \right\}$$

$$= \frac{n}{3} \left\{ 2(n+1)(2n-2) + 3 \right\}$$

$$= \frac{n}{3} \left\{ 4n^2 - 4n + 4n - 4 + 3 \right\}$$

$$= \frac{n}{3} \left\{ 4n^2 - 1 \right\}$$

EXAMPLE 7 Find the sum of squares of first *n* even natural numbers.

Solution The series is 2^2 , 4^2 , 6^2 , 8^2 , ... The term of the series are given by $(2m)^2 = 4m^2$ where m = 1, 2, 3, 4 ...

$$S_n = 4 \sum_{m=1}^{n} m^2 = \frac{4n(n+1)(2n+1)}{6} = \frac{2n(n+1)(2n+1)}{3}$$

EXAMPLE 8 Find the sum of first n terms of the series 1, 2x, $3x^2$, $4x^3$, ...

Solution
$$S_{n} = 1 + 2x + 3x^{2} + \dots + nx^{n-1}$$

$$xS_{n} = x + 2x^{2} + 3x^{3} + \dots + nx^{n}$$

$$S_{n} - xS_{n} = 1 + x + x^{2} + \dots + x^{n-1} - nx^{n}$$

$$S_{n} = \frac{1 + x + x^{2} + \dots + x^{n-1} - nx^{n}}{(1 - x)}$$

$$= \frac{1 + x + x^{2} + \dots + x^{n-1}}{1 - x} - \frac{nx^{n}}{1 - x}$$

$$= \frac{1(1 - x^{n})}{(1 - x)^{2}} - \frac{nx^{n}}{(1 - x)}$$

$$= \frac{1 - x^{n}}{(1 - x)^{2}} - \frac{nx^{n}}{(1 - x)}$$

EXAMPLE 9 Find the value of
$$\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right)\left(1 + \frac{1}{x+3}\right)$$
.

Solution The given expression = $\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right) \times \left(1 + \frac{1}{x+3}\right)$

$$= \frac{x+1}{x} \times \frac{x+2}{x+1} \times \frac{x+3}{x+2} \times \frac{x+4}{x+3} = \frac{x+4}{x}$$

EXAMPLE 10 Find the sum of the series $5^3 + 6^3 + \cdots + 10^3$.

Solution The sum of cubes of first n natural numbers

$$S_{n} = \left\{ \frac{n(n+1)}{2} \right\}^{2}$$

$$\therefore S_{10} = 1^{3} + 2^{3} + \dots + 10^{3}$$

$$S_{4} = 1^{3} + 2^{3} + \dots + 4^{3}$$

$$S_{10} - S_{4} = 5^{3} + 6^{3} + \dots + 10^{3}$$

$$= \left[\frac{10 \times 11}{2} \right]^{2} - \left[\frac{4 \times 5}{2} \right]^{2} = 5^{2} \times 11^{2} - 2^{2} \times 5^{2}$$

$$= 5^{2} \left[121 - 4 \right] = 5^{2} \times 117 = 2925$$

EXAMPLE 11 The sum $9 + 16 + 25 + 36 + \dots + 100$ is equal to _____. Solution S = 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100

i.e. sum of squares of first *n* natural numbers = $\frac{n(n+1)(2n+1)}{6}$

$$\therefore \quad \text{Required sum} = S_{10} - S_2$$

$$= \frac{10(10+1)(2\times10+1)}{6} - \frac{2(2+1)(2\times2+1)}{6}$$

$$= \frac{10\times11\times21}{6} - \frac{2\times3\times5}{6}$$

$$= 5\times11\times7 - 5$$

$$= 5(77-1) = 380$$

EXAMPLE 12 In the expansion of $\left(x^3 - \frac{3}{x^2}\right)^{15}$, the term free from x is ______.

Solution
$$\left(x^3 - \frac{3}{x^2}\right)^{15}$$

Let its rth term be independent of x.

$$(x^3)^{15-r+1} \cdot \left(\frac{1}{x^2}\right)^{r-1} = x^0$$
i.e.
$$3(15-r+1) + (r-1)(-2) = 0$$
i.e.
$$48 - 3r + 2 - 2r = 0$$
i.e.
$$50 - 5r = 0$$

$$\therefore r = 50/5 = 10$$

: The answer is 10th term.



Notes:

- 1. If the pth term of an AP is q and qth term of the AP is p, then (p + q)th term of the AP is zero.
- 2. If the pth term of an AP is x and the qth term of the AP is y, then the sum of its first (p + q) terms is

$$\frac{1}{2}(p+q)\left\{x+y+\frac{x+y}{p-q}\right\}$$

- 3. If the sum of n terms of an AP is m and the sum of m terms is n, then the sum of (m + n) terms is -(m + n).
- 4. The ratio of the sum of p arithmetic means to the sum of q arithmetic means inserted between any two numbers is p : q.
- 5. If a, b and I are the first, second and last terms of a GP respectively, then the sum of the GP is

$$\frac{bI - a^2}{(b - a)}$$

6. A GP of n terms has the first term a and common ratio r and S_n is the sum of the GP. If S'_n is the sum of the reciprocals of the terms, then

$$\frac{S_n}{S_n'} = a^2 r^{(n-1)}$$

7. If S_n , S_{2n} and S_{3n} are the sum of first n, 2n and 3n terms respectively of a GP, then

$$S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$$

- 8. A GP contains *n* terms and *n* is an odd number. The product of *n* terms of the GP is equal to the *n*th power of its middle term.
- 9. If a, b, c and d are in GP and x is any even number, then

$$\frac{a-bx}{a+bx} = \frac{b-cx}{b+cx} = \frac{c-dx}{c+dx}$$

10. If S_n denotes the sum of n terms of a GP, whose first term is a and common ratio is r, then

$$S_T = S_1 + S_2 + S_3 + \dots + S_n$$

= $\frac{n \cdot a}{1 - r} - \frac{ar(1 - r^n)}{(1 - r)^2}$

 S_T is the sum total of sum of G.P., with 1, 2, 3, ..., n terms.

i.e. $S_T = \sum_{i=1}^{n} S_i$, S_i is the sum of G.P. with number of terms = i.

- 11. If the first term of a GP is a, then the *n*th term is b and the product of first n terms is p, then $p^2 = (ab)^n$.
- 12. If S_n , S_{2n} , S_{3n} are the sums of first n, 2n, and 3n terms respectively of an AP, then $S_{3n} = 3(S_{2n} S_n)$.
- 13. The sum of first *n* odd integers/natural numbers = $\frac{n^2}{2}$.
- 14. The sum of first *n* even natural numbers = n(n + 1).
- 15. The sum of squares of first n natural numbers

$$\sum_{m=1}^{n} m^2 = \frac{n(n+1)(2n+1)}{6}.$$

16. The sum of first *n* natural numbers

$$\sum_{m=1}^{n} m = \frac{n(n+1)}{2}$$

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$$\sum_{m=1}^{n} m^3 = \left[\frac{n(n+1)}{2} \right]^2 = \left[\sum_{m=1}^{n} m \right]^2$$

18. The sum of squares of first *n* odd natural numbers

$$\sum_{m=1}^{n} (2m-1)^2 = \frac{n(4n^2-1)}{3} = \frac{n(2n+1)(2n-1)}{3}$$

19. The sum of squares of first *n* even natural numbers

$$\sum_{m=1}^{n} (2m)^2 = 4 \sum_{m=1}^{n} m^2 = \frac{2n(n+1)(2n+1)}{3}$$

20. The sum of first n terms of the series

$$\{1^2 - 2^2 + 3^2 - 4^2 + \cdots\} = \frac{1}{2} (-1)^{n+1} n(n+1)$$

21. The sum of first n terms of Arithmetico-Geometric Series

$$a, (a + d)r, (a + 2d)r^{2}, \dots, [a + (n-1)d]r^{n-1}$$

is
$$S_n = \frac{a - \{a + (n-1)d\}r^n}{1 - r} + \frac{dr(1 - r^{n-1})}{(1 - r)^2}$$

22. The sum of first n terms of the Arithmetico-Geometric Series

$$(1+2x+3x^2+\cdots+nx^{n-1})=\frac{1-x^n}{(1-x)^2}-\frac{nx^n}{1-x}$$

Sum of typical infinite series

23.
$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} = e$$

24.
$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = e^x$$

25.
$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \log_e(1 + x)$$
 where $-1 < x \le 1$

26.
$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots = -\log_e(1 - x)$$

27.
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \log 2$$

28.
$$x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right)$$
 where $-1 < x < 1$

29.
$$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \frac{1}{2} (e^x + e^{-x})$$

30.
$$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots = \frac{1}{2} (e^x - e^{-x})$$

31.
$$1 - x + x^2 - x^3 + \dots = \frac{1}{1+x}$$

32.
$$1 + x + x^2 + x^3 + \dots = \frac{1}{1 - x}$$

33.
$$1-2x+3x^2-4x^3+\cdots=\frac{1}{(1+x)^2}$$

33.
$$1-2x+3x^2-4x^3+\cdots=\frac{1}{(1+x)^2}$$
 34. $1+2x+3x^2+4x^3+\cdots=\frac{1}{(1-x)^2}$

(i) 1, 7, 17, 31, 49, ...

(ii) 12, 72, 232, 2592, ... (a) $2 \times 6^{n-1}$

(iii) 5, 14, 29, 50, 77, ... (a) $3n^3 + 1$

(a) 26, 38, 44, 52, 60, 68

(d) 26, 38, 44, 50, 62, 68

(i) 6, 12, 24, 48, ... (a) 192

(a) 14, 22

(a) 8400

(a) 45

(a) 6

common ratio of the GP.

(a) $2n^2 + 1$

1. Find the *n*th term of the following series:

(b) $2n^2 - 1$

(b) $2 \times 6^{n-2}$

(b) $3n^3 - 1$

number of terms in the AP and its common difference.

(b) 16, 20

(b) 7200

(b) 96

5. Write the seventh term of the following GP.

(b) 50

(b) 2

2. Insert 6 arithmetic means between 20 and 76.

EXERCISES

	(ii) 39366, 13122, 4374, 1458,
	(a) 27 (b) 54 (c) 108 (d) 216 (e) None of thes
	(iii) $21a, 84a^3 336a^5, 1344a^7$
	(a) a^{11} (b) a^{12} (c) a^{13} (d) None of these
6.	Find the sum of first 7 terms of a GP, whose first term is 1024 and the common ratio is $\frac{1}{2}$
	(a) 1016 (b) 508 (c) 2032 (d) 4064 (e) None of thes
7.	The sum of three numbers in a GP is 105 and their product is 8000. Find the numbers.
	(a) 5, 20, 40 (b) 5, 20, 80 (c) 4, 16, 64 (d) None of these
8.	The AM and GM of two numbers are 17 and 8 respectively. Find their Harmonic Mean and write the AP, GP and HP. Harmonic mean is
	(a) $\frac{16}{17}$ (b) $\frac{32}{17}$ (c) $\frac{64}{17}$ (d) $\frac{128}{17}$ (e) None of thes
9.	Find the sum of first n terms of the AP $\frac{n-1}{n}, \frac{n+1}{n}, \frac{n+2}{n}, \frac{n+3}{n}, \cdots$
	(a) $\frac{n-1}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{3(n-1)}{2}$ (d) $\frac{3(n+1)}{2}$ (e) None of thes
10.	If the sum of first n terms of two arithmetic progressions 3, 7, 11, 15, and 30, 33, 36, 39 are equal, then find the value of n .

(c) 55

(c) 24

11. The sum of first 20 terms of a GP is 1025 times the sum of its first 10 terms. Find the

(c) $n^2 + 1$

(c) 2×6^n

(c) $3n^3 + 2$

(b) 24, 36, 38, 50, 62, 68

(e) 28, 34, 46, 50, 62, 68 3. The first and last terms of an AP are 75 and 375 respectively. If its sum is 3600, find the

(c) 18, 18

(c) 7100

(c) 384

4. Find the sum of an AP of 40 terms whose first and last terms are 80 and 275.

(d) $n^2 - 1$

(d) $2 \times 6^{n+1}$

(d) $3n^3 - 2$

(d) 20, 16

(d) 4800

(d) 48

(d) 60

(d) 18

(e) None of these

(c) 28, 36, 44, 52, 60, 68

12. The sum of first *n* terms of the series 1, 1 + m, $1 + m + m^2$, $1 + m + m^2 + m^3$, will be: ______. [Hint: Try yourself]

13. Find the sum of $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$ to infinite terms.

Hint:
$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \text{ Put } x = \frac{1}{2}$$

(a) 1

(b) $\frac{3}{2}$

(c) 2

(d) $\frac{5}{2}$

(e) None of these

14. Find the sum of first n terms of the series $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + (9^3 - 8^3) + \dots$

[Hint: Try yourself]

15. Find the sum to n terms of the series $0.9 \pm 0.99 \pm 0.999 \pm ...$

[Hint: Try yourself]

16. Find the sum of n terms of the series $1 \cdot 2 \cdot 3 + 4 \cdot 5 \cdot 6 + 7 \cdot 8 \cdot 9 + \dots$

[Hint: Try yourself]

17. Find the sum of *n* terms of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6}$

[Hint: Try yourself]

18. Find the sum of infinite terms of a GP $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \cdots$

(a) $\frac{3}{4}$

(b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{9}{4}$

(e) None of these

19. Find the geometric mean between $\frac{3}{2}$ and $\frac{27}{2}$.

(b) $\frac{9}{2}$ (c) $\frac{11}{2}$

(d) $\frac{13}{2}$

18. (d)

(e) None of these



7. (b)

1. (i) (b), (ii) (c), (iii) (c)

2. (c) **10.** (c)

3. (b)

13. (c)

11. (b)

4. (c)

5. (i) (c), (ii) (b), (iii) (d)

19. (b)

6. (c)

Solutions with Necessary Explanation

9. (c)

- (i) nth term of the series is $2n^2 1$ where n = 1, 2, 3...
 - (ii) nth term of the series is 2×6^n where n=1, 2, 3...
 - (iii) *n*th term of the series is $3n^3 + 2$, where n = 1, 2, 3, ...
- 2. The given two numbers are 20 and 76.

Let six arithmetic means between 20 and 76 differ by a common difference of d between consecutive terms.

$$\therefore$$
 76 = 20 + 7d

$$\Rightarrow \qquad d = \frac{76 - 20}{7} = 8$$

.: Arithmetic means are 28, 36, 44, 52, 60 and 68

3. Let the number of terms and common difference of AP be n and d respectively.

$$\therefore a_1 = 75 \text{ and } a_n = 375$$

$$\Rightarrow a_n = a_1 + (n-1)d$$

$$375 = 75 + (n-1)d$$

$$\Rightarrow (n-1)d = 300$$
(1)

Given
$$S_n = \frac{n}{2} \{2a_1 + (n-1)d\} = 3600$$

$$\frac{n}{2}\left\{150 + (n-1)d\right\} = 3600\tag{2}$$

Equations (1) and (2)

⇒
$$\frac{n}{2} \{150 + 300\} = 3600$$

⇒ $n = \frac{2 \times 3600}{450} = 16$
∴ (1) ⇒ $d = \frac{300}{n-1} = \frac{300}{16-1} = 20$

4. Given $a_1 = 80$ and $a_{40} = 275$

$$∴ a_1 + 39d = 940$$

$$⇒ 80 + 39d = 275$$

$$∴ d = \frac{275 - 80}{39} = 5$$

$$S_{40} = \frac{n}{2} \{a_1 + a_{40}\} = \frac{40}{2} \{80 + 275\} = 7100$$

5. (i) 7th term of GP =
$$a \times r^6$$

= $6 \times 2^6 = 6 \times 64 = 384$

(ii) 7th term of GP =
$$39366 \times \left(\frac{1}{3}\right)^6 = 54$$

(iii) 7th term of GP =
$$21 \times a \times 4^6 \times a^{2\times 6} = 21 \times 4^6 \times a^{1+12} = 21 \times 4^6 \ a^{13} = 86016 \ a^{13}$$

6. Given
$$a = 1024$$
; $r = \frac{1}{2}$

$$S_7 = a \frac{(1 - r^7)}{(1 - r)} = \frac{1024 \left(1 - \left(\frac{1}{2}\right)^7 \right)}{1 - \frac{1}{2}}$$
$$= 1024 \times \frac{127}{128} \times 2$$

$$= 16 \times 127 = 2032$$

7. In the GP,
$$S_3 = 105$$
 (1)

$$\frac{a}{r} \times a \times ar = 8000 \tag{2}$$

$$(2) \Rightarrow a^3 = 8000 \Rightarrow a = 20$$

$$\therefore (1) \Rightarrow \frac{20}{r} + 20 + 20r = 105$$

$$\Rightarrow 20 \left(\frac{1}{r} + r\right) = 105 - 20 = 85$$

$$\Rightarrow r + \frac{1}{r} = \frac{85}{r} = \frac{17}{r} = 4\frac{1}{r}$$

$$\Rightarrow r + \frac{1}{r} = \frac{85}{20} = \frac{17}{4} = 4\frac{1}{4}$$

Here a = 20; r = 4

 \therefore numbers are $\frac{a}{r}$, a and ar.

i.e.
$$\frac{20}{4}$$
, 20 and 20×4

i.e. 5, 20 and 80.

8. Given AM = 17: GM = 8

Let a and b be the two numbers

:. AM =
$$\frac{a+b}{2} = 17$$
 or $a+b=34$ (1)

$$GM = \sqrt{ab} = 8 \quad \text{or} \quad ab = 64 \tag{2}$$

$$(a-b) = \sqrt{(a+b)^2 - 4ab} = \sqrt{(34)^2 - 4 \times 64}$$
$$= \sqrt{34^2 - 16^2} = \sqrt{(34+16)(34-16)}$$
$$= \sqrt{50 \times 18} = \sqrt{900} = 30$$

$$\therefore \qquad \qquad a-b=30 \tag{3}$$

Solving (1) and (3), $a = \frac{34 + 30}{2} = 32$;

$$b = \frac{34 - 30}{2} = 2$$

$$\therefore \text{ Harmonic Mean} = \frac{2ab}{a+b} = \frac{2 \times 32 \times 2}{32+2}$$
$$= \frac{2 \times 64}{34} = \frac{64}{17}$$

:. AP is 2, 17, 32 GP is 2, 8, 32

HP is
$$2, \frac{64}{17}, 32$$

Harmonic Mean = $\frac{64}{17}$

9.
$$S_n = \frac{n-1}{n} + 1 + \frac{n+1}{n} + \frac{n+2}{n} + \cdots$$
 up to *n* terms

Here
$$a = \frac{n-1}{n}$$
; $d = 1 - \frac{n-1}{n} = \frac{1}{n} = \frac{n+1}{n} - 1$

$$S_n = \frac{n}{2} \left\{ 2a + (n-1)d \right\} = \frac{n}{2} \left\{ \frac{2(n-1)}{n} + (n-1)\frac{1}{n} \right\}$$
$$= \frac{n}{2} \times \frac{3(n-1)}{n} = \frac{3(n-1)}{2}$$

10. Given two Arithmetic Progressions.

For the first AP = 3, 7, 11, 15 ... sum of n terms is given by

$$S_{n_1} = \frac{n}{2} \{ 2a + (n-1)d \}$$

$$= \frac{n}{2} \{ 2(3) + (n-1)4 \}$$

$$= \frac{n}{2} \{ 6 + 4n - 4 \}$$

$$= \frac{n}{2} \{ 4n + 2 \} = n(2n+1)$$

For the second AP = 30, 33, 36, 39 ... sum of n terms is given by

$$S_{n_2} = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$$

$$= \frac{n}{2} \left\{ 2 \times 30 + (n-1)3 \right\}$$

$$= \frac{n}{2} \left\{ 60 + 3n - 3 \right\}$$

$$= \frac{n}{2} \left\{ 57 + 3n \right\}$$

$$= \frac{3n(19+n)}{2}$$

Given that

Given that
$$S_{n_1} = S_{n_2}$$

 $\therefore n(2n+1) = \frac{3n}{2}(19+n)$

i.e.
$$(2n+1)2 = 3(19+n)$$

$$4n + 2 = 57 + 3n$$

$$\therefore 4n - 3n = 57 - 2 = 55$$

i.e.
$$n = 55$$

$$\therefore$$
 Value of $n = 55$

11.
$$S_{10} = \frac{a(r^{10} - 1)}{(r - 1)}; S_{20} = \frac{a(r^{20} - 1)}{(r - 1)} = 1025 S_{10}$$

i.e.
$$\frac{a(r^{20}-1)}{(r-1)} = 1025 \times \frac{a(r^{10}-1)}{(r-1)}$$

i.e.
$$(r^{20}-1) = 1025 (r^{10}-1)$$

$$\Rightarrow r^{20} - 1025r^{10} = 1 - 1025 = -1024$$

i.e.
$$r^{10}(r^{10} - 1025) = -1024$$

$$(1025 - r^{10})r^{10} = 1024 = 32^2 = 2^{10}$$

 $(2^{10} + 1 - r^{10})r^{10} = 2^{10}$

Solving by trial and error,

$$\Rightarrow$$
 $r = \pm 2$

12.
$$S_n = 1 + (1+m) + (1+m+m^2) + (1+m+m^2+m^3) + \cdots$$
 (1) $mS_n = m + (m+m^2) + (m+m^2+m^3) + \cdots$ (2)

$$mS_n = m + (m + m^2) + (m + m^2 + m^3) + \cdots$$

$$(1) - (2) \Rightarrow (1 - m)S_n = 1(1 - m) + (1 + m)(1 - m) + (1 + m + m^2)(1 - m) + \cdots$$

$$\begin{cases} (1-m)S_n = (1-m) + (1-m^2) + (1-m^3) + \dots + (1-m^n) \end{cases}$$

$$(1-m)S_n = 1 \times n - (m+m^2 + m^3 + \cdots + m^n)$$
$$= n - m \frac{(1-m^n)}{(1-m)}$$

i.e.
$$S_n = \frac{n}{1-m} - \frac{m(1-m^n)}{(1-m)^2}$$

$$\left[:: S_n = \frac{a(1-r^n)}{(1-r)}, \text{ if } r < 1 \right]$$

$$S_n = \frac{n}{1-m} + \frac{m(1-m^n)}{(1-m)^2}$$

13.
$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$$
$$= 1 + x + x^2 + x^3 + \cdots \quad \text{where } x = \frac{1}{2}$$
$$= \frac{1}{1 - x} = \frac{1}{1 - \frac{1}{2}} = 2$$

14.
$$(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \dots$$

$$= \sum_{m=1}^{n} (2m+1)^3 - (2m)^3 = \sum_{m=1}^{n} [(2m+1) - 2m][(2m+1)^2 + (2m+1)2m + (2m)^2]$$

$$= \sum_{m=1}^{n} 1 \times [4m^2 + 1 + 4m + 2m(2m+1) + 4m^2]$$

$$= \sum_{m=1}^{n} 12m^2 + 6m + 1$$

$$= 12\sum_{m=1}^{n} m^{2} + 6\sum_{m=1}^{n} m + \sum_{m=1}^{n} 1$$

$$= 12 \times \frac{n(n+1)(2n+1)}{6} + 6 \frac{n(n+1)}{2} + n$$

$$= 2n(n+1)(2n+1) + 3n(n+1) + n$$

$$= n(n+1) \left\{ 2(2n+1) + 3 \right\} + n$$

$$= n(n+1) \{4n+2+3\} + n$$

$$= n\{(n+1) (4n+5) + 1\}$$

$$= n\{4n^2 + 9n + 5 + 1\}$$

$$= 4n^3 + 9n^2 + 6n$$
15. $S_n = 0.9 + 0.99 + 0.999 + \cdots$

$$= 9\{0.1 + 0.11 + 0.111 + \cdots\}$$

$$= \frac{9}{10^{n+1}} \{10^n + 11 \times 10^{n-1} + 111 \times 10^{n-2} + \cdots\}$$

$$= \frac{1}{10^{n+1}} \{9 \times 10^n + 99 \times 10^{n-1} + 999 \times 10^{n-2} + \cdots\}$$

$$= \frac{1}{10^{n+1}} \{(10 - 1)10^n + (100 - 1)10^{n-1} + (1000 - 1)10^{n-2} + \cdots\}$$

$$= \frac{1}{10^{n+1}} \{n \times 10^{n+1} - (10^n + 10^{n-1} + 10^{n-2} + \cdots + 10^1)\}$$

$$= \frac{1}{10^{n+1}} \{n \times 10^{n+1} - \frac{10^n \left[1 - (1/10)^n\right]}{1 - \frac{1}{10}}\}$$

$$= \frac{1}{10^{n+1}} \{n \times 10^{n+1} - \frac{10^{n+1} - 10}{9}\}$$

$$= n - \frac{1 - 10^{-n}}{9}$$
16. $S_n = 1 \cdot 2 \cdot 3 + 4 \cdot 5 \cdot 6 + 7 \cdot 8 \cdot 9 + \cdots$

$$= \sum_{m=1}^{3n-2} m(m+1)(m+2) = \sum_{m=1}^{3n-2} m(m^2 + 3m + 2)$$

$$= \sum_{m=1}^{3n-2} m^3 + 3m^2 + 2m$$

$$= \left[\frac{(3n-2)(3n-1)}{2}\right]^2 + \frac{3(3n-2)(3n-1)(6n-3)}{6} + 2(3n-2)$$

$$= \frac{9n^2 - 9n + 2}{4} + \frac{3(9n^2 - 9n + 2)(2n-1)}{2} + 2(3n-2)$$

$$= \frac{9n^2 - 9n + 2}{4} \{6(2n-1) + 1\} + 6n - 4$$

$$= \frac{(9n^2 - 9n + 2)(12n-5) + 24n - 16}{4}$$

$$= \frac{108n^3 - 108n^2 + 24n - 45n^2 + 45n - 10 + 24n - 16}{4}$$

$$= \frac{108n^3 - 153n^2 + 93n - 26}{4}$$

$$= \frac{3(3n - 2)(n + 1)(9n^2 - 9n + 4)}{4}$$
17. $S_n = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots$

$$= \sum_{m=1}^{n} \frac{1}{m(m+1)(m+2)} = \frac{1}{m} \left(\frac{1}{m+1} - \frac{1}{m+2} \right)$$

$$= \sum_{m=1}^{n} \left(\frac{1}{m} - \frac{1}{m+1} \right) - \frac{1}{2} \left(\frac{1}{m} - \frac{1}{m+2} \right)$$

$$= \sum_{m=1}^{n} \frac{1}{2m} - \frac{1}{m+1} + \frac{1}{m+2}$$

$$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n} \right) + \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n+2} \right) - \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n+1} \right)$$

$$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n} \right) + \frac{1}{n+2} - \frac{1}{2}$$

$$= \sum \frac{1}{2n} - \frac{n}{2(n+2)} = \frac{1}{2} \sum \frac{1}{n} - \frac{n}{2(n+2)}$$
18. $S_n = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \cdots$

$$= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \quad \text{where } x = \frac{1}{3}$$

$$= \frac{1}{(1-x)^2} = \frac{1}{\left(1 - \frac{1}{3}\right)^2} = \frac{9}{4}$$

19. GM between $\frac{3}{2}$ and $\frac{27}{2}$ is given by $GM = \sqrt{\frac{3}{2} \times \frac{27}{2}} = \sqrt{\frac{81}{4}} = \frac{9}{2}$

Fractions

If any unit is divided into a number of equal parts, one or more of these parts is called a *fraction* of the unit. For example, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, etc. are fractions.

The numerator and denominator of a fraction are called its terms.

PROPER FRACTION: A proper fraction is one whose numerator is less than the denominator. For example, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, etc. are proper fractions.

The value a proper fraction is always less than 1.

IMPROPER FRACTION: A fraction, whose numerator is equal to or greater than the denominator, is called an improper fraction. For example, $\frac{11}{9}$, $\frac{7}{3}$, $\frac{13}{6}$, $\frac{19}{3}$, etc. are improper fractions.

The value of improper fraction is always more than 1.

MIXED FRACTIONS: An improper fraction can be expressed as a whole number and a proper fraction. This expression is called mixed fraction. For example, $2\frac{3}{4}$, $3\frac{5}{7}$, $7\frac{2}{9}$, etc. are mixed fractions.

LIKE FRACTIONS: Fractions in which denominators are the same are called like fractions. For example, $\frac{11}{17}$, $\frac{14}{17}$, $\frac{9}{17}$, etc. are like fractions.

fractions. For example, $\frac{3}{8}$, $\frac{4}{11}$, $\frac{5}{17}$, $\frac{7}{19}$, etc. are unlike fractions.

COMPOUND FRACTIONS: The fraction of a fraction is called a compound fraction. For example,

$$\frac{1}{3}$$
 of $\frac{4}{7} = \frac{1}{3} \times \frac{4}{7} = \frac{4}{21}$

$$\frac{3}{7}$$
 of $\frac{5}{11} = \frac{3}{7} \times \frac{5}{11} = \frac{15}{77}$

COMBINED OPERATIONS: In simplifying the fractions involving the various signs and brackets the following points should be remembered.

- 1. The operations of multiplication and division should be performed before those of addition and subtraction.
- 2. Each of the signs 'x' or '-' should be applied only to the number which immediately follows it. For example,

(i)
$$\frac{4}{5} \times \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{7}{12} \times \frac{24}{5} = \frac{\mathbf{56}}{\mathbf{25}}$$

(ii)
$$\frac{3}{7} \div \frac{5}{12} \times \frac{5}{24} = \frac{3}{7} \times \frac{12}{5} \times \frac{5}{24} = \frac{3}{14}$$

- 3. The operations within brackets are to be carried out first.
- 4. The rule of 'BODMAS' is applied for combined operations.

Complex Fractions: A complex fraction is one in which the numerator or denominator or both are fractions. For example, $\frac{3/5}{7}$, $\frac{7}{3/5}$, $\frac{5/9}{7/9}$, $\frac{1/2 + 3/5}{3/4 - 2/5}$ are complex fractions.

EXAMPLE 1 Simplify the following:

(a)
$$\frac{3/15}{2/5}$$

(b)
$$\frac{1/2 + 2/3}{3/4 - 2/9}$$

(c)
$$\frac{7}{5-8/3} \div \frac{3-\frac{2}{3-1/2}}{4-1/2}$$

Solution

(a)
$$\frac{3/15}{2/5} = \frac{3}{15} \times \frac{5}{2} = \frac{1}{2}$$

(b)
$$\frac{1/2 + 2/3}{3/4 - 2/9} = \frac{\frac{1 \times 3 + 2 \times 2}{6}}{\frac{3 \times 9 - 4 \times 2}{36}} = \frac{7/6}{19/36} = \frac{7}{6} \times \frac{36}{19} = \frac{42}{19}$$

(c)
$$\frac{7}{5-8/3} \div \frac{3 - \frac{2}{3-1/2}}{4-1/2}$$

$$= \frac{7}{\frac{15-8}{3}} \div \frac{3 - \frac{2}{\frac{(6-1)}{2}}}{\frac{(8-1)}{2}} = \frac{7 \times 3}{7} \div \frac{3 - \frac{4}{5}}{\frac{7/2}}$$

$$= 3 \div \frac{\frac{(15-4)}{5}}{\frac{7/2}} = 3 \div \frac{11}{5} \times \frac{2}{7}$$

$$= \left(3 \div \frac{11}{5}\right) \times \frac{2}{7} = \left(3 \times \frac{5}{11}\right) \times \frac{2}{7} = \frac{30}{77}$$

Continued Fraction: A fraction that contains an additional fraction in the numerator (or the denominator) is called the continued fraction.

EXAMPLE 2 Solve
$$2 + \frac{1}{1 + \frac{1}{2 + 2/3}}$$

Solution
$$2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3}}} = 2 + \frac{1}{1 + \frac{1}{8/3}} = 2 + \frac{1}{1 + \frac{3}{8}} = 2 + \frac{1}{11/8} = 2 + \frac{8}{11} = \frac{30}{11}$$

EXAMPLE 3 Solve
$$3 + \frac{2}{3 + \frac{1}{5 - \frac{4}{3}}}$$

Solution
$$3 + \frac{2}{3 + \frac{1}{5 - \frac{4}{3}}} = 3 + \frac{2}{3 + \frac{1}{11/3}} = 3 + \frac{2}{3 + \frac{3}{11}}$$
$$= 3 + \frac{2}{36/11} = 3 + \frac{2 \times 11}{36} = 3 + \frac{11}{18} = \frac{65}{18}$$



Note: To simplify a continued fraction begin at the bottom and work upwards.

EXAMPLE 4 Simplify
$$\frac{1}{2 + \frac{1}{2 + \frac{1}{1 + 1/3}}}$$

Solution
$$\frac{1}{2 + \frac{1}{2 + \frac{1}{1 + 1/3}}} = \frac{1}{2 + \frac{1}{2 + \frac{1}{4/3}}} = \frac{1}{2 + \frac{1}{2 + 3/4}}$$
$$= \frac{1}{2 + \frac{1}{11/4}} = \frac{1}{2 + \frac{4}{11}} = \frac{1}{\frac{22 + 4}{11}} = \frac{11}{26}$$

EXAMPLE 5 Simplify
$$3 + \frac{1}{5 + \frac{1}{4 + 2/7}}$$

Solution
$$3 + \frac{1}{5 + \frac{1}{4 + \frac{2}{7}}} = 3 + \frac{1}{5 + \frac{1}{\frac{30}{7}}} = 3 + \frac{1}{5 + \frac{7}{30}}$$
$$= 3 + \frac{1}{\frac{157}{30}} = 3 + \frac{30}{157} = \frac{501}{157}$$

EXAMPLE 6 Simplify
$$4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{4 + 1/3}}}$$

Solution
$$4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{4 + 1/3}}} = 4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{13/3}}} = 4 + \frac{1}{5 + \frac{1}{3 + 3/13}} = 4 + \frac{1}{5 + \frac{1}{\frac{42}{13}}}$$

$$= 4 + \frac{1}{5 + \frac{13}{42}} = 4 + \frac{1}{\frac{223}{42}} = 4 + \frac{42}{223} = 4 + \frac{42}{223}$$

EXAMPLE 7 A sum of money gets increased by its seventh part and amounts to $\stackrel{?}{\sim} 40$. Find the sum.

Solution Let the sum be S.

Then
$$S + \frac{S}{7} = 40$$

$$\Rightarrow \frac{8S}{7} = 40$$

$$\Rightarrow S = \frac{40 \times 7}{8} = 35$$
∴ Sum = ₹ 35

EXAMPLE 8 A train starts with full of passengers. At the first station it drops one-third of passengers and takes in 96 more. At the next, it drops one-half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 passengers left. With how many passengers did the train start?

Solution Let the number of passengers be x when the train started off.

 \therefore At the first station, the number of passengers = $x - \frac{x}{3} + 96 = \frac{2x}{3} + 96$

At the second station, the number of passengers = $\left(\frac{2x}{3} + 96\right)\frac{1}{2} + 12$

At the third station, the number of passengers left = $\frac{x}{3}$ + 48 + 12 = 248

$$\frac{x}{3} + 12 = 200$$

$$x = (200 - 12)3 = 564$$

DECIMAL FRACTION: Fraction in which the denominators are powers of 10 are called decimal fractions.

For example,

$$\frac{1}{10} = 0.1;$$
 $\frac{1}{100} = 0.01;$ $\frac{5}{1000} = 0.005;$ $\frac{11}{1000} = 0.011$ etc.

RULE

Converting a decimal into a vulgar fraction.

Write down the given number without decimal point in the numerator, and in the denominator write 1 followed by as many zeros as the number of figures after the decimal point in the given decimal fraction.

EXAMPLE 9 Reduce the following decimal fraction into vulgar fraction.

Solution

(i)
$$0.89 = \frac{89}{100}$$
 (ii) $2.078 = \frac{2078}{1000} = 2\frac{78}{1000}$ (iii) $0.00056 = \frac{56}{100000}$

Addition and Subtraction of Decimals



Write down the numbers under one another, placing the decimal points in one column. The number can now be added or subtracted as usual.

EXAMPLE 10 Add 4.076, 2.374, 0.56 and 15.

 Solution
 4.076

 2.374 0.56

 15.0 22.010

EXAMPLE 11 Add 2.057, 14.146, 0.078 and 12.

Solution 2.057 + 14.146 0.078 12.0 28.281

EXAMPLE 12 Subtract 17.078 from 24.35

Solution $24.350 - \frac{17.078}{7.272}$

Multiplication of Decimal Fraction by a Power of 10: Shift the decimal point to the right as many places as in the power of 10.

For example,

 $7.6342 \times 100 = 763.42;$ $0.0876 \times 1000 = 87.6;$ $0.0963 \times 10000 = 963;$

Multiplication of Decimal Fractions: Multiply the given numbers without considering the decimal point. In the product, decimal point is marked off by shifting as many places of decimal to the left of the number as the sum of number of decimals in the given numbers.

For example,

:.

$$0.3 \times 0.03 \times 0.003 = ?$$

 $3 \times 3 \times 3 = 27$
Sum of decimal places = $1 + 2 + 3 = 6$
 $0.3 \times 0.03 \times 0.003 = \mathbf{0.000027}$

Dividing a Decimal Fraction by a Counting Number: Divide the given number without considering the decimal point, by the given counting number. In the quotient so obtained mark the decimal point to give as many places of decimal as there are in the given decimal fraction.

For example,

Find $0.0234 \div 18$ $234 \div 18 = 13$

The given number contains 4 decimal places

 \therefore 0.0234 ÷ 18 = **0.0013**

Dividing a Decimal Fraction by a Decimal Fraction: Multiply both dividend and divisor by a suitable power of 10 to make divisor a whole number.

For example,

$$0.00088 \div 0.11 = ?$$

Solution
$$\frac{0.00088}{0.11} = \frac{0.00088}{0.11} \times \frac{100}{100} = \frac{0.088}{11} = \mathbf{0.008}$$

COMPARISON OF FRACTIONS: If the given fractions are to be arranged in ascending or descending order of magnitude, then convert each one of them in the decimal format and then arrange accordingly.

For example,

Arrange
$$\frac{3}{7}$$
, $\frac{7}{11}$ and $\frac{5}{9}$ in ascending order.

We have,
$$\frac{3}{7} = 0.428$$
; $\frac{5}{9} = 0.555...$; $\frac{7}{11} = 0.6363...$;

or
$$\frac{3}{7} < \frac{5}{9} < \frac{7}{11}$$

RECURRING DECIMAL: In a decimal fraction, if a digit or set of digits is repeated continuasly, then it is called a *recurring decimal*.

For example,

$$\frac{1}{3} = 0.333...;$$
 $\frac{1}{6} = 0.1666...;$ $\frac{2}{3} = 0.666....$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a *pure recurring decimal*.

For example,

$$\frac{1}{3} = 0.333...;$$
 $\frac{1}{9} = 0.111...;$ are pure recurring decimals.

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a *mixed recurring decimal*.

For example,

$$0.01633... = 0.016\overline{3}$$

Converting a Pure Recurring Decimal into Vulgar Fraction: Write the repeated figures only once in the numerator and write as many mines in the denominator as the number of repeating figures.

For example,

$$0.\overline{5} = \frac{5}{9}$$
; $0.0\overline{67} = \frac{67}{999}$; $0.\overline{059} = \frac{59}{999}$; $0.\overline{74} = \frac{74}{99}$ etc.

Converting a Mixed Recurring Decimal into Vulgar Fraction: In the numerator, write the difference between the number formed by all the digits after decimal point and that formed by digits which are not repeated. In the denominator, write as many mines as the number of repeating digits followed by as many zeros as the number of non-repeating digits.

For example,

$$0.\overline{19} = \frac{19 - 1}{90} = \frac{18}{90} = \frac{1}{5};$$
$$0.44\overline{65} = \frac{4465 - 44}{9900} = \frac{4421}{9900}$$

EXAMPLE 13 Arrange the fractions $\frac{3}{5}$, $\frac{4}{7}$, $\frac{8}{9}$ and $\frac{9}{11}$ in their descending order. [RBI]

Solution
$$\frac{3}{5} = 0.6$$
; $\frac{4}{7} = 0.5711$; $\frac{8}{9} = 0.88$; $\frac{9}{11} = 0.8181$
 $0.88 > 0.8181 > 0.6 > 0.5711$
or $\frac{8}{9} > \frac{9}{11} > \frac{3}{5} > \frac{4}{7}$

or

EXAMPLE 14 What value will replace the question mark in the following equation?

$$5172.49 + 378.352 + ? = 9318.678$$
 [BSRB]

Solution
$$? = 9318.678 - [5172.49 + 378.352]$$

= $9318.678 - 5550.842 = 3767.836$

EXAMPLE 15 Evaluate $2.5 \div 0.0005$

[MBA]

Solution
$$2.5 \div 0.0005 = \frac{2.5 \times 10000}{0.0005 \times 10000} = \frac{25000}{5} = 5000$$

EXAMPLE 16 Evaluate 136.09 ÷ 43.9

[Hotel Management]

Solution
$$\frac{136.09}{43.9} = \frac{136.09 \times 10}{43.9 \times 10} = \frac{1360.9}{439} = 3.1$$

EXAMPLE 17 Simplify:
$$\frac{0.05 \times 0.05 \times 0.05 + 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 - 0.05 \times 0.04 + 0.04 \times 0.04}$$
 [IGNOU]

Solution Given expression is of the form $\frac{a^3 + b^3}{a^2 - ab + b^2}$

where a = 0.05 and b = 0.04

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b = 0.05 + 0.04 = 0.09$$

EXAMPLE 18 Evaluate: 6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025

Solution Adding, we get

6202.5 620.25 62.025 6.2025 0.62025 6891.59775

Sum = 6891.59775



EXERCISES

What will come in the place of question mark (?) in the following?

1.
$$6\frac{1}{7} + 15\frac{2}{3} + 11\frac{1}{6} = 33\frac{1}{21} - ?$$

(a)
$$\frac{13}{14}$$

(a)
$$\frac{13}{14}$$
 (b) $1\frac{1}{42}$ (c) $\frac{1}{14}$ (d) $\frac{1}{7}$

(c)
$$\frac{1}{14}$$

(d)
$$\frac{1}{7}$$

(e) None of these

2.
$$57\frac{4}{7} + 29\frac{1}{21} + ? = 90\frac{3}{35}$$

(a)
$$4\frac{7}{15}$$
 (b) $4\frac{9}{15}$

(b)
$$4\frac{9}{15}$$

(c)
$$3\frac{1}{15}$$
 (d) $3\frac{7}{15}$

(d)
$$3\frac{7}{15}$$

(e) None of these

3.
$$12\frac{5}{7} + 23\frac{3}{5} - 14\frac{1}{3} = 28\frac{4}{7} - ?$$

(a)
$$3\frac{4}{105}$$

(a)
$$3\frac{4}{105}$$
 (b) $4\frac{62}{105}$

(c)
$$6\frac{62}{105}$$

(d)
$$4\frac{61}{105}$$

(e) None of these

4.
$$7^5 \div 7^3 \div 7^2 \times 7^4 \times \frac{1}{7^3} = 7^?$$

(e) None of these

5.
$$13\frac{4}{?} + 5\frac{3}{4}$$
 of $3\frac{1}{5} \div 4\frac{3}{5} = 17\frac{4}{5}$

(e) None of these

6.
$$47\frac{1}{17} \div 1\frac{49}{51} + 23\frac{5}{7} + ? = 67\frac{4}{9}$$

(b)
$$19\frac{46}{63}$$

(c)
$$20\frac{46}{63}$$

(e) None of these

7.
$$22\frac{2}{9} + 33\frac{4}{7} - ? = 28\frac{4}{45}$$

(a)
$$27\frac{37}{105}$$
 (b) $29\frac{74}{105}$ (c) $27\frac{74}{105}$ (d) $28\frac{74}{105}$

(b)
$$29\frac{74}{105}$$

(c)
$$27\frac{74}{105}$$

(d)
$$28\frac{74}{105}$$

(e) None of these

8.
$$1\frac{4}{7} + 1\frac{3}{5} + 1\frac{1}{3} = ?$$

(a)
$$5\frac{47}{105}$$
 (b) $4\frac{58}{105}$ (c) $4\frac{53}{105}$ (d) $5\frac{43}{105}$

(b)
$$4\frac{58}{105}$$

(c)
$$4\frac{53}{105}$$

(d)
$$5\frac{43}{105}$$

(e) None of these

9.
$$12\frac{1}{3} + 10\frac{5}{6} - 7\frac{2}{3} - 1\frac{4}{7} = ?$$

(a)
$$13\frac{13}{14}$$
 (b) $13\frac{11}{14}$ (c) $11\frac{13}{14}$ (d) $14\frac{11}{13}$

(b)
$$13\frac{11}{14}$$

(c)
$$11\frac{13}{14}$$

(d)
$$14\frac{11}{13}$$

(e) None of these

10.
$$4\frac{3}{4} + 2\frac{1}{8} + 7\frac{1}{4} + 3\frac{7}{8} + 11\frac{12}{13} = ?$$

(a)
$$27\frac{12}{13}$$
 (b) $28\frac{12}{13}$

(b)
$$28\frac{12}{13}$$

(c)
$$29\frac{12}{13}$$
 (d) $29\frac{11}{13}$

(d)
$$29\frac{11}{13}$$

(e) None of these

11.
$$7\frac{11}{18} + 13\frac{1}{9} + 5\frac{7}{9} - 16\frac{2}{3} = ?$$

(a)
$$9\frac{5}{6}$$

(a)
$$9\frac{5}{6}$$
 (b) $9\frac{5}{18}$ (c) $9\frac{1}{6}$

(c)
$$9\frac{1}{6}$$

(d)
$$8\frac{5}{6}$$

(e) $10\frac{5}{6}$

12.
$$1\frac{7}{9} + 3\frac{5}{8} - 2\frac{1}{18} + 4\frac{7}{16} - 9\frac{5}{18} + ? = 0$$

(a)
$$1\frac{71}{144}$$

(a)
$$1\frac{71}{144}$$
 (b) $1\frac{73}{144}$

(c)
$$1\frac{37}{144}$$

(d)
$$2\frac{71}{144}$$

(e) None of these

13.
$$425 \div 16.95 \times ? = 225$$

(e) 0.9

14.
$$198.995 \times 12.005 + 16.25 \times 6.95 = ?$$

(e) 2500

15.
$$1.2 \times 1.02 \times 1.002 = ?$$

(e) None of these

16.
$$6.4 - 1.6 \div 0.2 = ?$$

(e) None of these

17.
$$8.88 \times 88.8 \times 88 = ?$$

(e) None of these

18.
$$989.001 + 1.00982 \times 76.792 = ?$$

19.
$$375 + 2.832 - 1.001 + 1.803 = ?$$

(e) None of these

$$\frac{6}{11}$$
, $\frac{13}{17}$, $\frac{19}{27}$, $\frac{21}{23}$, $\frac{5}{7}$

(e) None of these

21. Find the smallest fraction which when added to
$$\frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8}$$
 gives a whole number.

(c)
$$37/40$$

(a) 0.4

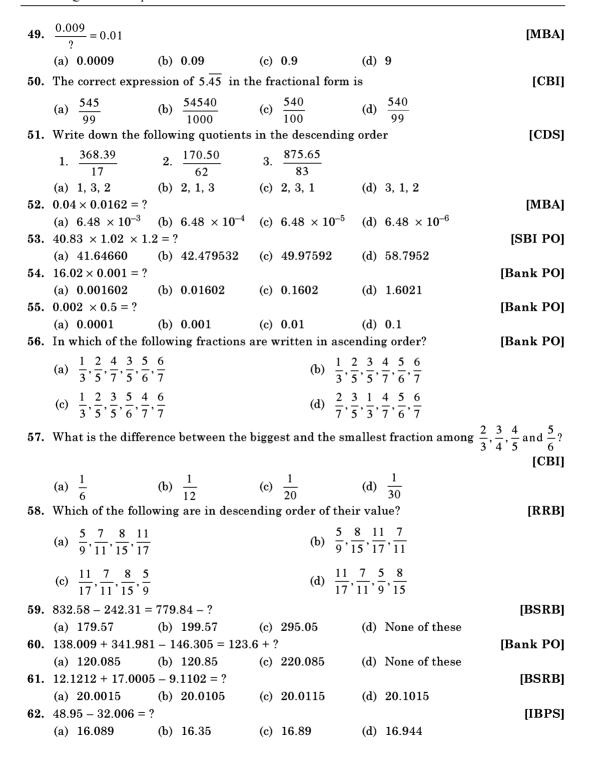
(b) 1.2

22. The value of
$$\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + (0.1)^2}$$
 is: [SSC] (a) 0.86 (b) 0.95 (c) 0.97 (d) 1.06 (23.3) -0.027 (d) 1.06 (23. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is: [SSC] (a) 0 (b) 1.6 (c) 2 (d) 3.4 (24. Simplify $\frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044}$ [SSC] (a) 0.004 (b) 0.4 (c) 1 (d) 2 (SSC] (a) 2 (b) 4 (c) 6 (d) 8 (SSC] (a) 2 (b) 4 (c) 6 (d) 8 (SSC] (a) 2 (b) 4 (c) 6 (d) 8 (SSC] (a) 2 (b) 4 (c) 6 (d) 6 (d) 6 (e) 6 (d) 6 (f) 6 (f) 6 (f) 6 (g) 6

(c) 1.4

(d) 4

35.	The value of	$\frac{3.157 \times 4126 \times 3.198}{63.972 \times 2835.121}$	is nearer to:		[CBI]	
	(a) 0.002	(b) 0.02	(c) 0.2	(d) 2		
36.	$\frac{0.0203 \times 2}{0.0073 \times 14.5}$	$\frac{.92}{5 \times 0.7} = ?$			[RRB]	
	(a) 0.8	(b) 1.45	(c) 2.40	(d) 3.25		
37.	The value of	$\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}$ is:			[SSC]	
	(a) 80	(b) 800	(c) 8000	(d) 80000		
38.	$\frac{(0.1667)(0.8)}{(0.2222)(0.6)}$	$\frac{333)(0.3333)}{667)(0.1250)}$ is app	roximately equal	to:	[MBA]	
		(b) 2.40	(c) 2.43	(d) 2.50		
39 .				$+ (0.22)^3 + \dots + (0.99)^3$	is close to: [SSC]	
	(a) 0.2695	(b) 0.3695	(c) 2.695	(d) 3.695		
40.		16134, then the valu		is:	[SSC]	
		(b) 0.16134		(d) 16134		
41.		$6 = 172$, then $29.94 \div$		(4) 170	[LIC]	
		(b) 1.72	(c) 11.2	(d) 172		
42.	$(0.\overline{09} \times 7.\overline{3})$ is				[SSC]	
	(a) $0.\overline{6}$	(b) $0.\overline{657}$	(c) $0.\overline{67}$	(d) $0.\overline{657}$		
43.	$3.\overline{87} - 2.\overline{59} =$?			[LIC AAO]	
	(a) 1.20	(b) $1.\overline{2}$	(c) $1.\overline{27}$	(d) $1.\overline{28}$		
44.	The value of	2.1 36 is:				
			(c) $2\frac{3}{22}$	(d) None of these		
45 .	The correct ex	expression of $6.\overline{46}$ in	the fractional for	rm is	[CBI]	
	(a) ${99}$	(b) $\frac{64640}{100}$	(c) $\frac{310}{100}$	(d) $\frac{1}{99}$		
46.	. The rational number for the recurring decimal 0.125125 is: [MBA]					
	(a) $\frac{63}{487}$	(b) $\frac{119}{993}$	(c) $\frac{125}{999}$	(d) None of these		
47.	7. When 0.232323 is converted into a fraction, then the result is:					
	(a) $\frac{1}{5}$	(b) $\frac{2}{9}$	(c) $\frac{23}{99}$	(d) $\frac{23}{100}$		
48.	If $\frac{144}{0.144} = \frac{14}{x}$	$\frac{.4}{.}$, then the value of	f x is:		[CBI]	
	(a) 0.0144	(b) 1.44	(c) 14.4	(d) 144		



1	 •	3
	١.	. 1

63.	617 + 6.017 + 0.63	17 + 6.0017 =	?		[MBA]
	(a) 6.2963	(b) 62.965	(c) 629.6357	(d) None of these	
64.	34.95 + 240.016 +	- 23.98 = ?			[Bank PO]
	(a) 298.0946	(b) 298.111	(c) 298.946	(d) 299.09	
65.	3889 + 12.952 - ?				[Bank PO]
	(a) 47.095	(b) 47.752	(c) 47.932	(d) 47.95	
66.	337.62 + 8.591 + 3	34.4 = ?			[SSC]
		(b) 380.511	(c) 380.611	(d) 426.97	
67.	The sum of first 2	20 terms of the	e series	[Ho	tel Management]
	$\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 6}$	l — + is			
	$5 \times 6 6 \times 7 7 \times$	× 8			
	(a) 0.16	(b) 1.6	(c) 16	(d) None of these	
68.	When 52416 is div	vided by 312, t	the quotient is 168.	What will be the quoti	
	divided by 0.0168	3?			tel Management]
	(a) 3.12	(b) 312	(c) 3120	(d) None of these	
69.	In which of the fo	llowing fraction	ons are written in a	scending order?	[NABARD]
	(2) $\frac{2}{3}$ $\frac{3}{7}$ $\frac{7}{9}$ $\frac{8}{8}$	3	3 2 9 7 8	(c) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}$	9 8
	$(a) \frac{1}{3}, \frac{1}{5}, \frac{1}{9}, \frac{1}{11}, \frac{1}{9}$	`	$\frac{5}{5}, \frac{7}{3}, \frac{7}{11}, \frac{7}{9}, \frac{7}{9}$	$(5) \frac{1}{5}, \frac{1}{3}, \frac{1}{9},$	11, 9
	. 8 9 7 2 3	3	. 8 9 7 3 2		
	(d) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$	- ((e) $\frac{1}{9}, \frac{1}{11}, \frac{1}{9}, \frac{1}{5}, \frac{1}{3}$		
70.	Find the value of	the following	expression upto for	ur places of decimals	
	г			1	
	$1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2}$	$\frac{1}{\sqrt{4}} + \frac{1}{1 \times 2 \times 4}$	$\frac{1}{8} + \frac{1}{1 \times 2 \times 4 \times 8 \times}$	[Hot	tel Management]
		4 1 1 2 2 4 7	X	10]	
	(a) 1.6414	(b) 1.6415	(c) 1.6416	(d) 1.6428	
71.	The denominator	of a fraction is	3 more than the n	umerator. If the numer	rator as well as the
denominator is increased by 4, the fraction becomes $\frac{4}{5}$. What was the original fra				riginal fraction?	
		ioreasea sy 1,	one muchon second	5 · · · · · · · · · · · · · · · · · · ·	[SBI PO]
	8	a. 5	10	7	
	(a) $\frac{3}{11}$	(b) $\frac{8}{8}$	(c) $\frac{10}{13}$	(d) $\frac{10}{10}$	
		1	14	35	
72 .	The product of two	o fractions is	$\frac{15}{15}$ and their quotie	ent is $\frac{35}{24}$. The greater	fraction is: [SSC]
	(a) $\frac{4}{5}$	(b) $\frac{7}{6}$	(c) $\frac{7}{4}$	(d) $\frac{7}{3}$	
70	J	6 - f - f +: :	4 - : 1 h 0	141	
13.	If the numerator of a fraction is increased by 2 and the denominator is increased by 3, the				
	fraction becomes $\frac{7}{9}$ and if both the numerator and the denominator are decreased by 1, the				
	fraction becomes	$\frac{4}{5}$. What is th	e original fraction?	?	[SBI PO]
	(a) $\frac{5}{6}$	(b) $\frac{9}{11}$	(c) $\frac{13}{16}$	(d) $\frac{17}{21}$	
	U	* *	10	- *	

74. In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is

 $\frac{2}{25}$. What is the value of fraction A?

[Bank PO]

(b) $\frac{1}{25}$ (c) $\frac{2}{5}$

(d) Data inadequate

75. The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by $1\frac{1}{4}$. Find the value of fraction [MBA]

(b) $2\frac{1}{4}$ (c) $3\frac{1}{4}$

(d) 6

76. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator of the fraction, the fraction becomes 1. The fraction is: [CBI]

(b) $\frac{2}{3}$

(c) $\frac{3}{4}$

(d) $\frac{3}{2}$



1. (c) **2.** (d) **3.** (c) **4.** (a) **5.** (c) **6.** (b) **7.** (c) 8. (c) **9.** (a) **10.** (c)

11. (a) 12. (a) **13.** (c) **14.** (e) **15.** (a) **16.** (e) 17. (e) 18. (c) **19.** (c) 20. (b)

21. (c) 22. (a) **23.** (c) 24. (c) **25.** (b) **26.** (c) **27.** (b) 28. (b) **29.** (d) **30.** (c)

31. (c) **32.** (d) **33.** (c) **34.** (d) **35.** (c) **36.** (a) **37.** (b) 38. (d) **39.** (c) **40.** (c)

41. (c) **42.** (a) **44.** (c) **45.** (d) **46.** (c) **47.** (c) 48. (a) **49.** (c) **50.** (d) **43.** (d)

51. (a) **52.** (b) **53.** (c) **54.** (b) **55.** (b) **56.** (a) **57.** (a) 58. (d) **59.** (d) **60.** (d)

61. (c) **62.** (d) **63.** (c) **64.** (c) **65.** (d) **66.** (c) **67.** (a) **68.** (c) **69.** (c) **70.** (c)

71. (a) **72.** (b) 74. (c) **75.** (b) **76.** (b) **73.** (a)



Solutions with Necessary Explanation

1.
$$\frac{1}{14}$$

2.
$$57\frac{4}{7} + 29\frac{1}{21} + ? = 90\frac{3}{35}$$

 $? = (90 - 57 - 29) + \frac{3}{35} - \frac{4}{7} - \frac{1}{21}$ or $=4+\frac{3\times3}{35\times3}-\frac{4\times15}{105}-\frac{5}{105}=4+\frac{9-65}{105}=4-\frac{56}{105}$ $=3\frac{49}{105}=3\frac{7}{15}$

3.
$$? = 28\frac{4}{7} + 14\frac{1}{3} - 23\frac{3}{5} - 12\frac{5}{7}$$

= $(28 + 14 - 23 - 12) + \left(\frac{4}{7} + \frac{1}{3} - \frac{3}{5} - \frac{5}{7}\right)$

$$= 7 - \frac{1}{7} + \frac{5 - 9}{15} = 7 - \frac{1}{7} - \frac{4}{15}$$
$$= 7 - \left(\frac{1 \times 15 + 4 \times 7}{105}\right) = 7 - \frac{43}{105} = 6\frac{62}{105}$$

4.
$$7^5 \div 7^3 \div 7^2 \times 7^4 \times \frac{1}{7^3} = 7^9$$

$$7^{5-3-2+4-3} = 7^{9-8} = 7^1$$

 \therefore Answer = 1

5.
$$13\frac{4}{?} + 5\frac{3}{4}$$
 of $3\frac{1}{5} \div 4\frac{3}{5} = 17\frac{4}{5}$

$$\Rightarrow 13\frac{4}{?} = 17\frac{4}{5} - \left(5\frac{3}{4} \text{ of } 3\frac{1}{5} \div 4\frac{3}{5}\right)$$

$$= 17\frac{4}{5} - \left(\frac{23}{4} \times \frac{16}{5} \div \frac{23}{5}\right) = 17\frac{4}{5} - \left(\frac{23}{4} \times \frac{16}{5} \times \frac{5}{23}\right) = 17\frac{4}{5} - 4 = 13\frac{4}{5}$$

6.
$$47\frac{1}{17} \div 1\frac{49}{51} + 23\frac{5}{7} + ? = 67\frac{4}{9}$$
 \Rightarrow $47\frac{1}{17} \times \frac{1}{1\frac{49}{51}} + 23\frac{5}{7} + ? = 67\frac{4}{9}$

$$\Rightarrow \frac{800}{17} \times \frac{1}{100} + 23\frac{5}{7} + ? = 67\frac{4}{9} \qquad \Rightarrow \qquad \frac{800}{17} \times \frac{51}{100} + \frac{166}{7} + ? = \frac{607}{9}$$

$$\Rightarrow ? = \frac{607}{9} - \frac{166}{7} - \frac{800}{17} \times \frac{51}{100} = \frac{607}{9} - \frac{166}{7} - 24 = \frac{607 \times 7 - 166 \times 9 - 24 \times 63}{63} = \frac{1243}{63} = 19 \frac{46}{63}$$

7.
$$22\frac{2}{9} + 33\frac{4}{7} - ? = 28\frac{4}{45}$$

$$\Rightarrow ? = 22\frac{2}{9} + 33\frac{4}{7} - 28\frac{4}{45} = 22 + 33 - 28 + \frac{2}{9} + \frac{4}{7} - \frac{4}{45}$$
$$= 27 + \frac{2 \times 35 + 4 \times 45 - 4 \times 7}{315} = 27 + \frac{70 + 180 - 4 \times 7}{315}$$

$$= 27 + \frac{70 + 180 - 28}{315} = 27 + \frac{222}{315} = 27 + \frac{74}{105} = 27 \frac{74}{105}$$

8.
$$? = 1 + 1 + 1 + \frac{4}{7} + \frac{3}{5} + \frac{1}{3} = 3 + \left(\frac{60 + 63 + 35}{105}\right) = 3 + \frac{158}{105} = 3 + 1 + \frac{53}{105} = 4\frac{53}{105}$$

9.
$$? = 12\frac{1}{3} + 10\frac{5}{6} - 7\frac{2}{3} - 1\frac{4}{7}$$

$$= (12+10-7-1) + \left(\frac{1}{3} + \frac{5}{6} - \frac{2}{3} - \frac{4}{7}\right) = 14 + \left(\frac{14+35-28-24}{42}\right) = 14 - \frac{3}{42}$$

$$= 14 - \frac{1}{14} = 13 \frac{13}{14}$$

10.
$$4\frac{3}{4} + 2\frac{1}{8} + 7\frac{1}{4} + 3\frac{7}{8} + 11\frac{12}{13}$$

$$= 4\frac{3}{4} + 7\frac{1}{4} + 2\frac{1}{8} + 3\frac{7}{8} + 11\frac{12}{13}$$

$$= (4 + 7 + 2 + 3 + 11) + \left(\frac{3}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{7}{8}\right) + \frac{12}{13}$$

$$= 27 + 1 + 1 + \frac{12}{13} = 29\frac{12}{13}$$
11. $(7 + 13 + 5 - 16) + \left(\frac{11}{18} + \frac{1}{9} + \frac{7}{9} - \frac{2}{3}\right) = 9 + \left(\frac{11 + 16 - 12}{18}\right) = 9 + \frac{15}{18} = 9\frac{5}{6}$
12. $1\frac{7}{9} + 3\frac{5}{8} - 2\frac{1}{18} + 4\frac{7}{16} - 9\frac{5}{18} + x = 0$
or $(1 + 3 - 2 + 4 - 9) + \frac{7}{9} + \frac{5}{8} + \frac{7}{16} - \frac{1}{18} - \frac{5}{18} + x = 0$

$$-3 + \left(\frac{7}{9} - \frac{6}{18}\right) + \frac{5}{8} + \frac{7}{16} + x = 0$$

 $-3 + \frac{14 - 6}{18} + \frac{10 + 7}{16} + x = 0$ \Rightarrow $-3 + \frac{4}{9} + \frac{17}{16} + x = 0$

 $-3 + \frac{64 + 153}{144} + x = 0$ \Rightarrow $-3 + \frac{217}{144} + x = 0$

$$\Rightarrow \qquad -3+1+\frac{73}{144}+x=0 \qquad \Rightarrow \qquad x=2-\frac{73}{144}=1\frac{71}{144}$$
13. $425 \div 16.95 \times ?=225$

or
$$\frac{425 \times ?}{17} = 225$$
 or $? = \frac{225 \times 17}{425} = 9$

14.
$$198.995 \times 12.005 + 16.25 \times 6.95 = ?$$

 $\therefore ? \approx 199 \times 12 + 16 \times 7 \approx 2388 + 112$
or $? = 2500$

15.
$$1.2 \times 1.02 \times 1.002 = ?$$

 $1.2 \times 1.02 \times 1.002 = 1.226448$

16.
$$6.4 - 1.6 \div 0.2 = 6.4 - 1.6 \times \frac{1}{0.2} = 6.4 - 8 = -1.6$$

.. None of these

 \Rightarrow

 \Rightarrow

17.
$$? = 8.88 \times 88.8 \times 88 = 69391.872$$

: None of these

18.
$$989.001 + 1.00982 \times 76.792 = ?$$

 \Rightarrow ? \approx $989 + 77 \approx 1066 \approx 1065$
19. $3.75 + 2.832 - 1.001 + 1.803 = 7.384$

20.
$$\frac{6}{11}, \frac{13}{17}$$

$$6 \times 17 < 11 \times 13$$

$$\therefore \frac{6}{11} < \frac{13}{17}$$

Then
$$\frac{6}{11}, \frac{19}{27}$$

$$6 \times 27 < 11 \times 19$$

$$\therefore \frac{6}{11} < \frac{19}{27}$$

$$\frac{6}{11}, \frac{21}{23}$$

$$6 \times 23 < 11 \times 21$$

$$\therefore \frac{6}{11} < \frac{21}{23}$$

$$\frac{6}{11}, \frac{5}{7}$$

$$6 \times 7 < 11 \times 5$$

$$\therefore \frac{6}{11} < \frac{5}{7}$$

$$\therefore$$
 Smallest fraction is $\frac{6}{11}$

21.
$$\frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8} = \frac{3}{40} < 1$$

To get whole a number, the number to be added

$$= 1 - \frac{3}{40} = \frac{37}{40}$$

22. The given expression is of the form
$$\frac{a^3 - b^3}{a^2 + ab + b^2}$$
 and is equal to $(a - b)$

Here
$$a = 0.96$$
; $b = 0.1$

$$a - b = 0.96 - 0.1 = 0.86$$

23.
$$\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09} = \frac{(2.3)^3 - (0.3)^3}{(2.3)^2 + (2.3)(0.3) + (0.3)^2}$$

$$\therefore \text{ Given expression} = \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

Here
$$a = 2.3$$
; $b = 0.3$

$$a - b = 2.3 - 0.3 = 2$$

24.
$$\frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044} = \frac{0.04 + 0.004}{0.044} = \frac{0.044}{0.044} = 1$$

25.
$$\frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61} = \frac{a^2 - b^2}{a - b} = a + b$$

Here
$$a = 2.39$$
; $b = 1.61$

$$\therefore$$
 $a+b=2.39+1.61=4$

26.
$$\frac{(2.644)^2 - (2.356)^2}{0.288} = \frac{(2.644)^2 - (2.356)^2}{(2.644 - 2.356)} = 2.644 + 2.356 = 5$$

27.
$$\frac{1.49 \times 14.9 - 0.51 \times 5.1}{14.9 - 5.1} = \frac{1.49 \times 1.49 \times 10 - 0.51 \times 0.51 \times 10}{10(1.49 - 0.51)}$$

$$= \frac{(1.49)^2 - (0.51)^2}{(1.49 - 0.51)} = 1.49 + 0.51 = 2$$

28.
$$\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1} = \frac{(4.2)^2 - (1.9)^2}{(4.2 - 1.9)(4.2 + 1.9)} = 1$$

29.
$$7.5 \times 7.5 + 37.5 + 2.5 \times 2.5 = (7.5)^2 + 2(7.5)(2.5) + (2.5)^2$$

= $(7.5 + 2.5)^2 = 10^2 = 100$

30. Given expression =
$$\frac{(0.051)^3 + (0.041)^3}{(0.051)^2 - (0.051)(0.041) + (0.041)^2}$$

is of the form
$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$$

where a = 0.051 and b = 0.041

$$\therefore$$
 Given expression = a + b = 0.051 + 0.041 = 0.092

31. Given expression =
$$\frac{(0.5)^3 + (0.3)^3}{(0.5)^2 + (0.3)^2 - (0.5)(0.3)} = 0.5 + 0.3 = 0.8$$

32. Given expression =
$$\frac{(2 \times 3.75)^3 + 1^3}{(7.5)^2 + 1^2 - (7.5)(1)} = \frac{(7.5)^3 + 1^3}{(7.5)^2 - (7.5)(1) + 1^2} = 7.5 + 1 = 8.5$$

33. Given expression =
$$\frac{10.3^3 + 1^3}{10.3^2 - (10.3)(1) + (1)^2} = 10.3 + 1 = 11.3$$

34. Given expression =
$$\frac{8-2.8}{1.3} = \frac{5.2}{1.3} = 4$$

35. Given expression
$$\simeq \frac{3.2 \times 4126 \times 3.2}{64 \times 2835} \simeq \frac{32 \times 4126 \times 32}{64 \times 2835 \times 100}$$

$$\simeq \frac{16 \times 4126}{2835 \times 100} \simeq \frac{66016}{2835 \times 100} = \frac{23.28}{100} \simeq 0.23 \simeq 0.2$$

36.
$$\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8$$

37.
$$\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5} = \frac{36 \times 48 \times 250}{12 \times 9 \times 5} = 4 \times 4 \times 50 = 800$$

38. Given expression =
$$\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$$

$$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{3}\right) \div \left(\frac{2}{9} \times \frac{2}{3} \times \frac{1}{8}\right) = \frac{1}{6} \times \frac{5}{6} \times \frac{1}{3} \times \frac{9}{2} \times \frac{3}{2} \times 8 = \frac{5}{2} = 2.5$$

39. Given
$$1^3 + 2^3 + \dots + 9^3 = 2025$$

$$\therefore (0.11)^3 + (0.22)^3 + \dots + (0.99)^3$$

$$= (0.11)^3 \{1^3 + 2^3 + \dots + 9^3\}$$

$$= (0.11)^3 \times 2025 = 0.001331 \times 2025 = 2.695$$

40. Given
$$\frac{1}{6.198} = 0.16134$$

$$\therefore \frac{1}{0.0006198} = \frac{10000}{6.198} = 10000 \times 0.16134 = 1613.4$$

41. Given
$$\frac{2994}{14.5} = 172$$

$$\therefore \frac{29.94}{1.45} = \frac{2994}{100} \times \frac{10}{14.5} = \frac{172 \times 10}{100} = 17.2$$

42.
$$0.\overline{09} \times 7.\overline{3} = ?$$

$$0.\overline{09} = \frac{9}{99} = \frac{1}{11};$$
 $7.\overline{3} = 7\frac{1}{3} = \frac{22}{3}$

$$\therefore 0.\overline{09} \times 7.\overline{3} = \frac{1}{11} \times \frac{22}{3} = \frac{2}{3} = \mathbf{0.\overline{6}}$$

43.
$$3.\overline{87} - 2.\overline{59} = ?$$

$$3.\overline{87} - 2.\overline{59} = 1.\overline{28}$$

44.
$$2.\overline{136} = 2 + 0.\overline{136}$$

$$=2+\frac{136-1}{990}=2+\frac{3}{22}=2\frac{3}{22}$$

45.
$$6.\overline{46} = 6 + \frac{46}{99} = \frac{640}{99}$$

46.
$$0.125125... = 0.\overline{125}$$

$$0.\overline{125} = \frac{125}{999}$$

47.
$$0.232323... = 0.\overline{23}$$

$$0.\overline{23} = \frac{23}{99}$$

48.
$$\frac{144}{0.144} = \frac{14.4}{x}$$

or
$$x = \frac{14.4 \times 0.144}{144} = \mathbf{0.0144}$$

49.
$$? = \frac{0.009}{0.01} =$$
0.9

50.
$$5.\overline{45} = 5 + 0.\overline{45}$$

$$0.\overline{45} = \frac{45}{99}$$

$$\therefore \text{ Given expression} = 5 + \frac{45}{99} = \frac{540}{99}$$

51. 1.
$$\frac{368.39}{17}$$
 = **21.67**

2.
$$\frac{170.50}{62}$$
 = 2.75

3.
$$\frac{875.65}{83} = 10.55$$

Hence answer is (a) 1, 3, 2

52.
$$0.04 \times 0.0162 = 4 \times 10^{-2} \times 162 \times 10^{-4}$$

= 648×10^{-6}
= 6.48×10^{-4}

53.
$$40.83 \times 1.02 \times 1.2 = ?$$

$$4083 \times 102 \times 12 = 4997592$$

$$\therefore 40.83 \times 1.02 \times 1.2 = 49.97592$$

54.
$$16.02 \times 0.001 =$$
0.01602

55.
$$0.002 \times 0.5 = 0.001$$

56.
$$\frac{1}{3} = 0.333...;$$
 $\frac{2}{5} = 0.4$

$$\frac{4}{7} = 0.57142...;$$
 $\frac{3}{5} = 0.6$

$$\frac{5}{6} = 0.8333...;$$
 $\frac{6}{7} = 0.8571...$

$$0.8571... > 0.8333... > 0.6 > 0.57142 > 0.4 > 0.333$$

i.e.
$$\frac{6}{7} > \frac{5}{6} > \frac{3}{5} > \frac{4}{7} > \frac{2}{5} > \frac{1}{3}$$

57.
$$\frac{2}{3} = 0.666...;$$
 $\frac{3}{4} = 0.75$

$$\frac{4}{5} = 0.8;$$
 $\frac{5}{6} = 0.8333...$

Required difference = 0.833... - 0.666...

$$=\frac{5}{6}-\frac{2}{3}=\frac{5-4}{6}=\frac{1}{6}$$

$$\begin{array}{lll} \mathbf{58.} & \frac{5}{9} = 0.555...; & \frac{7}{11} = 0.6363... \\ & \frac{8}{15} = 0.533...; & \frac{11}{17} = 0.64705... \\ & \therefore & 0.647 > 0.63 > 0.55 > 0.533... \\ & \mathrm{i.e.} & \frac{11}{17} > \frac{7}{11} > \frac{5}{9} > \frac{8}{15} \\ \\ \mathbf{59.} & 832.58 - 242.31 = 779.84 - ? \\ & \therefore & ? = 779.84 + 242.31 - 832.58 \\ & = 779.84 - (832.58 - 242.31) \\ & = 779.84 - 590.27 \\ & = 189.57 \\ \mathbf{60.} & 138.009 + 341.981 - 146.305 = 123.6 + ? \\ & \therefore & ? = 138.009 + 341.981 - 146.305 - 123.6 \\ & = 138.009 + 341.981 - (146.305 + 123.6) \\ & = 138.009 + 72.076 = 210.085 \\ \mathbf{61.} & 12.1212 + 17.0005 - 9.1102 \\ & = 17.0005 + 9.1102 \\ & = 17.0005 + 3.0110 = 20.0115 \\ \mathbf{62.} & 48.95 - 32.006 = 16.944 \\ \mathbf{63.} & 617 + 6.017 + 0.617 + 6.0017 = 629.6357 \\ \hline & 617.0 \\ & 6.017 \\ & 6.017 \\ \hline & 6.0017 \\ \hline & 629.6357 \\ \mathbf{64.} & 34.95 + 240.016 + 23.98 = 298.946 \\ & 34.95 \\ & 240.016 \\ & 23.98 \\ \hline & 298.946 \\ \hline \mathbf{65.} & 3889 + 12.952 - ? = 3854.002 \\ & \therefore & ? = 3889 + 12.952 - 3854.002 = 47.95 \\ & 3889.0 \\ & 12.952 \\ \hline & 3901.952 \\ \hline \end{array}$$

3854.002 47.950

66.
$$337.62 + 8.591 + 34.4 = 380.611$$

67.
$$\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \dots + \frac{1}{24 \times 25}$$
$$= \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \dots + \frac{1}{24} - \frac{1}{25}$$
$$= \frac{1}{5} - \frac{1}{25} = \frac{5 - 1}{25} = \frac{4}{25} = \frac{16}{100} = 0.16$$

68. Given
$$\frac{52416}{312} = 168$$

$$Let \ \frac{52.416}{0.0168} = x$$

or
$$x = \frac{52.416}{0.0168} = \frac{52416}{1000} \times \frac{10000}{168} = \frac{52416}{168} \times 10 = 312 \times 10 = 3120$$

69.
$$\frac{2}{3} = 0.666...;$$
 $\frac{3}{5} = 0.6$

$$\frac{7}{9} = 0.777...;$$
 $\frac{9}{11} = 0.8181...$

$$\frac{8}{9} = 0.888...$$

$$\therefore$$
 0.6 < 0.666... < 0.777... < 0.8181... < 0.888...

i.e.
$$\frac{3}{5} < \frac{2}{3} < \frac{7}{9} < \frac{9}{11} < \frac{8}{9}$$

70.
$$1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 4} + \frac{1}{1 \times 2 \times 4 \times 8} + \frac{1}{1 \times 2 \times 4 \times 8 \times 16}$$

= $1 + 0.5 + 0.125 + \frac{16 + 1}{1 \times 2 \times 4 \times 8 \times 16}$
= $1.625 + \frac{17}{1024} = 1.625 + 0.0166 = 1.6416$

71. Let the numerator be a and denominator = a + 3

$$\frac{a+4}{[(a+3)+4]} = \frac{4}{5}$$

$$\Rightarrow \qquad 5(a+4) = 4(a+7)$$

$$\Rightarrow \qquad a = 28 - 20 = 8$$

$$\therefore \qquad \text{Required fraction} = \frac{8}{8+3} = \frac{8}{11}$$

72. Let the two fractions be x and y.

$$\therefore \qquad xy = \frac{14}{15} \tag{1}$$

$$\frac{x}{y} = \frac{35}{24} \tag{2}$$

$$\frac{(1)}{(2)} \qquad \Rightarrow \qquad \frac{xy}{x/y} = \frac{14}{15} \div \frac{35}{24}$$

$$\Rightarrow y^2 = \frac{14}{15} \times \frac{24}{35}$$
$$= \frac{2}{5} \times \frac{8}{5} = \frac{16}{25}$$

$$\therefore \qquad \qquad y = \frac{4}{5}$$

$$\therefore$$
 (1) \Rightarrow $x = \frac{14}{15} \div y = \frac{14}{15} \times \frac{5}{4} = \frac{7}{6}$

 \therefore Greater fraction is $\frac{7}{6}$

73. Let $\frac{x}{y}$ be the fraction.

$$Now \qquad \frac{x+2}{y+3} = \frac{7}{9} \tag{1}$$

$$\frac{x-1}{y-1} = \frac{4}{5} \tag{2}$$

(1)
$$\Rightarrow$$
 9 $(x + 2) = 7(y + 3)$
 \Rightarrow 9x - 7y = 21 - 18 = 3

$$9x - 7y = 3 \tag{A}$$

(2)
$$\Rightarrow 5(x-1) = 4(y-1)$$
$$\Rightarrow 5x - 4y = 5 - 4 = 1$$
$$\Rightarrow 5x - 4y = 1$$
 (B)

Solving (A) and (B), we get

$$4 \times (A)$$
 \Rightarrow $36x - 28y = 12$ (C)

$$7 \times (B) \qquad \Rightarrow \qquad 35x - 28y = 7 \tag{D}$$

$$(C) - (D) \qquad \Rightarrow \qquad x = 5$$

∴ (B)
$$\Rightarrow y = \frac{5x - 1}{4} = \frac{25 - 1}{4} = 6$$

 \therefore Required fraction is $\frac{5}{6}$

74. Let the fractions B be x and A be 2x.

$$\therefore \qquad x \times 2x = \frac{2}{25}$$

$$\Rightarrow 2x^2 = \frac{2}{25}$$

$$\Rightarrow x^2 = \frac{1}{25}$$

$$\Rightarrow x = \frac{1}{5}$$

 \therefore Required value of fraction $A = 2 \times \frac{1}{5} = \frac{2}{5}$

75. Let $\frac{x}{y}$ be the fraction.

Then
$$x - y = 5$$
 (1)

$$\frac{x}{y+5} = \frac{x}{y} - \frac{5}{4} \tag{2}$$

$$\Rightarrow \frac{-x}{y+5} + \frac{x}{y} = \frac{5}{4}$$

$$\Rightarrow \qquad 4[x(y+5) - xy] = 5y(y+5) \tag{A}$$

Substitute value of y in (A) from (1), we get

$$4[x\{x\} - x(x-5)] = 5(x-5)x$$

$$\Rightarrow 4(x^2 - x^2 + 5x) = 5x^2 - 25x$$

$$\Rightarrow 20x + 25x = 5x^2$$

$$\Rightarrow 9x = x^2$$

$$\Rightarrow x = 9$$

$$\therefore (1) \Rightarrow y = x - 5 = 9 - 5 = 4$$

 \therefore Required fraction is $\frac{9}{4} = 2 \frac{1}{4}$

76. Let the fraction be $\frac{x}{y}$.

Then
$$\frac{x}{y+1} = \frac{1}{2} \tag{1}$$

$$\frac{x+1}{y} = 1\tag{2}$$

(1)
$$\Rightarrow 2x = y + 1$$

 $\Rightarrow 2x - y = 1$ (A)

(2)
$$\Rightarrow x + 1 = y$$

 $\Rightarrow y - x = 1$ (B)

Solving (A) and (B), we get

$$(A) + (B)$$
 \Rightarrow $x = 2$
 \therefore (B) \Rightarrow $y = x + 1 = 2 + 1 = 3$

$$\therefore \qquad \text{Required fraction} = \frac{2}{3}$$

Elementary Algebra I

ALGEBRAIC EXPRESSIONS: A number along with literal numbers and signs of fundamental operations is called an *algebraic expression*.

It may be **monomial** if it has only one term and **binominal** if it has two terms. It may be even polynomial if it has more than two terms. For example,

Monomial: +2x, -3y, etc.

Binomial: x + 2y, 2x + 3y, etc.

Polynomial: 2x + 3y + 4z, 3x - 2y + 5z, etc.

ADDITION AND SUBTRACTION OF POLYNOMIALS: Here the sum or difference of coefficients of like terms is performed. For example, if $3x^2 + 4y + 5z$ and $2x^2 - 3y + 4z$ are two polynomials, then

Sum of these two polynomials = $(3x^2 + 4y + 5z) + (2x^2 - 3y + 4z) = 5x^2 + y + 9z$.

Difference of these two polynomials = $(3x^2 + 4y + 5z) - (2x^2 - 3y + 4z) = x^2 + 7y + z$.

EXAMPLE 1 Find the sum of $(4x^2 + 3y - 2z)$, $(3x^2 - 4y + 5z)$ and $(5x^2 + 6y - 7z)$.

Solution Sum of polynomials = $(4x^2 + 3y - 2z) + (3x^2 - 4y + 5z) + (5x^2 + 6y - 7z)$ = $(4 + 3 + 5)x^2 + (3 - 4 + 6)y + (-2 + 5 - 7)z$ = $12x^2 + 5y - 4z$

EXAMPLE 2 If $f(x) = 2x^2 + 3y - 4z$, $g(x) = 5x^2 - 6y + 7z$ and $h(x) = 3x^2 - 8y + 5z$. Find f(x) + g(x) + h(x).

Solution $f(x) + g(x) + h(x) = 2x^2 + 3y - 4z + 5x^2 - 6y + 7z + 3x^2 - 8y + 5z$ = $\mathbf{10}x^2 - \mathbf{11}y + 8z$

FACTORIZATION OF POLYNOMIALS

For factorization of polynomials, factor theorem and remainder theorem are useful.

Factor Theorem: If (x-a) is a factor of the polynomial P(x), then the remainder obtained when P(x) is divided by (x-a) is given by P(a) = 0. For example, if (x+2) is a factor of $x^2 + 5x + 6$, then P(-2) = 0.

$$P(-2) = (-2)^2 + 5(-2) + 6 = 0$$
. Hence $(x + 2)$ is a factor of $x^2 + 5x + 6$.

EXAMPLE 3 Let $f(x) = x^3 - 12x^2 + 44x - 48$. Find out whether (x - 2) and (x - 3) are factors of f(x).

Solution

(a) If f(2) = 0, then (x - 2) is a factor of f(x) $f(2) = (2)^3 - 12(2)^2 + 44(2) - 48$

$$= 8 - 48 + 88 - 48$$

= 0

 \therefore (x-2) is a factor of f(x)

(b) If f(3) = 0, then (x - 3) is a factor of f(x)

$$f(3) = (3)^3 - 12(3)^2 + 44(3) - 48$$
$$= 27 - 108 + 132 - 48$$
$$= 3 \neq 0$$

 $f(3) \neq 0$, (x-3) is not a factor of f(x).

EXAMPLE 4 Find out whether (3x - 1) is a factor of $27x^3 - 9x^2 - 6x + 2$ by factor theorem.

Solution If (3x-1) is a factor of $f(x) = 27x^3 - 9x^2 - 6x + 2$, then $f\left(\frac{1}{3}\right) = 0$

$$f\left(\frac{1}{3}\right) = 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 2$$
$$= 1 - 1 - 2 + 2 = 0$$

 \therefore (3x - 1) is a factor of f(x).

Remainder Theorem: If P(x) is a polynomial in x and if it is divided by (x-a), then the remainder will be P(a). For example, when $P(x) = 3x^2 - 2x + 2$ is divided by (x-2), then the remainder is given by P(2).

Here
$$P(2) = 3(2)^2 - 2(2) + 2$$

= $3(4) - 4 + 2$
= $12 - 4 + 2 = 10$

EXAMPLE 5 Without using the division process, find the remainder when $x^3 + 4x^2 - 5x + 3$ is divided by x + 3.

Solution For x + 3 = 0, x = -3

:. Remainder will be
$$P(-3) = (-3)^3 + 4(-3)^2 - 5(-3) + 3$$

= $-27 + 36 + 15 + 3 = 27$

EXAMPLE 6 Find the remainder when $3x^3 - 4x^2 + 5x - 2$ is divided by (x + 3).

Solution Remainder will be P(-3)

$$P(-3) = 3(-3)^3 - 4(-3)^2 + 5(-3) - 2$$
$$= -81 - 36 - 15 - 2 = -134$$

EXAMPLE 7 If the expression $Px^3 + 4x^2 - 26$ and $2x^3 - 6x + P$ when divided by (x - 3) leave the same remainder, find the value of P.

Solution
$$P_{1}(x) = P(3)^{3} + 4(3)^{2} - 26$$

$$= 27P + 36 - 26$$

$$= 27P + 10$$

$$P_{2}(x) = 2(3)^{2} - 6(3) + P$$

$$= 54 - 18 + P$$

$$= 36 + P$$
∴
$$P_{1}(x) = P_{2}(x)$$

$$27P + 10 = 36 + P$$

$$27P - P = 36 - 10$$
⇒
$$26P = 26$$
∴
$$P = \frac{26}{26} = 1$$

EXAMPLE 8 Find the values of p and q when $px^3 + x^2 - 2x - q$ is exactly divisible by (x-1) and (x+1).

Solution When the given expression is exactly divisible by the given divisor, the remainder will be zero.

When it is divisible by (x-1),

$$P(1) = p(1)^{3} + (1)^{2} - 2(1) - q = 0$$

$$\Rightarrow \qquad p + 1 - 2 - q = 0$$

$$\Rightarrow \qquad p - q = 1$$
(1)

When it is divisible by (x + 1),

i.e.
$$P(-1) = p(-1)^3 + (-1)^2 - 2(-1) - q = 0$$
$$-p + 1 + 2 - q = 0$$
$$\Rightarrow \qquad p + q = 3 \tag{2}$$

Solving (1) and (2), we have

$$p = 2, q = 1$$

RULE

Conditions of Divisibility

1. $x^n + a^n$ is exactly divisible by (x + a) only when n is odd.

For example, $a^3 + b^3$ is exactly divisible by (a + b).

- 2. $x^n + y^n = (x + y) (x^{n-1} x^{n-2}y + x^{n-3} \cdot y^2 \dots + y^{n-1})$
- 3. $x^n + a^n$ is not exactly divisible by (x + a) when n is even.

For example, $a^4 + b^4$ is not exactly divisible by (a + b).

4. $x^n + a^n$ is never divisible by (x - a).

For example, $a^3 + b^3$ or $a^6 + b^6$ is not divisible by (a - b).

5. $x^n - a^n$ is exactly divisible by x + a when n is even.

For example, $x^4 - a^4$ is exactly divisible by x + a.

- 6. $x^n y^n = (x + y)(x^{n-1} x^{n-2} \cdot y + x^{n-3} \cdot y^2 \dots y^{n-1})$ when *n* is even.
- 7. $x^n a^n$ is exactly divisible by (x a) whether n is odd or even.

For example, $a^7 - b^7$ and $a^8 - b^8$ are exactly divisible by a - b.

8. $x^n - y^n = (x - y)(x^{n-1} + x^{n-2} \cdot y + \dots + y^{n-1})$ when *n* is either odd or even.

The 5th power of any single digit number has the same right-hand digit as the number itself.

 $x^3 - x$ is divisible by 3.

 $x^5 - x$ is divisible by 5.

 $x^7 - x$ is divisible by 7.

 x^{11} – x is divisible by 11

 $x^{13} - x$ is divisible by 13.

In general, if n is prime, then for any whole number, $x^n - x$ is divisible by n.

If n is odd, $n(n^2 - 1)$ is divisible by 24.

If n is an odd prime greater than 3, $n^2 - 1$ is divisible by 24. For example,

n = 11, $11^2 - 1 = 120 = 24 \times 5$, divisible by 24.

If n is odd, $2^n + 1$ is divisible by 3 and if n is even, $2^n - 1$ is divisible by 3.

If n is prime, then $n(n^4 - 1)$ is divisible by 30.

If *n* is odd, $2^{2n} + 1$ is divisible by 5 and if *n* is even, $2^{2n} - 1$ is divisible by 5.

If n is odd, $5^{2n} + 1$ is divisible by 13 and if n is even, $5^{2n} - 1$ is divisible by 13.

EXAMPLE 9 The expression $5^{2n} - 2^{3n}$ has a factor

(a) 3

(b) 7

(c) 10

(d) 17

(e) None of these

Solution $5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25)^n - (8)^n$

The above expression has a factor (25 - 8) = 17

17 is a factor of $5^{2n} - 2^{3n}$. Therefore, the answer is (d).

EXAMPLE 10 The last digit in the expansion of $(41)^n - 1$ when n is any positive integer is

(a) 2

(b) 1

(c) 0

(d) -1

(e) None of these

Solution The last digit in $(41)^n$ for any value of n is 1.

:. The last digit in $(41)^n - 1$ is 0. Hence the answer is (c).

EXAMPLE 11 Find the last two digits of the expression of $2^{12n} - 6^{4n}$ when n is any positive integer.

(e) None of these

Solution
$$(2^6)^{2n} - (6^2)^{2n} = (64)^{2n} - (36)^{2n}$$

:.
$$(64 + 36)$$
 is a factor of $2^{12n} - 6^{4n}$. (by rule 5)

i.e., last two digits of the expression must be **00** as the number is a multiple of 100. **Hence answer is (d).**

EXAMPLE 12 For all integral values of n, the expression $7^{2n} - 3^{3n}$ is a multiple of

Solution $7^{2n} - 3^{3n} = (7^2)^n - (3^3)^n$

$$= (49)^n - (27)^n$$

$$\therefore \qquad (49-27) = 22 \text{ is a factor of the expression } 7^{2n} - 3^{3n}. \quad \text{(by rule 7)}$$

Hence answer is (a).

EXAMPLE 13 What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make it exactly divisible by (3x - 1)?

Solution $3x - 1 = 0 \Rightarrow x = 1/3$

$$\text{Remainder is } f\left(\frac{1}{3}\right) = 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) - 5$$

$$= 1 - 1 - 2 - 5$$

$$= -7.$$

If we reduce remainder = -7 from the given expression, the expression will be exactly divisible by the given divisor.

EXAMPLE 14 What should be subtracted from $x^2 + 7x + 16$ to make it exactly divisible by (x + 2)?

Solution If $P(x) = x^2 + 7x + 16$ is divisible by (x + 2), P(-2) = 0

Here
$$P(-2) = (-2)^2 + 7(-2) + 16 = 6$$

 \div 6 is to be subtracted from the given polynomial.

➤ Theorem: For Zero of a Polynomial

Let $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$ be a polynomial with integral coefficients. If an integer k is a zero polynomial then k is a factor of a_n . For this value of k, f(k) = 0. Consider the following example:

$$f(y) = y^3 - 2y^2 + y + 4$$

Suppose k is an integral zero of polynomial f(y). Then by the theorem, k is a factor of a_n . Hence possible value of k are 1, -1, 2, -2, 4 and -4. Here we get f(-1) = 0 and f(k) for all other values are non zero.

 \therefore -1 is the zero of f(y).

EXAMPLE 15 Find integral zeros of the polynomial $f(x) = x^3 - 2x^2 + x + 4$.

Solution Let m be an integral zero of the polynomial f(x). Then by the theorem m is a factor of a_{n+1} , i.e., 4.

Hence possible values of m are 1, -1, 2, -2, 4 and -4.

Now.

$$f(1) = (1)^3 - 2(1)^2 + (1) + 4$$

$$= 1 - 2 + 1 + 4 = 4 \neq 0$$

$$f(-1) = (-1)^3 - 2(-1)^2 + (-1) + 4$$

$$= -1 - 2 - 1 + 4 = 0$$

(-1) is the zero of f(x).

Similarly, 2, -2, 4 and -4 are not the zeros of f(x). Thus, the only integral zero of f(x) is -1.

THEORY OF INDICES: The expression a^m means the product of m factors each equal to a. When m is a positive integer, it is called the exponent or index or the power of a.

• Any quantity raised to the power zero is always equal to 1.

For example,
$$a^0 = 1$$
 $(a \neq 0)$, $1^0 = 1$, $(-1)^0 = 1$, $\left(\frac{1}{2}\right)^0 = 1$

 Whenever any index or power is taken from the numerator to denominator or from the denominator to numerator, the sign of the power changes. For example,

$$a^{m} = \frac{1}{a^{-m}}; \frac{1}{a^{m}} = a^{-m};$$

$$\frac{a^m}{a^n} = a^m \cdot a^{-n} = a^{m-n}$$

LAW OF INDICES

In algebra, root problems are solved by laws of indices. For example,

$$\sqrt[2]{a} = a^{1/2}, \sqrt[4]{a} = a^{1/4}, \sqrt[4]{a^3} = a^{3/4}$$

RULE -

Laws of Indices:

 $1. \quad x^m \times x^n = x^{m+n}$

$$9 (\alpha^m)^n - \alpha^{mn}$$

$$m^n \qquad (m^n)$$

2.
$$(x^m)^n = x^{mn}$$
 3. $x^{m^n} = x^{(m^n)}$ 4. $(xy)^m = x^m \cdot y^m$

5.
$$\frac{x^m}{x^n} = x^{m-n}$$
, if $m > n$

5.
$$\frac{x^m}{x^n} = x^{m-n}$$
, if $m > n$ 6. $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$, if $n > m$

HCF: The highest common factor of two or more algebraic expressions can be determined by the following methods:

- 1. Factor method
- 2. Division method

In case of factor method, the factors of the given expressions are found separately. Then the maximum number of common factors are taken out and multiplied, the product becomes the HCF.

For example, find HCF of $8(x^2 - 5x + 6)$ and $12(x^2 - 9)$ is obtained as:

$$8(x^{2} - 5x + 6) = 4 \times 2 (x - 3) (x - 2)$$
$$12(x^{2} - 9) = 4 \times 3 (x - 3) (x + 3)$$
$$HCF = 4(x - 3)$$

The minimum multiplied quantities out of two or more algebraic expressions are called the LCM.

If only two expressions are given, first find their HCF by factor method or by division method, and apply the following formula to find their LCM:

$$LCM of two expressions = \frac{Product of two expressions}{Their HCF}$$

For example, LCM of $8(x^2 - 5x + 6)$ and $12(x^2 - 9)$ is obtained as:

LCM =
$$\frac{8(x^2 - 5x + 6) \times 12(x^2 - 9)}{4(x - 3)}$$
$$= 24(x - 2)(x + 3)(x - 3)$$
$$= 24(x^3 - 2x^2 - 9x + 18)$$



:.

1.	$27^{3n+1} \cdot 81^{-n}$
	$9^{n+5} \cdot 3^{3n-7}$

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) None of these

- 2. If x = 12, y = 4; then find the value of $(x + y)^{x/y}$. (a) 144
 - (b) 288
- (c) 256
- (d) 4096
- (e) None of these

- 3. If y varies as x and y = 7 when x = 2, find y when x = 5.
- (b) 9
- (d) 49
- (e) None of these
- 4. If x + 2 exactly divides $x^3 + 6x^2 + 11x + 6k$, find the value of k.
 - (a) 0

- (d) 2
- (e) None of these

5. What must be added to $\frac{x}{y}$ to make it $\frac{y}{x}$?

(a)
$$\frac{y^2 + x^2}{xy}$$
 (b) $\frac{y^2 - x^2}{xy}$ (c) $\frac{xy}{x^2 + y^2}$ (d) $\frac{xy}{y^2 - x^2}$ (e) None of these

(b)
$$\frac{y^2 - x^2}{xy}$$

(c)
$$\frac{xy}{x^2 + y^2}$$

d)
$$\frac{xy}{y^2-y^2}$$

- **6.** What is the square root of $(x^2 + 4x + 4)(x^2 + 6x + 9)$?
 - (a) $x^2 + 4x + 4$ (b) $x^2 + 6x + 9$ (c) $x^2 + 5x + 6$ (d) $x^2 + 5x + 4$

- (e) None of these

- 7. If $3^{x-y} = 27$ and $3^{x+y} = 243$, find the value of x.
 - (a) 3
- (b) 9
- (d) 2
- (e) None of these
- 8. If x = 9, $y = \sqrt{17}$, the value of $(x^2 y^2)^{-1/3}$ is equal to _____?
- (b) 4
- (d) 9
- (e) None of these

- 9. Find the value of $[x^{b+c}]^{b-c} \times [x^{c+a}]^{c-a} \times [x^{a+b}]^{a-b}$.
 - (a) 0
- (b) -1
- (d) 2
- (e) None of these

10.	Find the value of	$3^{n+4} - 6 \times 3^{n+1}$	
		3^{n+2}	•

(a) 3

(b) 2

(c) 5

(d) 7

(e) None of these

11. Find the least number which when divided by 3, 5, 6 and 11 in succession the remainders got were 1, 2, 4 and 9 respectively.

(a) 1688

(b) 6188

(c) 1866

(d) 6818

(e) None of these

12. Find the least number, which when divided by 11, 13, 17 and 19 in succession the remainders obtained were 7, 9, 14 and 15 respectively.

(a) 87455

(b) 84575

(c) 85745

(d) 84755

(e) None of these

13. Find the least number, which when divided by 2, 5, 7 and 11 in succession, remainders obtained were 1, 4, 6 and 10 respectively.

(a) 1358

(b) 5138

(c) 3158

(d) 1538

(e) None of these

14. Find the greatest 4 digit number, which when divided by 3, 7, 11 and 13 in succession, remainders obtained were 1, 5, 9 and 12 respectively.

(a) 7959

(b) 9579

(c) 5979

(d) 9759

(e) None of these

15. Find the greatest 4 digit number, which when divided by 2, 7, 11 and 13 in succession remainders got were 1, 6, 10 and 12 respectively.

(a) 6008

(b) 8060

(c) 8600

(d) 8006

(e) None of these

16. The values of a and b for which $3x^3 - ax^2 - 74x + b$ is a multiple of $x^2 + 2x - 24$ are:

(a) a = -5, b = 24

(b) a = 5, b = 24

(c) a = 13, b = 16

(d) a = -13, b = 16

(e) None of these

17. Find the values of a and b when $f(x) = 2x^3 + ax^2 - 11x + b$ is exactly divisible by (x-2)(x+3).

(a) a = 3, b = 6

(b) a = 3, b = -6

(c) a = -3, b = 6

(d) a = -3, b = -6

(e) None of these

18. If $f(x) = 4x^3 - 2x^2 + 5x - 8$ is divided by x - 2, what will be the remainder?

(a) 25

(b) 42

(c) 16

(d) 26

(e) None of these



1. (b)

2. (d)

3. (e)

4. (b)

5. (b)

6. (c)

7. (c)

8. (e)

11. (c)

12. (d)

13. (d)

14. (e)

15. (d)

16. (a)

17. (b) 18. (d)

9. (c) **10.** (d)



$^{\prime}$ Solutions with Necessary Explanation

1.
$$\frac{27^{3n+1} \cdot 81^{-n}}{9^{n+5} \cdot 3^{3n-7}} = \frac{(3^3)^{3n+1} \cdot (3^4)^{-n}}{(3^2)^{n+5} \cdot 3^{3n-7}} = \frac{3^{9n+3} \cdot 3^{-4n}}{3^{2n+10} \cdot 3^{3n-7}} = 3^{9n+3-4n-(2n+10+3n-7)} = 3^0 = 1$$

2. x = 12, y = 4

$$\therefore (x+y)^{x/y} = (12+4)^{12/4} = 16^3 = 4096.$$

3.
$$y \propto x$$
; $y = \frac{7}{2} \times 5 = \frac{35}{2}$

4.
$$P(x) = x^3 + 6x^2 + 11x + 6k$$

 $\therefore x + 2 \text{ is a factor of } P(x), P(-2) = 0.$

i.e.
$$P(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6k = 0$$

 $\Rightarrow \qquad -8 + 24 - 22 + 6k = 0$
 $\Rightarrow \qquad 6k = 8 + 22 - 24 = 6$
 $\therefore \qquad k = \frac{6}{6} = 1$

5. Let it be f(x, y)

Then
$$f(x, y) + \frac{x}{y} = \frac{y}{x}$$

$$f(x, y) = \frac{y}{x} - \frac{x}{y} = \frac{y^2 - x^2}{xy}$$

6.
$$\sqrt{(x^2+4x+4)(x^2+6x+9)} = \sqrt{(x+2)^2(x+3)^2} = (x+2)(x+3) = x^2+5x+6$$

7.
$$3^{x-y} = 27 = 3^3 \implies x - y = 3$$
 (1)

$$3^{x+y} = 243 = 3^5 \implies x + y = 5$$
 (2)

Solving (1) and (2)

$$x = 4$$

8.
$$x = 9$$
, $y = \sqrt{17}$

$$\therefore (x^2 - y^2)^{-1/3} = (9^2 - 17)^{-1/3} = (81 - 17)^{-1/3} = (64)^{-1/3} = 4^{-1} = \frac{1}{4}.$$

9.
$$[x^{b+c}]^{b-c} \cdot [x^{c+a}]^{c-a} \cdot [x^{a+b}]^{a-b} = x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2}$$

$$= x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)} = x^0 = \mathbf{1}$$

10.
$$\frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} = \frac{3^{n+4} - 2 \times 3^{n+2}}{3^{n+2}}$$
$$= \frac{3^{n+2} (3^2 - 2)}{3^{n+2}}$$
$$= 9 - 2 = 7$$

11. Each time, on division remainders got were less than the divisor by 2. Let the number be m.

$$m \div 3 = n$$
 remainder 1
 $n \div 5 = p$ remainder 2
 $p \div 6 = q$ remainder 4
 $q \div 11 = r$ remainder 9.
$$\Rightarrow \qquad q = 11r + 9 \qquad \Rightarrow \qquad p = 6q + 4$$
i.e. $p = 6(11r + 9) + 4$

$$\Rightarrow n = 5p + 2 \Rightarrow n = 5[6(11r + 9) + 4] + 2$$

$$m = 3n + 1 \Rightarrow m = 3 \times \{5[6(11r + 9) + 4] + 2\}$$

$$= 3 \times 5 \times 6 \times 11r + 3 \times 5 \times 6 \times 9 + 3 \times 5 \times 4 + 3 \times 2$$

= Last quotient × (Product of all divisors) + Last remainder

× (Product of divisors except last one) + Last but one remainder

× (Product of divisors except last two) + (Remainder before last but one)

 \times (Product of divisors except last three) + \cdots + Second remainder \times First divisor.

Required number can be obtained as shown below.

- \therefore Number is least, \therefore last quotient = 1.
- :. Required number = $1 \times 3 \times 5 \times 6 \times 11 + 9 \times 3 \times 5 \times 6 + 4 \times 3 \times 5 + 2 \times 3$ = 990 + 810 + 60 + 6 = 1866
- 12. Follow the above rule,

Required number =
$$1 \times 11 \times 13 \times 17 \times 19 + 15 \times 11 \times 13 \times 17 + 14 \times 11 \times 13 + 9 \times 11$$

= $46189 + 36465 + 2002 + 99 = 84755$

13. Required number =
$$1 \times 2 \times 5 \times 7 \times 11 + 10 \times 2 \times 5 \times 7 + 6 \times 2 \times 5 + 4 \times 2$$

= $770 + 700 + 60 + 8 = 1538$

14. Least number =
$$1 \times 3 \times 7 \times 11 \times 13 + 12 \times 3 \times 7 \times 11 + 9 \times 3 \times 7 + 5 \times 3 + 1$$

= $3003 + 2772 + 189 + 15 = 5979 + 1 = 5980$

Greatest number = $5980 + (3 \times 7 \times 11 \times 13) = 8983$

15. Least number =
$$1 \times 2 \times 7 \times 11 \times 13 + 12 \times 2 \times 7 \times 11 + 10 \times 2 \times 7 + 6 \times 2$$

= $2002 + 1848 + 140 + 12 = 4002$

:. Greatest number of 4 digit = 4002 + 2(2002) = 8006

16. Given
$$f(x) = 3x^3 - ax^2 - 74x + b$$

$$x^2 + 2x - 24 = (x + 6)(x - 4)$$

For (x + 6) to be factor of f(x), f(-6) = 0

$$f(-6) = 3(-6)^3 - a(-6)^2 - 74(-6) + b = -648 - 36a + 444 + b = 0$$

$$\Rightarrow -36a + b - 204 = 0$$
(1)

For (x-4) to be factor of f(x), f(4)=0

$$f(4) = 3(4)^3 - a(4)^2 - 74(4) + b = 0$$

$$\Rightarrow$$
 192 - 16a - 296 + b = 0

$$\Rightarrow \qquad -16a + b - 104 = 0 \tag{2}$$

$$(1) \Rightarrow b = 204 + 36a \tag{A}$$

$$(2) \Rightarrow b = 104 + 16a \tag{B}$$

Equating (A) and (B),

$$36a + 204 = 16a + 104$$
⇒
$$20a = -100$$
⇒
$$a = -5$$
∴ (B) ⇒
$$b = 104 + 16(-5)$$

$$= 104 - 80 = 24$$

$$a = -5, b = 24$$

17. Given $f(x) = 2x^3 + ax^2 - 11x + b$

For f(x) to be divisible by (x-2)(x+3),

$$f(2) = 0 \text{ and } f(-3) = 0$$

$$f(2) = 2(2)^3 + a(2)^2 - 11(2) = b = 0$$

$$\Rightarrow 16 + 4a - 22 + b = 0$$
i.e.
$$4a + b - 6 = 0$$

$$f(-3) = 0 \Rightarrow 2(-3)^3 + a(-3)^2 - 11(-3) + b = 0$$
(1)

$$\Rightarrow \qquad -54 + 9a + 33 + b = 0$$

$$\Rightarrow \qquad 9a + b - 21 = 0 \tag{2}$$

$$(1) \Rightarrow b = 6 - 4a \tag{A}$$

$$(2) \Rightarrow b = 21 - 9a \tag{B}$$

Equating (A) and (B),

$$6 - 4a = 21 - 9a$$

$$\therefore$$
 9*a* - 4*a* = 21 - 6 = 15

$$\therefore$$
 5a = 15

$$\therefore \qquad \qquad a = \frac{15}{5} = 3$$

$$\therefore \qquad (A) \Rightarrow b = 6 - 4 \times 3 = -6$$

$$\therefore a = 3, b = -6$$

18. Given $f(x) = 4x^3 - 2x^2 + 5x - 8$. If f(x) is divided by (x-2), remainder will be f(2).

$$\therefore \quad \text{remainder} = f(2) = 4(2)^3 - 2(2)^2 + 5(2) - 8$$
$$= 32 - 8 + 10 - 8 = 26$$

Elementary Algebra II

LINEAR EQUATIONS

Equations with One Variable: A statement of equality which contains only one unknown quantity or variable is called an equation. The graph of such equation is a straight line, whose abscissa x or ordinate y satisfies the given equation.

Root or solution: Any value of the variable that makes the statement of equation true is called the *root of the equation*.

EXAMPLE 1 Ashok's father is four times as old as him. Four years ago, his father was six times as old as he was then. Find their present ages.

Solution Let the present age of Ashok be x years, and the present age of his father be 4x years.

Four years ago, the age of father was 4x - 4 and age of Ashok was (x - 4).

i.e.
$$4x-4=6(x-4)$$

$$\Rightarrow 4x-4=6x-24$$
i.e.
$$2x=20$$

$$\therefore x=10 \text{ years}$$
i.e. Age of Ashok = 10 years
$$Age \text{ of father} = 40 \text{ years}$$

QUADRATIC EQUATION: An expression of form $ax^2 + bx + c$, $(a \ne 0)$ where a, b and c are real numbers is called a quadratic expression in x. Corresponding equation of expression is $ax^2 + bx + c = 0$. In other words, an equation in which the highest power of the variable is 2, is called a quadratic equation.

For example, the equation of the type $3x^2 + 6x + 9 = 0$ denotes a quadratic equation.

SOLVING QUADRATIC EQUATIONS: A quadratic equation can be solved either by factor method or by formula. When solved, it gives two values for the variable, which are called roots of the equation.

By Factor Method: In this method, find the factors of the given equation making the right-hand side equal to zero. Then by equating the factors to zero, we get values of the variable.

For example, the factors of equation $(x^2 - 5x + 6) = 0$ is obtained by the factor method. On factorizing, (x - 2)(x - 3) = 0

i.e. (x-2)(x-3)

and

$$x-2=0$$
 \Rightarrow $x=2$
 $x-3=0$ \Rightarrow $x=3$

 \therefore Solution is x = 2 and x = 3.

By Formula: $ax^2 + bx + c = 0$ is the standard form of a quadratic equation. The roots of the equation are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $b^2 - 4ac$ is called the *discriminant (D)*. The two values of x obtained from a quadratic equation are called the *roots of the equation*.

If α and β are the roots of the equation, the sum of roots of the equation $\alpha + \beta = \frac{-b}{a}$

and product of roots = $\alpha \beta = \frac{c}{a}$.

For a quadratic equation $ax^2 + bx + c = 0$,

- (i) The roots will be equal if $b^2 4ac = 0$.
- (ii) The roots will be unequal and real if $b^2 4ac > 0$.
- (iii) The roots will be unequal and imaginary if $b^2 4ac < 0$.

FACTORIZATION OF QUADRATIC EQUATIONS: The quadratic polynomial $ax^2 + bx + c$ can be factorized only if there exists two numbers α and β such that

(i) Sum of roots =
$$\alpha + \beta = \frac{-b}{a}$$

(ii) Product of roots =
$$\alpha\beta = \frac{c}{a}$$

EXAMPLE 2 Factorize $2x^2 + 11x + 5$.

Solution Here a = 2; b = 11; c = 5

$$\therefore \quad \text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = \frac{-11}{2}$$

Product of roots =
$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

 $2(\alpha + \beta) = -11$ (1)

$$2\alpha \times 2\beta = 10\tag{2}$$

$$2\alpha = -1 \text{ and } 2\beta = -10$$
or
$$\alpha = \frac{-1}{2} \text{ and } \beta = -5$$
i.e.
$$2x^2 + 11x + 5 = \left(x + \frac{1}{2}\right)(x+5) \times 2 = (2x+1)(x+5)$$

CONDITIONS FOR FACTORIZATION OF A QUADRATIC EQUATION: All quadratic equations cannot be factorized. For that, following conditions are to be tested:

- (i) If $b^2 4ac > 0$, then the quadratic equation can be factorized.
- (ii) If $b^2 4ac < 0$, then the quadratic equation cannot be factorized.

Some important formulae that are used in basic operations and in finding the factors of an expression are given below:

lmportant Formulae

- 1. $a^2 b^2 = (a + b)(a b)$
- 2. $(a + b)^2 = a^2 + 2ab + b^2$
- 3. $(a-b)^2 = a^2 2ab + b^2$

- 4. $a^3 + b^3 = (a + b) (a^2 ab + b^2)$
- 5. $a^3 b^3 = (a b) (a^2 + ab + b^2)$
- 6. $a^3 + b^3 = (a + b)^3 3ab (a + b)$
- 7. $a^3 b^3 = (a b)^3 + 3ab (a + b)$
- 8. $(a + b)^2 = (a b)^2 + 4ab$

- 9. $(a-b)^2 = (a+b)^2 4ab$
- 10. $a^3 + b^3 + c^3 3abc = (a + b + c)(a^2 + b^2 + c^2 ab bc ca)$

$$= \frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

- 11. $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(b+c)(c+a)(a+b)$
- 12. $a^2 + b^2 = (a+b)^2 2ab$
- 13. $a^2 + b^2 = (a b)^2 + 2ab$
- 14. If a + b + c = 0 then the value of $a^3 + b^3 + c^3$ is 3abc
- 15. $x^2 + (a + b)x + ab = (x + a)(x + b)$
- 16. $a^2(b+c) + b^2(c+a) + c^2(a+b) + 3abc = (a+b+c)(ab+bc+ca)$
- 17. ab(a + b) + bc(b + c) + ca(c + a) + 2abc = (a + b)(b + c)(c + a)
- 18. $a^2(b-c) + b^2(c-a) + c^2(a-b) = -(a-b)(b-c)(c-a)$
- 19. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Conditions for Maximum and Minimum Values of Quadratic Expression

 $ax^2 + bx + c$ (= y suppose) For minimum value of y, a > 0.

Minimum value of $y = \frac{-D}{4a}$ (which is possible when $x = \frac{-b}{2a}$)

For maximum value of y, a < 0

Maximum value of $y = \frac{-D}{4a}$ (which is possible when $x = \frac{b}{2a}$)

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Solution Minimum value of the expression = $\frac{-D}{4a} = \frac{-49}{4 \times 2} = -6.125$

$$x = \frac{-b}{2a} = \frac{-(-5)}{2 \times 2} = \frac{5}{4} = 1.25$$

EXAMPLE 4 A certain number of tennis balls were purchased for $\stackrel{?}{\stackrel{\checkmark}}$ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by $\stackrel{?}{\stackrel{\checkmark}}$ 15. Find the number of balls purchased.

Solution Let the number of balls purchased be x.

$$\therefore \quad \text{Cost price of a ball} = \frac{450}{r} = \frac{450}{r+5} + 15$$

$$\Rightarrow$$
 $x = 10$ or -15

Cost price of a ball =
$$\frac{450}{10}$$
 = 45

 \therefore Number of balls purchased = 10

FORMULATING A QUADRATIC EQUATION FROM GIVEN ROOTS: Whenever the roots of the quadratic equations are given, the quadratic equation is given by

$$x^2$$
 – (Sum of roots) x + Products of roots

For example, if roots are given as 2 and 3, then the quadratic equation will be

$$x^2 - (3+2)x + 3 \times 2 = 0$$

i.e.

$$x^2 - 5x + 6 = 0$$



Notes:

- 1. A quadratic equation $ax^2 + bx + c = 0$ will have reciprocal roots, if a = c.
- 2. When a quadratic equation $ax^2 + bx + c = 0$ has one root equal to zero, then c = 0.
- 3. When the roots of the quadratic equation $ax^2 + bx + c = 0$ are negative reciprocals of each other, then c = a.
- 4. When both the roots are equal to zero, b = 0 and c = 0.
- 5. When one root is infinite, then a = 0 and when both the roots are infinite, then a = 0 and b = 0.
- 6. When the roots are equal in magnitude but opposite in sign, then b = 0.
- 7. If two quadratic equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ have a common root (i.e. one root common), then $(bc_1 b_1c)$ $(ab_1 a_1b) = (ca_1 c_1a)^2$.
- 8. If they have both the roots common, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.
- 9. The square root of any negative number is an imaginary number.

For example,
$$\left(\sqrt{-3}, \sqrt{-5}, \sqrt{-25}, \sqrt{-0.04}, \sqrt{-3^4}, \sqrt{-\frac{17}{15}} \text{ etc.}\right)$$

For problem solving purposes, take $\sqrt{-1} = i$

So $\sqrt{-25} = \sqrt{(-1) \times 25} = \sqrt{-1} \times \sqrt{25} = 5i$, which is an imaginary number.

EXAMPLE 5 Find the discriminant of the following quadratic equations. Also find the nature of roots of the equation. Verify them.

(i)
$$x^2 - 5x + 4 = 0$$

(ii)
$$3x^2 + 4x - 3 = 0$$

(iii)
$$x^2 + 5x + 6 = 0$$

(iv)
$$-5x^2 + 7x = 0$$

(v)
$$2x^2 - 6x + 7 = 0$$
.

Solution

(i) In $x^2 - 5x + 4 = 0$, a = 1, b = -5, c = 4. $b^2 - 4ac = (-5)^2 - 4 \times 1 \times 4 = 25 - 16 = 9 > 0$ and a perfect square.

.. Roots will be distinct and rational numbers.

Here roots are:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{9}}{2 \times 1} = \frac{5 \pm 3}{2} = \frac{5 + 3}{2} \text{ or } \frac{5 - 3}{2}$$

$$= 4 \text{ or } 1$$

(ii) In $3x^2 + 4x - 3 = 0$, a = 3, b = 4, c = -3

:. $b^2 - 4ac = (4)^2 - 4 \times 3 \times (-3) = 16 + 36 = 52 > 0$ and not a perfect square.

.. Roots will be distinct and irrational numbers.

Roots are $\frac{-4 \pm \sqrt{52}}{2 \times 3} = \frac{-4 \pm 2\sqrt{13}}{6} = \frac{-4 + 2\sqrt{13}}{6} \text{ or } \frac{-4 - 2\sqrt{13}}{6}$ $= \frac{-2 + \sqrt{13}}{2} \text{ or } \frac{-2 - \sqrt{13}}{2}$

(iii) In $x^2 + 5x + 6 = 0$, a = 1, b = 5, c = 6

:. $b^2 - 4ac = (5)^2 - 4 \times 1 \times 6 = 25 - 24 = 1 > 0$, a perfect square

:. Roots will be distinct and rational numbers.

∴ Roots are
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{1}}{2 \times 1} = \frac{-5 \pm 1}{2}$$

= $\frac{-5 + 1}{2}$ or $\frac{-5 - 1}{2}$
= -2 or -3

(iv) $\text{In } -5x^2 + 7x = 0$, a = -5, b = 7, c = 0

 $b^2 - 4ac = 7^2 - 4(-5)$ (0) = $7^2 = 49 > 0$, a perfect square.

.. Roots will be distinct and rational numbers.

Here roots are
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{49}}{2 \times (-5)} = \frac{-7 \pm 7}{-10}$$
$$= \frac{-7 + 7}{-10} \text{ or } \frac{-7 - 7}{-10}$$
$$= \mathbf{0} \text{ or } \frac{7}{5}$$

(v) In
$$2x^2 - 6x + 7 = 0$$
, $a = 2$, $b = -6$, $c = 7$

$$b^2 - 4ac = (-6)^2 - 4(2)$$
 (7) = $36 - 56 = -20 < 0$

.. Roots will be imaginary.

EXAMPLE 6 Form a quadratic equation whose roots are (i) 3 and -5 (ii) 2 and 7; also verify them.

Solution

(i) The quadratic equation will be

$$x^{2} - (3 + (-5))x + 3 \times (-5) = 0$$
$$x^{2} + 2x - 15 = 0$$

(ii) The quadratic equation will be

$$x^{2} - (2+7)x + 2 \times 7 = 0$$
$$x^{2} - 9x + 14 = 0$$

EXAMPLE 7

i.e.,

i.e.,

(i) If one of the roots of the equation $x^2 - 19x + 88 = 0$ is 8, find the other root.

(ii) If one of the roots of the equation $4x^2 - 27x + 18 = 0$ is 6, find the other root.

Solution

(i) Here the product of roots $\frac{c}{a} = \frac{88}{1} = 88$

Given one of the roots = 8

$$\therefore \text{ Second root} = \frac{88}{8} = 11$$

or

Sum of the roots
$$\frac{-b}{a} = \frac{19}{1} = 19$$

 \therefore One root is 8, second root will be 19 - 8 = 11

(ii) Here the product of roots $\frac{c}{a} = \frac{18}{4} = \frac{9}{2}$

Given one of the roots = 6

$$\therefore \text{ Second root} = \frac{9/2}{6} = \frac{3}{4}$$

or

Sum of the roots =
$$\frac{-b}{a} = \frac{27}{4}$$

 \therefore One of the roots = 6,

Second root will be
$$\frac{27}{4} - 6 = \frac{3}{4}$$

EXAMPLE 8 Find the roots of the equations (i) $2x^2 + 3x - 5 = 0$ (ii) $x^2 - 8x + 7 = 0$.



Note: Whenever we get a + b + c = 0, one of the roots will always be 1.

Solution

 \Rightarrow

(i) Here a + b + c = 2 + 3 + (-5) = 0i.e. one of the roots is 1

Here the sum of the roots = $\frac{-b}{a} = \frac{-3}{2}$

$$\therefore \text{ Other root} = \frac{-3}{2} - 1 = \frac{-5}{2}$$

So the required roots are 1 and $\frac{-5}{2}$

(ii) Here a + b + c = 1 + (-8) + 7 = 0i.e. one of the roots is 1.

Here, the sum of the roots = $\frac{-b}{a} = \frac{8}{1} = 8$

 \therefore The other root = 8 - 1 = 7

So the required roots are 1 and 7.

EXAMPLE 9 A motorcycle travels 20 km an hour faster than a cycle over a journey of 600 km. The cycle takes 15 hours more than the motorcycle. Find their speed.

Solution Let the speed of cycle be x km/h

:. Speed of motorcycle =
$$(20 + x) \text{ km/h}$$

Time taken by the cycle =
$$\frac{\text{Distance}}{\text{Speed}} = \frac{600}{x}$$

Time taken by motorcycle = $\frac{600}{r+20} = \frac{600}{r} - 15$

i.e.
$$\frac{600}{x+20} = \frac{600-15x}{x}$$

i.e.
$$\frac{40}{x+20} = \frac{40-x}{x}$$

$$\Rightarrow \qquad 40x = (40 - x)(x + 20)$$

$$\Rightarrow 40x = 40x + 800 - x^2 - 20x$$

$$x^{2} + 20x - 800 = 0$$

$$x = \frac{-20 \pm \sqrt{(20)^{2} - 4 \times 1 \times -800}}{2 \times 1}$$

$$= \frac{-20 \pm \sqrt{400 + 4 \times 800}}{2}$$

$$= \frac{-20 \pm 60}{2} = \frac{-20 + 60}{2} \text{ or } \frac{-20 - 60}{2}$$
$$= \frac{40}{2} \text{ or } \frac{-80}{2}$$
$$= 20 \text{ or } -40$$

 $\therefore x = 20 \text{ km/h} \text{ and } x + 20 = 40 \text{ km/h}$

 \therefore Speed of cycle = 20 km/h

Speed of motorcycle = x + 20 = 40 km/h.

EXAMPLE 10 Solve the equation $3^{2x+1} - 3^x = 3^{x+3} - 3^2$.

Solution
$$3^{2x+1} - 3^x = 3^{x+3} - 3^2$$

$$\Rightarrow 3^{2x} \times 3^1 - 3^x = 3^x \times 3^3 - 3^2$$
i.e.
$$3 \times (3^x)^2 - 3^x = 3^3 \times 3^x - 3^2$$
Put
$$3^x = k$$
Now
$$3k^2 - k = 27k - 9$$
i.e.
$$3k^2 - 28k + 9 = 0$$

$$\therefore k = \frac{28 \pm \sqrt{(-28)^2 - 4(3)(9)}}{2 \times 3}$$

$$= \frac{28 \pm \sqrt{(28)^2 - 108}}{6} = \frac{28 \pm \sqrt{676}}{6}$$

$$= \frac{28 \pm 26}{6} = \frac{28 + 26}{6} \text{ or } \frac{28 - 26}{6}$$

$$= 9 \text{ or } \frac{1}{3}$$

When $3^x = 9$, x = 2 $3^x = \frac{1}{2}$, $3^x = 3^{-1}$ or x = -1

 \therefore Roots are 2 and -1.

EXAMPLE 11 For the equation $-9x^2 + 12x - m = 0$ be a perfect square of a linear expression, what will be the value of m?

Solution A linear expression is of the form ax + b = 0, $a \ne 0$, a and b are constant. A quadratic equation whose roots are a and b is given by (x - a) (x - b) = 0.

If $\alpha = \beta$, then the equation becomes $(x - \alpha)^2 = 0$. For both the roots to be equal, we have D = 0.

 $\therefore -9x^2 + 12x - m = 0$ can be a perfect square of a linear expression if discriminant = 0

or
$$b^2 - 4ac = (12)^2 - 4(-9)(-m) = 0$$

 $\Rightarrow 144 - 36 m = 0$

$$\Rightarrow \qquad m = \frac{144}{36} = \mathbf{4}$$

(Examples 12–16) In each question one or more equation(s) is /are provided. On the basis of these you have to find out the relation between p and q.

Give answer (1) if p = q

Give answer (2) if p > q

Give answer (3) if q > p

Give answer (4) if $p \ge q$ and

Give answer (5) if $q \ge p$

12. I.
$$2p + \frac{5}{2} = p + 3$$

II.
$$q - \frac{5}{2} = 1$$

13. I.
$$\frac{p}{2} - \frac{p}{3} = 1$$

II.
$$q^2 + 36 = 12q$$

14. I.
$$\frac{p}{5} - \frac{2}{7} = 0$$

II.
$$q^2 = 2q - 1$$

15. I.
$$p^2 + 3 = 12$$

II.
$$3q - 5 = 1 + q$$

16. I.
$$7p^2 - 8p + 1 = 0$$

II.
$$\frac{5q}{2} - \frac{q}{4} = \frac{1}{8}$$

Solution

12. I.
$$2p - p = 3 - \frac{5}{2} \implies p = \frac{1}{2}$$

II.
$$q = 1 + \frac{5}{2} = 3.5$$
 Therefore $q > p$.

∴ Answer (3)

13. I.
$$\frac{p}{2} - \frac{p}{3} = 1 \implies \frac{p}{6} = 1 : p = 6$$

II.
$$q^2 - 12q + 36 = 0 \implies (q - 6)^2 = 0$$

$$\therefore q = 6$$
 Therefore $p = q$.

:. Answer (4)

14. I.
$$\frac{p}{5} - \frac{2}{7} = 0 \implies p = \frac{5 \times 2}{7} = \frac{10}{7}$$

II.
$$q^2 - 2q + 1 = 0 \implies (q - 1)^2 = 0 \implies q = 1$$

Therefore p > q.

∴ Answer (2)

15. I.
$$p^2 = 12 - 3 = 9$$
 : $p = \pm 3$

II.
$$3q - 5 = 1 + q$$

$$2q = 6$$
 : $q = 3$

Therefore $q \ge p$.

∴ Answer (5)

16. I.
$$p = \frac{8 \pm \sqrt{8^2 - 4(7)(1)}}{2 \times 7}$$

 $= \frac{8 \pm \sqrt{64 - 28}}{2 \times 7} = \frac{8 \pm 6}{2 \times 7}$
 $= \frac{8 + 6}{14} \quad \text{or} \quad \frac{8 - 6}{14}$
 $= \mathbf{1} \quad \text{or} \quad \frac{1}{7}$
II. $\frac{5q}{2} - \frac{q}{4} = \frac{1}{8}$
 $\Rightarrow \frac{9q}{4} = \frac{1}{8} \Rightarrow q = \frac{1}{18} \therefore p > q$

(Examples 17-21) In each question one or more equation(s) is/are provided. On the basis of these, you have to find out relation between p and q.

Give answer (1) if p = q

:. Answer (2)

(2) if
$$p > q$$

(3) if
$$q > p$$

(4) if
$$p \ge q$$
 and

(5) if
$$q \ge p$$

17. (i)
$$\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$$

18. (i)
$$p-7=0$$

(ii)
$$3q^2 - 10q + 7 = 0$$

19. (i)
$$4p^2 = 16$$

(ii)
$$q^2 - 10q + 25 = 0$$

19. (i)
$$4p^- = 16$$
 (ii) $q^- - 10q + 25 = 20$. (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$

(ii)
$$q^2 - 2q + 1 = 0$$

21. (i)
$$q^2 - 11q + 30 = 0$$
 (ii) $2p^2 - 7p + 6 = 0$

(ii)
$$2p^2 - 7p + 6 = 0$$

Solution

17. (i)
$$\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$$

 $\Rightarrow 3p = 13q$ or $\frac{p}{q} = \frac{13}{3}$

$$\therefore p > q$$
. The answer is (2).

18. (i)
$$p-7=0 \implies p=7$$
; (ii) $3q^2-10q+7=0$
 $\Rightarrow 3q(q-1)-7q+7=0 \implies (3q-7)(q-1)=0$
 $\Rightarrow q=\frac{7}{3} \text{ or } 1$
 $\therefore p > q$.

The answer is (2).

19. (i)
$$4p^2 = 16 \implies p = \pm 2$$
; (ii) $q^2 - 10q + 25 = 0 \implies (q - 5)^2 = 0 \implies q = 5$. $\therefore q > p$.

The answer is (3).

20. (i)
$$4p^2 - 5p + 1 = 0$$

$$\Rightarrow \qquad p = \frac{5 \pm \sqrt{5^2 - 4 \times 4 \times 1}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{8}$$

$$= \frac{5 \pm 3}{8} = \frac{5 + 3}{8} \quad \text{or} \quad \frac{5 - 3}{8}$$

$$= 1 \quad \text{or} \quad \frac{1}{4}$$

(ii)
$$q^2 - 2q + 1 = 0 \implies (q - 1)^2 = 0 \implies q = 1$$

 $\therefore q \ge p$.

The answer is (5).

21. (i)
$$q^2 - 11q + 30 = 0 \implies (q - 6)(q - 5) = 0 \implies q = 6$$
 or 5

(ii)
$$2p^2 - 7p + 6 = 0$$

$$\Rightarrow p = \frac{7 \pm \sqrt{7^2 - 4 \times 2 \times 6}}{2 \times 2} = \frac{7 \pm \sqrt{49 - 48}}{4}$$
$$= \frac{7 \pm 1}{4} = \frac{7 + 1}{4} \quad \text{or} \quad \frac{7 - 1}{4} = 2 \quad \text{or} \quad \frac{3}{2}$$

$$\therefore q > p$$
.

The answer is (3).

Conditions for the maximum and the minimum values of the quadratic expression $ax^2 + bx + c$ (= y, suppose).

For the minimum value of y, the condition is a > 0 (if a < 0, y has no minimum value).

The minimum value of $y = \frac{-D}{4a}$ which is possible when $x = \frac{b}{2a}$.

EXAMPLE 22 If x be real, find the maximum value of $-2x^2 + x + 3$ and also find the corresponding value of x.

Solution The corresponding equation is $-2x^2 + x + 3 = 0$.

$$D = (1)^2 - 4(-2)(3) = 25.$$

 \therefore The required maximum value of the given expression = $\frac{-D}{4a} = \frac{-25}{4 \times (-2)} = \frac{25}{8}$ and

corresponding value of $x = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4}$

Explanation:
$$-2x^2 + x + 3 = y \tag{1}$$

or $-2x^2 + x + (3 - y) = 0$

$$D = (+1)^2 - 4(-2)(3 - y)$$

$$= 1 + 8(3 - y) = 1 + 24 - 8y = 25 - 8y$$

Given that *x* is real, so $D \ge 0$ or $25 - 8y \ge 0$ or $25 \ge 8y$.

$$\therefore y \le \frac{25}{8}$$
 i.e. maximum value of y is $\frac{25}{8}$.

Substituting
$$y = \frac{25}{8}$$
 in (1), we get $-2x^2 + x + 3 = \frac{25}{8}$

$$\Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow$$
 $(4x-1)^2 = 0$ or $(4x-1) = 0$

$$\therefore x = \frac{1}{4}.$$

EXAMPLE 23 A certain number of balls were purchased for $\stackrel{?}{\stackrel{\checkmark}}$ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by $\stackrel{?}{\stackrel{\checkmark}}$ 15. Find the number of balls purchased.

(a) 15

(b) 20

(c) 10

(d) 25

(e) None of these

Solution Let the number of balls purchased be x. Then comparing price of balls, in both conditions

$$\frac{450}{x} = \frac{450}{x+5} + 15$$
⇒
$$\frac{30}{x} = \frac{30}{x+5} + 1 = \frac{30+x+5}{x+5} = \frac{35+x}{x+5}$$
i.e.
$$30(x+5) = x(35+x)$$
⇒
$$30x + 150 = 35x + x^2$$
⇒
$$x^2 + 5x - 150 = 0$$
i.e.
$$(x+15)(x-10) = 0$$
∴
$$x = 10 \text{ or } -15$$

 \cdot No. of balls is a +ve value, x = 10.

∴ Cost price of a ball =
$$\frac{450}{r} = \frac{450}{10} = ₹ 45$$

When we get 5 balls more for the same amount, the cost price of a ball = $\frac{450}{x+5}$ which is less by 15 compared to the actual cost price.



Short cut:

$$\frac{450}{x} = \frac{450}{x+5} + 15$$

$$\frac{30}{x} = \frac{30}{x+5} + 1$$

 \Rightarrow

Put the value of x such that the ratio on the left side is greater then the right by 1.

 $\therefore x = 10 \text{ will satisfy.}$

EXAMPLE 24 A businessman knows that the price of a commodity will increase by ₹ 5 per packet. He bought some packets of this commodity for ₹ 4500. If he bought this packet on the new price then he gets 10 packets less. What is the number of packets bought by him?

(a) 90

(b) 100

(c) 50

(d) 125

(e) None of these

Solution Let the businessman bought x packets.

Then the cost price of a packet = $\frac{4500}{x}$.

When he get 10 packets less then cost price of a packet = $\frac{4500}{r-10}$

Here

$$\frac{4500}{x} + 5 = \frac{4500}{x - 10}$$

From the choices given above, number x and x-10 both should divide 4500 completely. ∴ 100 is the correct answer.



EXERCISES

1. Roots of $2kx^2 + 5kx + 2 = 0$ are equal if k is equal

(a) 9/25

(b) 16/25

(c) 7/25

(d) 18/25

(e) None of these

2. The area of a rectangle is same as that of a circle of radius $\frac{\sqrt{35}}{11}$ cm. If the length of the rectangle exceeds its breadth by 3 cm, find the dimensions of rectangle.

(a) 6×3 cm

(b) 5×2 cm

(c) 7×4 cm

(d) 4×1 cm

(e) None of these

3. The surface area of a pipe, open at both ends is equal to 628 m². Difference between its radius and its length is 15 m. If the pipe was closed at one end, what amount of water it can hold?

(a) 1750 m^3 (b) 7150 m^3 (c) 1570 m^3 (d) 7510 m^3

(e) None of these

4. The roots of $2kx^2 + 5kx + 2 = 0$ are equal if k is equal to ______.

(b) -4/5

(c) 16/25

(d) -16/25

(e) None of these

5. What is the condition that one of the roots of the equation is double the other in $ax^2 + bx + c = 0?$

(a) $ab^2 = 2ac$ (b) $b^2 = 18ac$ (c) $2b^2 = 9ac$ (d) $b^2 = 4ac$ (e) None of these

6. The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero.

(a) $a^2 + b^2$ (b) $-(a^2 + b^2)$ (c) $\frac{1}{2}(a^2 + b^2)$ (d) $-\frac{1}{2}(a^2 + b^2)$ (e) None of these

7. Find the roots of the equation, $\sqrt{3y+1} = \sqrt{y-1}$.

(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) 1

(d) 0

(e) None of these

8. If α and β are the roots of the equation $x^2 + 3\alpha x + c = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value of α .

(a) $\sqrt{\frac{2+5c}{9}}$ (b) $\sqrt{\frac{5+2c}{9}}$ (c) $\sqrt{\frac{2-5c}{9}}$ (d) $\sqrt{\frac{5-2c}{9}}$ (e) None of these

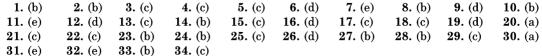
	(a) 0	(b) −1	(c) +1	(d) ±1	(e) None of these						
10.	If $x + \frac{1}{x} = 2$, then	the value of x^2 +	$\frac{1}{1^2}$ will be								
	(a) 1		(c) 3	(d) 4	(e) None of these						
11.	If $x + \frac{1}{x} = 3$, then	the value of x^8 +	will be								
	11. If $x + \frac{1}{x} = 3$, then the value of $x^8 + \frac{1}{x^8}$ will be										
12	(a) 3 If $x + y = 3$, $xy = 2$	(b) 9 then find the val	(c) 27	(d) 81	(e) None of these						
14.	(a) 2	(b) 3	(c) 4	(d) 7	(e) None of these						
13.	If $x - \frac{1}{x} = \sqrt{21}$, th	en the value of $\left(x\right)$		ll be	`,						
	(a) 151	(b) 511	(c) 115	(d) 165	(e) None of these						
14.	If $a+b+c=0$, th										
	(a) 2 <i>abc</i>	(b) 3 <i>abc</i>	(c) $a^2b^2c^2$	(d) $2a^2b^2c^2$	(e) None of these						
15.	If $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, $y = \frac{1}{3}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$ find the va	lue of $x^2 + y^2$.								
	(a) 7		(c) 14	(d) 49	(e) None of these						
16.	Find the combined	d product of $(a^4 + b^4)$	$(b^4)(a^2+b^2)(a+b)$	(a-b)							
	(a) $a^4 - b^4$	(b) $a^4 + b^4$	(c) $a^8 + b^8$	(d) $a^8 - b^8$	(e) None of these						
17.	Find the value of	$x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x^3}$	$\frac{1}{x} = a$.								
			(c) $a^3 + 3a$	(d) $3a^3 + a$	(e) None of these						
18.	If $x = 2^{1/3} - 2^{-1/3}$, f	ind value of $2x^3$ +	6 <i>x</i> .								
	(a) 1	(b) 2	(c) 3	(d) 4	(e) None of these						
19.	Find the value of	$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{1}{(a-b)^2}$	$\frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-b)^2}{(a-b)^2}$	$\frac{(-c)^2}{(a)(b-c)}$.							
		(b) 1		(d) 3	(e) None of these						
			` ,	` ,							
20.	If $a+b+c=0$, th	en find the value	of $\frac{1}{b^2 + c^2 - a^2} + \frac{1}{c^2}$	$\frac{1}{a^2 + a^2 - b^2} + \frac{1}{a^2 + b^2}$	$\frac{1}{c^2-c^2}$.						
	(a) 0	(b) 1	(c) 2	(d) 3	(e) None of these						
21.	If $x + y + z = 0$, the	en find the value o	of $\frac{(x+y)(y+z)(z)}{xyz}$	$\frac{(x+x)^2}{2}$.							
	(a) 0	(b) 1	(c) -1	(d) 2	(e) None of these						
22 .	If $a + b + c = 0$, th	en find the value	of $\frac{a^4 + b^4 + c^4}{b^2c^2 + c^2a^2 + a^2b}$	₃ 2 ·							
	(a) 0		(c) 2	(d) 3	(e) None of these						

9. If α and β are the roots of the equation $x^2 + 3\alpha x + 2\alpha^2 = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value

of a.

23. If x + y = 2z, find the value of $\frac{x}{x-z} + \frac{z}{y-z}$. (a) 0 (b) 1 (d) 3 (e) None of these **24.** If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$ then find value of $z + \frac{1}{y}$. (a) 0 (d) 3 (e) None of these **25.** If a = b = c, then find the value of $\frac{(a+b+c)^2}{a^2+b^2+c^2}$. (d) 4 (e) None of these **26.** Find the value of $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$, if a = x + y, b = x - y and c = 2x - 1. (a) $(2x+2y+1)^2$ (b) $(2x-2y+1)^2$ (c) $(2x-2y-1)^2$ (d) $(1-2x-2y)^2$ (e) None of these **27.** Find the value of $x^3 + y^3 + z^3 - 3xyz$, when x + y + z = 16 and xy + yz + zx = 78. (a) 176 (c) 716 (d) 532 (e) None of these **28.** $\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}=?$ (b) $a^3 + b^3 + c^3 - 3abc$ (c) $a^3 + b^3 + c^3 + 3abc(a + b + c)$ (a) $a^3 + b^3 + c^3 + 3abc$ (d) 3abc **29.** Find the value of $x^3 + y^3 + z^3 - 3xyz$ when x = 89, y = 87, z = 84. (c) 4940 (b) 19 (e) None of these **30.** If x = a(b - c), y = b(c - a) and z = c(a - b), then $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = ?$ (c) 3xyzabc (d) 3 (a) $\frac{3xyz}{zhz}$ (b) $\frac{xyz}{zhz}$ (e) None of these **31.** When $x + \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$. (c) 9 (d) 27 (e) None of these (a) 3 (b) 6 (c) 9 (d) 27 (e) N 32. Find the value of $a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ when a = 17, b = 15 and c = 13. (d) 361 (b) 121 (c) 225 (e) None of these **33.** When a = -5, b = -6 and c = 10 find the value of $\frac{a^3 + b^3 + c^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$. (a) -1(b) 1 (c) 2 **34.** Find the value of $(a+b+c)^3 - 3(b+c)(c+a)(a+b)$ if a=5, b=3 and c=2. (c) 160 (a) 120 (b) 180 (d) 240 (e) None of these





V

$\mathcal S$ olutions with Necessary Explanation

- 1. Roots of $2kx^2 + 5kx + 2 = 0$ are equal
 - \therefore discriminant = 0

i.e.
$$(5k)^2 - 4(2k)(2) = 0$$

i.e.
$$25k^2 - 16k = 0$$

$$\therefore$$
 $(25k - 16)k = 0$

$$\Rightarrow \qquad \qquad k = \frac{16}{25}$$

- **2.** Let the breadth of rectangle be b
 - \therefore length = b + 3
 - \therefore Area of rectangle = $b(b+3) = \pi \left(\sqrt{\frac{35}{11}}\right)^2$

$$b^2 + 3b = \frac{22}{7} \times \frac{35}{11} = \frac{70}{7} = 10$$

i.e.
$$b^2 + 3b = 10$$

i.e.
$$b^2 + 3b - 10 = 0$$

$$(b + 5) (b - 2) = 0$$

$$\therefore$$
 Length = $b + 3 = 2 + 3 = 5$ cm

Hence dimension of rectangle is 5×2 cm.

b = 2 cm

3. Let r be the radius of the pipe

$$\therefore 2\pi rl = 628 \text{ m}^2$$

Substituting value of l in terms of r, we get

$$l-r = 15 \text{ m or } l = (15+r)$$

$$\therefore 2\pi r (15 + r) = 628 = 200\pi \,\mathrm{m}^2$$

$$(15 + r)r = 100$$

$$\Rightarrow$$
 $r = 5 \text{ cm}$

:.
$$l = 15 + r = 20 \text{ cm}$$

$$\therefore$$
 Volume of the pipe = $\pi r^2 l = \pi(5)^2 \times 20$

$$= 500\pi = 1570 \text{ m}^{3]}$$

4. Roots of $2kx^2 + 5kx + 2 = 0$ are equal if discriminant = 0.

i.e.
$$(5k)^2 - 4(2k)(2) = 0$$

$$25k^2 - 16k = 0$$

$$\Rightarrow$$
 $25k - 16 = 0$

$$k=rac{16}{25}$$

5. Let one root of the equation $ax^2 + bx + c = 0$ be α .

Then other root be 2α .

:.

$$\therefore \quad \text{Sum of roots} = \frac{-b}{a} = \alpha + 2\alpha = 3\alpha \tag{1}$$

and product of roots =
$$\frac{c}{a} = \alpha(2\alpha) = 2\alpha^2$$
 (2)

(1)
$$\Rightarrow$$
 $\alpha = \frac{-b}{3a}$

(2)
$$\Rightarrow$$
 $\alpha^2 = \frac{c}{2a} = \left(\frac{-b}{3a}\right)^2 = \frac{b^2}{9a^2}$

i.e.
$$2b^2 = 9ac$$

6.
$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$$
 (1)

Let the roots of eqn. be α and $-\alpha$

 \therefore Sum of roots = $\alpha - \alpha = 0$

Product of roots = $\alpha(-\alpha) = -\alpha^2$

(1)
$$\Rightarrow$$
 $c(x+b)+c(x+a)=(x+a)(x+b)$

i.e.
$$x^2 + x(a + b - 2c) + (ab - bc - ca) = 0$$

$$\therefore \qquad \text{Sum of roots} = -(a+b-2c) = 0 \implies a+b = 2c$$

Product of roots = (ab - bc - ca) = ab - c(a + b)

$$= ab - \frac{(a+b)^2}{2}$$

$$=\frac{-1}{2}(a^2+b^2)$$

7.
$$\sqrt{3y+1} = \sqrt{y-1}$$

Squaring both sides,

$$3y + 1 = y - 1$$

$$3y - y = -1 - 1 = -2$$

$$2y = -2$$

$$\therefore \qquad \qquad y = \frac{-2}{2} = -1$$

$$\therefore y = -1 \implies \sqrt{3y+1} = \sqrt{3(-1)+1} = \sqrt{-2}$$
 which is not a real number.

Hence no real root exists.

8. Given α and β are roots of equation $x^2 + 3\alpha x + c = 0$.

$$\alpha^2 + \beta^2 = 5; a = ?$$

Here
$$\alpha + \beta = -3\alpha$$
 (1)

$$\alpha\beta = c \tag{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$a + p = (a + p) - 2ap$$

$$= (-3a)^2 - 2c$$

$$\Rightarrow \qquad = 9a^2 - 2c = 5$$

$$\therefore \qquad a = \sqrt{\frac{5 + 2c}{9}}$$

9. In the above equation, put
$$c = 2a^2$$

10.
$$x + \frac{1}{x} = 2$$
; $\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 2^2 - 2 = 2$

11.
$$x + \frac{1}{x} = 3$$
; $\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$
 $x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 7^2 - 2 = 49 - 2 = 47$
 $x^8 + \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)^2 - 2 = 47^2 - 2 = 2207$.

12.
$$x + y = 3$$
; $xy = 2$; $\Rightarrow (x - y)^2 = (x + y)^2 - 4xy = 3^2 - 4(2) = 1 \Rightarrow x - y = 1$
 $\Rightarrow x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$
 $= (x - y)[(x + y)^2 - xy]$
 $= 1 \times [3^2 - 2]$
 $= 7$

13.
$$x - \frac{1}{x} = \sqrt{21}$$
, $\left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) = ?$
 $\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 = 21 + 4 = 25$
 $\Rightarrow \qquad x + \frac{1}{x} = \sqrt{25} = 5$
 $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 5^2 - 2 = 23$

$$\therefore \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) = 23 \times 5 = 115$$

14.
$$a+b+c=0$$
;

$$\therefore a^3+b^3+c^3=(a+b+c)^3-3(b+c)(c+a)(a+b)$$

$$=0^3-3(-a)(-b)(-c)$$

15.
$$x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$
; $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$
$$x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 + 2\sqrt{3}}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

:.

$$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = \mathbf{2} - \sqrt{3}$$
$$\mathbf{x}^2 + \mathbf{y}^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 2(2)^2 + 2(\sqrt{3})^2$$
$$= 8 + 6 = \mathbf{14}$$

16.
$$(a^4 + b^4) (a^2 + b^2) (a + b) (a - b)$$

= $(a^4 + b^4) (a^2 + b^2) (a^2 - b^2)$
= $(a^4 + b^4) (a^4 - b^4)$
= $(a^4)^2 - (b^4)^2 = a^8 - b^8$

17. Given
$$x - \frac{1}{x} = a$$
; $x^3 - \frac{1}{x^3} = ?$

$$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$$
i.e.
$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \qquad a^3 = x^3 - \frac{1}{x^3} - 3(a)$$
i.e.
$$x^3 - \frac{1}{x^3} = a^3 + 3a$$

18. Given
$$x = 2^{1/3} - 2^{-1/3}$$
; $2x^3 + 6x = ?$
 $2x^3 + 6x = 2x(x^2 + 3)$

Let
$$x = a - \frac{1}{a}$$

$$2x^{3} + 6x = 2\left\{\left(a - \frac{1}{a}\right)^{3} + 3\left(a - \frac{1}{a}\right)\right\}$$

$$= 2\left\{a^{3} - \frac{1}{a^{3}} - 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right) + 3\left(a - \frac{1}{a}\right)\right\}$$

$$= 2\left\{a^{3} - \frac{1}{a^{3}}\right\} = 2\left\{(2^{1/3})^{3} - \frac{1}{(2^{1/3})^{3}}\right\}$$

$$= 2\left\{2 - \frac{1}{2}\right\} = 3$$

19.
$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)} = \frac{(a-b)^3 + (b-c)^3 + (a-c)^2 \cdot (c-a)}{(a-b)(b-c)(c-a)}$$
$$= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

20.
$$a+b+c=0$$

$$\frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2} + \frac{1}{a^2+b^2-c^2} = ?$$

$$a+b+c=0 \implies a+b=(-c)$$

or
$$(a+b)^2 = (-c)^2$$

or $a^2 + 2ab + b^2 = c^2$
or $a^2 + b^2 - c^2 = -2ab$
Similarly,
 $a^2 + c^2 - b^2 = -2ac$
and $b^2 + c^2 - a^2 = -2bc$

.. Given expression becomes

$$\frac{1}{-2bc} + \frac{1}{-2ac} + \frac{1}{-2ab} = \frac{-(a+b+c)}{2abc} = \mathbf{0}$$

21.
$$x + y + z = 0$$
, $\frac{(x + y)(y + z)(z + x)}{xyz} = ?$
i.e. $\frac{(-z)(-x)(-y)}{xyz} = \frac{-xyz}{xyz} = -1$

22.
$$a+b+c=0$$
,
$$\frac{a^4+b^4+c^4}{b^2c^2+c^2a^2+a^2b^2} = ?$$
$$(a+b+c) = 0$$
$$(a+b+c)^2 = 0$$
$$\Rightarrow \qquad a^2+b^2+c^2 = -2(ab+bc+ca)$$

Squaring both sides,

$$(a^{2} + b^{2} + c^{2})^{2} = [-2(ab + bc + ca)]^{2}$$

$$a^{4} + b^{4} + c^{4} + 2a^{2}b^{2} + 2b^{2}c^{2} + 2c^{2}a^{2} = 4[a^{2}b^{2} + b^{2}c^{2} + a^{2}c^{2} + 2abc(a + b + c)]$$

$$\therefore \frac{a^{4} + b^{4} + c^{4}}{b^{2}c^{2} + c^{2}a^{2} + a^{2}b^{2}} = \frac{4[a^{2}b^{2} + b^{2}c^{2} + a^{2}c^{2} + 2abc(a + b + c)] - 2(a^{2}b^{2} + b^{2}c^{2} + c^{2}a^{2})}{(b^{2}c^{2} + c^{2}a^{2} + a^{2}b^{2})}$$

$$= (4 - 2) + 8abc (0)$$

23.
$$x + y = 2z$$
; $\frac{x}{x - z} + \frac{z}{y - z} = ?$

$$\frac{x}{x - z} + \frac{z}{y - z} = \frac{x}{z - y} + \frac{z}{y - z}$$

$$= \frac{x}{z - y} - \frac{z}{z - y}$$

$$= \frac{x - z}{z - y} = 1$$

24.
$$x + \frac{1}{y} = 1$$
; $y + \frac{1}{z} = 1$; $z + \frac{1}{x} = ?$

$$x + \frac{1}{y} = 1 \qquad \Rightarrow \qquad x = 1 - \frac{1}{y}$$
or
$$\frac{1}{x} = \frac{1}{1 - \frac{1}{x}} = \frac{y}{y - 1}$$

$$y + \frac{1}{z} = 1 \qquad \Rightarrow \qquad \frac{1}{z} = 1 - y$$
or
$$z = \frac{1}{1 - y}$$

$$z + \frac{1}{z} = \frac{1}{z} + \frac{y}{z} = \frac{y}{z} = \frac{1}{z} + \frac{y}{z} = \frac{1}{z} + \frac{y}{z} = \frac{y}{z} =$$

$$\therefore z + \frac{1}{x} = \frac{1}{1 - y} + \frac{y}{y - 1} = \frac{1}{1 - y} + \frac{-y}{1 - y} = \frac{1 - y}{1 - y} = \mathbf{1}$$

25.
$$a = b = c;$$
 $\frac{(a+b+c)^2}{a^2+b^2+c^2} = ?$

$$\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{(3a^2)}{3a^2} = \frac{9a^2}{3a^2} = 3$$

26.
$$a = x + y$$
; $b = x - y$; $c = 2x - 1$;
 $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc = (b - a - c)^2$
 $= [x - y - (x + y) - (2x - 1)]^2$
 $= [x - y - x - y - 2x + 1]^2$
 $= [-2x - 2y + 1]^2$

27.
$$x + y + z = 16$$
; $xy + yz + zx = 78$;
 $x^3 + y^3 + z^3 - 3xyz = (x + y + z) [(x^2 + y^2 + z^2 - (xy + yz + zx))]$
 $= 16[100 - 78] \quad [\because x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)]$
 $= 352 = 16^2 - 2(78) = 256 - 156 = 1001$

28. By standard identity,

$$a^{3} + b^{3} + c^{3} - 3abc = \frac{1}{2}(a + b + c) \{(a - b)^{2} + (b - c)^{2} + (c - a)^{2}\}$$

 \therefore Answer is $a^3 + b^3 + c^3 - 3abc$

29.
$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) \{(x - y)^2 + (y - z)^2 + (z - x)^2 \}$$

Given x = 89, y = 87, z = 84

$$\therefore RHS = \frac{1}{2} (89 + 87 + 84) \{ (89 - 87)^2 + (87 - 84)^2 + (84 - 89)^2 \}$$

$$= \frac{1}{2} \times 260 \times \{ 2^2 + 3^2 + (-5)^2 \}$$

$$= 130 \times 38 = 3800 + 1140 = 4940$$

30. Given x = a(b - c), y = b(c - a), z = c(a - b)

$$\therefore \qquad \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{x}\right)^3 = (b-c)^3 + (c-a)^3 + (a-b)^3$$
If
$$p + q + r = 0, p^3 + q^3 + r^3 = 3pqr$$

$$\therefore \qquad \text{RHS} = 3(b-c)(c-a)(a-b)$$

$$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c} = \frac{3xyz}{abc}$$

31. Given
$$x + \frac{1}{x} = 3$$

$$\therefore x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$$

32. Given
$$a = 17$$
, $b = 15$ and $c = 13$

$$a^{2} + b^{2} + c^{2} - 2ab - 2bc - 2ca = ?$$

$$= (a + b + c)^{2} - 4ab - 4bc - 4ca$$

$$= (a + b + c)^{2} - 4b(a + c) - 4ac$$

$$= (17 + 15 + 13)^{2} - 4 \times 15(17 + 13) - 4 \times 17 \times 13$$

$$= 45^{2} - 60 \times 30 - 4 \times 17 \times 13$$

$$= 45(45 - 40) - 4 \times 17 \times 13 = 45 \times 5 - 68 \times 13 = 225 - 884 = -659$$

33. Given
$$a = -5$$
, $b = -6$, $c = 10$

$$\therefore \frac{(a^3 + b^2 + c^3 - 3abc)}{(ab + bc + ca - a^2 - b^2 - c^2)} = -(a + b + c) = -[-5 + (-6) + 10] = 1$$

34. Given
$$a = 5$$
, $b = 3$, $c = 2$

$$\therefore (a+b+c)^3 - 3(b+c)(c+a)(a+b) = a^3 + b^3 + c^3$$
i.e. RHS = $5^3 + 3^3 + 2^3 = 125 + 27 + 8 = 160$

Partnership

Partnership is an association of two or more persons who put their money together inorder to carry on a certain business. Person entered into such an association are called **Partners**. It is of two kinds: (i) Simple partnership, and (ii) Compound partnership.

SIMPLE PARTNERSHIP: If the capital of the partners is invested for the same period, the partnership is called the simple partnership. In such a partnership, profit and loss is distributed among the partners in the ratio of their investments.

For example, if A invests \overline{x} and B invest \overline{x} y, then their profit or loss at the end of a year will be in the ratio x : y.

COMPOUND PARTNERSHIP: If the capital of the partners is invested for different lengths of time, the partnership is called the compound partnership.

If a group of n persons invested different amounts for different periods, then their profit ratio is

$$At_1$$
: Bt_2 : Ct_3 : Dt_4 ...: Xt_n

[Here the first person invested an amount A for t_1 years, second person invested an amount B for t_2 years, and so on]. When investments are of different periods, then equivalent capitals are calculated for a unit of time by taking (capital x number of units of time).

If A invests $\not\in x$ for t_1 months and B invest $\not\in y$ for t_2 months in a year, then A's profit or loss or B's profit or loss = $xt_1 : yt_2$



Note: A partner who manages the business is known as a working partner and are who simply invests the money is a sleeping partner.

EXAMPLE 1 Three partners A, B, and C invest ₹ 1600, ₹ 1800, and ₹ 2300 respectively in a business. How should they divide the profit of ₹ 1938?

Solution The profit should be divided in the ratios of the capital, i.e. 16:18:23.

$$16 + 18 + 23 = 57.$$
∴ A's share = $\frac{16}{57}$ of ₹ 1938 = ₹ 544

B's share = $\frac{18}{57}$ of ₹ 1938 = ₹ 612

C's share = $\frac{23}{57}$ of ₹ 1938 = ₹ 782

EXAMPLE 2 X, Y, and Z enter into partnership. X advances ₹ 12,000 for 4 months, Y ₹ 14,000 for 8 months and Z ₹ 10,000 for 10 months. They gain ₹ 5850 together. Find the share of each.

Solution ₹ 12000 in 4 months earns a profit equal to profit for ₹ 48000 in 1 month.

Similarly, profit earned by $\ref{14000}$ in 8 months = profit earned by $\ref{14000} \times 8$ in 1 month.

Similarly, profit earned by $\ref{10000}$ in 10 months = profit earned by $\ref{10000} \times 10$ in 1 month.

 \therefore Profit is divided in the ratio 48000, 112000, and 100000, i.e. in the ratio of 12, 28 and 25. Now 12 + 28 + 25 = 65

∴ A's share =
$$\frac{12}{65} \times 5850 = ₹1080$$

B's share = $\frac{28}{65} \times 5850 = ₹2520$
C's share = $\frac{25}{65} \times 5850 = ₹2250$



Note: In compound partnership, the ratio of profits is directly proportional to both money and time, so they are multiplied together to get the corresponding shares in the ratio of profits.

EXAMPLE 3 A started a business with ₹ 2000. B joined him after 3 months with ₹ 4000. C put a sum of ₹ 10,000 in the business for 2 months only. At the end of the year, the business gave a profit of ₹ 5600. How should the profit be divided among them?

Solution Ratio of their profits (A's: B's: C's)

$$= 2 \times 12 : 4 \times 9 : 10 \times 2$$

$$= 6 : 9 : 5$$
Now,
$$6 + 9 + 5 = 20$$
∴
A's share = $\frac{6}{20} \times 5600 = ₹ 1680$
B's share = $\frac{9}{20} \times 5600 = ₹ 2520$
C's share = $\frac{5}{20} \times 5600 = ₹ 1400$

EXAMPLE 4 A and B entered into a partnership for a year. A contributed ₹ 1500 and B ₹ 2000. After 4 months they admitted C, who contributed ₹ 2250. If B withdraw his contribution after 9 months, how would they share a profit of ₹ 900 at the end of the year?

Solution A's share: B's share: C's share

=
$$1500 \times 12 : 2000 \times 9 : 2250 \times 8$$

= $15 \times 12 : 20 \times 9 : 22.5 \times 8$
= $180 : 180 : 180$
= $1 : 1 : 1$

∴ each of them got $\frac{900}{3} = ₹ 300$

EXAMPLE 5 A and B entered into a speculation. A put in $\stackrel{?}{\sim}$ 500 and B put in $\stackrel{?}{\sim}$ 450. At the end of 4 months A withdrew half of his capital and at the end of 6 months B withdrew half of his capital, then C entered with a capital of $\stackrel{?}{\sim}$ 700. At the end of 12 months, in what ratio would the profit be divided?

Solution A's share: B's share: C's share

$$= \left(500 \times 4 + \frac{500}{2} \times 8\right) : \left(450 \times 6 + \frac{450}{2} \times 6\right) : 700 \times 6$$

$$= 500 \times 8 : 450 \times 9 : 700 \times 6$$

$$= 80 : 81 : 84$$

.. Profit will be divided in the ratio 80:81:84

Important Formula

Formula for compound partnership

$$\frac{A's \ capital \times A's \ time \ in \ partnership}{B's \ capital \times B's \ time \ in \ partnership} = \frac{A's \ profit}{B's \ profit}$$

EXAMPLE 6 A began a business with $\stackrel{?}{\sim} 4500$ and was joined afterwards by B with $\stackrel{?}{\sim} 3,000$. When did B join if the profits at the end of the year were divided in the ratio 2:1? **Solution** If B joined the business for x months

$$\therefore \frac{4500 \times 12}{3000 \times x} = \frac{2}{1}$$
or
$$3000 \times x \times 2 = 4500 \times 12 \times 1$$
or
$$x = \frac{45 \times 6}{30} = 9 \text{ months}.$$

 \therefore B joined after (12-9) = 3 months.

EXAMPLE 7 A and B entered into a partnership with their capitals in the ratio 7:9. At the end of 8 months, A withdrew his capital. If they receive the profits in the ratio 8:9, find how long B's capital was used.

Solution If B's capital was used for x months

$$\therefore \frac{7 \times 8}{9 \times x} = \frac{8}{9} \quad \text{or} \quad x = \frac{7 \times 8 \times 9}{9 \times 8} = 7 \text{ months}$$

.. B's capital was used for 7 months.

EXAMPLE 8 A, B and C invested capitals in the ratio 2:3:5, the timing of their investments being in the ratio 4:5:6. In what ratio would their profit be disbursed?

Solution If three investments are in the ratio a:b:c and duration of their investments be in the ratio x:y:z, then the profit would be disbursed in the ratio ax:by:cz.

$$\therefore \qquad \text{Required ratio} = 2 \times 4 : 3 \times 5 : 5 \times 6 = 8 : 15 : 30$$

EXAMPLE 9 A, B, and C are partners. A receives 2/5 of the profit and B and C share the remaining profit equally. A's income is increased by $\stackrel{?}{\sim} 220$ when the profit rises from 8% to 10%. Find the capitals invested by A, B, and C.

Solution

A's share =
$$(10 - 8)\% = ₹ 220$$

∴ $100\% = ₹ 11000$

∴ A's capital = ₹ 11000

i.e., $\frac{2}{5}$ of share = ₹ 11000

 $\frac{3}{5}$ of share = $11000 \times \frac{3}{2} = ₹ 16500$

∴ B's and C's capitals are ₹ 8250 each.

EXAMPLE 10 Two partners invest ₹ 125000 and ₹ 85000 respectively in a business and agree that 60% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets ₹ 300 more than the other, find the total profit made in the business.

Solution The difference counts only due to the 40% of the profit distributed among them according to their investments.

Let the total profit be \mathbb{Z} x

Then 40% of profit = 0.4x

Ratio of profit = 125000 : 85000 = 25 : 17

$$\therefore \qquad \text{Share of first partner} = \frac{25}{25 + 17} \times (40\% \text{ of } x)$$

$$= \frac{25}{42} \times \frac{4}{10} x = \frac{5}{21} x.$$

$$\text{Share of second partner} = \frac{17}{25 + 17} (40\% \text{ of } x) = \frac{17}{42} \times \frac{4}{10} x = \frac{17x}{105}$$

$$\text{Difference in share} = \frac{5x}{21} - \frac{17x}{105} = 300$$
or
$$\frac{(25x - 17x)}{105} = 300$$

$$8x = 300 \times 105$$

$$x = \frac{300 \times 105}{8} = \text{ } 3937.50$$



Short cut: Ratio of profit = 125000 : 85000 = 25 : 17

$$\therefore \qquad \text{Total profit} = 300 \left(\frac{100}{40} \right) \left(\frac{25 + 17}{25 - 17} \right) = ₹ 3937.50$$

EXAMPLE 11 A, B, and C are partners in a business. A, whose money has been used for 4 months, claims 1/8 of the profit. B, whose money has been used for 6 months, claims 1/3 of the profit. C had invested ₹ 1560 for 8 months. How much did A and B contribute?

Solution Ratio of their shares in profit
$$=\frac{1}{8}:\frac{1}{3}:\left(1-\frac{1}{8}-\frac{1}{3}\right)$$

 $=\frac{1}{8}:\frac{1}{3}:\frac{13}{24}=3:8:13$

Let contribution of A and B be a and b.

$$a \times 4 : b \times 6 : 1560 \times 8 = 3 : 8 : 13.$$

$$\therefore \frac{a \times 4}{3} = \frac{1560 \times 8}{13} \quad \text{or} \quad a = \frac{3}{4} \times \frac{1560 \times 8}{13}$$

i.e.

$$a = 3 \times 120 \times 2 = ₹720$$

∴ Contribution of A = ₹ 720

Contribution of B =
$$\left(\frac{720 \times 8}{3}\right) \times \frac{4}{6} = 80 \times 8 \times 2 = ₹ 1280$$

Solution 30% of the profit will be distributed among the partners proportional to their investment.

 \therefore Ratio of profit = 50000 : 70000 = 5 : 7.

Let x be the total profit.

Then difference of their profit =
$$\frac{5-7}{5+7}(0.3x) = 90$$

∴ Total profit = ₹ 1800



Short cut: Ratio of profit = 50000: 70000 = 5:7

$$\therefore \qquad \text{Total profit} = 90 \times \left(\frac{100}{30}\right) \left(\frac{5+7}{7-5}\right) = \text{ } \boxed{1800}$$

EXAMPLE 13 A, B, and C invested capitals in the ratio 2:3:4. At the end of the business term, they received the profits in the ratio 3:6:10. Find the ratio of periods for which they contributed their capitals.

Solution If investments are in the ratio x:y:z and profits in the ratio a:b:c, then the

ratio of periods =
$$\frac{a}{x} : \frac{b}{y} : \frac{c}{z}$$

$$\therefore \qquad \text{Required ratio} = \frac{3}{2} : \frac{6}{3} : \frac{10}{4}$$

$$= \frac{3}{2} \times 12 : \frac{6}{3} \times 12 : \frac{10}{4} \times 12 = 18 : 24 : 30 = 3 : 4 : 5$$

EXAMPLE 14 In a partnership, A invested 1/6 of the capital for 1/6 of the time, B invested 1/3 of the capital for 1/3 of the time, and C invested the rest of the capital for the whole period. At the end of the period, they earned a profit of $\stackrel{?}{\stackrel{\checkmark}}$ 4600. Find the share of B.

Solution C invested $1 - \left(\frac{1}{6} + \frac{1}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$ part of the capital

Ratio of profit =
$$\frac{1}{6} \times \frac{1}{6} : \frac{1}{3} \times \frac{1}{3} : \frac{1}{2} \times 1$$

= $\frac{1}{36} : \frac{1}{9} : \frac{1}{2} = 1 : 4 : 18$

∴ B's share =
$$\left(\frac{4}{1+4+18}\right)4600 = \frac{4}{23} \times 4600 = ₹800$$

EXAMPLE 15 P, Q and R enter into a partnership. P invests 3 times as much as Q invests and Q invests two-thirds of what R invests. At the end of the year, the profit earned is $\stackrel{?}{\underset{?}{$\sim}}$ 8800. What is the share of Q?

Solution Let the investment made by R be $\mathfrak{T} x$.

$$\therefore \qquad \text{Investment by Q} = \frac{2}{3}x$$

$$\therefore \qquad \text{Investment by P = } 3 \times \frac{2}{3} x = 2x$$

:. Ratio of investments of P, Q and R is

$$2x: \frac{2x}{3}: x = 6:2:3$$

∴ Share of Q =
$$\frac{2}{6+2+3} \times 8800$$

= $\frac{2}{11} \times 8800 = ₹ 1600$

EXAMPLE 16 X, Y and Z start a business each investing ₹ 30000. After 6 months X withdrew ₹ 5000, Y withdrew ₹ 6000 and Z invests ₹ 6000 more. At the end of the year, a total profit of ₹ 17500 was recorded. Find the share of each.

Solution Ratio of capitals of X, Y and Z

$$= 30000 \times 6 + 25000 \times 6 : 30000 \times 6 + 24000 \times 6 : 30000 \times 6 + 36000 \times 6$$
55000 × 6 : 54000 × 6 : 66000 × 6 = 55 : 54 : 66

∴ Share of $X = ₹ \left(17500 \times \frac{55}{175} \right) = ₹ 5500$

Share of $Y = ₹ \left(17500 \times \frac{54}{175} \right) = ₹ 5400$

Share of $Z = ₹ \left(17500 \times \frac{66}{175} \right) = ₹ 6600$

EXAMPLE 17 P invested ₹ 64000 in a business. Q joined him with ₹ 48000 after a few months. If the total profit was divided among them in the ratio 2: 1 towards the end of year. After how may months did Q join.

Solution Let Q joined the business after x months.

 \therefore Q invested his money for (12 - x) months.

$$\frac{64000 \times 12}{48000 \times (12 - x)} = \frac{2}{1}$$

$$\Rightarrow \qquad 48000 (12 - x) \times 2 = 64000 \times 12$$

$$\Rightarrow \qquad 6(12 - x) = 4 \times 12$$

$$\Rightarrow \qquad 6x = 6 \times 12 - 4 \times 12$$

$$\Rightarrow \qquad = 24$$
or
$$x = \frac{24}{6} = 4 \text{ months}$$

EXAMPLE 18 A, B, and C enter into a partnership. They invest ₹ 40000, ₹ 80000 and ₹ 120000 respectively. At the end of the first year, B withdraws ₹ 40000, while at the end of the second year, C withdraws ₹ 80000. In what ratio will the profit be shared at the end of 3 years? [Hotel Management]

Solution Ratio of shares of profit between A, B and C is

$$(40000 \times 36) : (80000 \times 12) + (40000 \times 24) : (1,20,000 \times 24) + (40000 \times 12)$$

= $4 \times 36 : (8 \times 12 + 4 \times 24) : (12 \times 24 + 4 \times 12)$
= $4 \times 36 : 4 \times 48 : 4 \times 84 = 3 : 4 : 7$

(e) None of these

? EXERCISES

(a) 1:1

	(a) 6:8:9	(b) 8:6:9	(c) 1:1:1	(d) 1:2:3	(e) None of these			
3.	A and B enter into a partnership; A puts in $\stackrel{?}{\sim} 50$ and B puts in $\stackrel{?}{\sim} 45$. At the end of 4 months, A withdraws half his capital and at the end of 5 months B withdraws 1/2 of his; C then enters with a capital of $\stackrel{?}{\sim} 70$; at the end of 12 months, the profit of concern is $\stackrel{?}{\sim} 254$; how can the profit be divided among A, B and C?							
	=	_		30 (d) 80, 98 and	76 (e) None of these			
4.	Three partners altogether invested ₹ 114000 in a business. At the end of the year, one got ₹ 337.50, the second ₹ 1125.00 and the third ₹ 675.00 as profit. How much amount did each invest? What is the percentage of profit?							
	(a) 75%	(b) 3.75%	(c) 1.875%	(d) 15%	(e) None of these			
5.		hey receive profit	s in the ratio of 5	: 9, find how long	the end of 8 months, B's capital was used?			
	(a) 9 months	(b) 8 months	(c) 10 months	(d) 12 months				
6.	months investing	g an amount equ s equal to₹98000	al to that of Q. A 0. What was the a	t the end of the	joined them after six year, 20% profit was y R? [SBI PO]			
	(a) ₹ 105000	(b) ₹ 175000	(c) ₹ 210000	(d) Data inade	equate			
	(e) None of these	e						
7.	amount of ₹ 1000 invested another	00 and Rajeev joi additional amou ill be Rajeev's sha	ined him with an .nt of ₹ 10000 and are in the profit of	amount of ₹ 350 Jatin joined the	nvested an additional 00. In 2001, Shekhar m with an amount of at the end of 3 years [SBI PO]			
	(a) ₹ 45000	(b) ₹ 50000	(c) ₹ 70000	(d) ₹ 75000	(e) None of these			
	(a) \ 40000							
8.	A started busines year, profit is div	vided in the ratio	2:3. What is B's co	ntribution in the	s his partner. After a			
	A started busines year, profit is div (a) ₹7500	rided in the ratio (b) ₹8000	2:3. What is B's co (c) ₹8500	ontribution in the (d) ₹9000	s his partner. After a capital? [SSC]			
	A started busines year, profit is div (a) ₹7500 A, B and C jointly A would invest ₹ A wants to be the earned was ₹740	rided in the ratio (b) ₹8000 y thought of engage 6500 for 6 mont working membe 00. Calculate the	2:3. What is B's co (c) ₹8500 ging themselves in hs, B₹8400 for 5 r for which he was share of B in the p	ontribution in the (d) ₹ 9000 a business ventu months and C, ₹ to receive 5% of crofit.	s his partner. After a			
	A started busines year, profit is div (a) ₹ 7500 A, B and C jointly A would invest ₹ A wants to be the	rided in the ratio (b) ₹8000 y thought of engage 6500 for 6 mont e working membe	2:3. What is B's co (c) ₹8500 ging themselves in hs, B₹8400 for 5 r for which he was	ontribution in the (d) ₹ 9000 a business ventu months and C, ₹ to receive 5% of	s his partner. After a capital? [SSC] re. It was agreed that 10000 for 3 months. the profits. The profit			
9.	A started busined year, profit is divided (a) ₹ 7500 A, B and C jointly A would invest ₹ A wants to be the earned was ₹ 740 (a) ₹ 1900	rided in the ratio (b) ₹8000 y thought of engage 6500 for 6 mont e working membe 00. Calculate the s (b) ₹2660	2:3. What is B's co (c) ₹8500 ging themselves in hs, B₹8400 for 5 r for which he was share of B in the p (c) ₹2800	ontribution in the (d) ₹ 9000 a business ventu months and C, ₹ to receive 5% of profit. (d) ₹ 2840	s his partner. After a capital? [SSC] re. It was agreed that 10000 for 3 months. the profits. The profit			
9.	A started busines year, profit is div (a) ₹7500 A, B and C jointly A would invest ₹ A wants to be the earned was ₹740 (a) ₹1900 A, B and C enter half of his capital share?	rided in the ratio (b) ₹8000 y thought of engage 6500 for 6 mont e working membe 00. Calculate the s (b) ₹2660 red into a partner l and after 10 mon	2:3. What is B's co (c) ₹8500 ging themselves in hs, B₹8400 for 5 r for which he was share of B in the p (c) ₹2800 ship in the ratio ½	intribution in the (d) ₹ 9000 a business ventumonths and C, ₹ to receive 5% of the order of the following the fo	s his partner. After a capital? [SSC] re. It was agreed that 10000 for 3 months. the profits. The profit [MBA]			
9.	A started busines year, profit is div (a) ₹7500 A, B and C jointly A would invest ₹ A wants to be the earned was ₹740 (a) ₹1900 A, B and C enter half of his capital	rided in the ratio (b) ₹8000 y thought of engage 6500 for 6 mont e working membe 00. Calculate the s (b) ₹2660 red into a partner	2:3. What is B's co (c) ₹8500 ging themselves in hs, B₹8400 for 5 r for which he was share of B in the p (c) ₹2800 ship in the ratio ½	intribution in the (d) ₹ 9000 a business ventumonths and C, ₹ to receive 5% of the order of the following the fo	s his partner. After a capital? [SSC] re. It was agreed that 10000 for 3 months. the profits. The profit [MBA] months, A withdraws ong them. What is B's			

1. How should a profit of ₹ 450 be divided between two partners, one of whom has contributed

2. A and B enter into a partnership for a year. A contributes ₹ 1500 and B ₹ 2000. After 4 months, they admit C, who contributes ₹ 2250. If B withdraws his contribution after 9 months,

(d) 2:3

(c) 2:1

₹ 1200 for 5 months and the other ₹ 750 for 4 months?

how would they share a profit of ₹ 900 at the end of the year?

(b) 1:2

(a) 6 months

(b) 9 months

11.	A, B and C entered into a partnership in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$. After 4 months, A increases his						
	share by 50%. If the total profit at the end of one year be ₹21600, then B's share in the profit is: [LIC AAO]						
	(a) ₹2100 (b) ₹2400 (c) ₹3600 (d) ₹4000						
12.	A and B entered into partnership with capitals in the ratio 4:5. After 3 months, A withdrew						
	$\frac{1}{4}$ of his capital and B withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months was						
	₹ 760. A's share in the profit is: [AAO]						
	(a) ₹330 (b) ₹360 (c) ₹380 (d) ₹430						
13.	In a business, A and C invested amounts in the ratio 2:1, whereas the ratio between amounts invested by A and B was 3:2. If ₹ 1,57,300 was their profit, how much amount did B receive? [Bank PO]						
	(a) $\stackrel{?}{\sim} 24200$ (b) $\stackrel{?}{\sim} 36300$ (c) $\stackrel{?}{\sim} 48400$ (d) $\stackrel{?}{\sim} 72600$						
14.	Arun, Kamal and Vinay invested ₹ 8000, ₹ 4000 and ₹ 8000 respectively in a business. Arun left after six months. If after eight months, there was a gain of ₹ 4005, then what will be the share of Kamal? [RRB]						
	(a) $\stackrel{?}{$\times$} 890$ (b) $\stackrel{?}{$\times$} 1335$ (c) $\stackrel{?}{$\times$} 1662$ (d) $\stackrel{?}{$\times$} 1780$						
15.	Kamal started a business investing $\stackrel{?}{\stackrel{\checkmark}}$ 9000. After five months, Sameer joined with a capital of $\stackrel{?}{\stackrel{\checkmark}}$ 8000. If at the end of the year, they earn a profit of $\stackrel{?}{\stackrel{\checkmark}}$ 6970, then what will be the share of Sameer in the profit? [RRB]						
	(a) ₹ 1883.78 (b) ₹ 2380 (c) ₹ 3690 (d) ₹ 3864						
16	A, B and C enter into a partnership investing ₹ 35000, ₹ 45000 and ₹ 55000 respectively.						
10.	The respective shares of A, B and C in an annual profit of ₹ 40500 are: [CBI]						
	(a) ₹ 10500, ₹ 13500, ₹ 16500 (b) ₹ 11500, ₹ 13000, ₹ 16000						
	(c) ₹ 11000, ₹ 14000, ₹ 15500 (d) ₹ 11500, ₹ 12500, ₹ 16500						
17.	Simran started a software business by investing ₹ 50000. After six months, Nanda joined her with a capital of ₹ 80000. After 3 years, they earned a profit of ₹ 24500. What was Simran's share in the profit? [Bank PO]						
	(a) ₹ 9423 (b) ₹ 10250 (c) ₹ 12500 (d) ₹ 14000 (e) None of these						
18.	 18. A and B start a business with investments of ₹ 5000 and ₹ 4500 respectively. After 4 months, A takes out half of his capital. After two more months, B takes out one-third of his capital while C joins them with a capital of ₹ 7000. At the end of a year, they earn a profit of ₹ 5080. Find the share of each member in the profit. [Bank PO] (a) A - ₹ 1400, B - ₹ 1900, C - ₹ 1780 (b) A - ₹ 1600, B - ₹ 1800, C - ₹ 1680 (c) A - ₹ 1800, B - ₹ 1500, C - ₹ 1780 (d) A - ₹ 1680, B - ₹ 1600, C - ₹ 1800 						
19.	Three partners shared the profit in a business in the ratio 5:7:8. They had partnerd for 14						
	months, 8 months and 7 months respectively. What was the ratio of their investments? [Hotel Management]						
	(a) 5:7:8 (b) 28:49:64 (c) 38:28:21 (d) None of these						
20.	A and B are partners in a business. A contributes $\frac{1}{4}$ of the capital for 15 months and B						
	received $\frac{2}{3}$ of the profit. For how long B's money was used? [SSC]						

(c) 10 months

(d) one year

- 21. A began a business with ₹85000. He was joined afterwards by B with ₹42500. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3:1? [NIFT]
 - (a) 4 months
- (b) 5 months
- (c) 6 months
- (d) 8 months
- 22. A and B started a business in partnership investing ₹ 20000 and ₹ 15000 respectively. After six months, C joined them with ₹ 20000. What will be B's share in the total profit of ₹ 25000 earned at the end of 2 years from the starting of the business? [SBI PO]
 - (a) ₹ 7500
- (b) ₹ 9000
- (c) ₹9500
- (d) ₹ 10000
- (e) None of these
- 23. Aman started a business investing ₹ 70000. Rakhi joined him after six months with an amount of ₹ 1,05,000 and Sagar joined them with ₹ 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Aman, Rakhi and Sagar respectively, 3 years after Aman started the business?

 [Bank PO]
 - (a) 7:6:10

(b) 12:15:16

(c) 42:45:56

- (d) Cannot be determined
- (e) None of these
- 24. Satheesh started a business investing ₹ 60000. After 3 months, Sunil joined him with a capital of ₹ 75000. After another 6 months, Subhash joined them with a capital of ₹ 105000. At the end of the year, they made a profit of ₹ 19000. Find the share of each.
 - (a) ₹8500, ₹7000, ₹3500
- (b) ₹8000, ₹7500, ₹3500
- (c) ₹8000, ₹6500, ₹4500
- (d) ₹8500, ₹6500, ₹4000
- 25. A and B started a partnership business investing some amount in the ratio of 3:5. C joined them after six months with an amount equal to that of B. In what proportion should the profit at the end of one year be distributed among A, B and C? [Bank PO]
 - (a) 3:5:2

(b) 3:5:5

(c) 6:10:5

- (d) Data inadequate
- (e) None of these



- 1. (c) 2. (c) 3. (b) 4. (c) 5. (d) 6. (c) 7. (b) 8. (d) 9. (b) 10. (b)
- 11. (d) 12. (a) 13. (c) 14. (a) 15. (b) 16. (a) 17. (e) 18. (b) 19. (d) 20. (c)
- 21. (d) 22. (a) 23. (b) 24. (b) 25. (c)



Solutions with Necessary Explanation

- 1. Ratio of profit = $1200 \times 5 : 750 \times 4 = 2 : 1$
 - ∴ Ist partner gets $450 \times \frac{2}{2+1} = ₹300$
 - ∴ 2nd partner gets $450 \times \frac{1}{2+1} = ₹ 150$
- 2. A's share: B's share: C's share

 $= 1500 \times 12 : 2000 \times 9 : 2250 \times 8$

= 18000 : 18000 : 18000 = 1 : 1 : 1

: Share of A = Share of B = Share of C

i.e. profit of ₹ 900 is divided into 3 equal parts equal to ₹ 300

3. A's share: B's share: C's share

 $= (50 \times 4 + 25 \times 8) : (45 \times 5 + 22.5 \times 7) : 70 \times 7$

= 400:382.5:490 = 800:765:980

= 160:153:196

Total profit = ₹ 254

$$\therefore \qquad \text{Profit of A} = \frac{160}{160 + 153 + 196} \times 254 = \frac{160}{509} \times 254 = \approx 80$$

$$\text{Profit of B} = \frac{153}{509} \times 254 \approx 76$$

$$\text{Profit of C} = \frac{196}{509} \times 254 \approx 98$$

4. Total investment = ₹ 114000

Ratio of investment =
$$337.50:1125:675$$

$$= 1: \frac{10}{3}: 2$$

$$= 3:10:6$$

$$\therefore \qquad \text{Percentage profit} = \frac{337.5 + 1125 + 675}{114000} \times 100\%$$
$$= \frac{2137.5}{1140} \% = 1.875\%$$

5. Ratio of capitals = 5:6

Let the ratio of time = 8:x

ratio of profits
$$= 5:9$$

$$5 \times 8 : 6 \times x = 5 : 9$$

or
$$40: 6x = 5: 9$$
 or $\frac{40}{6x} = \frac{5}{9}$

$$\therefore \qquad x = \frac{40 \times 9}{6 \times 5} = 4 \times 3 = 12 \text{ months}$$

6. Let the total profit be $\not\in x$.

$$\therefore 20\% \text{ of } x = 98000$$

$$\Rightarrow x = 98000 \times \frac{100}{20} = 490000$$

Ratio of investments between P and Q = 5: 6. R joined 6 months after starting the business. Let the capital of P, Q and R be $\stackrel{?}{\sim} 5y$, 6y and 6y respectively.

R's investment =
$$6y = 76 \times 35000$$

7. Shekhar: Rajeev: Jatin

:.

=
$$(25000 \times 12 + 35000 \times 12 + 45000 \times 12) : 35000 \times 24 : 35000 \times 12$$

= $105000 \times 12 : 35000 \times 24 : 35000 \times 12$
= $3 : 2 : 1$

∴ Rajeev's share =
$$\frac{2}{3+2+1}$$
 × 150000 = ₹50000

- 8. Let B's contribution be ξx .
 - \therefore A: B = 3500 × 12: $x \times 7 = 2:3$

$$\therefore \frac{3500 \times 12}{7x} = \frac{2}{3}$$

or

$$x = \frac{3500 \times 12 \times 3}{7 \times 2}$$

9. Ratio of investments between A, B and C

$$= 6500 \times 6:8400 \times 5:10000 \times 3$$

$$=65 \times 6:84 \times 5:100 \times 3$$

$$=390:420:300=13:14:10$$

Profit earned = ₹ 7400

- ∴ A receives 5% of profit = $\frac{5}{100} \times 7400 = ₹370$
- ∴ Net profit to be shared = 7400 370 = ₹ 7030
- ∴ Share of profit to B = $\frac{14}{37}$ × 7030 = 14 × 190 = ₹ 2660
- 10. Ratio of investment between A, B and C at the start of business

$$=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$$

Let their investments be ₹ 6k, ₹ 4k and ₹ 3k..

Intestment by $A = 6k \times 2 + 3k \times 10 = 42k$

Intestment by $B = 4k \times 12 = 48k$

Intestment by $C = 3k \times 12 = 36k$

Ratio between investments = 42k : 48k : 36k = 7 : 8 : 6

Share of profit to B from ₹ 378

$$= \frac{8}{7+8+6} \times 378 = \frac{8}{21} \times 378$$
$$= 8 \times 18 = ₹ 144$$

11. Ratio of investments between A, B and C

$$=\frac{7}{2}:\frac{4}{3}:\frac{6}{5}=7\times15:4\times10:6\times6=105:40:36$$

Let their investments be ₹ 105k, ₹ 40k and ₹ 36k respectively at the beginning After 4 months, investment by $A = 1.5 \times 105k = 157.5k$

Total profit = ₹ 21600.

.. Ratio of investments

$$= (105k \times 4 + 157.5k \times 8) : 40k \times 12 : 36k \times 12$$

$$= (420k + 1260k) : 490k : 36 \times 12k$$

$$= 1680k : 480k + 36 \times 12k = 140 : 40 : 36 = 35 : 10 : 9$$

∴ Share of B =
$$\frac{10}{35+10+9} \times 21600$$

= $\frac{10 \times 21600}{54}$
= ₹ 4000

- 12. Initial ratio of investment = 4 : 5. Let the initial investment of A and B be 4k and 5k respectively.
 - :. Ratio of investments between A and B

$$(4k \times 3 + 3k \times 7) : (5k \times 3 + 4k \times 7) = 33k : 43k = 33 : 43$$

$$\therefore \text{ Share of profit to A} = \frac{33}{33+43} \times 760$$

$$=\frac{33}{76}$$
×760 = ₹ 330

13. Ratio of investments between A and C = 2 : 1 = 6 : 3

Ratio of investments between A and B = 3: 2 = 6: 4

Ratio of investments between A, B and C = 6:4:3

Total profit = ₹ 157300

∴ Share of B =
$$\frac{4}{6+4+3} \times 157300 = 4 \times 12100 = ₹48400$$

14. Arun: Kamal: Vinay

$$= (8000 \times 6) : (4000 \times 8) : (8000 \times 8)$$

$$8 \times 6 : 4 \times 8 : 8 \times 8 = 3 : 2 : 4$$

Total gain = ₹ 4005

:. Share of Kamal =
$$\frac{2}{3+2+4} \times 4005 = \frac{2}{9} \times 4005$$

15. Kamal : Sameer =
$$9000 \times 12 : 8000 \times 7$$

= $9 \times 12 : 8 \times 7 = 27 : 14$

∴ Share of Sameer =
$$\frac{14}{41} \times 6970 = 14 \times 170 = ₹ 2380$$

∴ Share of A =
$$\frac{7}{27} \times 40500 = 7 \times 1500 = ₹10500$$

Share of B =
$$\frac{9}{27}$$
 × 40500 = ₹13500

Share of C =
$$\frac{11}{27}$$
 × 40500 = 11 × 1500 = ₹ **16500**

- 17. Simran : Nanda = $50000 \times 36 : 80000 \times 30 = 5 \times 36 : 8 \times 30 = 3 : 4$ Total profit = ₹ 24500
 - ∴ Simran's share of profit = $\frac{3}{7} \times 24500 = ₹ 3 \times 3500 = ₹ 10500$
- 18. A : B : C = $(5000 \times 4 \times 2500 \times 8) : (4500 \times 6 + 3000 \times 6) : 7000 \times 6$ = 40000 : 45000 : 42000 = 40 : 45 : 42

Total profit = ₹5080

∴ Share of profit to A =
$$\frac{40}{127} \times 5080 = ₹ 1600$$

Share of profit to B =
$$\frac{45}{127} \times 5080 = ₹1800$$

Share of profit to
$$C = \frac{42}{127} \times 5080 = ₹1680$$

19. Let x, y and z be their investments.

$$x \times 14 : y \times 8 : z \times 7 = 5 : 7 : 8$$

$$\therefore \frac{14x}{7z} = \frac{5}{8}$$

or
$$16x = 5z$$

or
$$z = \frac{16}{5}x$$

$$\frac{14x}{8y} = \frac{5}{7}$$

$$\Rightarrow$$
 14 × 7 x = 40 y

or
$$y = \frac{98}{40}x$$

$$\therefore x: y: z = x: \frac{98}{40}x: \frac{16}{5}x$$
$$= x: \frac{49}{20}x: \frac{16}{5}x = 20:49:64$$

20. Let total capital be $\not \in x$.

Let B's money was used for y months.

$$\therefore A: B = \left(\frac{x}{4} \times 15\right) : \frac{3x}{4} \times y = 1:2$$

i.e.
$$15x : 3xy = 1 : 2$$

or
$$5: y = 1:2$$

or
$$y = 5 \times 2 = 10$$
 months

21. Let B was in the business for x months.

$$\therefore$$
 A: B = 85000 × 12: 42500 × x = 3: 1

i.e.
$$24: x = 3:1$$

or
$$\frac{24}{x} = \frac{3}{1}$$
or
$$x = \frac{24 \times 1}{3} = 8 \text{ months}$$

22. A : B : C =
$$(20000 \times 24)$$
 : (15000×24) : (20000×18) = 48 : 36 : 36 = 4 : 3 : 3

Total profit = ₹ 25000

∴ Share of profit to B =
$$\frac{3}{4+3+3} \times 25000 = ₹7,500$$

24. Satheesh: Sunil: Subhash

$$=60000 \times 12: 75000 \times 9: 105000 \times 3$$

= $720: 9 \times 75: 315 = 144: 135: 63 = 16: 15: 7$

Total profit = ₹ 19000

∴ Share of Satheesh =
$$\frac{16}{38} \times 19000 = ₹8000$$

Share of Sunil = $\frac{15}{38} \times 19000 = ₹7500$

Share of Subhash =
$$\frac{7}{38}$$
 ×19000 = ₹ **3500**

- **25.** Let the investment by A and B be $\stackrel{?}{\sim} 3x$ and $\stackrel{?}{\sim} 5x$.
 - \therefore Investment by $C = \mathbb{Z} 5x$

$$\therefore A:B:C = (3x \times 12): (5x \times 12): (5x \times 6)$$
$$= 36:60:30=6:10:5$$

Ratio by which profit is to be shared = 6:10:5

Simple Interest

When an amount is deposited in a bank or lent to a person for an interest of r% per annum, the amount of interest will be calculated for the total number of days lent or deposited at the specified rate using the formula.

$$I = \frac{PRN}{100} \text{ and } A = P\left(1 + \frac{RN}{100}\right)$$

where P = Principal amount

R = Rate of interest per annum

N = Number of years (period of deposit)

A = Total amount accrued at the end of the period

(The interest thus accrued is called the simple interest.)

When time is given in days (D), convert it into years by dividing it by 365 $\left[\text{i.e.} \frac{D}{365}\right]$

$$I = \frac{PR}{100} \times \frac{D}{365}$$

If I_1 and I_2 are interests for 2 different periods, find the rate/annum.

$$I = \frac{P_1 R N_1}{100} + \frac{P_2 R N_2}{100}$$
$$= \frac{R}{100} (P_1 N_1 + P_2 N_2)$$
$$R = \frac{100I}{(P_1 N_1 + P_2 N_2)}$$

:.

If R_1 and R_2 are rate of interest for same duration, principal being different, duration is given by

$$N = \frac{100I}{P_1 R_1 + P_2 R_2}$$

Find the annual instalment that will discharge a debt of \mathcal{F} A due in t years at R% per annum simple interest.

Annual instalment required
$$x = \frac{100A}{100N + R \frac{(N-1)N}{2}}$$

where A = Total amount

N =Number of years/instalments

R = Rate of interest per annum

$$x = \frac{A}{N + \frac{R}{100} [1 + 2 + 3 + \dots (N - 1)]}$$

A sum of money becomes n times in t years at simple interest. Find the rate of interest per annum in terms of n and t.

$$r = \frac{100(n-1)}{t}\%$$

In what time will an investment at r% per annum simple interest become n times?

Period of investment
$$t = \frac{100(n-1)}{r}$$
 years

Investment =
$$\frac{\text{Additional income} \times 100}{\text{Time} \times \text{Difference in rate of interest}}$$

The difference in interests on a certain sum at r_1 % for t_1 years and r_2 % for t_2 years is ₹ I. Find the sum.

$$P = \frac{100I}{|r_1t_1 - r_2t_2|}$$

If a certain sum of money amounts to \overline{A}_1 in t_1 years at t_1 , it would amount to

₹
$$A_2$$
 in $t_2 = \frac{1}{r} \left\{ \frac{A_2}{A_1} (100 + rt_1) - 100 \right\}$ years.

EXAMPLE 1 The simple interest on a sum of money is 1/9 of the principal, and the number of years is equal to the rate per cent per annum. Find the rate per cent.

Solution Let Principal = P, Time = t years, rate = t%

$$\frac{P \times t \times t}{100} = \frac{P}{9}$$

$$\frac{t^2}{100} = \frac{1}{9} \quad \text{or} \quad t^2 = \frac{100}{9}$$

$$t = \frac{10}{3} = 3\frac{1}{3}$$

$$\therefore \text{ Rate per cent} = 3\frac{1}{3}\% \quad (\because \text{ rate} = t\%)$$



:.

Short cut: Rate = time =
$$\sqrt{100 \times \frac{1}{9}} = \frac{10}{3} = 3\frac{1}{3}$$

Rate =
$$3\frac{1}{3}\%$$
; time = $3\frac{1}{3}$ years.

EXAMPLE 2 What annual payment will discharge a debt of ₹ 770 due in 5 years, the rate of interest being 5% p.a.?

Solution Let annual payment be *P* rupees.

Interest accrued in 4 years at 5% p.a. along with the amount credited

$$A = P\left(1 + \frac{NR}{100}\right) = P\left(\frac{100 + 4 \times 5}{100}\right) = \frac{120P}{100}$$

Similarly in 3 years, amount =
$$P\left(\frac{100 + 3 \times 5}{100}\right) = \frac{115P}{100}$$

Similarly in 2 years, amount =
$$P\left(\frac{100 + 2 \times 5}{100}\right) = \frac{110P}{100}$$

Similarly in 1 year, amount =
$$P\left(\frac{100 + 1 \times 5}{100}\right) = \frac{105P}{100}$$

Total amount accrued in 4 years along with the last annual payment of $\overline{\epsilon}$ P.

$$= \frac{120P}{100} + \frac{115P}{100} + \frac{110P}{100} + \frac{105P}{100} + P = 770 \quad \therefore \quad \frac{550P}{100} = 770$$

$$P = \frac{770 \times 100}{550} = 140$$

Hence, the annual payment = ₹ 140.

Trence, the annual payment 140.

of interest r% per annum is $\frac{100A}{100t + \frac{rt(t-1)}{2}}$.

EXAMPLE 3 What annual payment will discharge a debt of ₹848 in 4 years at 4% p.a.?

 \triangleright Theorem: The annual payment that will discharge a debt of $\not\in A$ due in t years at the rate

Solution By the theorem: Annual payment = $\frac{100A}{100N + \frac{R(N-1)N}{2}}$

Here A = ₹ 848; N = 4 years; R = 4%.

∴ Annual payment = $\frac{848 \times 100}{4 \times 100 + \frac{4(3)(4)}{2}} = ₹ 200$

EXAMPLE 4 The rate of interest for the first two years is 3% per annum, for the next three years is 8% per annum and for the period beyond 5 years 10% per annum. If a man gets ₹ 1,520 at simple interest for six years, how much money will he deposit?

Solution Let the deposit be ₹ 100.

Then interest for first 2 years @ 3% p.a = ₹ 6.

Interest for next 3 years @ 8% p.a. = ₹ 24.

Interest for the last year = ₹ 10.

Total interest = ₹ 40

∴ Deposited amount when interest is ₹ 1520 = $\frac{100}{40}$ × 1520 = ₹ 3800



Short cut: Deposited amount or Principal =
$$\frac{\text{Interest} \times 100}{(r_1t_1 + r_2t_2 + r_3t_3 + \cdots)}$$
$$= \frac{1520 \times 100}{3 \times 2 + 8 \times 3 + 10 \times 1}$$
$$= \frac{1520 \times 100}{40} = 3800$$

EXAMPLE 5 A sum of money doubles itself in 10 years at simple interest. What is the rate of interest?

Solution Let the sum be ₹ 100.

After 10 years amount = ₹ 200

∴ Interest accrued
$$I = 200 - 100 = ₹ 100$$

$$\therefore \qquad \text{Rate } r = \frac{100I}{Pt} = \frac{100 \times 100}{100 \times 10} = 10\%$$



Short cut: Rate =
$$\frac{100 \text{ (Multiple number of Principal } - 1)}{\text{Time}}$$

Rate =
$$\frac{100(2-1)}{10}$$
 = 10%

EXAMPLE 6 A sum of money becomes four times at simple interest rate at 5% per annum in what time?

Solution Time = $\frac{100(\text{Multiple number of principal} - 1)}{\text{Rate}}$ $= \frac{100(4-1)}{5} = 60 \text{ years}$

EXAMPLE 7 A certain sum of money amounts to ₹ 756 in 2 years and to ₹ 873 in 3.5 years. Find the sum and the rate of interest.

Solution
$$P + SI$$
 for 2 years = 756 (1)

$$P + SI \text{ for } 3.5 \text{ years} = 873$$
 (2)

$$I = \frac{Prt}{100} 9$$

:. SI for 1.5 years = 873 - 756 = 117

i.e.,
$$r = \frac{I \times 100}{P \times t} = \frac{117 \times 100}{P \times 1.5}$$

$$Pr = \frac{11700}{1.5} = 7800$$

$$\therefore \qquad \text{For 2 years, interest} = \frac{P \times 2 \times r}{100} = 756 - P$$
 i.e.
$$\frac{2 \times 7800}{100} = 756 - P$$

$$\therefore (3) \Rightarrow r = \frac{7800}{P} = \frac{7800}{600} = 13\% \text{ p.a.}$$

EXAMPLE 8 A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched ₹ 300 more. Find the sum.

Solution Let the sum be x deposited at y% per annum.

For simple interest deposit for 2 years,
$$I_1 = \frac{x \times 2 \times y}{100}$$

When the interest rate is 3% more (i.e. y + 3)

$$I_2 = \frac{x \times 2 \times (y+3)}{100} = 300 + I_1$$
i.e.
$$I_2 - I_1 = \frac{2x}{100} [(y+3) - y] = 300$$

$$\Rightarrow \qquad 3x = \frac{100 \times 300}{2} = ₹ 15000$$
or
$$x = ₹ 5000$$



Short cut: Sum =
$$\frac{\text{More interest} \times 100}{\text{Time} \times \text{More rate}} = \frac{300 \times 100}{2 \times 3} = ₹5000$$

Theorem: A sum X is lent out in n parts in such a way that the interest on the first part at r_1 % for t_1 years, the interest on the second part at r_2 % for t_2 years, the interest on the third part at r_3 % for t_3 years and so on are equal. The ratio in which the sum was divided in n parts is given by $\frac{1}{r_1t_1}:\frac{1}{r_2t_2}:\frac{1}{r_3t_3}:\cdots:\frac{1}{r_nt_n}$.

EXAMPLE 9 A sum of ₹ 26000 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another part at 9% for 6 years. Find the two sums.

Solution Interest =
$$\frac{P_1 \times 5 \times 10}{100} = \frac{P_2 \times 6 \times 9}{100}$$

or $\frac{P_1}{P_2} = \frac{6 \times 9}{5 \times 10} = \frac{27}{25} = 27:25$

:. Ist part =
$$26000 \times \frac{27}{(27+25)} =$$
₹ 13500

$$\therefore$$
 2nd part = 26000 - 13,500 = ₹ 12500

Using the above shortcut,

1st part : 2nd part =
$$\frac{1}{r_1 t_1}$$
: $\frac{1}{r_2 t_2}$
= $\frac{1}{10 \times 5}$: $\frac{1}{9 \times 6}$ = $\frac{1}{50}$: $\frac{1}{54}$ = 54 : 50 = 27 : 25

EXAMPLE 10 A certain sum amounted to ₹575 at 5% in a time in which ₹750 amounted to ₹840 at 4%. If the rate of interest is simple, find the sum.

Solution Interest = 840 - 750 = 90

:. Time =
$$\frac{90 \times 100}{750 \times 4} = 3 \text{ years}$$

$$Sum = \frac{100 \times Amount}{100 + rt} = \frac{100 \times 575}{100 + 5 \times 3} = ₹ 500$$



Short cut: Sum =
$$\frac{100 \times \text{Amount}}{100 + rt}$$

EXAMPLE 11 A person lent a certain sum at 4% per annum simple interest, and in 8 years the interest amounted to ₹ 340 less than the sum lent. Find the sum lent.

Solution Let the sum be $\mathfrak{T} x$.

$$\therefore \qquad \qquad \text{Interest} = \frac{Prt}{100} = \frac{x \times 4 \times 8}{100} = \frac{32x}{100}$$

$$= x - 340$$

$$\therefore \qquad \qquad x - \frac{32x}{100} = 340$$

$$\Rightarrow \qquad \qquad \frac{68x}{100} = 340$$

$$\Rightarrow \qquad \qquad x = \frac{340 \times 100}{68} = ₹ 500$$



Short cut: Sum =
$$\frac{100}{100 - 8 \times 4} \times 340 = ₹ 500$$

i.e.
$$Sum = \frac{100}{100 - rt} \times (Difference in Principal and Interest)$$

Theorem: When different amounts mature at the same amount at simple rate of interest, the ratio of amounts invested is in inverse ratio of $(100 + \text{rate} \times \text{time})$. That is, the ratio in which

amounts are invested is
$$\frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$$

EXAMPLE 12 Two equal amounts of money are deposited in two banks each at 15% per annum for 3.5 years and 5 years respectively. If the difference between their interests is ₹ 144, find each sum.

Solution Let the sum be \mathbb{Z} x.

Then
$$\frac{x \times 15 \times 5}{100} - \frac{x \times 15 \times 7}{200} = 144$$
∴
$$\frac{x}{200} (150 - 105) = 144$$
i.e.
$$x = \frac{200 \times 144}{45} = ₹ 640$$

Short cut: Two equal amounts are deposited at r_1 % and r_2 % for t_1 years and t_2 years respectively. If difference in interest is I_d , then

Sum =
$$\frac{I_d \times 100}{r_1 t_1 - r_2 t_2}$$

Sum = $\frac{144 \times 100}{15 \times 5 - 15 \times 3.5} = \frac{144 \times 100}{22.5} = ₹ 640$

Here,

EXAMPLE 13 Some amount out of ₹7000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹1600, find the sum lent at 6% per annum.

Solution Let the amount be \mathbb{Z} x that was lent at 6% per annum simple interest.

$$\therefore \frac{x \times 6 \times 5}{100} + \frac{(7000 - x) \times 4 \times 5}{100} = 1600$$
i.e.,
$$\frac{3x}{10} + \frac{7000 - x}{5} = 1600$$
or
$$3x + 2(7000 - x) = 16000$$
or
$$x + 14000 = 16000$$
or
$$x = 16000 - 14000 = ₹ 2000$$

By method of Alligation:

Overall rate of interest =
$$\frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$$

$$\frac{6\%}{32}$$

$$\frac{\frac{32}{7}\%}{\frac{4}{7}\%}$$
 $\frac{10}{7}\%$

 \therefore Ratio of two amounts = 2:5.

∴ Amount lent at
$$6\% = 7000 \times \frac{2}{7} = ₹ 2000$$

EXAMPLE 14 A person borrows ₹ 5,000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}$ % p.a. for 2 years. Find his gain in the transaction per year. [SSC]

Solution

Gain in 2 years =
$$\left(5000 \times \frac{25}{4} \times \frac{1}{100} \times 2\right) - \left(5000 \times 4 \times \frac{2}{100}\right)$$

= $5000 \times \frac{2}{100} \times \left(\frac{25}{4} - 4\right) = 5000 \times \frac{2}{100} \times \left(\frac{25 - 16}{4}\right)$
= $100 \times \frac{9}{4} = ₹ 225$

:. Gain in 1 year =
$$\frac{225}{2}$$
 = 112.50

EXAMPLE 15 A sum of $\stackrel{?}{\stackrel{?}{\stackrel{?}{?}}}$ 12,500 amounts to $\stackrel{?}{\stackrel{?}{\stackrel{?}{?}}}$ 15,500 in 4 years at the rate of simple interest. What is the rate of interest? [Bank PO]

Solution Let R% be the rate of interest p.a.

$$A = P + \frac{PNR}{100}$$
or
$$I = \frac{PNR}{100}$$

$$\therefore 15500 - 12500 = \frac{12500 \times 4 \times R}{100}$$
or
$$R = \frac{3000 \times 100}{50000} = 6\%$$

EXAMPLE 16 A sum fetched a total simple interest of ₹ 4016.25 at the rate of 9% p.a. in 5 years. What is the sum? [NABARD]

Solution Let P be the sum.

$$P \times 5 \times \frac{9}{100} = 4016.25$$

$$P = \frac{4016.25 \times 100}{9 \times 5} = \left(\frac{401625}{9 \times 5}\right)$$

$$= ₹ \left(\frac{401625}{45}\right) = ₹ 8925$$

EXAMPLE 17 Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers representing the rate of interest in per cent and time in years be equal, then time, for which the principal is lent out is: [RRB]

Solution Let P be the principal and N be the period in years

$$\therefore \qquad \text{Simple interest} = \frac{P \times N \times R}{100} = \frac{9}{16}P$$
or
$$\frac{NR}{100} = \frac{9}{16}$$

Given that N = R.

$$\therefore \frac{N^2}{100} = \frac{9}{16}$$

Taking square root on both sides,

$$\frac{N}{10} = \frac{3}{4}$$

$$N = \frac{10 \times 3}{4} = 7\frac{1}{2} \text{ years}$$

or

EXAMPLE 18 The difference between the simple interest received from two different sources on ₹ 1500 for 3 years is ₹ 13.50. The difference between their rates of interest is: [RRB]

Solution Let R_1 and R_2 be the rates of interest.

$$\therefore \frac{1500 \times 3 \times R_1}{100} - \frac{1500 \times 3 \times R_2}{100} = 13.50$$

$$\therefore R_1 = R_2 = 13.50 \times 100 = 1350$$

$$\therefore R_1 - R_2 = \frac{13.50 \times 100}{1500 \times 3} = \frac{1350}{4500} = \mathbf{0.3\%}$$

? EXERCISES

- 1. If ₹ 640 amounts to ₹ 832 in 2 years, what will ₹ 860 amount to in 4 years at the same rate of interest per annum?
 - (a) ₹ 1736
- (b) ₹ 1376
- (c) ₹3176
- (d) ₹3716
- (e) None of these
- 2. A sum of ₹25400 was lent out in two parts, one at 12% and the other at $12\frac{1}{2}$ %. If the total annual income is 3124.2, the money lent at 12% is ______.
 - (a) 15240
- (b) 25400
- (c) 10160
- (d) 31242
- (e) None of these
- 3. A sum of ₹ 26000 was lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on the other part at 9% for 6 years. The sum lent out at 10% is
 - (a) 12500
- (b) 13500
- (c) 26000
- (d) 39000
- (e) None of these

3	22
.1	32.

4.	A sum of money of	wil	l double itse	lf ir	n 16 years at	sim	iple interest	at a	an yearly rate
	(a) 25%	(b)	$12\frac{1}{2}\%$	(c)	$6\frac{1}{4}\%$	(d)	$8\frac{1}{3}\%$	(e)	None of these
5.	A sum of money p	ut at	simple intere	st tr	ebles itself in 1	.5 ye	ears. The rate	per o	cent per annum
	(a) $6\frac{1}{3}\%$	(b)	$8\frac{1}{3}\%$	(c)	$13\frac{1}{3}\%$	(d)	$12\frac{1}{3}\%$	(e)	None of these
6.	₹ 8000 amounts t 3%, to how much			s at	simple interes	t. If	the interest r	ate	is increased by
	(a) 9400		9600	(c)	9720	(d)	9920	(e)	None of these
7.	A man invested 1 income is ₹ 561, t		_ -			the	remainder at	10%	6. If the annual
	(a) 4400	(b)	5500	(c)	6600	(d)	5800	(e)	None of these
8.	If the interest on	₹ 120	000 be more th	an 1	the interest on	₹ 10	0000 by ₹ 500	in 3	years, the rate
	per cent is								
	(a) $6\frac{1}{4}\%$	(b)	$12\frac{1}{2}\%$	(c)	$8\frac{1}{3}\%$	(d)	$16\frac{1}{3}\%$	(e)	None of these
9.	A sum was put at rate, it would have							en p	ut at 1% higher
	(a) ₹ 600	(b)	₹ 800	(c)	₹ 1200	(d)	₹ 1600	(e)	None of these
10.	A sum of money be per annum is			in 5	years at a cert	ain			
	(a) 8% The difference bet	(b)	10%	(c)	12%	(d)	$12\frac{1}{2}\%$	(e)	None of these
11.	The difference bet is ₹ 25. The differ					diffe	erent banks or	ւ₹ 5	,000 for 2 years
	(a) 3%	(b)	2%	(c)	1/2%	(d)	0.25%	(e)	None of these
12.	Two equal amoun	ıts ar	e deposited in	ı two	o banks, each a	at 15	5% per annum	, for	$3\frac{1}{2}$ years and
	5 years respective	ly. If	the difference	betv	ween their inte	rest	s is ₹ 1440, eac	h su	m is
	(a) ₹ 4600	(b)	₹ 5000	(c)	₹ 6400	(d)	₹ 7200		
13.	The rate of interenext 4 years and 8	3% ре	er annum for t	he p	eriod beyond 6	yea	rs . If the sim		
	by the sum for a t							<i>(</i>)	NI 641
	(a) ₹ 1000	` '	₹ 2000		₹ 3000				None of these
L 4.	The simple intererate per cent per							ears	s is equal to the
	(a) $1\frac{1}{2}\%$		_		_		_		
15.	A lent ₹ 6000 to B ₹ 900 as simplex							ltoge	ether from both

16. Divide ₹ 2379 into 3 parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is: [CBI] (c) ₹818

(c) 4%

(a) ₹ 759

(b) ₹ 792

(b) 3%

(d) 828

(d) 5%

(e) None of these

[Bank PO]

(c) ₹8000

18.	A money lender f	finds that due to a	fall in the annual	rate of interest from 8% to	$7\frac{3}{4}$ %, his
	yearly income dir	minishes by ₹ 61.5	0. His capital is:		[SSC]
	(a) ₹ 22400	(b) ₹ 23800	(c) ₹ 24600	(d) ₹ 26000	
19.	If the annual rat	e of simple interes	st increases from	10% to $12\frac{1}{2}$ %, a man's yea	arly income
	increases by ₹ 12	50. His principal (in ₹) is:		[SSC]
	(a) ₹ 45000	(b) ₹ 50000	(c) ₹ 60000	(d) ₹ 65000	
20.	amount at the ra	te of 20% p.a. sim	ple interest. The t	10% p.a. simple interest a total interest earned at the find the total amount investigation.	end of one
	(a) ₹ 20000	(b) ₹ 22000	(c) ₹ 24000	(d) ₹ 25000 (e) No	ne of these
21.				ars on simple interest at th m as interest. The rate of i	
	(a) 5%	(b) 7%	(c) $7\frac{1}{8}\%$	(d) 10%	
22.	If after 4 years, A C is:	A altogether receive	es ₹ 1120 as intere	same time at 7% p.a. simplest from B and C, then the	
	(a) ₹ 700	(b) ₹ 1500	(c) ₹ 4000	(d) ₹ 6500	
23.	At what rate per amount in 10 yea		vill the simple int	erest on a sum of money b	pe $\frac{2}{5}$ of the [SSC]
	(a) 4%	(b) $5\frac{2}{3}\%$	(c) 6%	(d) $6\frac{2}{3}\%$	
24.	At what rate per	cent of simple inte	erest will a sum of	money double itself in 12	years? [SSC]
	(a) $8\frac{1}{4}\%$	(b) $8\frac{1}{3}\%$	(c) $8\frac{1}{2}\%$	(d) $9\frac{1}{2}\%$	
25.		lent out at simple eriod of 5 years. Th		to ₹ 720 after 2 years and	l to ₹ 1,020 [SSC]
	(a) ₹ 500	(b) ₹ 600	(c) ₹ 710	(d) ₹ 700	
26.		amounts to₹9,800 t. The rate of inter		₹ 12,005 after 8 years at th	e same rate [SSC]
	(a) 5%	(b) 8%	(c) 12%	(d) 15%	

17. David invested certain amount in three different schemes A, B and C with the rate of interest 10% p.a., 12% p.a. and 15% p.a. respectively. If the total interest accrued in one year was ₹ 3200 and the amount invested in scheme C was 150% of the amount invested in scheme A and 240% of the amount invested in scheme B, what was the amount invested in

(b) ₹6500

(e) None of these

scheme B?

(a) ₹5000

(d) Cannot be determined

(a) ₹8000

(e) None of these

28.	A sum invested at 5% simple interest per annum grows to ₹ 504 in 4 years. The same amount					
	at 10% simple in	terest per annum	in $2\frac{1}{2}$ years will	grow to:	[CDS]	
	(a) ₹ 420	(b) ₹ 450	(c) ₹ 525	(d) ₹ 550		
29.	In how many yea	ars, ₹ 150 will pro	duce the same int	erest @8% as ₹ 800 pro	duce in 3 years	
	$@4\frac{1}{2}\%$?				[RRB]	
	(a) 6	(b) 8	(c) 9	(d) 12		
30.		e ratio of simple ars and that of 9 y		y certain amount at th	[Bank PO]	
	(a) 1:3	(b) 1:4	(c) 2:3	(d) Data inadequate		
	(e) None of thes					
31.				imple interest. If the ra		
			1₹800 become in	· ·	[Bank PO]	
	(a) ₹ 1020.80	(b) ₹ 1025	(c) ₹ 1052	(d) Data inadequate	:	
	(e) None of thes				•	
32.			uld it have earned	fter 7 years. Had the in?	iterest been 2% [Bank PO]	
	(a) ₹ 35	(b) ₹ 245	(c) ₹ 350	(d) Cannot be deter	mined	
	(e) None of thes					
33.	What is the pres			5% simple interest per	annum? [CBI]	
	(a) ₹ 112	(b) ₹ 118.8 0	(c) ₹ 120	(d) ₹ 122		
34.				a. simple interest. After pal amount borrowed b		
	(a) ₹ 2000	(b) ₹ 10000	(c) ₹ 15000	(d) ₹ 20000		
35.				r as many years as the riod, what was the rate		
	(a) 3.6	(b) 6	(c) 18	(d) Cannot be deter	mined	
	(e) None of thes	e				
36.	p.a. for the next t	hree years, and at	the rate of 14% p.	or the first two years, a a. for the period beyond ears, how much money	five years. If he	
					[Bank PO]	
	(a) ₹ 10000	(b) ₹ 11000	(c) ₹ 12000	(d) ₹ 13000 (e)	None of these	
37.	At what rate per	cent per annum v	will a sum of mone	y double in 16 years?	[RRB]	
	(a) $5\frac{1}{4}\%$	(b) $6\frac{1}{4}\%$	(c) $7\frac{1}{4}\%$	(d) $8\frac{1}{4}\%$ (e)	None of these	
38.	38. A sum of ₹ 1,550 is lent out into two parts, one at 8% and another one at 6%. If the total					
			oney lent at each		[LIC AAO]	
	(a) ₹600 950	(b) ₹ 650 900	(c) ₹ 700 850	(d) ₹550 1 000 (e)	None of these	

27. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond 8 years. If the total interest paid by him

(c) ₹ 12000

(d) Data inadequate

at the end of eleven years is ₹8,160, how much money did he borrow?

(b) ₹ 10000

- 39. The simple interest on a sum of money is $\frac{4}{9}$ of the principal. Find the rate per cent and time, if both are numerically equal. [SSC]
 - (a) $6\frac{1}{3}$ %, 6 years 4 months
- (b) $6\frac{2}{3}\%$, 6 years 8 months
- (c) $5\frac{2}{3}$ %, 5 years 8 months
- (d) $5\frac{1}{3}$ %, 5 years 4 months
- (e) None of these
- 40. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes:

 [NIFT]
 - (a) 10%
- (b) 10.25%
- (c) 10.5%
- (d) None of these



- 1. (b) 2. (c) 3. (b) 4. (c) 5. (c) 6. (d) 7. (c) 8. (c) 9. (c) 10. (c)
- 11. (d) 12. (c) 13. (b) 14. (b) 15. (d) 16. (d) 17. (a) 18. (c) 19. (b) 20. (a)
- 21. (d) 22. (b) 23. (a) 24. (b) 25. (b) 26. (c) 27. (a) 28. (c) 29. (c) 30. (c)
- 31. (c) 32. (d) 33. (c) 34. (e) 35. (b) 36. (c) 37. (b) 38. (b) 39. (b) 40. (b)



Solutions with Necessary Explanation

- 1. Rate of interest = $\frac{192 \times 100}{640 \times 2} = 15\%$
 - ∴ Interest on ₹ 860 = $\frac{860 \times 15 \times 4}{100}$ = ₹ 516
 - ∴ Amount = 860 + 516 = ₹ 1376



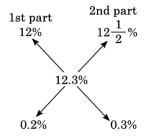
Short cut: Interest on ₹ 640 for 2 years is ₹ 192.

∴ Interest on ₹ 860 for 4 years

$$= 192 \left\lceil \frac{860}{640} \right\rceil \times \left\lceil \frac{4}{2} \right\rceil = ₹516$$

- . Amount = 860 + 516 = ₹ 1376
- **2.** Overall rate of interest = $\frac{3124.2}{25400} \times 100 = 12.3\%$
 - \therefore The sum will be divided in the ratio 0.2:0.3 or 2:3.
 - ∴ The sum lent at $12\% = 25400 \times \frac{2}{5} = ₹10160$

The sum lent at $12\frac{1}{2}\% = 25400 \times \frac{3}{5} = ₹ 15240$





Short cut: Ratio of two parts = r_2t_2 : r_1t_1

$$= 9 \times 6 : 10 \times 5 = 54 : 50 = 27 : 25$$

∴ Sum lent out at
$$10\% = 26000 \times \frac{27}{(27 + 25)} = ₹ 13500$$

4. The rate of interest =
$$\frac{100(2-1)}{16} = 6\frac{1}{4}\%$$

5. The rate of interest =
$$\frac{100(3-1)}{15} = \frac{200}{15} = 13\frac{1}{3}\%$$

6. The rate of interest =
$$\frac{1200 \times 100}{8000 \times 3} = 5\%$$

New rate becomes 5 + 3 = 8%

$$\therefore \qquad \text{Interest} = \frac{8000 \times 8 \times 3}{100} = \text{ } 1920$$

$$∴ Amount = 8000 + 1920 = ₹ 9920$$



Short cut: Increase in interest = $\frac{8000 \times 3 \times 3}{100} = ₹720$

Increased amount =
$$9200 + 720 = ₹ 9920$$

7. Let the capital be ₹ 120

Then total interest = 7% of
$$\frac{120}{3}$$
 + 8% of $\frac{120}{4}$ + 10% of remainder.
= 7% of 40 + 8% of 30 + 10% of 50
= 2.8 + 2.4 + 5 = ₹ 10.2

∴ Actual capital =
$$\frac{120}{10.2} \times 561 = ₹6600$$



Short cut: Capital = $\frac{561}{\left(7\% \text{ of } \frac{1}{3} + 8\% \text{ of } \frac{1}{4} + 10\% \text{ of } \frac{5}{12}\right)}$ $= \frac{561 \times 100}{\frac{7}{2} + \frac{8}{4} + \frac{25}{6}} = \frac{561 \times 100 \times 12}{28 + 24 + 50} = ₹6600$

8. Let the rate of interest be x %

Then
$$\frac{12000 \times 3x}{100} = \frac{10000 \times 3x}{100} + 500$$

$$\therefore \qquad 360x = 300x + 500$$

$$60x = 500$$

$$x = \frac{500}{60} = 8\frac{1}{3}\%$$



Short cut: Rate =
$$\frac{\text{Difference in interest} \times 100}{\text{Time (Difference in principal)}}$$

$$=\frac{500\times100}{3(12000-10000)}=\frac{25}{3}=8\frac{1}{3}\%$$

9. Let the sum deposited be P at r% per annum

$$\therefore \frac{P \times r \times 2}{100} = \frac{P(r+1) \times 2}{100} - 24$$

i.e.
$$\frac{2P}{100} - 24 = 0$$

$$\mathbf{or}$$

$$P = \frac{24 \times 100}{2} = 7 1200$$

∴ The sum deposited = ₹ 1200



Short cut: Sum =
$$\frac{\text{(Difference in interests)} \times 100}{\text{Time} \times \text{Difference in rate}}$$

$$=\frac{24 \times 100}{2 \times 1}$$
 = ₹ 1200

10. Rate =
$$\frac{100\left(\frac{8}{5} - 1\right)}{\text{Time}} = \frac{100 \times \frac{3}{5}}{5} = \frac{300}{25} = 12\%$$

 $Sum = \frac{Difference in interest \times 100}{Time \times Difference in rate}$ 11.

i.e.
$$5000 = \frac{25 \times 100}{2 \times r}$$

i.e.
$$x = \frac{2500}{5000 \times 2} = \frac{1}{4} = 0.25 \%$$

12. Sum = $\frac{\text{Difference in interests} \times 100}{\text{Rate} \times \text{Difference in times}} = \frac{1440 \times 100}{15 \times 1.5} = ₹ 6400$

13. Sum =
$$\frac{\text{Interest} \times 100}{r_1 t_1 + r_2 t_2 + r_3 t_3 + \cdots} = \frac{1120 \times 100}{4 \times 2 + 6 \times 4 + 8 \times 3} = \frac{1120 \times 100}{56} = ₹ 2000$$

14. Let the rate of interest be r%.

$$\therefore$$
 time = r years

$$\therefore \frac{S}{16} = \frac{S \times r \times r}{100}$$

or
$$r^{2} = \frac{100}{16} = \frac{25}{4}$$
 or
$$r = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}\%$$

15. Let the rate of interest be r% per annum.

Then
$$\frac{6000 \times 2 \times r}{100} + \frac{1500 \times 4 \times r}{100} = 900$$
i.e.
$$120r + 60r = 900$$

$$\therefore \qquad r = \frac{900}{180} = 5\%$$

- **16.** x, y and z be the three parts such that x + y + z = 2379
 - .. As the three amounts with interest are separately equal,

$$x + \frac{x \times 2 \times 5}{100} = y + \frac{y \times 3 \times 5}{100} = z + \frac{z \times 4 \times 5}{100}$$
i.e.
$$\frac{11x}{10} = \frac{23y}{20} = \frac{6z}{5} = k$$
or
$$x = \frac{10k}{11}; \ y = \frac{20k}{23}; \ z = \frac{5k}{6};$$
∴
$$x + y + z = \frac{10k}{11} + \frac{20k}{23} + \frac{5k}{6} = 2379$$

$$\Rightarrow \qquad 1380k + 1320k + 1265k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow \qquad 3965k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow \qquad k = \frac{2379 \times 11 \times 23 \times 6}{3965} = \frac{3 \times 11 \times 23 \times 6}{5}$$
∴
$$x = \frac{10k}{11} = \frac{10 \times 3 \times 11 \times 23 \times 6}{11 \times 5} = ₹828$$

17. Let the amount invested in scheme B be x.

$$\therefore$$
 Investment in scheme $C = \frac{240}{100} \times x = 2.4x$

$$\therefore$$
 Investment in scheme A = $\frac{240}{100} \times x \times \frac{100}{150} = 1.6x$

Total interest accrued = $1.6x \times \frac{10}{100} + x \times \frac{12}{100} + 2.4x \times \frac{15}{100} = 3200$

i.e.,
$$0.16x + 0.12x + 0.36x = 3200$$
or
$$0.64x = 3200$$
or
$$x = \frac{3200}{0.64} = 5000$$

i.e. Investment in scheme B = ₹ 5000

18. Let the capital be x.

19. Let his principal (in ₹) be x.

$$x \times (12\frac{1}{2} - 10)\% = 1250$$
or
$$x = \frac{1250}{2\frac{1}{2}\%}$$

$$= \frac{1250}{2.5} \times 100 = ₹ 50000$$

20. Let the total amount invested be x.

21. Let the rate of simple interest be x%.

$$\therefore 5000 \times 2 \times \frac{x}{100} + 3000 \times 4 \times \frac{x}{100} = 2200$$
i.e.
$$100x + 120x = 2200$$

$$x = \frac{2200}{100 + 120} = 10\%$$

22. Let the sum lent to C be x.

$$\therefore 2500 \times \frac{7}{100} \times 4 + x \times \frac{7}{100} \times 4 = 1120$$

$$\Rightarrow 700 + \frac{7x}{25} = 1120$$

$$\Rightarrow \frac{7x}{25} = 1120 - 700$$

$$\Rightarrow \qquad x = \frac{420 \times 25}{7}$$
$$= ₹ 1500$$

23. Let x be the rate per cent per annum and P be the principal.

$$\therefore \qquad P \times 10 \times \frac{x}{100} = \frac{2}{5}P$$

$$\Rightarrow \qquad \qquad x = \frac{2}{5} \times \frac{100}{10} = 4\%$$

24. Let x be the rate per cent per annum and P be the principal.

$$\therefore \qquad P \times 12 \times \frac{x}{100} = P$$

$$\Rightarrow \qquad \qquad x = \frac{100}{12}\% = 8\frac{1}{3}\%$$

25. Let the sum be P and rate be x%.

$$P \times 2 \times \frac{x}{100} + P = 720$$
or
$$P\left(1 + \frac{x}{50}\right) = 720 \tag{1}$$

$$P \times 7 \times \frac{x}{100} + P = 1020$$
or
$$P\left(1 + \frac{7x}{100}\right) = 1020 \tag{2}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\left(1 + \frac{x}{50}\right)}{\left(1 + \frac{7x}{100}\right)} = \frac{720}{1020}$$

$$\Rightarrow \frac{\left(\frac{100 + 2x}{100}\right)}{\left(\frac{100 + 7x}{100}\right)} = \frac{720}{1020}$$

$$\Rightarrow x = 10\%$$

$$\therefore (1) \Rightarrow P = \frac{720}{(1 + \frac{x}{50})} = \frac{720 \times 50}{(1 + \frac{10}{50})} = 7600$$

26. Let the rate of interest be x% and sum be $\not\in P$.

$$P + P \times 5 \times \frac{x}{100} = 9800$$
or
$$P\left(1 + \frac{x}{20}\right) = 9800$$

$$P + P \times \frac{8x}{100} = 12005$$
or
$$P\left(1 + \frac{8x}{100}\right) = 12005$$

$$\left(\frac{1}{1} + \frac{8x}{100}\right) = \frac{12005}{9800}$$

$$\frac{\left(\frac{1 + \frac{8x}{100}\right)}{\left(1 + \frac{x}{20}\right)} = \frac{12005}{9800}$$

$$\frac{\frac{(100 + 8x)/100}{(20 + x)/20} = \frac{12005}{9800}$$

$$\frac{100 + 8x}{100 + 5x} = \frac{12005}{9800} = \frac{2401}{1960} = \frac{343}{280}$$

$$\frac{4(25 + 2x)}{5(20 + x)} = \frac{49}{40}$$
i.e.
$$\left(\frac{25 + 2x}{20 + x} = \frac{49}{32}\right)$$

x = 12%

27. Let the amount borrowed by Nitin be $\not\in x$.

$$\therefore \quad x \times 3 \times \frac{6}{100} + x \times \frac{5 \times 9}{100} + x \times \frac{3 \times 13}{100} = 8160$$
i.e.
$$x \left(\frac{18}{100} + \frac{45}{100} + \frac{39}{100} \right) = 8160$$
or
$$x \times \frac{102}{100} = 8160$$
or
$$x = \frac{8160 \times 100}{102} = 8000$$

. Amount borrowed by Nitin = ₹8000

28. Let P be the sum.

or

 \Rightarrow

$$P + P \times 4 \times \frac{5}{100} = 504$$

$$\frac{6P}{5} = 504$$
(1)

$$P + P \times \frac{5}{2} \times \frac{10}{100} = \frac{5P}{4}$$

$$= \frac{5}{4} \times \left(\frac{5 \times 504}{6}\right) \quad \text{[value of } P \text{ from Eq. (1)]}$$

$$= 5 \times 5 \times \frac{504}{24} = 25 \times 21 = ₹525$$

29. Let the number of years be N.

$$150 \times N \times \frac{8}{100} = 800 \times 3 \times \frac{9}{200}$$

$$\Rightarrow 12N = 108$$
 or
$$N = 9 \text{ years}$$

30. $N_1 = 6$ years; $N_2 = 9$ years; $R_1 = R_2$;

$$\frac{PN_1R_1}{100} \div \frac{PN_2R_2}{100} = \frac{N_1R_1}{N_2R_2} = \frac{N_1}{N_2} \text{ (as } R_1 = R_2)$$

$$\frac{6}{9} = \frac{2}{3}$$

$$N_1 : N_2 = 2 : 3$$

31.
$$800 \times 3 \times \frac{x}{100} = 956 - 800 = 156$$
or
$$24x = 156$$
or
$$2x = 13$$

or
$$2x = 13$$

or $x = \frac{13}{2}\% = 6\frac{1}{2}\%$

:. Increased rate =
$$\left(6\frac{1}{2} + 4\right) = 10\frac{1}{2}\% = \frac{21}{2}\%$$

∴ Amount =
$$800 \times 3 \times \frac{21}{200} + 800 = 252 + 800 = ₹ 1052$$



:.

Short cut:
$$x = \frac{156}{800 \times 3} \times 100$$

= $\frac{156}{24} = 6\frac{1}{2}\%$

.. New amount =
$$800 + 800 \times 3 \times \frac{10\frac{1}{2}}{100}$$

= $800 + 252 = ₹ 1052$

32. Interest/year =
$$\frac{1750}{7}$$
 = ₹ 250

Hence interest cannot be determined as data is inadequate.

33. Le the present worth be $\not\in x$.

$$\therefore \qquad x\left(1 + \frac{2 \times 5}{100}\right) = 132$$

$$\Rightarrow \qquad x = \frac{132 \times 100}{100 + 10}$$

$$\Rightarrow \qquad x = \frac{1320}{11}$$

$$\Rightarrow \qquad x = 120$$

∴ Present worth = ₹ 120

34. Let the principal amount borrowed be $\overline{\xi}$ x.

$$x \times 3 \times \frac{12}{100} = 5400$$
or
$$x = 7 \cdot 15000$$

35. Let the loan period and interest be x.

$$\therefore 1200 \times x \times \frac{x}{100} = 432$$
or
$$x^2 = \frac{432}{12} = 36$$
or
$$x = 6\%$$

:. Rate of interest = 6%

36. Let the amount borrowed be \overline{x} .

$$\therefore x \times \left(2 \times \frac{6}{100} + 3 \times \frac{9}{100} + 4 \times \frac{14}{100}\right) = 11,400.$$

$$x \times \frac{95}{100} = 11,400$$
or
$$x = \frac{11400 \times 20}{19} = ₹ 12000$$

∴ Amount borrowed = ₹ 12000

37. Let P be the principal and x be rate of interest.

$$P\left(1 + \frac{16 \times x}{100}\right) = 2P$$
or
$$\frac{16x}{100} = 1$$
or
$$x = \frac{100}{16}\% = 6\frac{1}{4}\%$$

 \therefore Rate percent = $6\frac{1}{4}\%$

38. Let amouts be x and (1550 - x).

$$\therefore x \times \frac{8}{100} + (1550 - x) \times \frac{6}{100} = 106$$

$$8x - 6x + 1550 \times 6 = 10600$$

or

- :. Amounts are 650 and 900
- **39.** Let principal be P and rate per cent and time be x each.

$$\therefore \qquad P \times x \times \frac{x}{100} = \frac{4}{9}P$$

 \mathbf{or}

$$x^2 = \frac{400}{9}$$

 \mathbf{or}

$$x = \frac{20}{3} = 6\frac{2}{3}$$

- \therefore Rate per cent = $6\frac{2}{3}\%$ and period = $6\frac{2}{3}$ years = 6 years and 8 months.
- **40.** Let the amount lent be ₹ 100.
 - ∴ Total amount with interest at the end of 6 months = $100 + 100 \times \frac{5}{100} = ₹ 105$
 - : Amount at the end of 12 months

$$= 105 + 105 \times \frac{5}{100}$$
$$= 105 + 5.25$$

=110.25

 \therefore Total effect = 110.25 - 100 = 10.25%

Compound Interest

When an amount P is deposited in bank for a certain rate of interest (r% p.a.) on the condition that every year interest will be calculated and added to the principal. The way of calculating interest in this fashion is called compounding. The interest thus accrued is called compound interest.

$$A = P \left(1 + \frac{R}{100} \right)^N$$

where P = Principal amount (initial)

R =Rate of interest per annum

N = Duration of deposit in years

A = Amount accrued at the end of the period

N can also be denoted in terms of half year/quarter, etc. Then the corresponding rate will

be
$$\frac{R}{2}$$
 or $\frac{R}{4}$.

An article worth (P) at present $\mathcal{F}A$, t years hence is given by

$$P = \frac{A}{\left(1 + \frac{r}{100}\right)^t}$$

A sum at a rate of interest compounded yearly becomes $\mathcal{F}A_1$ in n years and $\mathcal{F}A_2$ in (n+1) years, then

$$r = \frac{100(A_2 - A_1)}{A_1}\%$$
 p.a. and $P = A_1 \left(\frac{A_1}{A_2}\right)^n$

A sum of loan amount A is to be paid back in n equal instalments of R x each. If the rates of interest is r% per annum compounded annually, then

$$x = \frac{Ar}{100 \left[1 - \left(\frac{100}{100 + r} \right)^n \right]}$$

An amount A due n years hence is to be paid in n equal annual instalments of R x each. If the rate of interest is r% per annum compounded annually, then

$$x = \frac{Ar}{100 \left[1 - \left(\frac{100 + r}{100} \right)^n \right]}$$

Money that was deposited at Compound Interest (CI) for which interest is calculated at the end of each year or a fixed period and the interest that was due to the depositor is not paid and the same is added to the sum deposited and the amount so obtained is the principal for the next period. The same process continues till the term of the deposit ends and the difference between the final amount and the initial principal is called Compound Interest (CI).

Important Formulae

Let the Principal = P, Time = t years and Rate = r% p.a.

Case I: When interest is compounded annually:

$$Amount = P \left[1 + \frac{r}{100} \right]^t$$

Case II: When interest is compounded half-yearly:

Amount =
$$P \left[1 + \frac{r/2}{100} \right]^{2t} = P \left[1 + \frac{r}{200} \right]^{2t}$$

Case III: When interest is compounded quarterly:

Amount =
$$P \left[1 + \frac{r/4}{100} \right]^{4t} = P \left[1 + \frac{r}{400} \right]^{4t}$$

Case IV: When the rate of interest is r_1 %, r_2 % and r_3 % for 1st year, 2nd year and 3rd year respectively, then

Amount =
$$P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$$

(a) To find the rate:

EXAMPLE 1 At what rate percentage compound interest does a sum become nine-fold in 2 years?

Solution Let the sum be \mathbb{Z} and rate be r% per annum, then

$$9x = x \left(1 + \frac{r}{100}\right)^2$$
or
$$9 = \left(1 + \frac{r}{100}\right)^2$$
or
$$3 = 1 + \frac{r}{100}$$

or
$$\frac{r}{100} = 2$$

$$\therefore r = 200\%$$



Short cut: If a certain sum becomes m times in t years, rate of compound interest r is

equal to $100[(m)^{1/t} - 1]$

Here,
$$r = 100[(9)^{1/2} - 1] = 100(3 - 1) = 200\%$$

EXAMPLE 2 At what rate percentage (compound interest) will a sum of money become eight times in three years?

Solution Rate% =
$$[(8)^{1/t}-1] \times 100$$

= $[(8)^{1/3}-1] \times 100$
= $[2-1] \times 100 = 100\%$

EXAMPLE 3 At what rate per cent compounded yearly will ₹80000 amount to ₹88200 in 2 years?

Solution We have

$$80000 \left(1 + \frac{r}{100}\right)^2 = 88200$$
or
$$\left(1 + \frac{r}{100}\right)^2 = \frac{88200}{80000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$$
or
$$\left(1 + \frac{r}{100}\right) = \frac{21}{20}$$

$$\therefore \qquad r = \frac{100}{20} = 5\%$$

EXAMPLE 4 The difference between compound interest and simple interest on an amount of ₹ 15000 for 2 years in ₹ 96. What is the rate of interest per annum? [RBI]

Solution

$$\left[15000\left(1 + \frac{R}{100}\right)^2 - 15000\right] - \frac{15000 \times R \times 2}{100} = 96$$

$$\Rightarrow \qquad 15000\left[\left(1 + \frac{R}{100}\right)^2 - 1 - \frac{R}{50}\right] = 96$$

$$\Rightarrow \qquad 15000\left[1 + \left(\frac{R}{100}\right)^2 + \frac{2R}{100} - 1 - \frac{2R}{100}\right] = 96$$

$$\Rightarrow \qquad 15000 \times \left(\frac{R}{100}\right)^2 = 96$$

or
$$R^{2} = \frac{96 \times 100^{2}}{15000}$$
or
$$R = \sqrt{\frac{96 \times 10^{4}}{15 \times 10^{3}}}$$

$$= \sqrt{\frac{960}{15}} = \sqrt{64} = 8\%$$

EXAMPLE 5 At what rate of compound interest per annum will a sum of ₹ 1500 become ₹ 1749.60 in 2 years?

Solution Let x% be the rate of compound interest per annum.

$$\therefore 1500 \left(1 + \frac{x}{100}\right)^2 = 1749.60$$

$$\Rightarrow \left(1 + \frac{x}{100}\right)^2 = \frac{1749.60}{1500} = 1.1664$$

$$\Rightarrow x = 8\%$$

EXAMPLE 6 Determine the effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly. [SSC]

Solution Let the principal be ₹ 100.

∴ Amount at the end of one year

$$= 100 \times \left(1 + \frac{3}{100}\right)^{2}$$
$$= 100 \times (1.03)^{2} = 100 \times 1.0609 = 106.09$$

- \therefore Effective rate of interest = 6.09%
- (b) To find time:

EXAMPLE 7 In what time will ₹ 30000 amount to 34347 at 7% conpound interest.

Solution Let the period be t years.

[LIC AAO]

 $\therefore \quad \text{Compound interest} = 30000 \left(1 + \frac{7}{100} \right)^t = 34347$

i.e.
$$\left(1 + \frac{7}{100}\right)^{t} = \frac{34347}{30000}$$
$$\left(\frac{107}{100}\right)^{t} = \frac{11449}{10000} = \left(\frac{107}{100}\right)^{2}$$
$$\therefore \qquad t = 2 \text{ years}$$

EXAMPLE 8 In how many years will a sum of ₹ 1600 at 10% per annum compound interest, compounded half yearly become ₹ 1944.81?

Solution Let t be the number of years.

- \therefore Number of half years = 2t.
- \therefore Amount at the end of t years is:

$$1600 \left(1 + \frac{5}{100}\right)^{2t} = 1944.81$$

$$\Rightarrow 1600 \times \left(\frac{105}{100}\right)^{2t} = 1600 \times \left(\frac{21}{20}\right)^{2t} = 1944.81$$

$$\therefore \left(\frac{21}{20}\right)^{2t} = \frac{1944.81}{1600} = \frac{194481}{160000}$$

$$\left(\frac{21}{20}\right)^{t} = \sqrt{\frac{194481}{160000}} = \frac{441}{400}$$

$$\Rightarrow \left(\frac{21}{20}\right)^{2} = \frac{441}{400}$$

$$\therefore t = 2 \text{ years}$$

EXAMPLE 9 In what time will ₹ 390625 amount to ₹ 456976 at 4% compound interest?

Solution Here
$$A = P\left(1 + \frac{r}{100}\right)^t$$

i.e. $390625\left(1 + \frac{4}{100}\right)^t = 456976$
 $\therefore \left(1 + \frac{1}{25}\right)^t = \frac{456976}{390625}$
i.e. $\left(\frac{26}{25}\right)^t = \frac{456976}{390625}$
i.e. $\left(\frac{26}{25}\right)^t = \left(\frac{26}{25}\right)^4$
 $\therefore t = 4$

The required time is 4 years.

EXAMPLE 10 A sum placed at compound interest doubles itself in 4 years. In how many years will it amount to eight times itself?

Solution If P is the principal and r, rate of interest per annum,

$$P\left(1 + \frac{r}{100}\right)^4 = 2P$$

$$\therefore \left(1 + \frac{r}{100}\right)^4 = 2$$

Cubing both sides,

$$\left(1 + \frac{r}{100}\right)^{12} = 2^3 = 8$$

or

∴

$$P\left(1 + \frac{r}{100}\right)^{12} = 8P$$

.. The required time to become the sum eight times itself = 12 years



Short cut: P becomes 2P in 4 years

2P becomes 4P in next 4 years

4P becomes 8P in next 4 years

P becomes 8P in 4 + 4 + 4 = 12 years

EXAMPLE 11 Find the least number of complete years in which a sum of money at 20% p.a. Compound interest will be more than double.

Solution We have,

$$P\left(1+\frac{20}{100}\right)^t > 2P$$

$$\therefore \qquad \left(\frac{6}{5}\right)^t > 2$$

By trial,

$$\left(\frac{6}{5}\right)^4 > 2$$

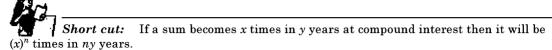
.. The required time is 4 years.

EXAMPLE 12 A sum of money at compound interest amounts to four times itself in 4 years. In how many years will it be 16 times of itself?

Solution Let the sum be ξx .

Then,
$$4x = x \left(1 + \frac{r}{100}\right)^4$$
or
$$4 = \left(1 + \frac{r}{100}\right)^4$$
Squaring,
$$(4)^2 = \left[\left(1 + \frac{r}{100}\right)^4\right]^2$$
or
$$16 = \left[1 + \frac{r}{100}\right]^8$$
or
$$16x = x \left[1 + \frac{r}{100}\right]^8$$

.. The sum will be 16 times in 8 years.



: If sum becomes 4 times in 4 years, it will be $(4)^2$ times in $2 \times 4 = 8$ years.

To find the SI, if the CI is given, and vice versa

EXAMPLE 13 If the CI on a certain sum for 2 years at 3% be ₹ 1015, what would be the SI?

Solution CI on 1 rupee =
$$\left(1 + \frac{3}{100}\right)^2 - 1 = \left(\frac{103}{100}\right)^2 - 1$$

= $\frac{609}{10000}$
SI on 1 rupee = $\frac{2 \times 3}{100} = \frac{6}{100}$

$$\therefore \frac{\text{SI}}{\text{CI}} = \frac{6}{100} \times \frac{10000}{609} = \frac{200}{203}$$

∴ SI =
$$\frac{200}{203}$$
 of CI = $\frac{200}{203} \times 1015 = ₹1000$



Short cut: Simple interest =
$$\frac{rt}{100\left[\left(1 + \frac{r}{100}\right)^{t} - 1\right]} \times \text{Compound Interest.}$$

Find sum and rate if CI and SI are given

EXAMPLE 14 The compound interest on a certain sum for 2 years is $\stackrel{?}{\sim}$ 408 and simple interest is $\stackrel{?}{\sim}$ 400. Find the rate of interest per annum and the sum.

Solution Difference between the simple interest and compound interest for 2 years is the interest on the first year's interest.

First year's SI =
$$\frac{400}{2}$$
 = ₹ 200
CI - SI = $408 - 400$ = ₹ 8

Interest on ₹ 200 for 1 year = ₹ 8

∴ Interest on ₹ 100 for 1 year = ₹ 4

$$\therefore \qquad \text{Principal is given by } P = \frac{100 \times I}{rt} = \frac{100 \times 400}{4 \times 2} = \text{ } \textbf{5000}$$



Short cut: (for 2 years only)

Rate =
$$\frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100$$

= $\frac{2 \times 8}{400} \times 100 = 4\%$
Sum = $\frac{400 \times 100}{4 \times 2} = ₹5000$

Division of Sum

EXAMPLE 15 Divide ₹ 5204 between X and Y, so that the share of X at the end of 7 years may be equal to the share of Y at the end of 9 years, compound interest being at 4%.

Solution We have

:.

(Share of X at present)
$$\left(1 + \frac{4}{100}\right)^7 = \text{(Share of Y at present)} \left(1 + \frac{4}{100}\right)^9$$

$$\frac{\text{Share of X at present}}{\text{Share of Y at present}} = \left(1 + \frac{4}{100}\right)^2 = \left(\frac{26}{25}\right)^2 = \frac{676}{625}$$

Dividing ₹ 5204 in the ratio 676 : 625.

Present share of X =
$$\frac{676}{676 + 625}$$
 × 5204 = ₹ **2704**

Present share of Y =
$$\frac{625}{676 + 625}$$
 × 5204 = ₹ **2500**

When difference between SI and CI is given

Theorem: When the difference between the compound interest and simple interest on a certain sum of money for 2 years at r% per annum is ξ x, then the sum is given by:

$$Sum = \frac{(CI \sim SI) \times 100 \times 100}{Rate \times Rate} = x \left(\frac{100}{r}\right)^{2}$$

When the sum is given, difference in SI and CI is calculated by

Difference =
$$Sum \times \left(\frac{r}{100}\right)^2$$

EXAMPLE 16 The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 years is ₹ 15. Find the sum.

EXAMPLE 17 Find the difference between the compound interest and the simple interest for the sum of $\overline{\xi}$ 1500 at 10% per annum for 2 years.

Solution Difference = Sum
$$\times \left(\frac{r}{100}\right)^2 = 1500 \left(\frac{10}{100}\right)^2 = ₹ 15$$

Theorem: If the difference between CI and SI on a certain sum for 3 years at r% is $\sqrt[3]{x}$, then the sum will be $\frac{\text{Difference} \times (100)^3}{r^2(300+r)}$ and if sum is given, difference will be $\text{sum} \times \frac{r^2(300+r)}{(100)^3}$.

EXAMPLE 18 If the difference between CI and SI on a certain sum of money for 3 years at 5% per annum is ₹ 1220, find the sum.

Solution Sum =
$$\frac{\text{Difference} \times (100)^3}{r^2(300+r)} = \frac{1220 (100)^3}{5^2(300+5)} = ₹ 160000.$$

EXAMPLE 19 Find the difference between CI and SI on ₹ 8000 for 3 years at 2.5% per annum.

Solution Difference = Sum ×
$$\frac{r^2(300+r)}{(100)^3} = \frac{8000 \times 2.5 \times 2.5 (300+2.5)}{100 \times 100 \times 100}$$

= $\frac{8 \times 25 \times 25 \times 3025}{100 \times 100 \times 100} = \frac{121}{8} = ₹ 15.125$

EXAMPLE 20 The simple interest on a sum at 4% per annum for 2 years is ₹80. Find the compound interest on the same sum for the same period.

Solution Rate =
$$\frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100$$

or Difference in CI and SI = $\frac{\text{Rate} \times \text{SI}}{2 \times 100} = \frac{4 \times 80}{2 \times 100} = 1.6$
∴ CI = 80+ 1.6 = ₹ 81.6

EXAMPLE 21 The compound interest on a certain sum of money for 2 years at 10% per annum is $\stackrel{?}{\stackrel{\checkmark}}$ 420. Find the simple interest at the same rate and for the same time.

Solution We have,
$$SI = \frac{rt}{100\left[\left(1 + \frac{r}{100}\right)^t - 1\right]} \times CI$$
When $t = 2$,
$$SI = \frac{2r}{100\left[1 + \frac{r^2}{r^2} + \frac{2r}{r^2} + \frac{1}{r^2}\right]} \times CI$$

$$SI = \frac{2r}{100 \left[1 + \frac{r^2}{100^2} + \frac{2r}{100} - 1 \right]} \times CI$$
$$= \frac{2r \times CI \times 100}{(r^2 + 200r)}$$

$$=\frac{200r\times CI}{r(r+200)}$$

Substituting,



Note: When CI is given and SI is asked, the above formulae may be used.

When relationship between CI and SI is asked for 3 years

$$SI = \frac{rt}{100 \left[\left(1 + \frac{r}{100} \right)^t - 1 \right]} \times CI$$

$$= \frac{3r}{100\left[\left(1 + \frac{r}{100}\right)^3 - 1\right]} \times CI$$

EXAMPLE 22 If the simple interest on a certain sum of money for 3 years at 5% is ₹ 150, find the corresponding compound interest.

$$SI = \frac{3r}{100\left[\left(1 + \frac{r}{100}\right)^3 - 1\right]} \times CI$$

i.e.
$$150 = \frac{3 \times 5}{100 \left[\left(1 + \frac{5}{100} \right)^3 - 1 \right]} \times CI$$

$$\therefore \qquad \text{CI} = \frac{150}{15} \times 100 \left[\left(1 + \frac{5}{100} \right)^3 - 1 \right] = 1000 \left[\left(\frac{21}{20} \right)^3 - 1^3 \right] \\
= 1000 \frac{(21 - 20) \left[21^2 + 21(20) + 20^2 \right]}{8000} \\
= \frac{441 + 420 + 400}{8} = \frac{1261}{8} = ₹ 157.62$$

EXAMPLE 23 An amount of money grows up to $\stackrel{?}{\sim}$ 4840 in 2 years and up to $\stackrel{?}{\sim}$ 5324 in 3 years on compound interest. Find the rate per cent.

Solution We have,

CI for 3rd year =
$$5324 - 4840 = ₹484$$

355

$$Rate = \frac{484 \times 100}{4840} = 10\%$$

(i.e., ₹ 484 is the interest accrued on the principal amount ₹ 4840 at the end of 2nd year.)

Short cut: Rate =
$$\frac{[\text{Difference of amount after } n \text{ years and } (n+1) \text{ years}] \times 100}{\text{Amount after } n \text{ years}}$$

$$\therefore \qquad \text{Rate} = \frac{[\text{Difference of amount after } 2 \text{ years and } 3 \text{ years}] \times 100}{\text{Amount after 2 years}}$$

$$= \frac{(5324 - 4840) \times 100}{4840} = 10\%$$

EXAMPLE 24 Find the compound interest on ₹ 19375 in 2 years, the rate of interest being 4% for the first year and 8% for the second year.

Solution After first year the amount =
$$19375\left(1 + \frac{4}{100}\right) = 19375\left(\frac{104}{100}\right)$$

After second year the amount =
$$19375 \left(\frac{104}{100}\right) \left(\frac{108}{100}\right)$$

= 19375
$$\left(\frac{26}{25}\right) \left(\frac{27}{25}\right)$$
 = ₹ 21762

$$∴ Compound interest = 21762 - 19375 = ₹ 2387$$

EXAMPLE 25 A sum of ₹ 4800 becomes ₹ 6000 in 4 years at a certain rate of compound interest. What will be the sum after 12 years?

Solution
$$4800 \left(1 + \frac{r}{100}\right)^4 = 6000$$
or
$$\left(1 + \frac{r}{100}\right)^4 = \frac{6000}{4800} = \frac{5}{4}$$
∴
$$\left(1 + \frac{r}{100}\right)^{12} = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$$
∴
$$4800 \left(1 + \frac{r}{100}\right)^{12} = 4800 \times \frac{125}{64} = ₹ 9375$$

∴ Amount after 12 years will be ₹ 9375



Short cut: Required amount =
$$\frac{(6000)^{12/4}}{(4800)^{12/4-1}} = \frac{(6000)^3}{(4800)^2} = ₹9375$$



Note: If a sum x becomes y in t_1 years at compound rate of interest, then after t_2 years the

sum becomes
$$\frac{y^{t_2/t_1}}{x^{t_2/t_1-1}}$$
 rupees.

EXAMPLE 26 Find the compound interest on ₹ 100000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year.

Solution The compound interest on \mathfrak{T} x in t years, if the rate of interest is r_1 % for the first year, r_2 % for the second year... and r_t % for the tth year, is given by

$$x\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\cdots\left(1 + \frac{r_r}{100}\right) - x$$
Here, Compound interest = $100000\left(1 + \frac{4}{100}\right)\left(1 + \frac{5}{100}\right)\left(1 + \frac{6}{100}\right) - 10000$

$$= 100000\left(\frac{26}{25}\right)\left(\frac{21}{20}\right)\left(\frac{53}{50}\right) - 100000$$

$$= 4 \times 26 \times 21 \times 53 - 100000 = 115752 - 100000$$

$$= ₹ 15752$$

EXAMPLE 27 What sum of money at compound interest will amount to ₹ 224952 in 3 years, if the rate of interest is 3% for the first year, 4% for the second year and 5% for the third year?

Solution
$$A = P\left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right)$$
Here,
$$224952 = P\left(1 + \frac{3}{100}\right) \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right)$$

$$= P(1.03) (1.04) (1.05)$$
∴
$$P = \frac{224952}{(1.03) (1.04) (1.05)} = ₹ 200000$$



Short cut: By rule of fraction;

Principal = 224952
$$\left(\frac{100}{103}\right) \left(\frac{100}{104}\right) \left(\frac{100}{105}\right) = ₹ 200000$$

EXAMPLE 28 Anil deposits ₹ 20000 at the rate of 16% compounded annually and Ashok deposits an equal amount at the rate of 15% compounded half yearly. If both deposit their money for $1\frac{1}{2}$ years only, calculate which deposit earns better interest.

Solution Amount after $1\frac{1}{2}$ years in both the cases will be $20000\left(1+\frac{16}{100}\right)^{3/2}$ and

$$20000 \left(1 + \frac{7.5}{100}\right)^3$$
 respectively.

Comparing both quantities,

$$20000 \left(1 + \frac{16}{100}\right)^{3/2} \quad \Leftrightarrow \quad 20000 \left(1 + \frac{7.5}{100}\right)^{3}$$
$$\left(1 + \frac{16}{100}\right)^{3/2} \quad \Leftrightarrow \quad \left(1 + \frac{7.5}{100}\right)^{3}$$

or

Squaring both sides,

$$\left(1 + \frac{16}{100}\right)^3 \quad \Leftrightarrow \quad \left(1 + \frac{7.5}{100}\right)^6$$

Taking cube roots on both sides,

$$\left(1 + \frac{16}{100}\right)^{1} \iff \left(1 + \frac{7.5}{100}\right)^{2}$$

$$\left(1 + \frac{16}{100}\right) \iff 1 + 2\left(\frac{7.5}{100}\right) + \left(\frac{7.5}{100}\right)^{2}$$

$$1 + \frac{16}{100} \iff 1 + \frac{15}{100} + \left(\frac{7.5}{100}\right)^{2}$$

$$\frac{1}{100} \iff \left(\frac{7.5}{100}\right)^{2}$$

$$\frac{1}{100} > \left(\frac{7.5}{100}\right)^{2}$$

$$1 > \frac{7.5^{2}}{100}$$

i.e.

 \mathbf{or}

:. First quantity is higher than the second.

EXAMPLE 29 A sum of money is lent at compound interest rate of 20% per annum for 2 years. It would fetch ₹ 482 more if the interest is compounded half-yearly. Find the sum.

Solution Let the sum be P.

Compound interest when interest is compounded yearly = $P\left[1 + \frac{20}{100}\right]^2 - P$

Compounded interest when interest is compounded half-yearly = $P \left[1 + \frac{10}{100} \right]^4 - P$

Difference in interests is given by

$$P\left[1 + \frac{10}{100}\right]^4 - P\left[1 + \frac{20}{100}\right]^2 = 482$$
i.e.
$$P(1.1)^4 - P(1.2)^2 = 482$$
i.e.
$$P(1.21^2 - 1.2^2) = 482$$
∴
$$P = \frac{482}{(1.21 + 1.2)(1.21 - 1.2)}$$

$$= \frac{482}{2.41 \times 0.01} = ₹ 20000$$

EXAMPLE 30 In what time will ₹ 1000 become ₹ 1331 at 10% per annum compounded annually? [SSC]

Solution
$$P = \overline{7} 1000$$
; $A = \overline{7} 1331$; $R = 10\%$; $t = ?$

$$A = P\left(1 + \frac{r}{100}\right)^{t}$$

$$1331 = 1000\left(1 + \frac{10}{100}\right)^{t}$$
or
$$\left(\frac{110}{100}\right)^{t} = \frac{1331}{1000}$$
or
$$\left(\frac{11}{10}\right)^{t} = \frac{1331}{1000}$$
or
$$\left(\frac{11}{10}\right)^{3} = \frac{1331}{1000}$$

$$t = 3 \text{ years}$$



- 1. If 64000 is deposited at 5% compound interest for a period of 18 months, interest being compounded half-yearly, find the amount.
 - (a) 69821
- (b) 68921
- (c) 62891
- (d) 62981
- (e) None of these
- 2. An amount of ₹ 12500 is deposited for a period of 2 years at an interest of 4% per annum. Find the difference between simple interest and compound interest for the deposit.
 - (a) ₹ 10
- (b) ₹ 20
- (c) ₹50
- (d) ₹40
- (e) None of these
- 3. An amount of ₹8000 was deposited at simple interest for 3 years at 7.5% per annum. How much more would have gained had it been deposited at the same rate per cent compound interest?
 - (a) ₹ 135.75
- (b) ₹ 138.75
- (c) ₹ 138.375
- (d) ₹ 138.753
- (e) None of these

ISSC1

(e) None of these

(e) None of these

(d) ₹ 3000

(d) ₹ 20000

	4 years. The rate of interest per annum is: [SSC					
	(a) $2\frac{1}{2}\%$	(b) 4%	(c) 5%	(d) $6\frac{2}{3}\%$		
7.	The difference be				te of 10% per annum s is ₹ 124.05. What is [SBI PO]	
	(a) ₹ 6000	(b) ₹8000	(c) ₹ 10000	(d) ₹ 12000	(e) None of these	
8.		_	erest and compount at 4% per annum is	_	unded annually on a ₹) is: [SSC]	
	(a) 625	(b) 630	(c) 640	(d) 650		
9.					r annum is half the im placed on simple [SSC]	
	(a) ₹ 1550	(b) ₹ 1650	(c) ₹ 1750	(d) ₹ 2000		
10.			m for 2 years at 1 ame rate for the sa		is ₹ 510, the simple is. [SSC]	
	(a) ₹ 400	(b) ₹ 450	(c) ₹ 460	(d) ₹ 480		
		_		1		
11.	The principal the compounded ann		1913 in 3 years at	$6\frac{1}{4}\%$ per annun	a compound interest [SSC]	
	(a) ₹ 3096	(b) ₹ 4076	(c) ₹ 4085	(d) ₹ 4096		
12.	The compound in		at 7% per annum	is ₹ 4347. The per	riod (in years) is: [LIC AAO]	
	(a) 2	(b) $2\frac{1}{2}$	(c) 3	(d) 4		
13.	What will be the sum of ₹ 1000 aft		en simple and com	pound interest @ :	10% per annum on a [Bank PO]	
	(a) ₹31	(b) ₹ 32.10	(c) ₹ 40.40	(d) ₹ 64.10	(e) None of these	
14.	Find the compour	nd interest on ₹ 56	25 for 9 months at	16% per annum co	mpounded quarterly. [RRB]	
	(a) ₹ 1851	(b) ₹ 1941	(c) ₹ 1951	(d) ₹ 1961		
15.	A bank offers 5% compound interest calculated on half-yearly basis. A customer deposited 1600 each on 1st January and 1st July of a year. At the end of the year, the amount has would have gained by way of interest is: [NDA]					
	(a) ₹ 120	(b) ₹ 121	(c) ₹ 122	(d) ₹ 123		
16.	What will be the annum?	compound interes	st on a sum of ₹ 250	000 after 3 years a	t the rate of 12% per [SBI PO]	
	(a) ₹ 9000.30	(b) ₹ 9720	(c) ₹ 10123.20	(d) ₹ 10483.20	(e) None of these	

4. A certain sum at compound interest amounts in two years to ₹ 2809 and in three years to

5. Difference between simple interest and compound interest on a certain sum for 3 years at

6. A sum of money invested at compound interest amounts to ₹800 in 3 years and to ₹840 in

(c) ₹ 2500

(c) ₹ 18000

₹ 2977.54. Find the rate of interest and the original sum.

(b) ₹ 2000

(b) ₹ 16000

5% per annum is ₹ 122. Find the sum.

(a) ₹ 14000

17.					years at compound naturity of the fixed [Bank PO]	
	(a) ₹8600	(b) ₹8620	(c) ₹8800	(d) ₹8840	(e) None of these	
18.	Sam invested ₹ 1	.5000 at 10% per a	annum for one yea	ır. If the interest i	s compounded half-	
	yearly, then the a	amount received by	y Sam at the end o	of the year will be:	[SBI PO]	
	(a) ₹ 16500	(b) ₹ 16525.50	(c) ₹ 16537.50	(d) ₹ 18150	(e) None of these	
19.	What is the differ	rence between the	compound interes	sts on ₹ 5000 for 1	$1\frac{1}{2}$ years at 4% per	
	annum compound	ded yearly end hal	f-yearly?		[SSC]	
	(a) ₹ 2.04	(b) ₹ 3.06	(c) ₹ 4.80	(d) ₹8.30		
20.	What annual pay		rge a debt of ₹ 10	025 due in 2 year	s at the rate of 5% [SSC]	
	(a) ₹ 550	(b) ₹ 551.25	(c) ₹ 560	(d) ₹ 560.75		
21.		laced at compound e same rate of inte			will amount to eight otel Management]	
	(a) 7 years	(b) 10 years	(c) 15 years	(d) 20 years		
22.	Mr. Dua invested money in two schemes A and B offering compound interest at 8% peannum and 9% per annum respectively. If the total amount of ineterest accrued through two schemes together in two years was ₹ 4818.30 and the total amount invested was ₹ 2700 What was the amount invested in scheme A. [Bank PC					
	(a) ₹ 12000	` '	₹ 13500	(c) ₹ 15000)	
	(d) Cannot be de	` '	None of these			
23.	On a sum of money, the simple interest for 2 years is ₹ 660, while the compound interest is ₹ 696.30, the rate of interest being the same in both the cases. The rate of interest is: [Hotel Management]					
	(a) 10%	(b) 10.5%	(c) 12%	(d) None of thes	se	
24.		There is 60% increase in an amount in 6 years at simple interest. What will be the compoun nterest of ₹ 12000 after 3 years at the same rate? [SIDBI				
	(a) ₹2160	(b) ₹ 3120	(c) ₹3972	(d) ₹ 6240	(e) None of these	
25.				t 10% per annum : f the rate per cent	is ₹ 525. The simple per annum is: [CBI]	
	(a) ₹ 400	(b) ₹ 500	(c) ₹ 600	(d) ₹800		
26.	If the simple inte	erest on a sum of i	noney for 2 years	` '	is ₹ 50, what is the e time? [CBI]	
	(a) ₹ 51.25	(b) ₹ 52	(c) ₹ 54.25	(d) ₹60		
27.		r of complete year nore than doubled		of money put ou	t at 20% compound [NIFT]	
	(a) 3	(b) 4	(c) 5	(d) 6		
28.	crued on an amount per annum? [Bank PO]					
	(a) 12%	(b) 13%	(c) 14%	(d) 15%	(e) None of these	

29. In how many years will a sum of ₹800 at 10% per annum compounded semi-annually become ₹ 926.10? [Section Officer's]

(a) $1\frac{1}{2}$

(b) $1\frac{1}{2}$ (c) $2\frac{1}{3}$ (d) $2\frac{1}{3}$

30. A man borrows ₹ 3825 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years into equal yearly instalments. How much will be each instalment?

(a) ₹ 1275

31. A sum of ₹ 15000 deposited at compound interest becomes double after 5 years. After 20 years, it will become: **ISSCI**

(a) ₹ 120000

(b) ₹ 150000

(c) ₹ 155000

(d) ₹ 240000

32. The difference between compound interest and simple interest on a sum for 2 years at 10% per annum, when the interest is compounded annually, is ₹ 20. If the interest were compounded half yearly, the difference in two interests will be:

(a) ₹ 31.82

(b) ₹ 31.01

(c) ₹ 32.81

(d) ₹ 32.18

33. A man borrows ₹ 25000 at 20% compound interest. At the end of every year he pays ₹ 5000 as part repayment. How much does he still owe after three such instalments?

(a) ₹ 25000

(b) ₹ 12864

(c) ₹ 15600

(d) None of these

34. A sum of money borrowed and paid back in two annual instalments of ₹882 each allowing 5% compound interest. The sum borrowed was: [AIMATS]

(a) ₹ 1620

(b) ₹ 1640

(c) ₹ 1680

(d) ₹ 1700

35. What will be the compound interest on a sum of $\overline{\varsigma}$ 50000 after 4 years at the rate of 10% per annum?

(a) ₹ 20000

(b) ₹ 21205

(c) ₹ 22205

(d) ₹ 23205



1. (b)

2. (b) **3.** (c)

32. (b)

4. (c)

5. (b) **6.** (c) 7. (b)

8. (a)

10. (d)

11. (d)

12. (a) 13. (d) 14. (c) 15. (b)

16. (c)

17. (e)

18. (c) 19. (a) 20. (b)

21. (c)

22. (a) 23. (d) 24. (c)

26. (a)

27. (b)

28. (d)

29. (b) **30.** (c)

9. (c)

31. (d)

33. (a)

34. (b)

25. (b) 35. (d)

\mathcal{S} olutions with Necessary Explanation

1. Amount = 64000
$$\left(1 + \frac{2.5}{100}\right)^3 = 64000 \left(\frac{41}{40}\right)^3 = ₹ 68921$$

2. Difference = Sum ×
$$\left(\frac{r}{100}\right)^2$$
 = 12500 $\left(\frac{4}{100}\right)^2$ = $\frac{12500}{625}$ = ₹ 20

3. Difference =
$$\frac{\text{Sum} \times r^2(300 + r)}{(100)^3} = \frac{8000 \times (7.5)^2 (300 + 7.5)}{(100)^3} = ₹ 138.375$$

4. Difference = 2977.54 - 2809 = ₹ 168.54

:. Rate of interest =
$$\frac{168.54 \times 100}{2809} = 6\%$$

Let x be the amount deposited

$$\therefore 2809 = x \left(1 + \frac{6}{100} \right)^2$$
or
$$x = \frac{2809 \times 100^2}{106^2} = 2809 \left(\frac{50}{53} \right)^2 = ₹ 2500$$

R = 5%

 $P\left|\left(\frac{21}{20}\right)^4 - \frac{6}{5}\right| = 124.05$

5. Sum =
$$\frac{\text{Difference} \times 100^3}{r^2(300+r)} = \frac{122 \times 100^3}{5^2 + (300+5)} = \frac{122}{61} \times \left(\frac{100}{5}\right)^3 = 2 \times 8000 =$$
₹ 16000

$$A1 = P\left(1 + \frac{R}{100}\right)^{3} = 800$$
or
$$P\left(1 + \frac{R}{100}\right)^{3} = 800$$
(1)
Similarly
$$P\left(1 + \frac{R}{100}\right)^{4} = 840$$

$$\left(1 + \frac{R}{100}\right) = \frac{840}{800} = 1.05$$

7. Let P be the principal.

or

:.

S.I. on first year =
$$P \times \frac{10 \times 1}{100} = \frac{P}{10}$$

S.I. on second year = $P \times \frac{10 \times 1}{100} = \frac{P}{10}$
Total S.I. = $2 \times \frac{P}{10} = \frac{P}{5}$ (1)
C.I. for 2 year = $P\left(1 + \frac{R}{200}\right)^4 - P$
= $P\left[\left(1 + \frac{10}{200}\right)^4 - 1\right]$
= $P\left[\left(1 + \frac{5}{100}\right)^4 - 1\right]$ (2)
(2)–(1) $\Rightarrow P\left[\left(1 + \frac{5}{100}\right)^4 - 1 - \frac{1}{5}\right] = 124.05$

or
$$P\left[\frac{1,94,481}{1,60,000} - \frac{6}{5}\right] = 124.05$$
or
$$P = \frac{12405}{100} \times \frac{1,60,000}{1,94,481 - 1,92,000}$$

8. Let the sum be P.

$$P\left[\left(1 + \frac{4}{100}\right)^2 - 1\right] - P \times \frac{2 \times 4}{100} = 1$$
or
$$P\left[\left(\frac{26}{25}\right)^2 - 1 - \frac{2}{25}\right] = 1$$
or
$$P = \frac{1 \times (25)^2}{(26)^2 - (25)^2 - 50} = \frac{(25)^2}{(51)! - 50} = (25)^2 = ₹ 625$$

9. Let the sum be P.

S.I. for 3 years =
$$\frac{P \times 3 \times 8}{100}$$
 (1)

C.I. (for ₹ 4000 @ 10% p.a for 2 year) =
$$4000 \left(1 + \frac{10}{100}\right)^2 - 4000$$

= $4000 \left[\left(\frac{11}{10}\right)^2 - 1\right] = 4000 \times \left(\frac{11^2 - 10^2}{10^2}\right)$
= $40 \times 21 = 840$ (2)

.: From (1) and (2),

$$P \times \frac{3 \times 8}{100} = \frac{1}{2} \times \text{C.I.} = \frac{1}{2} \times 840 = 420$$
$$P = \frac{420 \times 100}{24} = 70 \times 25 = ₹1750$$

10. Let the principal amount be P.

$$P\left[\left(1 + \frac{12(1/2)}{100}\right)^2 - 1\right] = ₹510$$
i.e.
$$P\left[\left(1 + \frac{1}{8}\right)^2 - 1\right] = 510$$
i.e.
$$P\left[\left(\frac{9}{8}\right)^2 - 1\right] = 510$$
or
$$P = \frac{510 \times (8)^2}{(9)^2 - (8)^2} = \frac{510 \times (8)^2}{(9 + 8) \times 1} = 30 \times 64 = ₹1920$$

$$\therefore S.I. = 1920 \times 2 \times \frac{25}{200} = ₹480$$

11. Let the principal be P.

$$P\left(1 + \frac{6\frac{1}{4}}{100}\right)^{3} = 4913$$
or
$$P\left(1 + \frac{25}{400}\right)^{3} = 4913$$
or
$$P = \frac{4913}{\left(1 + \frac{1}{16}\right)^{3}} = \frac{4913 \times 16^{3}}{17^{3}} = ₹ 4096$$

12. Let the period of deposit be N years.

$$\therefore 30000 \left[\left(1 + \frac{7}{100} \right)^{N} - 1 \right] = 4347$$
i.e. $30000 \left(\frac{107}{100} \right)^{N} - 30000 = 4347$
or $30000 \left(\frac{107}{100} \right)^{N} = 34347$

$$\therefore \left(\frac{107}{100} \right)^{N} = \frac{34347}{30000} = \frac{11449}{10000} = \left(\frac{107}{100} \right)^{2}$$

$$\therefore N = 2 \text{ years}$$

13. S.I. an ₹ 1000 @ 10% p.a. after 4 years

$$=\frac{1000 \times 4 \times 10}{100}$$
 = ₹ 400

C.I. on ₹ 1000 @ 10% p.a. after 4 years.

$$= 1000 \left[\left(1 + \frac{10}{100} \right)^4 - 1 \right] = 1000 \left[\left(\frac{11}{10} \right)^4 - 1 \right]$$

$$= \frac{1000}{10^4} (11^4 - 10^4) = \frac{1000}{10000} (121 + 100)(11 + 10)1$$

$$= \frac{1}{10} \times 221 \times 21 = \frac{4641}{10} = \text{ ₹ 464.10}$$

Hence C.I.– S.I. = 464.10 - 400 = ₹ 64.10

14. Compound interest on ₹ 15625 for 9 months @ 16% per annum

$$= 15625 \left[\left(1 + \frac{16}{100 \times 4} \right)^3 - 1 \right] = 15625 \left[\left(1 + \frac{1}{25} \right)^3 - 1 \right]$$
$$= 15625 \times \left[\frac{26^3}{25^3} - 1 \right] = 25 \times 625 \times \left[\frac{26^3}{25^3} - 1 \right]$$

$$= 26^3 - 25^3 = (26^2 + 25^2 + 26 \times 25)(26 - 25)$$
$$= 676 + 625 + 650 = ₹1951$$

15. Total interest at the end of the year

$$= 1600 \left(1 + \frac{5}{200} \right)^{2} + 1600 \left(1 + \frac{5}{200} \right) - 3200$$

$$= 1600 \left[\left(1 + \frac{1}{40} \right)^{2} - 1 \right] + 1600 \times \frac{1}{40}$$

$$= 1600 \times \frac{(41^{2} - 40^{2})}{40^{2}} + 40 = (41 + 40)(41 - 40) + 40$$

$$= 81 + 40 = ₹ 121$$

16. C.I. after 3 years =
$$25000 \left[\left(1 + \frac{12}{100} \right)^3 - 1 \right] = 25000 \left[\left(\frac{28}{25} \right)^3 - 1 \right]$$

= $\frac{25000}{25^3} (28^3 - 25^3) = \frac{40}{25} (28 - 25)(28^2 + 25^2 + 28 \times 25)$
= $\frac{40 \times 3}{25} (784 + 625 + 700) = \frac{24}{5} \times 2109$
= $4.8 \times 2109 = ₹ 10123.20$

17.
$$A = P \left(1 + \frac{R}{100} \right)^{N}$$
$$= 8000 \left(1 + \frac{5}{100} \right)^{2} = 8000 \times \left(\frac{21}{20} \right)^{2}$$
$$= 20 \times 21^{2} = \text{₹ 8820}$$

18.
$$A = P \left(1 + \frac{R}{100} \right)^{N}$$

$$= 15000 \left(1 + \frac{10}{200} \right)^{2} = 15000 \left(1 + \frac{5}{100} \right)^{2}$$

$$= 15000 \times \left(\frac{21}{20} \right)^{2} = \frac{15000 \times 441}{400}$$

$$= 37.5 \times 441 = ₹ 16537.50$$

19. Difference in C.I. =
$$5000 \left[\left(1 + \frac{2}{100} \right)^3 - 1 \right] - 5000 \left[\left(1 + \frac{4}{100} \right) \left(1 + \frac{2}{100} \right) - 1 \right]$$

= $5000 \left[\left\{ \left(\frac{51}{50} \right)^3 - 1 \right\} - \left(\frac{26}{25} \times \frac{51}{50} - 1 \right) \right]$
= $5000 \left[\left(\frac{51^3 - 50^3}{50^3} \right) - \frac{(13 \times 51 - 25^2)}{25^2} \right]$

$$= 5000 \left[\frac{51^2 + 50^2 + 51(50)}{50^3} - \frac{(663 - 625)}{25^2} \right]$$

$$= 5000 \left[\frac{2601 + 2500 + 2550}{50^3} - \frac{38}{25^2} \right] = 5000 \left[\frac{7651}{50^3} - \frac{38}{25^2} \right]$$

$$= \frac{5000}{50^3} [7651 - 38 \times 4 \times 50] = \frac{2}{50} [7651 - 7600]$$

$$= \frac{2}{50} \times 51 = ₹ 2.04$$

20. Let each annual instalment be $\not\in x$.

$$\frac{x}{\left(1 + \frac{5}{100}\right)^2} + \frac{x}{\left(1 + \frac{5}{100}\right)} = 1025$$
or
$$x \times \left(\frac{20}{21}\right)^2 + x\left(\frac{20}{21}\right) = 1025$$
or
$$\frac{400x}{441} + \frac{20x}{21} = 1025$$

$$\Rightarrow \frac{400x + 420x}{441} = 1025$$
or
$$x = \frac{1025 \times 441}{820} = ₹ 551.25$$

Hence each annual instalment = ₹ 551.25

21.
$$2P = P\left(1 + \frac{R}{100}\right)^5$$
 (1)

Let the number of years at which amount becomes 8 times be N.

$$\therefore 8P = P\left(1 + \frac{R}{100}\right)^N \tag{2}$$

$$\left(1\right) \Rightarrow \left(1 + \frac{R}{100}\right)^5 = 2\tag{3}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{8P}{2P} = \frac{P\left(1 + \frac{R}{100}\right)^N}{P\left(1 + \frac{R}{100}\right)^5}$$

on substitution from (3), we get,

$$4 = \frac{\left(1 + \frac{R}{100}\right)^N}{2}$$
or
$$\left(1 + \frac{R}{100}\right)^N = 8$$
(4)

(3)
$$\Rightarrow$$
 $\left(1 + \frac{R}{100}\right)^5 = 2$
 \therefore (4) \Rightarrow $\left(1 + \frac{R}{100}\right)^N = 2^3 = \left[\left(1 + \frac{R}{100}\right)^5\right]^3 = \left(1 + \frac{R}{100}\right)^{15}$
 \therefore $N = 15 \text{ years}$

- **22.** Let the amount invested in scheme A be \overline{x} .
 - \therefore Amount in sheeme B = (27000 x)
 - .. C.I. for 2 years.

$$= x \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right] + (27000 - x) \left[\left(1 + \frac{9}{100} \right)^2 - 1 \right] = 4818.30$$
i.e.
$$x \left[\left(\frac{27}{25} \right)^2 - 1 \right] + (27000 - x) \left[\left(\frac{109}{100} \right)^2 - 1 \right] = 4818.30$$
or
$$\frac{x}{25^2} [(27)^2 - (25)^2] + \frac{(27000 - x)}{100^2} [(109)^2 - (100)^2] = 4818.30$$
or
$$\frac{x}{25^2} \times 52 \times 2 + \frac{27000 - x}{10000} \times 209 \times 9 = 4818.30$$
or
$$\frac{x}{100^2} [104 \times (4)^2 - 1881] = 4818.30 - 1881 \times 2.7$$

$$-\frac{217x}{100^2} = -260.4$$
or
$$x = \frac{260.4 \times (100)^2}{217} = ₹ 12000$$

23. Let the rate of interest be R%.

S.I. =
$$\frac{P \times 2 \times R}{100} = 660$$
 (1)

C.I. =
$$P\left[\left(1 + \frac{R}{100}\right)^2 - 1\right] = 696.30$$
 (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{\left[\left(1 + \frac{R}{100}\right)^2 - 1\right]}{\frac{2R}{100}} = \frac{696.30}{660}$$

$$\Rightarrow \frac{100}{2R} \left(\frac{R^2}{100^2} + \frac{2R}{100}\right) = 1.055$$

$$\Rightarrow \frac{R}{200} + 1 = 1.055$$
or
$$R = (1.055 - 1) \times 200 = 11\%$$

24. S.I. for 6 years =
$$\frac{P \times 6 \times R}{100} = 0.6P$$

or
$$R = \frac{0.6 \times 100}{6} = 10\%$$

C.I. for 3 years for ₹ 12000 @ 10% p.a.

$$= 12000 \left[\left(1 + \frac{10}{100} \right)^3 - 1 \right] = 12000 \left[\left(\frac{11}{10} \right)^3 - 1 \right]$$

$$= 12000 \times \frac{(11)^3 - (10)^3}{10^3} = 12 \times (11 - 10) \left[(11)^2 + (10)^2 + 110 \right]$$

$$= 12 \times 1 \times 331 = 3972$$

25. C.I. =
$$P\left[\left(1 + \frac{10}{100}\right)^2 - 1\right] = 525$$
 (1)

S.I. =
$$\frac{P \times 4 \times 5}{100} = \frac{P}{5}$$
 (2)

Substituting from (1), we get

S.I. =
$$\frac{P}{5} = \frac{525}{\left[\left(\frac{11}{10}\right)^2 - 1\right] \times 5}$$

= $\frac{105 \times 10^2}{(11^2 - 10^2)} = \frac{10500}{21 \times 1} = ₹500$

26. S.I. =
$$P \times 2 \times \frac{5}{100} = 50$$

 \Rightarrow

$$P = ₹ 500$$
C.I. = $P\left[\left(1 + \frac{5}{100}\right)^2 - 1\right] = 500 \times \left[\left(\frac{21}{20}\right)^2 - 1\right]$

$$= 500 \times \frac{(21^2 - 20^2)}{20^2}$$

$$= \frac{500 \times 41 \times 1}{400} = \frac{205}{4} = ₹ 51.25$$

27. Let the number of year be N.

$$P\left(1 + \frac{20}{100}\right)^{N} > 2P$$
or
$$\left(1 + \frac{20}{100}\right)^{N} > 2$$

$$\left(\frac{6}{5}\right)^{N} > 2$$

$$\therefore \qquad N = 4 \text{ years}$$

28. Let rate of interest be R% p.a.

$$S.I. = \frac{18000 \times 2 \times R}{100} \tag{1}$$

C.I. =
$$18000 \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right]$$
 (2)

(2)-(1)
$$\Rightarrow$$
 18000 $\left[\left(1 + \frac{R}{100} \right)^2 - 1 - \frac{2R}{100} \right] = 405$

or
$$18000 \left[\left(\frac{R}{100} \right)^2 \right] = 405$$

or
$$\left(\frac{R}{100}\right)^2 = \frac{405}{18000}$$

$$= \frac{45}{2000}$$

$$= \frac{9}{400}$$

or
$$\frac{R}{100} = \frac{3}{20}$$

or
$$R = 15\%$$

29. Let the required number of years be N.

$$\therefore 800 \left(1 + \frac{10}{200}\right)^{2N} = 926.10 (1)$$
i.e. $800 \left(\frac{105}{100}\right)^{2N} = 926.10$

$$800 \left(\frac{21}{20}\right)^{2N} = 926.10$$

$$\therefore$$
 2N = 3 years

or
$$N=1\frac{1}{2}$$
 years

31.

30. Let each instalment be $\overline{\xi}$ x.

$$\frac{x}{\left(1 + \frac{4}{100}\right)^2} + \frac{x}{\left(1 + \frac{4}{100}\right)} = 3825$$
i.e.
$$\frac{x}{\left(\frac{26}{25}\right)^2} + \frac{x}{\left(\frac{26}{25}\right)} = 3825$$
or
$$x + x \times \frac{26}{25} = 3825 \times \left(\frac{26}{25}\right)^2$$
i.e.
$$x \times \frac{51}{25} = 3825 \times \frac{(26)^2}{(25)^2}$$

$$x = 3825 \times \frac{(26)^2}{(25)^2} \times \frac{25}{51}$$

$$= \frac{153 \times (26)^2}{51} = 3 \times (26)^2 = ₹ 2028$$

$$15000 \left(1 + \frac{R}{100}\right)^5 = 30000$$
or
$$\left(1 + \frac{R}{100}\right)^5 = 2$$

$$(1)$$

$$\therefore 15000 \left(1 + \frac{R}{100}\right)^{20} = 15000 \left[\left(1 + \frac{R}{100}\right)^5\right]^4$$

= $15000 \times 2^4 = 2,40,000$ ∴ Amount after 20 years = ₹ 2,40,000

32. C.I. for 2 years @ 10% p.a. interest

$$= P \left[\left(1 + \frac{10}{100} \right)^2 - 1 \right] \tag{1}$$

S.I. =
$$\frac{P \times 2 \times 10}{100} = \frac{P}{5}$$
 (2)

(1)-(2)
$$\Rightarrow P\left[\left(\frac{11}{10}\right)^2 - 1 - \frac{1}{5}\right] = 20$$

or $P\left(\frac{(11)^2 - (10)^2 - 20}{100}\right) = 20$

or
$$P = \frac{20 \times 100}{1} = \mathbf{₹2000}$$

:. If interest is compounded half yearly,

C.I. – S.I. =
$$P\left[\left(1 + \frac{5}{100}\right)^4 - 1\right] - \frac{P}{5}$$

$$= P\left[\left(1 + \frac{5}{100}\right)^4 - 1 - \frac{1}{5}\right]$$

$$= 2000 \left[\left(\frac{21}{20}\right)^4 - 1 - \frac{1}{5}\right] = 2000 \left[\frac{(21)^4 - (20)^4}{(20)^4} - \frac{1}{5}\right]$$

$$= 2000 \left[\frac{(21^2 + 20^2)(21^2 - 20^2)}{20^4} - \frac{1}{5}\right] = 2000 \left[\frac{841 \times 41 \times 1}{20^4} - \frac{1}{5}\right]$$

$$= 2000 \left[\frac{34481}{1,60,000} - \frac{1}{5}\right] = \frac{2000}{1,60,000} [34481 - 32000]$$

$$= \frac{1}{80} \times 2481 = 31.0125$$

∴ Difference of interest = ₹ 31.01

33. Balance amount =
$$25000 \left(1 + \frac{20}{100}\right)^3 - 5000 \left(1 + \frac{20}{100}\right)^2 - 5000 \left(1 + \frac{20}{100}\right) - 5000$$

= $25000 \times \left(\frac{6}{5}\right)^3 - 5000 \left(\frac{6}{5}\right)^2 - 5000 \left(\frac{6}{5}\right) - 5000$
= $200 \times 216 - 200 \times 36 - 6000 - 5000$
= $43200 - 7200 - 6000 - 5000 = ₹25000$

34. Annual instalment = ₹882, R = 5%

$$\therefore \text{ Required amount} = \frac{882}{\left(1 + \frac{5}{100}\right)^2} + \frac{882}{\left(1 + \frac{5}{100}\right)}$$

i.e., Amount borrowed
$$= \frac{882}{\left(\frac{21}{20}\right)^2} + \frac{882}{\left(\frac{21}{20}\right)}$$
$$= 882 \left[\left(\frac{20}{21}\right)^2 + \left(\frac{20}{21}\right) \right] = 882 \left[\frac{20^2 + 20 \times 21}{21^2} \right]$$

35. C.I. =
$$50000 \left[\left(1 + \frac{10}{100} \right)^4 - 1 \right]$$

= $50000 \left[\left(\frac{11}{10} \right)^4 - 1 \right] = \frac{50000}{10^4} [11^4 - 10^4]$
= $5 \times [(11^2 + 10^2)(11^2 - 10^2)] = 5 \times [221 \times 21] = 23205$

∴ Compound interest = ₹23205.

Time and Work

If the number of men engaged to do a certain work is increased in the ratio a: b, then the time required to do the same work will get changed in the ratio b:a.

Important Formulae

1. If W represents an amount of work, P represents the number of persons doing the job and T represents the time period of doing work, then

$$\frac{PT}{W} = \text{constant}$$

2. A can do a work in D_1 days and B can do the same work in D_2 days. Then A and B if work together, then the number of days required to complete the work is

$$\frac{D_1D_2}{D_1+D_2}$$

3. A, B, and C can do a work in D_1 , D_2 and D_3 days respectively working alone. Time taken to complete the work if they work together is given by

$$\frac{D_1 D_2 D_3}{D_1 D_2 + D_2 D_3 + D_3 D_1}$$
 days

4. A and B together can do a piece of work in D days. If A can do it alone in D_1 days, in how many days can B do it alone?

(A + B)'s 1 day's work =
$$\frac{1}{D}$$

A's 1 day's work = $\frac{1}{D_1}$

B's 1 day's work = $\frac{1}{D} - \frac{1}{D_1} = \frac{D_1 - D}{DD_1}$ ٠.

 \therefore B alone can do this work in $\frac{DD_1}{D_1-D}$ days

5. A and B can do a piece of work in x days, B and C in y days, C and A in z days. How long will each take to do the same work separately?

$$(A + B)$$
's 1 day's work = $\frac{1}{x}$

$$(B + C)$$
's 1 day's work = $\frac{1}{v}$

$$(C + A)$$
's 1 day's work = $\frac{1}{z}$

:.
$$\{(A + B) + (B + C) + (C + A)\}$$
's 1 day's work = $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

or
$$2(A + B + C)$$
's 1 day's work = $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

$$\therefore \qquad (A + B + C)'s \ 1 \ day's \ work = \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

$$\therefore \qquad \text{A's 1 day's work} = (A + B + C)'s 1 \text{ day's work}$$

$$- (B + C)'s 1 \text{ day's work}$$

$$=\frac{1}{2}\left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\right)-\frac{1}{r}$$

$$=\frac{1}{2}\left(\frac{1}{x}-\frac{1}{y}+\frac{1}{z}\right)$$

$$=\frac{yz-xz+xy}{2xyz}$$

 \therefore A alone can do the work in $\frac{2xyz}{xy + yz - zx}$ days

B alone can do the work in $\frac{2xyz}{yz + zx - xy}$ days

C alone can do the work in $\frac{2xyz}{zx + xy - yz}$ days

EXAMPLE 1 A and B can do a work in 15 days, B and C in 20 days, C and A in 12 days. In how many days can they complete it if:

(i) They work together and (ii) they work separately.

Solution

(i) If they work together, the number of days required to complete the work

$$=\frac{2xyz}{xy+yz+zx}=\frac{2(15)(20)(12)}{(15)(20)+(20)(12)+(12)(15)}$$

$$= \frac{10 \times 12}{5 + 4 + 3} = \frac{120}{12} = 10 \text{ days}$$

(ii) If they work separately

A alone can complete the work in $\frac{2xyz}{xy + yz - zx}$ days

$$= \frac{2(15)(20)(12)}{(15)(20) + (20)(12) - (12)(15)} = \frac{10 \times 12}{5 + 4 - 3}$$
$$= \frac{120}{6} = 20 \text{ days}$$

B alone can complete the work in $\frac{2xyz}{yz + zx - xy}$ days

$$= \frac{2(15)(20)(12)}{(20)(12) + (12)(15) - (15)(20)} = \frac{10 \times 12}{4 + 3 - 5}$$
$$= \frac{120}{2} = 60 \text{ days}$$

C alone can complete the same work in $\frac{2xyz}{zx + xy - yz}$ days

$$= \frac{2(15)(20)(12)}{(12)(15) + (15)(20) - (20)(12)} = \frac{10 \times 12}{3 + 5 - 4}$$
$$= \frac{120}{4} = 30 \text{ days}$$

EXAMPLE 2 A group of boys can do a certain piece of work in 40 days. If the group had 10 more boys, the work could be done in 5 days less. How many boys are there in the group? **Solution** The original number of workers in the group

 $= \frac{Number of additional workers \times Number of days taken by second group}{Number of less days}$

$$=\frac{10\times35}{5}=70 \text{ numbers}$$

EXAMPLE 3 A group of 20 boys took a contract to paint a shopping complex in 16 days. After 10 days they added 5 more boys and finished the work 2 days before the schedule. By how many days the schedule would have been delayed if these 5 boys were not added.

Solution Number of additional days needed

 $= \frac{\text{Number of additional workers} \times \text{Number of days taken by the second group}}{\text{Original number of workers in the group}}$ $= \frac{5 \times 4}{20} = 1 \text{ day}$

i.e. in all 15 days (i.e. 1 day in advance as per the schedule).

Important Formula

N men start to do a work in D days. After d days, n more men had to join in order to complete the work in scheduled time. If n men had not joined them, the number of additional days needed to complete the work is given by

Over and above
$$D$$
 days $\frac{n \times (D-d)}{N}$ days

EXAMPLE 4 50 men or 80 women can finish a job in 50 days. A contractor deploys 40 men and 48 women for this work, but after every duration of 10 days, 5 men and 8 women are removed till the work is completed. In how many days can the said work be completed? **Solution** 50 men can finish the job in 50 days.

 \therefore 1 man can finish it in 50×50 days or part of work done by 1 man/day = 1/2500. Similarly 80 women can finish the job in 50 days.

 \therefore 1 woman can finish it in 80×50 days.

Part of work done by a woman/day = 1/4000

At the start 40 men and 48 women were engaged.

$$50 \text{ men} = 80 \text{ women}$$

or
$$5 \text{ men} = 8 \text{ women}$$

$$\therefore \qquad \text{At the start the number of men engaged} = 40 + \frac{48 \times 5}{8} = 70$$

∴ After first 10 days work, part of work completed =
$$10 \times 70 \times \frac{1}{2500} = \frac{7}{25}$$

$$\therefore \qquad \text{The remaining part of work} = 1 - \frac{7}{25} = \frac{18}{25}$$

Then 5 men and 8 women are removed.

∴ After second 10 days, part of work completed =
$$10 \times 60 \times \frac{1}{2500} = \frac{6}{25}$$

$$\therefore \qquad \text{The remaining part of work} = \frac{18}{25} - \frac{6}{25} = \frac{12}{25}$$

Then 5 men and 8 women are removed

$$\therefore \qquad \text{After third 10 days, part of work completed} = 10 \times 50 \times \frac{1}{2500} = \frac{5}{25}$$

$$\therefore \qquad \text{The remaining part of work} = \frac{12}{25} - \frac{5}{25} = \frac{7}{25}$$

Then 5 men and 8 women are removed.

$$\therefore \qquad \text{After fourth 10 days, part of work completed} = 10 \times 40 \times \frac{1}{2500} = \frac{4}{25}$$

$$\therefore \qquad \text{The remaining part of work} = \frac{7}{25} - \frac{4}{25} = \frac{3}{25}$$

Then 5 men and 8 women are removed.

$$\therefore \qquad \text{After fifth 10 days, part of work completed} = 10 \times 30 \times \frac{1}{2500} = \frac{3}{25}$$

:. The remaining part of work = Nil

i.e. work got completed in 50 days.

If N_1 persons can do W_1 work in D_1 days and N_2 persons can do W_2 work in D_2 days then we have a very general formula in the relationship of $N_1D_1W_2 = N_2D_2W_1$.

or

$$\frac{N_1D_1}{W_1} = \frac{N_2D_2}{W_2}$$

From the above formula, we can arrive at the following results.

Important Results

- 1. The more men, the less days and conversely the more days the less men.
- 2. The more men, the more work and conversely the more work, the more men.
- 3. The more days, the more work and conversely the more work, the more days.

If we include working hours (T_1, T_2) also, then the above relationship becomes

$$\frac{N_1 D_1 T_1}{W_1} = \frac{N_2 D_2 T_2}{W_2}$$

If efficiency is also included as E_1 and E_2 for the persons in two groups, then

$$\frac{N_1 D_1 T_1 E_1}{W_1} = \frac{N_2 D_2 T_2 E_2}{W_2}$$

EXAMPLE 5 A can do a piece of work in 6 days. How many days will he take to complete four such works?

Solution From point 3 noted above, the more work, the more days.

Here $D_1 = 6$ days, $W_1 = 1$, $D_2 = ?$ $W_2 = 4$

$$\therefore W_2D_1 = W_1D_2$$

 \Rightarrow

$$D_2 = \frac{W_2 D_1}{W_1}$$
= $\frac{4 \times 6}{1}$ = 24 days

EXAMPLE 6 Ten men can do a piece of work in 12 days. How many men are needed to complete the work in 20 days?

Solution To do a work in 12 days, 10 men are needed.

To do a work in 1 day, 10×12 men are needed.

 \therefore To do the work in 20 days, $\frac{10 \times 12}{20} = 6$ men are needed.



Short cut: $N_1D_1 = N_2D_2$

Here $N_1 = 10$; $D_1 = 12$ days; $N_2 = ?$; $D_2 = 20$ days

$$N_2 = \frac{N_1 D_1}{D_2} = \frac{10 \times 12}{20} = 6$$

RULE OF FRACTIONS: To do the work in 20 days, we need less number of men than 10. So multiply 10 with a fraction which is less than 1.

$$\therefore \text{ Required number} = 10 \times \frac{12}{20} = 6$$

EXAMPLE 7 30 men can plant 60 trees in 5 hours. If 5 men leave the job, how many trees will be planted in 10 hours?

Solution 30 men working 5 hours plant 60 trees

or 1 man working 1 hour, number of trees planted =
$$\frac{60}{30 \times 5} = \frac{2}{5}$$
 trees

25 men working 10 hours, number of trees planted = $\frac{60 \times 25 \times 10}{30 \times 5}$ = 100 trees



:.

Short cut: $WN_1D_1W_2 = N_2D_2W_1$

$$30 \times 5 \times W_2 = 25 \times 10 \times 60$$

$$W_2 = \frac{25 \times 10 \times 60}{30 \times 5} = 100 \text{ trees}$$

There were 30 men, but when 5 men leaves the job, we are left with 25 men. As the number of men is reduced, less number of trees will be planted by them. So, 60 would be multiplied with less than one. And the fraction used is 25/30. As the number of hours increases, the more number of trees will be planted. So the previous product is multiplied by a fraction

more than one, i.e.
$$\left(\frac{10}{5}\right)$$
.

∴ Required number of trees to be planted =
$$60 \times \left(\frac{25}{30}\right) \times \left(\frac{10}{5}\right) = 100$$
 trees

EXAMPLE 8 5 men can prepare 20 items in 8 days working 6 hours a day. Then in how many days can 9 men prepare 30 items working 8 hours a day?

Solution We have,
$$\frac{N_1 D_1 T_1}{W_1} = \frac{N_2 D_2 T_2}{W_2}$$

i.e. $\frac{5 \times 8 \times 6}{20} = \frac{9 \times D_2 \times 8}{30}$
∴ $D_2 = \frac{5 \times 8 \times 6 \times 30}{20 \times 9 \times 8} = 5 \text{ days}$



Short cut: Required number of days,

$$\begin{split} D_2 &= \text{First number of days } (D_1) \times \left(\frac{N_1}{N_2}\right) \times \left(\frac{W_2}{W_1}\right) \times \left(\frac{T_1}{T_2}\right) \\ &= 8 \times \left(\frac{5}{9}\right) \times \left(\frac{30}{20}\right) \times \left(\frac{6}{8}\right) = 5 \text{ days} \end{split}$$

Theorem: If A can do a piece of work in 'x' days and B can do it in 'y' days, then A and B together can do the same work in $\frac{xy}{x+y}$ days.

EXAMPLE 9 A can do a piece of work in 10 days and B can do the same work in 15 days. How long will they take if both work together?

Solution A can do 1/10 part of the work in 1 day.

B can do 1/15 part of the work in 1 day.

- \therefore A and B together can do (1/10 + 1/15) part of the work in 1 day.
- \therefore Time taken to complete the work, when both A and B work together = $\frac{1}{1/10 + 1/15}$

$$=\frac{10\times15}{10+15}=\frac{150}{25}=6$$
 days



Short cut: A and B can do the work in $\frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6$ days

Theorem: If A, B, and C can do a work in x, y, and z days respectively, then all of them working together can finish the work in $\frac{xyz}{xy + yz + zx}$ days.

EXAMPLE 10 If A can do a piece of work in 10 days, B can do the same work in 15 days and C can do the same work in 30 days, how long will they take to complete the work when they work together?

Solution By the above theorem, time taken to complete the work

$$= \frac{10 \times 15 \times 30}{10 \times 15 + 15 \times 30 + 30 \times 10}$$
$$= \frac{1 \times 15 \times 30}{15 + 45 + 30} = \frac{30}{1 + 3 + 2} = \frac{30}{6} = 5 \text{ days}$$

Theorem: If Sunil and Suresh together can do a piece of work in x days and Sunil alone can do it in y days, then Suresh alone can do the same work in $\frac{xy}{y-x}$ days.

Solution Anil and Manoj can do 1/6 part of the work in 1 day.

Anil alone can do 1/8 part of the work in 1 day.

- :. Manoj alone can do (1/6-1/8) part of the work in 1 day.
- \therefore Manoj alone can do the same work in $\frac{1}{(1/6-1/8)} = \frac{8\times 6}{8-6} = \frac{48}{2} = 24$ days



Short cut: Manoj alone can do the same work in $\frac{xy}{y-x}$ days.

$$=\frac{8\times6}{8-6}=24~\mathrm{days}$$

EXAMPLE 12 P and Q can do a piece of work in 12 days, Q and R in 15 days, R and P in 20 days. How long would each take separately to do the same work?

Solution P and Q can complete the work in 12 days.

Q and R can complete the work in 15 days.

R + P can complete the work in 20 days.

- $\therefore 2(P+Q+R) \text{ can complete the work in } \frac{12\times15\times20}{12\times15+15\times20+20\times12} = 5 \text{ days}$
- \therefore P, Q and R can do the work in $5 \times 2 = 10$ days

P and Q can do the work in 12 days

Q and R can do the work in 15 days

R and P can do the work in 20 days

 \therefore P alone can do the work in $\frac{15 \times 10}{15 - 10} = \frac{150}{5} = 30 \text{ days}$

$$[\, \cdot \cdot \cdot \, P = (P+Q+R) - (Q+R)]$$

Similarly Q alone can do the work in $\frac{20 \times 10}{20 - 10} = \frac{200}{10} = = 20 \text{ days}$

Similarly R alone can do the work in $\frac{12 \times 10}{12 - 10} = \frac{120}{2} = 60 \text{ days}$

EXAMPLE 13 A and B together can do a piece of work in 16 days which B and C together can do in 12 days. After A has been working at it for 2 days and B for 7 days, C takes up and finishes it alone. In how many days could each to the work by himself?

Solution A and B can do 1/16 part of the work in 1 day.

B and C can do 1/12 part of the work in 1 day.

A's 2 day's + B's 7 day's + C's 16 day's work = 1

or A's 2 days + B's 2 day's + B's 5 day's + C's 5 day's + C's 11 day's work = 1

 \therefore (A + B)'s 2 day's + (B + C)'s 5 day's + C's 11 day's work = 1

 $\frac{2}{16} + \frac{5}{12} + \text{ C's } 11 \text{ day's work} = 1$

:. C's 11 day's work =
$$1 - \left(\frac{2}{16} + \frac{5}{12}\right) = \frac{11}{24}$$

:. C's 1 day's work =
$$\frac{11}{24 \times 11} = \frac{1}{24}$$

B's 1 day's work =
$$\frac{1}{12} - \frac{1}{24} = \frac{1}{24}$$

A's 1 day's work =
$$\frac{1}{16} - \frac{1}{24} = \frac{1}{48}$$

:. A, B, and C can do the work in 48, 24 and 24 days respectively when they work alone.

EXAMPLE 14 If 3 men or 4 women can complete a certain job in 43 days, how long will 7 men and 5 women take to reap it?

Solution 3 men can do 1/43 part of job in 1 day.

4 women can do 1/43 part of job in 1 day.

 \therefore 1 man can do $1/(43 \times 3)$ part of job in 1 day.

1 woman can do $1/(43 \times 4)$ part of job in 1 day.

∴ (7 men + 5 women) can do
$$\left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4}\right)$$
 part of the job in 1 day

$$\therefore \text{ Time taken to complete the job by 7 men and 5 women} = \frac{1}{\left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4}\right)}$$

$$= \frac{43 \times 3 \times 4}{7 \times 4 + 5 \times 3} = \frac{43 \times 3 \times 4}{28 + 15}$$

$$= \frac{43 \times 3 \times 4}{43} = 12 \text{ days}$$

Another method:

$$3 \text{ men} = 4 \text{ women}$$

$$\therefore$$
 1 man = 4/3 women

$$\therefore$$
 7 men = 28/3 women

$$\therefore$$
 7 men + 5 women = $28/3 + 5 = 43/3$ women

Now we have,

4 women do the job in 43 days.

$$\therefore$$
 43/3 women do the same job is $\frac{4 \times 43}{43/3} = 4 \times 3 = 12$ days



Short cut: Required number of days =
$$\frac{1}{\left[\frac{7}{43\times3} + \frac{5}{43\times4}\right]} = \frac{43\times3\times4}{7\times4 + 5\times3} = 12 \text{ days}$$

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Solution Let the number of men be x

(x + 8) men can finish the work in (60 - 10) = 50 days

i.e.
$$x \times 60 = (x + 8) \times 50$$

On simplifying the equation,

$$\Rightarrow$$

$$10 \times x = 50 \times 8$$

$$\Rightarrow$$

$$x = \frac{50 \times 8}{10} = 40 \text{ men}$$

Another method:

Let the original number of men be x.

x men can do the job in 60 days.

(x + 8) men can do the same job in (60 - 10) = 50 days.

 \therefore Work done by 8 men in 50 days is same as what x men can do in 10 days.

$$\therefore 8 \times 50 = x \times 10$$

$$x = \frac{8 \times 50}{10} = 40$$



Short cut: Original number of workers

_ Number of more workers × Number of days taken by second group

Number of less days

$$= \frac{8 \times (60 - 10)}{10} = \frac{8 \times 50}{10} = 40 \text{ men}$$

EXAMPLE 16 X is twice as good a workman as Y. Together, they finish the work in 14 days. In how many days can it be done by each separately?

Solution Let Y finish the work in 2n days.

Therefore X can finish the same work in n days

$$\therefore$$
 (X + Y) can finish the work in $\frac{2n \times n}{2n + n} = 14$

$$\frac{2n^2}{3n} = 14$$

$$n = 21$$

.. X finishes the work in 21 days and Y finishes the same work in 42 days when they work independently.



Short cut: (2 + 1) men - one man work in 14 days

- $1 \text{ man one day work for } 14 \times 3 = 42 \text{ days}$
- \therefore 2 men one day work for 42/2 = 21 days

EXAMPLE 17 A and B can do a work in 45 days and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave?

Solution If A and B work together, time taken to complete the work

$$=\frac{40\times45}{40+45}=\frac{40\times45}{85}=\frac{40\times9}{17}$$

When B works alone for 23 days, the part of the work completed = $23 \times \frac{1}{40} = \frac{23}{40}$

- \therefore Remaining work = $1 \frac{23}{40} = \frac{17}{40}$ part
- .. Time taken to complete the above part of the work when A and B work together

$$=\frac{40\times9}{17}\times\frac{17}{40}=9 \text{ days}$$



Short cut: Required number of days =
$$\frac{40 \times 45}{40 + 45} \left(\frac{40 - 23}{40} \right) = 9 \text{ days}$$

EXAMPLE 18 A group of men decided to do a work in 10 days. But five of them were absent. If the rest of the group did the work in 12 days, find the original number of men.

Solution Let the original number of men be n. Here number of man days is constant.

i.e.
$$10 \times n = 12(n-5)$$

$$n = \frac{12 \times 5}{(12 - 10)} = 30 \text{ men}$$

EXAMPLE 19 A person decided to build a farmhouse in 40 days. He employed 100 men in the beginning and 100 more after 35 days and completed the work in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

Solution Let the time taken to complete the work by 100 men be t days.

:. Work done by 100 men in 35 days + Work done by 200 men in (40-35) = 5 days = 1

or
$$\frac{100 \times 35 + 200 \times 5}{100t} = 1$$

$$\Rightarrow \frac{45}{t} = 1 \quad \text{or} \quad t = 45 \text{ days}$$

: If additional men were not employed, the work would have completed in (45 - 40) = 5 days behind the schedule.



Short cut: 200 men do the rest of the work in 40-35=5 days

- \therefore 100 men can do the rest of the work in $\frac{5 \times 200}{100} = 10$ days
- \therefore Required number of days = 10 5 = 5 days

EXAMPLE 20 P can complete a work in 25 days and Q can complete the same work in 20 days. If they work together and P leaves after 5 days, in how many days Q will finish the rest of the work.

Solution Part of the work done by P and Q in 5 days = $5\left[\frac{1}{25} + \frac{1}{20}\right] = 5 \times \frac{9}{100} = \frac{9}{20}$

$$\therefore \qquad \text{Rest of the work} = 1 - \frac{9}{20} = \frac{11}{20}$$

Q will do the rest of work in
$$\frac{20}{1} \times \frac{11}{20} = 11$$
 days

EXAMPLE 21 30 men working 7 hours a day can complete a work in 20 days. In how many days will 21 men working 8 hours a day do the same work?

Solution We have $M_1D_1T_1 = M_2D_2T_2$

$$D_2 = \frac{M_1 D_1 T_1}{M_2 T_2} = \frac{30 \times 20 \times 7}{21 \times 8} = 25 \text{ days}$$

EXAMPLE 22 X takes as much time as Y and Z together take to complete a job. X and Y together can complete the job in 10 days. Z alone can complete the same job in 15 days. In how many days can Y alone do the same job?

Solution Given (X + Y) together can complete the job in 10 days.

Z alone can complete the same job in 15 days.

:. Time taken to complete the job when all of them work together = $\frac{15 \times 10}{15 + 10} = 6$ days

But time taken by X is same as Y and Z together. \therefore Y and Z can complete the job in $2 \times 6 = 12$ days

∴ Y alone can complete the work in
$$\frac{15 \times 12}{15 - 12} = 60$$
 days [∴ Y = (Y + Z) – Z]

EXAMPLE 23 Ashok can do a piece of work in 16 days. Raman can do the same work in 8 days while Satish can do it in 32 days. All of them worked together when they started, but Raman left after 2 days. Ashok left 3 days before the completion of the work. How long would work last?

Solution Let the work lasted for t days.

Then Raman's 2 days' work + Ashok's (t-3) days' work + Satish's t days' work = 1

or
$$\frac{2}{8} + \frac{t - 3}{16} + \frac{t}{32} = 1$$
i.e.
$$\frac{1}{4} + \frac{t - 3}{16} + \frac{t}{32} = 1$$

i.e.
$$\frac{8 + 2(t - 3) + t}{32} = 1$$

i.e.
$$3t + 2 = 32$$

$$3t = 32 - 2 = 30$$

:.

:.

$$t = \frac{30}{3} = 10 \text{ days}$$

EXAMPLE 24 P and Q undertake to do a job for ₹91. P can do it alone in 13 days and Q can do it alone in 7 days. If with the assistance of a boy they finish the work in 2 days then the boy gets ₹_

Solution P's 2 days' work + Q's 2 days' work + Boy's 2 days' work = 1

i.e.
$$\frac{2}{13} + \frac{2}{7} + \text{Boys 2 day's work} = 1$$

i.e.
$$\frac{2}{13} + \frac{2}{7} + \text{Boys 2 day's work} = 1$$

 \therefore Boy's 2 day's work

Boy's 2 day's work =
$$1 - \left(\frac{2}{13} + \frac{2}{7}\right)$$

= $1 - \left(\frac{14 + 26}{91}\right)$
= $\frac{91 - 40}{91} = \frac{51}{91}$

Ratio of shares =
$$\frac{2}{13} : \frac{2}{7} : \frac{51}{91}$$

= $2 \times 7 : 2 \times 13 : 51$
= $14 : 26 : 51$

Boy's share =
$$\frac{51}{14 + 26 + 51} \times 91 = \frac{51}{91} \times 91 = ₹51$$

EXERCISES

- 1. P and Q working together can complete a certain work in 6 days. Q alone can do it in 8 days. In how many days P alone could finish the same work?
 - (a) 12 days
- (b) 16 days
- (c) 18 days
- (d) 24 days
- (e) None of these
- 2. A certain sum of money is sufficient to pay wages for 21 days of X or 28 days of Y. The money is sufficient to pay the wages of both for _____ days.
- (b) 12 days
- (c) 16 days
- (d) 20 days
- (e) None of these
- 3. If 3 men or 5 women can reap a field in 43 days, how long will 5 men and 6 women take to reap it?
 - (a) 10 days
- (b) 12 days
- (c) 15 days
- (d) 18 days
- (e) None of these
- 4. 3 men and 4 boys can complete a certain work in 8 days, while 4 men and 4 boys can complete the same work in 6 days. Then 2 men and 4 boys will finish it in ____ days.
 - (a) 24 days
- (b) 8 days
- (c) 6 days
- (d) 4 days
- (e) None of these
- 5. 2 men and 3 women can complete a work in 10 days, while 4 men can complete it in 10 days. In how many days will 3 men and 3 women complete the work?
 - (a) 6 days
- (b) 8 days
- (c) 10 days
- (d) 12 days
- (e) None of these

6.	P and Q can do a piece of work in 30 and 40 days respectively. They began the work together but P left after some days and Q finished the remaining work in 12 days. After how many days did P leave?						
	(a) 8 days (b) 10 days (c) 12 days (d) 14 days (e) None of these						
7.	P is twice as good a workman as Q, and together they finish a work in 16 days. In how many days can it be done by each separately?						
	(a) 16 days, 24 days (b) 24 days, 48 days (c) 20 days, 40 days						
	(d) 24 days, 16 days (e) None of these						
8.	Ram, Rajan and Rangan can do a certain work in 6, 12 and 24 days respectively. In what time will they altogether do it?						
	(a) 6/7 days (b) 12/7 days (c) 24/7 days (d) 48/7 days (e) None of these						
9.	P and Q complete a certain work in 30 days, Q and R in 40 days while R and P in 60 days. How long will they take to finish it together?						
	(a) 40/3 days (b) 80/3 days (c) 160/3 days (d) 20/3 days (e) None of these						
10.	P, Q and R can finish a work in 10, 12 and 15 days respectively. If Q stops after 2 days, how long would it take P and R to finish the same work?						
	(a) 6 days (b) 4 days (c) 3 days (d) 2 days (e) None of these						
11.	If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be [SSC]						
	(a) 4 days (b) 5 days (c) 6 days (d) 7 days						
12.	Twenty four men can complete a work in sixteen days. Thirty-two women can complete the same work in twenty-four days. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days? [Bank PO]						
	(a) 16 (b) 24 (c) 36 (d) 48 (e) None of these						
13.	10 women can complete a work in 7 days and 10 children take 14 days to complete the work How many days will 5 women and 10 children take to complete the work? [Bank PO]						
	(a) 3 (b) 5 (c) 7						
	(d) Cannot be determined (e) None of these						
14.	4 men and 6 women can complete a work in 8 days, while 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete it? [SSC]						
	(a) 35 (b) 40 (c) 45 (d) 50						
15.	If 12 men and 16 boys can do a piece of work in 5 days; 13 men and 24 boys can do it in 4 days then the ratio of the daily work done by a man to that of a boy is [SSC]						
	(a) 2:1 (b) 3:1 (c) 3:2 (d) 5:4						
16.	Three men, four women and six children can complete a work in seven days. A woman does double the work a man does and a child does half the work a man does. How many women alone can complete this work in 7 days? [SBI PO]						
	(a) 7 (b) 8 (c) 12						
	(d) Cannot be determined (e) None of these						
17.	A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day? [RRB]						
	(a) 12 days (b) 15 days (c) 16 days (d) 18 days						
18.	Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman?						

(a) 3:4

(b) 4:3

(c) 5:3

(d) Data inadequate

be completed in 3 days?

(e) None of these

(b) 18

(a) 15

	complete the sam		110W many days w	vill be required for one woman alone to [Bank PO]	
	(a) 90	(b) 125	(c) 145	(d) 150 (e) None of these	
21.	A man, a woman a	and a boy can com	plete a job in 3, 4 aı	nd 12 days respectively. How many boys	3
	must assist 1 ma	n and 1 woman t	o complete the job	in $\frac{1}{4}$ of a day? [SSC]	J
	(a) 1	(b) 4	(c) 19	(d) 41	
22.	_		After they have we complete the remain	orked for 6 days, 6 more men join them. ining work? [RRB]	
	(a) 2 days	(b) 3 days	(c) 4 days	(d) 5 days (e) None of these	_
23.	10 men can compl	lete a piece of wor	k in 15 days and 15	5 women can complete the same work in ner, in how many days will the work get [SBI PO]	ı t
	(a) 6	(b) $6\frac{1}{3}$	(c) $6\frac{2}{3}$	(d) $7\frac{2}{3}$	
24.			days and B alone in Il be completed in	n 12 days. Starting with A, they work on [SSC]	
	(a) 12 days	(b) 13 days	(c) $13\frac{5}{7}$ days	(d) $13\frac{3}{4}$ days	
25 .	A, B and C are en	aployed to do a pi	ece of work for ₹52	29. A and B together are supposed to do)
	$\frac{19}{23}$ of the work a	nd B and C toget	her $\frac{8}{23}$ of the worl	k. What amount should A be paid?	
	23		23	[CBI]	J
	(a) ₹ 315	(b) ₹ 345	(c) ₹ 355	(d) ₹ 375	
26.	the work only for			lone can complete it in 20 days. If B does days A and B together will complete the	
	work?			[RRB]	
	(a) 10 days	(b) 11 days	(c) 15 days	(d) 20 days]
27.	(a) 10 days A can do a piece o before the complet	f work in 14 days tion of the work, A	which B can do in 2 leaves off. The total	(d) 20 days 21 days. They begin together but 3 days al number of days to complete the work is: [RRB]] s :
27.	(a) 10 days A can do a piece o before the complet	f work in 14 days tion of the work, A	which B can do in 2	(d) 20 days 21 days. They begin together but 3 days al number of days to complete the work is: [RRB]] s :
	(a) 10 days A can do a piece o before the complet (a) $6\frac{3}{5}$ A alone can do a p	f work in 14 days tion of the work, A (b) $8\frac{1}{2}$ piece of work in 6	which B can do in a leaves off. The total (c) $10\frac{1}{5}$ days and B alone in	(d) 20 days 21 days. They begin together but 3 days of number of days to complete the work is: [RRB] (d) 13 \frac{1}{2} In 8 days. A and B under took to do it for in 3 days. How much is to be paid to C?	s :]
	(a) 10 days A can do a piece o before the complet (a) $6\frac{3}{5}$ A alone can do a p	f work in 14 days tion of the work, A (b) $8\frac{1}{2}$ piece of work in 6	which B can do in a leaves off. The total (c) $10\frac{1}{5}$ days and B alone in	(d) 20 days 21 days. They begin together but 3 days of number of days to complete the work is: [RRB] (d) 13 \frac{1}{2} 2 n 8 days. A and B under took to do it for	s :]
28.	 (a) 10 days A can do a piece of before the complete (a) 6 3/5 A alone can do a propertion of the state of the complete (a) ₹ 3200. With the state of the can do a propertion of the can do a pr	f work in 14 days tion of the work, A (b) 8 \frac{1}{2} Diece of work in 6 help of C, they co (b) ₹ 400 Diece of work in 4 haves after some days after which A	which B can do in a leaves off. The total (c) 10 1/5 days and B alone in simpleted the work in (c) ₹ 600 days and 40 days ays and then B con left the work was:	(d) 20 days 21 days. They begin together but 3 days of number of days to complete the work is: [RRB] (d) 13 \frac{1}{2} In 8 days. A and B under took to do it for in 3 days. How much is to be paid to C? [SSC] (d) ₹800 respectively. They began to do the work mpleted the remaining work in 23 days. [Bank PO]	s :]
28.	 (a) 10 days A can do a piece of before the complete (a) 6 3/5 A alone can do a presentadore de la presentada de la present	f work in 14 days tion of the work, A (b) 8 \frac{1}{2} Diece of work in 6 help of C, they co (b) ₹ 400 Diece of work in 4 days	which B can do in 2 leaves off. The total (c) 10 1/5 days and B alone in the sumpleted the work in the sum of the sum	 (d) 20 days 21 days. They begin together but 3 days all number of days to complete the work is: [RRB] (d) 13 ½ (e) n 8 days. A and B under took to do it for in 3 days. How much is to be paid to C? [SSC] (d) ₹ 800 respectively. They began to do the work impleted the remaining work in 23 days. 	s :]

19. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days, 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, if it is to

20. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man

(c) 22

[SBI PO]

(d) Data inadequate

30.		k in 3 days while D : and get ₹ 150. Wl				ork in 2 days. Both of them finish? [SSC]	
	(a) ₹ 30	(b) ₹ 60	(c)	₹ 70	(d)	₹ 75	
31.						worked for 16 days, B finishes the B finish the whole work alone? [CBI	
	(a) 30 days	(b) 40 days	(c)	60 days	(d)	70 days	
32.		rk in 24 days, B in er 3 days. The ren				vs. B and C start the work but are by A in: [SSC	_
	(a) 5 days	(b) 6 days	(c)	10 days	(d)	$10\frac{1}{2}$ days	
33.		Iow long B alone v	woul	d take to do th	e w		
	(a) 23 days	(b) 37 days	(c)	$37\frac{1}{2}$ days	(d)	40 days	
34.	_					d together for 20 days and then I k. In how many days A alone car [SSC	n
	(a) 40	(b) 50	(c)	54	(d)	60	
35.		rk in 18 days and l n how many days,				t in 15 days. B worked for 10 days remaining work? [Bank PO	
	(a) 5	(b) $5\frac{1}{2}$	(c)	6	(d)	8	
36.	in 10 hours, while me the work. Approxi	e machine R can p nachine P is closed imately at what ti	print at 1 me w	them in 12 ho 1.00 a.m. and t vill the work be	ours he r e fin	-	t e
	(a) 11.30 a.m.	(b) 12.00 Noon		12.30 p.m.		01.00 p.m.	
37.						ectively. X started the work alone work. How long did the work last [Bank PO	?
	(a) 6 days	(b) 10 days	(c)	15 days	(d)	20 days	
38.	C and A in 20 day	rs. They all work to to finish the work?	ogeth	ner for 10 days	whe	n do the same work in 24 days and en B and C leave. How many days [CBI	s
	(a) 18 days	(b) 24 days		30 days	` '	36 days	
39.		r 2 days B had to ompleted in:	leav	e and A alone		ively. They started doing the worl apleted the remaining work. The [SSC]	e
	(a) 8 days	(b) 10 days	(c)	12 days	(d)	15 days	
40.			ays. A	A is $1\frac{3}{4}$ times	as e	fficient as B. The same job can be	
	done by A alone in	ı:				[SSC]	1

(a) $9\frac{1}{3}$ days (b) 11 days (c) $12\frac{1}{4}$ days (d) $16\frac{1}{3}$ days

(b) 20 days

(b) $22\frac{1}{2}$ days

B. Working together, they can do it in:

fraction of the work that is left is:

(a) 15 days

(a) 20 days

	(a) $\frac{1}{4}$	(b) $\frac{1}{10}$	(c) $\frac{7}{15}$	(d) $\frac{8}{15}$	
44.		icient than B. How ne could have done		ey, working together, take to compl [Hotel Manageme	
	(a) 11 days	(b) 13 days	(c) $20\frac{3}{17}$ days	(d) None of these	
45.			20 days. Tanya is lo the same piece o	25% more efficient than Sakshi. Tof work is [Hotel Management	_
	(a) 15	(b) 16	(c) 18	(d) 25	
46.		vork in 8 days, B ar days. A and C toge		ne work in 12 days. A, B and C togetl [RR	
	(a) 4 days	(b) 6 days	(c) 8 days	(d) 12 days	
47.	8 days working 10		oth P and Q work to	day. Q can complete the same work ogether, working 8 hours a day, in h [Bank P	ow
	(a) $5\frac{5}{11}$	(b) $5\frac{6}{11}$	(c) $6\frac{5}{11}$	(d) $6\frac{6}{11}$	
48.	together, they can	n finish the work in	n 2 days. B can do t		_
	(a) 4 days	(b) 6 days	()	•	
49.	together they will	complete work in:	:	C and A in 20 days. If A, B and C wo [SS	
	(a) 5 days	(b) $7\frac{5}{6}$ days	(c) 10 days	(d) $15\frac{2}{3}$ days	
50.	•		•	6 days and B can do the same job in . Then C alone can do the job in: [SS	
	(a) $9\frac{1}{5}$ days	(b) $9\frac{2}{5}$ days	(c) $9\frac{3}{5}$ days	(d) 10 days	
51.	A man can do a pi what time can the		ys, but with the he	elp of his son, he can do it in 3 days. [SS	
	(a) $6\frac{1}{2}$ days	(b) 7 days	(c) $7\frac{1}{2}$ days	(d) 8 days	
52.			S can do the same w same work they can	vork in half the time taken by A. Then finish in a day?	
	(a) $\frac{1}{6}$	(b) $\frac{1}{9}$	(c) $\frac{2}{5}$	(d) $\frac{2}{7}$	

41. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do it in: [SSC]

(c) 25 days

(c) 25 days

42. A is thrice as good a workman as B and therefore is able to finish a job in 60 days less than

43. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the

(d) 30 days

(d) 30 days

[SSC]

[SSC]

53. A, B and C can complete a piece of work in 24, 6 and 12 days respectively. Working together, they will complete the same work in: [CBI]

(a)
$$\frac{1}{24}$$
 day

(a) $\frac{1}{24}$ day (b) $\frac{7}{24}$ day (c) $3\frac{3}{7}$ days (d) 4 days

54. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work? [RRB]

(a) 5 days

(b) 6 days

(c) 8 days

(d) 9 days

55. A tyre has two punctures. The first puncture along would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat? [DMRC]

(a) $1\frac{1}{2}$ minutes (b) $3\frac{1}{2}$ minutes (c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes



1. (d) **2.** (b) **3.** (c) **4.** (e) **5.** (b) **6.** (c) **7.** (b) 8. (c) 9. (b) **10.** (c)

12. (b) **13.** (c) 14. (b) 15. (a) 16. (a) 17. (b) 11. (a) 18. (b) 19. (a) **20.** (e)

21. (d) 22. (a) **23.** (c) **24.** (c) **25.** (b) **26.** (c) **27.** (c) 28. (b) **29.** (c) **30.** (b)

31. (c) **32.** (c) **33.** (c) **34.** (d) **35.** (c) **36.** (d) **37.** (a) 38. (a) 39. (b) **40**. (b)

41. (c) **42.** (b) **43.** (d) **44.** (b) **45.** (b) **46.** (c) **47.** (a) 48. (b) **49.** (c) **50.** (c)

52. (a) **51.** (c) **53.** (c) **54.** (b) **55.** (c)



Solutions with Necessary Explanation

- 1. Required number of days = $\frac{6 \times 8}{8 6}$ = 24 days.
- 2. As the given sum is for 21 days or 28 days of wages of X or Y, the sum will be completely divisible by 21 and 28. So let the sum be LCM of 21 and 28 = 84.

X gets
$$\frac{84}{21}$$
 = ₹ 4/day

Y gets
$$\frac{84}{28}$$
 = ₹ 3/day

- ∴ (X + Y) together gets ₹ 7/day
- ∴ ₹ 84 is sufficient for $\frac{84}{7}$ = 12 days to pay both of them.



Short cut: Number of days = $\frac{\text{Multiplication of number of days}}{\text{Addition of number of days}}$

$$=$$
 $\frac{21 \times 28}{21 + 28} = 12 \text{ days}$

3. 3 men = 5 women

$$5 \text{ men} = \frac{5}{3} \times 5 = \frac{25}{3} \text{ women}$$

$$5 \text{ men} + 6 \text{ women} = \frac{25}{3} + 6 = \frac{43}{3} \text{ women}$$

5 women can reap in 43 days

$$W_1 D_1 = W_2 D_2$$

$$\therefore 43/3 \text{ women can reap is } \frac{43 \times 5}{43/3} = 5 \times 3 = 15 \text{ days}$$



Short cut: Required number of days =
$$\frac{1}{\left[\frac{5}{3 \times 43} + \frac{6}{5 \times 43}\right]} = 15 \text{ days}$$

4. 3 men + 4 boys complete the work in 8 days

Equation (2) and (1)
$$\Rightarrow$$
 1 man complete the work in $\frac{8 \times 6}{8 - 6} = 24$ days (3)

$$\therefore$$
 3 men can complete the work in $\frac{24}{3} = 8$ days

(1)

Equations (1) and (4) \Rightarrow boys do not work.

 \therefore 2 men + 4 boys complete in $\frac{24}{2}$ = 12 days.

5. 4 men complete in 10 days

$$\therefore$$
 2 men complete in $2 \times 10 = 20$ days

$$(2)$$

Equations (2) and (1)
$$\Rightarrow$$
 3 women complete in $\frac{10 \times 20}{20 - 10} = 20$ days (3)

Equation (1)
$$\Rightarrow$$
 3 men complete in $\frac{20 \times 2}{3} = \frac{40}{3}$ days (4)

Equations (3) + (4)
$$\Rightarrow$$
 3 men + 3 women complete in $\frac{20 \times 40/3}{20 + 40/3} = \frac{20 \times 40}{100} = 8 \text{ days}$

6. Work done by Q alone in 12 days = $\frac{12}{40} = \frac{3}{10}$ work

Work done by P and Q together =
$$1 - \frac{3}{10} = \frac{7}{10}$$
 work

P and Q together can do the work in $\frac{40 \times 30}{40 + 30} = \frac{120}{7}$ days

:. The number of days worked by P and Q together to complete $\frac{7}{10}$ work = $\frac{120}{7} \times \frac{7}{10} = 12$ days



Short cut: Required number of days =
$$\frac{30 \times 40}{30 + 40} \times \frac{40 - 12}{40} = 12 \text{ days}$$

- 7. Let Q do the work in 2n days
 - \therefore P does the work in n days
 - :. P and Q together can do it in $\frac{2n \times n}{2n + n} = \frac{2n^2}{3n} = \frac{2n}{3} = 16 \text{ days}$
 - $\therefore n = \frac{3 \times 16}{2} = 24 \text{ days}$

P completes in 24 days and Q completes in 48 days when worked independently.

- 8. Required number of days = $\frac{6 \times 12 \times 24}{6 \times 12 + 12 \times 24 + 24 \times 6} = \frac{24}{1 + 4 + 2} = \frac{24}{7}$ or $3\frac{3}{7}$ days
- 9. 2(P+Q+R) will do the work in $\frac{30\times40\times60}{30\times40+40\times60+60\times30} = \frac{120}{2+4+3} = \frac{120}{9} = \frac{40}{3}$ days.
 - \therefore P + Q + R will do the work in $2 \times \frac{40}{3} = \frac{80}{3}$ or $26\frac{2}{3}$ days
- 10. Part of work completed by P, Q, and R in 2 days = $2\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right) = 2 \times \left(\frac{6+5+4}{60}\right) = \frac{1}{2}$ work

:. If Q is withdrawn P and R have to complete the remaining half portion of the work.

Time taken to do 1 work by $P + R = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ days}$

- \therefore 1/2 work will be completed in 6/2 = 3 days.
- 11. Let the part of work done per day by a man = x and that by a boy = y

$$\therefore \qquad 6x + 8y = \frac{1}{10} \tag{1}$$

$$26x + 48y = \frac{1}{2} \tag{2}$$

Solving (1) and (2), we get

$$x = \frac{1}{100}$$
 and $y = \frac{1}{200}$

:. 1 days's work of 15 men and 20 boys

$$= 15 \times \frac{1}{100} + 20 \times \frac{1}{200} = \frac{15 + 10}{100} = \frac{25}{100} = \frac{1}{4}$$

 \therefore 15 men and 20 boys will complete the work in 4 days.

12. 1 man's 1 day's work =
$$\frac{1}{16 \times 24}$$
 (1)

$$1 \text{ woman's } 1 \text{ day's work} = \frac{1}{32 \times 24}$$
 (2)

∴ 16 men's + 16 women's 1 day's work

$$= 16 \times \frac{1}{16 \times 24} + 16 \times \frac{1}{32 \times 24} = \frac{1}{24} + \frac{1}{48}$$
$$= \frac{1}{16}$$

$$\therefore$$
 12 day's work = $12 \times \frac{1}{16} = \frac{3}{4}$

$$\therefore$$
 Remaining work = $1 - \frac{3}{4} = \frac{1}{4}$

Let x more men be required for 2 days work to complete

$$\therefore (16+x) \times \frac{2}{16 \times 24} + 16 \times \frac{2}{32 \times 24} = \frac{1}{4}$$

i.e.
$$2(16 + x) + 16 = \frac{16 \times 24}{4}$$
$$= 16 \times 6$$

$$\therefore 2x + 32 + 16 = 16 \times 6$$

$$2x = 16 \times 6 - 32 - 16$$
= 48

$$\therefore \qquad x = \frac{48}{2} = 24$$

 \therefore Required number of men for 2 day's work = 24

13. : 1 woman's work/day =
$$\frac{1}{10 \times 7} = \frac{1}{70}$$

1 child's work/day =
$$\frac{1}{10 \times 14} = \frac{1}{140}$$

∴ 5 women's + 10 children's work/day =
$$5 \times \frac{1}{70} + 10 \times \frac{1}{140}$$

= $\frac{5}{70} + \frac{5}{70}$
= $\frac{10}{70} = \frac{1}{7}$

.. 7 days will be required to complete the work.

14. Let part of work/day by men be x and that by women be y.

$$\therefore \qquad 4x + 6y = \frac{1}{8} \tag{1}$$

$$3x + 7y = \frac{1}{10} \tag{2}$$

Solving (1) and (2), we get

$$x = \frac{11}{400}$$
; $y = \frac{1}{400}$;

.. Part of work done by 10 women/day is given by

$$10 \times \frac{1}{400} = \frac{1}{40}$$

.. The work can be completed in 40 days.

$$\therefore 12x + 16y = \frac{1}{5} \tag{1}$$

$$13x + 24y = \frac{1}{4} \tag{2}$$

Solving (1) and (2), we get

$$x = \frac{1}{100}; \quad y = \frac{1}{200};$$

$$x: y = \frac{1}{100}: \frac{1}{200} \text{ or } 2:1$$

16. Let part of work done/day by a man be x.

 \therefore Part of work done/day by a woman = 2x;

Part of work done/day by a child = $\frac{x}{2}$;

$$\therefore \qquad 3x + 4(2x) + 6\left(\frac{x}{2}\right) = \frac{1}{7}$$

$$\Rightarrow$$
 14x = $\frac{1}{7}$ or $x = \frac{1}{14 \times 7} = \frac{1}{98}$

$$\therefore \qquad 2x = 2 \times \frac{1}{98} = \frac{1}{49}$$

$$\therefore$$
 Work done by a woman for 7 days = $7 \times \frac{1}{49} = \frac{1}{7}$

:. Number of women required to complete the work = 7

17. Part of work done by A/day = $\frac{1}{20}$

Part of work done by B/day = $\frac{1}{30}$

Part of work done by $C/day = \frac{1}{60}$

.. Part of work done by A, B and C for first 3 days

$$=\left(\frac{3}{20}+\frac{1}{30}+\frac{1}{60}\right)=\frac{9}{60}+\frac{2}{60}+\frac{1}{60}=\frac{12}{60}=\frac{1}{5}$$

 \therefore Number of days required to complete the work = 5×3 days = 15 days

18. Part of work done/day by a woman = $\frac{1}{20 \times 16}$;

Part of work done/day by a man = $\frac{1}{16 \times 15}$;

Ratio of capacities from man to woman

$$= \frac{1}{16 \times 15} \div \frac{1}{20 \times 16} = \frac{1}{16 \times 15} \times 20 \times 16$$
$$= 4:3$$

19. 1 day's work by a man = $\frac{1}{12 \times 4}$

1 day's work by a woman = $\frac{1}{15 \times 4}$

Part of work done by 6 men for 2 days = $\left(6 \times \frac{1}{12 \times 4}\right) \times 2 = \frac{1}{4}$

 \therefore Remaining work = $1 - \frac{1}{4} = \frac{3}{4}$

Let number of women required for 3 days be x.

$$x \times \frac{1}{15 \times 4} \times 3 = \frac{3}{4}$$
or
$$x = 15$$

20. $10x + 15y = \frac{1}{6}$ (1)

1 day's 1 man's work =
$$x = \frac{1}{100}$$
 (2)

$$y = \left(\frac{1}{6} - 10 \times \frac{1}{100}\right) \frac{1}{15} = \left(\frac{1}{6} - \frac{1}{10}\right) \frac{1}{15}$$
$$= \left(\frac{5 - 3}{30}\right) \times \frac{1}{15} = \frac{1}{225}$$

- :. 225 days are required for a woman to complete the work.
- 21. Let x, y and z be part of work done/day by a man, woman and a boy respectively.

$$\therefore x = \frac{1}{3}; \ y = \frac{1}{4}; \ z = \frac{1}{12}$$

Let N be the number of boys to assist 1 man and 1 woman for $\frac{1}{4}$ of a day.

$$\therefore \left(1 \times \frac{1}{3} + 1 \times \frac{1}{4} + N \times \frac{1}{12}\right) \frac{1}{4} = 1$$

i.e.
$$\frac{1}{12} + \frac{1}{16} + \frac{N}{48} = 1$$

$$N = \left(1 - \frac{1}{12} - \frac{1}{16}\right) \times 48 = \frac{48 - 4 - 3}{48} \times 48 = 41$$

22. Part of work done by a man/day = $\frac{1}{12 \times 9}$

Let the number of days taken to complete the remaining work be x days.

Part of work done during first 6 days = $6 \times \left(12 \times \frac{1}{12 \times 9}\right)$

$$=\frac{6}{9}=\frac{2}{3}$$

 \therefore Remaining work = $1 - \frac{2}{3} = \frac{1}{3}$

Number of men working after 6 days = 12 + 6 = 18

$$\therefore 18 \times \frac{1}{12 \times 9} \times x = \frac{1}{3}$$

$$x = \frac{12 \times 9}{18 \times 3} = 2$$

$$\therefore$$
 Required number of days = 2

23. Part of work done by man/day =
$$\frac{1}{10 \times 15}$$

Part of work done by woman/day =
$$\frac{1}{15 \times 12}$$

Let number of days required be x

$$\therefore \qquad \left(10 \times \frac{1}{10 \times 15} + 15 \times \frac{1}{15 \times 12}\right) x = 1$$

$$\mathbf{or}$$

$$x = \frac{1}{\frac{1}{15} + \frac{1}{12}}$$

$$=\frac{15\times12}{15+12}=\frac{20}{3}=6\frac{2}{3}$$
 days

24. Part of work done by A per day =
$$\frac{1}{16}$$

Part of work done by B per day =
$$\frac{1}{12}$$

$$\therefore$$
 For every two days, work done = $\frac{1}{16} + \frac{1}{12}$

$$=\frac{3+4}{48}=\frac{7}{48}$$

Let total number of days required to complete the work be x days.

$$\therefore \frac{7}{48} \times \frac{x}{2} = 1$$

$$x = \frac{48 \times 2}{7}$$

$$=\frac{96}{7}=13\frac{5}{7}$$
 days

25. Part of work by A + B =
$$\frac{19}{23}$$

$$\therefore$$
 Part of work by C alone = $1 - \frac{19}{23} = \frac{4}{23}$

Part of work by A alone =
$$1 - \frac{8}{23} = \frac{15}{23}$$

:. Part of work by B alone =
$$1 - \left(\frac{4}{23} + \frac{15}{23}\right) = \frac{4}{23}$$

∴ Amount should be paid to
$$A = 529 \times \frac{15}{23}$$

= $23 \times 15 = ₹345$

26. Part of work done by A and B/day =
$$\frac{1}{12}$$

Part of work done by A alone/day = $\frac{1}{20}$

:. Part of work done by B alone/day =
$$\frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{1}{30}$$

.. Part of work done by A for full day and B for half day
$$=\frac{1}{20} + \frac{1}{60} = \frac{3+1}{60} = \frac{1}{15}$$

Let the total number of days required to complete the work be x days.

$$\therefore \frac{1}{15} \times x = 1$$

$$\therefore x = 15 \text{ days}$$

27. Part of work done/day by
$$A = \frac{1}{14}$$

Part of work done/day by B = $\frac{1}{21}$

Let the total number of days required to complete the work be x days.

$$\frac{1}{14} + \frac{1}{21}(x - 3) + \frac{1}{21} \times 3 = 1$$
i.e.
$$\frac{3+2}{42}(x - 3) + \frac{1}{7} = 1$$
or
$$(x - 3) = \frac{6}{7} \times \frac{42}{5}$$

$$= 7.2$$

$$x = 7.2 + 3$$

$$= 10.2$$

$$= 10\frac{1}{5} \text{ days}$$

28. Part of work/day by
$$A = \frac{1}{6}$$

Part of work/day by $B = \frac{1}{8}$

Time taken by A and B to complete the work = $\frac{8 \times 6}{8+6} = \frac{48}{14} = \frac{24}{7}$

Time taken by A, B and C = 3 days

∴ Time taken by C alone =
$$\frac{24/7 \times 3}{24/7 - 3} = \frac{72/7}{(24 - 21)/7}$$

= $\frac{72}{7} \times \frac{7}{3}$
= 24 days

:. Part of work by C/day =
$$\frac{1}{24}$$

 $\frac{1}{6}: \frac{1}{8}: \frac{1}{24} = 4:3:1$

∴ Amount to be paid to
$$C = 3200 \times \frac{1}{4+3+1}$$
$$= \frac{3200}{8}$$
$$= ₹400$$

29. Part of work done/day by A =
$$\frac{1}{45}$$

Part of work done/day by
$$B = \frac{1}{40}$$

Let the number of days after which A left be x.

$$\therefore \qquad \left(\frac{1}{45} + \frac{1}{40}\right)x + \frac{1}{40} \times 23 = 1$$
i.e.
$$\left(\frac{8+9}{360}\right)x = 1 - \frac{23}{40}$$

$$= \frac{17}{40}$$

$$\therefore \qquad x = \frac{17}{40} \times \frac{360}{17} = 9 \text{ days}$$

30. Share of Kim =
$$\frac{2}{2+3}$$
 × 150 = ₹ **60**

31. Let the number of days required for B to finish the whole work be
$$x$$
.

$$(A + B) \rightarrow finishes is 30 days.$$

A for 16 days and B for 44 days \rightarrow total 60 days.

.. B finish the whole work in 60 days.

32. Part of work by A alone/day =
$$\frac{1}{24}$$

Part of work by B alone/day =
$$\frac{1}{9}$$

Part of work by C alone/day =
$$\frac{1}{12}$$

Let the number of days taken by A to complete the remaining work be x.

$$\therefore \qquad \left(\frac{1}{9} + \frac{1}{12}\right) 3 + \frac{1}{24} \times x = 1$$

$$x = \left[1 - \left(\frac{3}{9} + \frac{3}{12}\right)\right] \times 24 = \left[1 - \left(\frac{1}{3} + \frac{1}{4}\right)\right] \times 24$$

$$\therefore \qquad = \left[1 - \left(\frac{4+3}{12}\right)\right] \times 24 = \frac{12-7}{12} \times 24 = \mathbf{10 \ days}$$

33. Part of work completed in 20 days by $A = \frac{4}{5}$

Remaining work =
$$1 - \frac{4}{5} = \frac{1}{5}$$

Let x be the part of work/day by B

:. Time taken by B alone to complete the work

$$=\frac{1}{2/75}=\frac{75}{2}=37\frac{1}{2}$$
 days

34. Part of work done/day by A + B = $\frac{1}{30}$

Let part of work done/day by A alone be x.

$$\therefore \frac{1}{30} \times 20 + x \times 20 = 1$$

$$\therefore x = \left(1 - \frac{20}{30}\right) \frac{1}{20} = \frac{1}{3} \times \frac{1}{20} = \frac{1}{60}$$

 \therefore A alone can finish the work in $\frac{1}{1/60} = 60 \text{ days}$

35. Part of work done/day by $A = \frac{1}{18}$

Part of work done/day by $B = \frac{1}{15}$

Let the number of days taken by A to finish remaining work be x.

$$\therefore \frac{1}{15} \times 10 + \frac{1}{18} \times x = 1$$

$$\therefore x = \left(1 - \frac{10}{15}\right) 18 = \frac{1}{3} \times 18 = 6 \text{ days}$$

Part of work done/hr by machine $Q = \frac{1}{10}$

Part of work done/hr by machine $R = \frac{1}{12}$

$$\therefore \left(\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right) 2 + \left(\frac{1}{10} + \frac{1}{12}\right) x = 1$$

$$x = \left\{1 - \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right)\right\} \div \left(\frac{1}{10} + \frac{1}{12}\right) = \left\{1 - \left(\frac{15 + 12 + 10}{60}\right)\right\} \div \left(\frac{6 + 5}{60}\right)$$
$$= \frac{23}{60} \times \frac{60}{11} = \frac{23}{11} = 2\frac{1}{11} \text{ hours}$$

 $\therefore 11 + 2\frac{1}{11} = approximately 1 p.m.$

37.
$$\left(\frac{1}{20} + \frac{1}{12}\right) 4 + \frac{1}{12} \times x = 1$$

$$\therefore \qquad x = \left\{1 - \left(\frac{1}{5} + \frac{1}{3}\right)\right\} \times 12 = \left\{1 - \left(\frac{3+5}{15}\right)\right\} \times 12$$
$$= \left(\frac{15-8}{15}\right) \times 12 = \frac{7\times4}{5} = 5.6 \approx 6 \text{ days}$$

38.
$$(A + B) = \frac{1}{30}$$
; $(B + C) = \frac{1}{24}$; $(C + A) = \frac{1}{20}$;

$$(A + B + C) = \frac{1}{2} \left(\frac{1}{30} + \frac{1}{24} + \frac{1}{20} \right)$$

$$= \frac{1}{2} \left(\frac{4 + 5 + 6}{120} \right) = \frac{15}{240} = \frac{1}{16}$$

Let x days be required by A to finish the remaining work

$$\therefore \frac{1}{16} \times 10 + \left(\frac{1}{16} - \frac{1}{24}\right) \times x = 1$$

$$x = \left(1 - \frac{10}{16}\right) \div \left(\frac{1}{16} - \frac{1}{24}\right)$$
$$= \frac{6}{16} \times \frac{1}{\frac{3-2}{48}} = \frac{6}{16} \times \frac{48}{1} = 18 \text{ days}$$

39.
$$\left(\frac{1}{15} + \frac{1}{10}\right) \times 2 + \frac{1}{15} \times x = 1$$

$$\therefore x = \left\{1 - 2\left(\frac{2}{30} + \frac{3}{30}\right)\right\} \times 15 = \frac{2}{3} \times 15 = 10 \text{ days}$$

40. A + B
$$\rightarrow \frac{1}{7}$$

Let part of work by B/day be x

$$\therefore$$
 Part of work by A/day = $\frac{7}{4}x$

$$\therefore \qquad x + \frac{7}{4}x = \frac{1}{7}$$

or
$$x = \frac{1}{7} \div \left(1 + \frac{7}{4}\right)$$
$$= \frac{1}{7} \times \frac{4}{11} = \frac{4}{77}$$

$$\therefore$$
 Part of work by A/day = $\frac{7}{4} \times \frac{4}{77} = \frac{1}{11}$

A alone can complete the work in $\frac{1}{1/11} = 11 \text{ days}$.

41. A + B
$$\rightarrow \frac{1}{10}$$

or

$$C \rightarrow \frac{1}{50}$$

$$A = B + C = ?$$

Let x be the number of days required for the work by B alone.

$$\therefore$$
 Part of work by B alone = $\frac{1}{x}$

$$A \to \frac{1}{10} - \frac{1}{x} = \frac{1}{x} + \frac{1}{50} \to B + C$$

$$\frac{1}{x} + \frac{1}{x} = \frac{1}{10} - \frac{1}{50}$$

$$\frac{2}{x} = \frac{5-1}{50}$$

$$\frac{2}{x} = \frac{4}{50}$$

$$x = \frac{2 \times 50}{4} = 25 \text{ days}$$

42. Let number of days taken by B to complete a work be
$$x$$
.

$$\therefore$$
 Number of days taken by A for completing the same work = $\frac{x}{3} = x - 60$

$$\therefore \frac{2}{3}x = 60$$

or
$$x = \frac{60 \times 3}{2} = 90 \text{ days}$$

Time taken by B = 90 days

Time taken by A = 30 days.

:. Time taken by A and B together to complete the work

$$= \frac{30 \times 90}{30 + 90} = \frac{30 \times 90}{120} = 22\frac{1}{2} \, \mathbf{d} \, \mathbf{ays}$$

43. Fraction of work left = $1 - \left(\frac{1}{15} + \frac{1}{20}\right)4$

$$=1-\frac{(4+3)}{60}\times 4=1-\frac{7}{15}=\frac{8}{15}$$

- **44.** B completes x part of a work in one day.
 - \therefore B completes the same work in $\frac{1}{x}$ days
 - \therefore A completes 1.3x part of work in a day
 - \therefore A completes the work in $\frac{1}{1.3x}$ days

But

$$\frac{1}{1.3x} = 23$$

or

$$x = \frac{1}{23 \times 1.3} \tag{1}$$

Time taken by A and B together $= \frac{\frac{1}{x} \times \frac{1}{1.3x}}{\frac{1}{x} + \frac{1}{1.3x}}$ $= \frac{1}{1.3x^2} = \frac{1}{1.3x^2}$

$$=\frac{\frac{1}{1.3x^2}}{\frac{2.3x}{1.3x^2}} = \frac{1}{2.3x}$$

- \therefore From (1), required time = 13 days.
- **45.** Part of work done/day by Sakshi = $\frac{1}{20}$

Part of work done/day by Tanya = $\frac{5}{4} \times \frac{1}{20} = \frac{1}{16}$

- \therefore Number of days taken by Tanya to complete the work = 16 days.
- **46.** Part of work/day by A + B = $\frac{1}{8}$

Part of work/day by B + C = $\frac{1}{12}$

Part of work/day by $A + B + C = \frac{1}{6}$

Part of work/day by A + C = Part of work done by (A + B + C) — Part of work done by B alone

$$= \frac{1}{6} - \left\{ \frac{1}{6} - \left[2 \times (A + B + C) - (A + B) - (B + C) \right] \right\}$$

$$= \frac{1}{6} - \left\{ \frac{1}{6} - \left(2 \times \frac{1}{6} - \frac{1}{8} - \frac{1}{12} \right) \right\} = \frac{1}{3} - \frac{1}{8} - \frac{1}{12}$$

$$= \frac{8 - 3 - 2}{24} = \frac{3}{24} = \frac{1}{8}$$

A and C together will complete the job in $\frac{1}{1/8} = 8$ days

47. Quantity of work by P/hr = $\frac{1}{12 \times 8} = \frac{1}{96}$;

Quantity of work by Q/hr = $\frac{1}{10 \times 8} = \frac{1}{80}$;

Let the required number of days to complete the work be x. Then

$$\left(\frac{1}{96} + \frac{1}{80}\right) 8 \times x = 1$$

$$x = \frac{1}{8 \times \left(\frac{1}{96} + \frac{1}{80}\right)}$$

$$= \frac{1}{8 \times \left(\frac{5+6}{480}\right)} = \frac{480}{8 \times 11} = \frac{60}{11} = 5\frac{5}{11} \, days$$

48. Let the time required to complete the work by B alone be $\frac{x}{2}$

Time required by A = x

٠:.

Time required by $C = \frac{x}{3}$

.. Time taken by A, B and C when they work together = $\frac{1}{\frac{1}{r} + \frac{2}{r} + \frac{3}{r}} = 2$

or
$$\frac{x}{6} = 2$$
 or
$$x = 12 \text{ days}$$

 \therefore B takes x/2 = 6 days to complete the work.

49. $(A + B) \rightarrow part of work done/day \rightarrow \frac{1}{12}$

$$(B + C) \rightarrow part of work done/day \rightarrow \frac{1}{15}$$

$$(C + A) \rightarrow part of work done/day \rightarrow \frac{1}{20}$$

$$(A+B+C) \rightarrow part~of~work~done/day \rightarrow \frac{1}{2} \bigg(\frac{1}{12} + \frac{1}{15} + \frac{1}{20}\bigg)$$

$$=\frac{1}{2}\left(\frac{5+4+3}{60}\right)=\frac{1}{10}$$

 \therefore A + B + C will complete the work in 10 days.

50. Part of work by A/day =
$$\frac{1}{16}$$

Part of work by B/day =
$$\frac{1}{12}$$

Part of work by
$$A + B + C/day = \frac{1}{4}$$

∴ Part of work by C alone/day
$$= \frac{1}{4} - \left(\frac{1}{16} + \frac{1}{12}\right)$$

 $= \frac{1}{4} - \left(\frac{3+4}{48}\right) = \frac{12-7}{48} = \frac{5}{48}$

$$\therefore$$
 C alone can do the job in $\frac{48}{5} = 9\frac{3}{5}$ days

51. Let the number of days required by son to do the work alone be x.

$$\frac{1}{5} + \frac{1}{x} = \frac{1}{3}$$
or
$$x = \frac{1}{\frac{1}{3} - \frac{1}{5}} = \frac{15}{5 - 3} = 7\frac{1}{2} \, days$$

52. Work done/day by
$$A = \frac{1}{18}$$

Work done/day by
$$B = \frac{1}{9}$$

When they work together, part of work done/day = $\frac{1}{18} + \frac{1}{9} = \frac{1}{6}$

53. Required time to complete the work =
$$\frac{abc}{ab+bc+ca}$$

$$= \frac{24 \times 6 \times 12}{24 \times 6 + 6 \times 12 + 12 \times 24}$$
$$= \frac{24}{2 + 1 + 4} = \frac{24}{7} = 3\frac{3}{7} \text{ days}$$

54. Required number of days =
$$\frac{10 \times 15}{10 + 15}$$

$$=\frac{150}{25}=6 \text{ days}$$

55. Required time
$$=\frac{9\times6}{9+6} = \frac{54}{15} = \frac{18}{5} = 3.6 \text{ minutes}$$

$$=3\frac{3}{5}$$
 minutes

Work and Wages

This chapter deals with the calculation of man-hours for a certain work and calculation of total wages paid for a work according to the principle "Total man-days for a certain work remains constant".

For example, if X number of persons complete a certain job in x days and Y number of persons complete the same job in y days, then

$$\mathbf{X} \cdot \mathbf{x} = \mathbf{Y} \cdot \mathbf{y}$$

EXAMPLE 1 A can do certain piece of work in d_1 days and B in d_2 days. Both of them do the work together. If the total amount paid for the work is \mathbb{Z} M, how much would each get?

Solution A's daily work =
$$\frac{1}{d_1}$$

B's daily work = $\frac{1}{d_2}$

A's share: B's share = $\frac{1}{d_1}$: $\frac{1}{d_2} = d_2$: d_1
 \therefore A's share = $₹\left(\frac{d_2}{d_2 + d_1}\right)M$

B's share = $₹\left(\frac{d_1}{d_2 + d_1}\right)M$

EXAMPLE 2 The wages of 20 boys for 15 days is ₹ 9000. If the daily wage of a man is one and half times that of a boy, then how many men must work for 30 days to earn ₹ 13500?

Solution Total wage of a group = Number of persons in the group \times One person's daily wage \times Number of days.

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Number of persons in the group × Number of days

∴ One boy's daily wage = $\frac{9000}{20 \times 15}$ = ₹ 30

One man's daily wage = $1.5 \times 30 = ₹45$

∴ Number of men in the group = $\frac{\text{Total wages of group of men}}{\text{One man's daily wage} \times \text{Number of days}}$ = $\frac{13500}{45 \times 30} = 10$

? EXERCISES

- 1. A person can do a work in 20 days. With the help of a boy he can do the same work in 12 days. If they get ₹ 500 for that work, what is the share of that boy?
 - (a) ₹ 100
- (b) ₹ 200
- (c) ₹ 250
- (d) ₹ 300
- (e) None of these
- 2. Two person's agreed to do a work for ₹ 2000. One alone could do it in 6 days the other in 8 days. With the assistance of a boy they finish it in 3 days. How should the money be divided?
 - (a) 1:2:3
- (b) 2:3:1
- (c) 1:3:4
- (d) 4:3:1
- (e) None of these
- 3. The wages for 12 women amount to ₹ 30240 in 24 days. How many men must work 16 days to receive ₹ 10080, the daily wages of a man being equal to that of a woman?
 - (a) 4
- (b) 6
- (c) 8
- (d) 10
- (e) None of these
- 4. If 3 men and 4 boys can earn ₹ 4200 in 7 days, and 11 men and 13 boys can earn ₹ 16600 in 8 days, in what time will 7 men with 9 boys earn ₹ 19250?
 - (a) 10 days
- (b) 12 days
- (c) 14 days
- (d) 16 days
- (e) None of these
- 5. Two persons P and Q together complete a piece of work which it would have taken them respectively 12 and 18 days to complete if they worked separately. They received a payment of ₹ 1200. Find their shares.
 - (a) 500, 700
- (b) 700, 500
- (c) 480, 720
- (d) 720, 480
- (e) None of these



- 1. (b
- 2. (d)
- **3.** (b)
- **4.** (c)
- **5.** (d)



${f S}$ olutions with Necessary Explanation

- 1. The boy can do the work in $\frac{12 \times 20}{20 12} = 30$ days
 - \therefore Man's share : boy's share = 30 : 20 = 3 : 2
 - ∴ Boy's share = $\frac{2}{3+2} \times 500 = ₹ 200$

2. 1st man's 3 day's work =
$$\frac{1}{6} \times 3 = \frac{3}{6} = \frac{1}{2}$$

2nd man's 3 day's work =
$$\frac{1}{8} \times 3 = \frac{3}{8}$$

Boy's 3 day's work =
$$1 - \left(\frac{3}{6} + \frac{3}{8}\right) = 1 - \frac{3 \times 4 + 3 \times 3}{24} = 1 - \frac{21}{24} = \frac{3}{24} = \frac{1}{8}$$

... Their shares will be in the ratio
$$\frac{3}{6}:\frac{3}{8}:\frac{1}{8}=3\times4:3\times3=4:3:1$$

∴ Share for 1st man =
$$\frac{4}{4+3+1} \times 2000 = ₹ 1000$$

Share for 2nd man =
$$\frac{3}{4+3+1}$$
 × 2000 = ₹ 750

Boy's share =
$$\frac{1}{4+3+1} \times 2000 =$$
₹ 250

3. Wages of a woman/day =
$$\frac{30240}{12 \times 24}$$
 = ₹ 105

Wages of a man/day = ₹ 105

$$\therefore \qquad \text{Number of men} = \frac{\text{Total wage}}{\text{Number of days} \times \text{Wage of man/day}} = \frac{10080}{16 \times 105} = 6$$

4. 3 men+ 4 boy in 1 day earn
$$\frac{4200}{7} = 600$$
 (1)

11 men + 13 boy in 1 day earn ₹
$$\frac{16600}{8}$$
 = ₹ 2075 (2)

Equations (1) and (2)
$$\Rightarrow \frac{3 \text{ men } + 4 \text{ boy}}{600} = \frac{11 \text{ men } + 13 \text{ boy}}{2075}$$

 $\Rightarrow \frac{3 \text{ men } + 4 \text{ boy}}{2000} = \frac{11 \text{ men } + 13 \text{ boy}}{2000}$

$$\Rightarrow \frac{3 \text{ men} + 4 \text{ boy}}{24} = \frac{11 \text{ men} + 13 \text{ boy}}{83}$$

or men
$$(3 \times 83 - 11 \times 24) = \text{boy } (13 \times 24 - 4 \times 83)$$

$$\frac{\text{men}}{\text{boy}} = \frac{-20}{-15} = \frac{4}{3}$$

$$\therefore (1) \qquad \Rightarrow 3 \text{ men} + 4 \text{ boy} = 3 \text{ men} + 4 \times \frac{3}{4} \text{ men}$$

∴ 7 men + 9 boys earns in 1 day =
$$\left[7 + 9\left(\frac{3}{4}\right)\right] \times 100 = ₹ 1375$$

∴ 7 men + 9 boys earn ₹ 19250 in
$$\frac{19250}{1375}$$
 = 14 days.

5. Wages are distributed in inverse proportion of a number of days. Hence money will be divided between P and Q in the ratio 18:12.

∴ P gets
$$\frac{18}{18+12} \times 1200 = ₹720$$
 and Q gets $\frac{12}{18+12} \times 1200 = ₹480$

Pipes and Cistern

In this chapter, all problems are related to the pipe flow, calculation of filling and emptying time for a water tank or flush tank, etc.

Important Formulae

1. If a pipe can fill a tank in x hours, and another pipe can fill the same tank in y hours, when both pipes are open, time taken to completely fill the tank

$$=\frac{xy}{x+y}$$

2. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, the net part filled in 1 hour, when both pipes are open simultaneously is given by

$$\left(\frac{1}{x} - \frac{1}{y}\right) = \frac{y - x}{xy}$$

3. When both the pipes are open simultaneously, time taken to fill the tank completely

$$\frac{xy}{y-x}$$
 hours

4. A pipe can fill a tank in x hours. But due to leakage in its bottom, the tank in now filled in y hours. Then the time taken by the leakage to empty the full tank

$$\frac{xy}{y-x}$$
 hours

5. A pipe can fill a tank in x hours and another one fills it in y hours. A third pipe can empty the full tank in z hours. When all the three pipes are open simultaneously the net part of the tank filled in 1 hour

$$=\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \frac{yz + zx - xy}{xyz}$$

6. When all the three pipes are open, time taken to fill the tank

$$= \frac{xyz}{yz + zx - xy} \text{ hours}$$

EXAMPLE 1 A pipe can fill a tank in 20 hours. Due to a leak in the bottom of the tank, it is filled in 25 hours. If the tank is full, how much time will the leak take to empty it?

Solution Time taken =
$$\frac{xy}{y-x} = \frac{25 \times 20}{25-20} = 100 \text{ hours}$$

EXAMPLE 2 8 taps are fitted to a water tank. Some of them are water taps to fill the tank and the remaining are outlet taps used to empty the tank. Each water tap can fill the tank in 12 hours and each outlet tap can empty it in 36 hours. On opening all the taps, tank is filled in 3 hours. Find the number of water taps.

Solution Let the number of inlet water taps be x.

 \therefore Number of outlet taps = 8 - x

The tank can be filled by each tap in 12 hours.

 \therefore Part filled by x taps in 1 hour = x/12

The tank can be emptied by each outlet tap in 36 hours.

$$\therefore$$
 Part emptied by $(8-x)$ outlet taps in 1 hour = $\frac{8-x}{36}$

.. Net part filled in 1 hour =
$$\frac{x}{12} - \frac{8-x}{36} = \frac{3x - (8-x)}{36} = \frac{4x - 8}{36} = \frac{x - 2}{9}$$

 \therefore Tank will be filled in $\frac{9}{x-2}$ hours on opening all the taps together

$$\frac{9}{x-2} = 3$$
or
$$3(x-2) = 9 \Rightarrow x = 5$$

 \therefore Number of water taps = 5

EXAMPLE 3 Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the same tank in 40 hours. If all the pipes are opened together, what will be the time needed to make the tank fill?

Solution Net part filled in 1 hour =
$$\left(\frac{1}{20} + \frac{1}{30} - \frac{1}{40}\right) = \frac{7}{120}$$

... The tank will be filled in $\frac{120}{7}$ hours i.e. $17\frac{1}{7}$ hours.



Short cut: Time required to completely fill the tank is $\frac{xyz}{yz + zx - xy}$ hours

$$= \frac{20 \times 30 \times 40}{30 \times 40 + 40 \times 20 - 20 \times 30} = \frac{120}{7} = 17\frac{1}{7} \text{ hours}$$

EXAMPLE 4 Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes.

Solution A can fill 1/24 part of tank in a minute.

B can fill 1/32 part of tank in a minute.

Let after x minutes B be closed.

$$\frac{1}{24} + \frac{1}{32} x + \frac{1}{24} (18 - x) = 1$$

x = 8 minutes



Short cut: Pipe B should be closed after $\left(1 - \frac{18}{24}\right) \times 32 = 8$ minutes

EXAMPLE 5 Three pipes A, B, and C can fill a cistern in 6 hours. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hours. Find the time in which the cistern can be filled by pipe C.

Solution Part of the cistern that was filled by A, B, and C together in 1 hour = 1/6.

 \therefore For the first 2 hours, filled portion = $2 \times \frac{1}{6} = \frac{1}{3}$

Part of cistern that was filled by A and B together in 1 hour = 1/8

$$\therefore \left(1 - \frac{1}{3}\right) = \frac{2}{3}$$
 part should be filled in 8 hours

 \therefore A + B can fill the cistern in $\frac{8 \times 3}{2}$ = 12 hours

 \therefore Time in which cistern can be filled by pipe C alone = $\frac{12 \times 6}{12 - 6} = 12$ hours.



EXERCISES

- 1. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?
 - (a) 4 hours
- (b) 6 hours
- (c) 3 hours
- (d) 2 hours
- (e) None of these
- 2. If two pipes function simultaneously, the reservoir will be filled in 6 hours. One pipe fills the reservoir 5 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
 - (a) 8 hours
- (b) 10 hours
- (c) 12 hours
- (d) 9 hours
- (e) None of these
- 3. A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
 - (a) 5000 litres
- (b) 4680 litres
- (c) 6840 litres
- (d) 8640 litres
- (e) None of these
- 4. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is filled in 20 minutes. How long will the waste pipe take to empty a full cistern?
 - (a) 5 minutes
- (b) 10 minutes (c) 15 minutes
- (d) 20 minutes
- (e) None of these

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will the tank be completely filled?

	(a) 18 minutes	(b) 16 minutes	(c)	11 minutes	(d)	8 minutes	(e) None of these
6.							inutes respectively.
					$\mathbf{em}_{\mathbf{j}}$	pty state if B	is used for half the
	time and A and B				(1)	20 : 4	[DMRC]
_		(b) 20 minutes					1
7.							nd a waste pipe can n fill the tank in 15
	minutes. The capa						[Bank PO]
	(a) 60 gallons	(b) 100 gallons	(c)	120 gallons	(d)	180 gallons	
8.						_	vely. Both the pipes e total time required [UPSC]
	(a) 10 min 20 sec	(b) 11 min 45 se	c (c)	12 min 30 sec	(d)	14 min 40 se	ec
9.		d B can fill the re					gether for 2 hours, C er of hours taken by [LIC AAO]
	(a) 10	(b) 12	(c)	14	(d)	16	
10.	A tank is filled by the	hree pipes with u	nifor	m flow. The fir	st tw	o pipes opera	ting simultaneously
			-				nird pipe alone. The
	pipe. The time req						lower than the third [MBA]
	(a) 6 hours	(b) 10 hours	(c)	15 hours	(d)	30 hours	
11.	One pipe can fill a the tank in 36 min						he two pipes can fill ne tank in: [CBI]
	(a) 81 minutes	(b) 108 minutes	(c)	144 minutes	(d)	192 minutes	3
12.	minutes respective discharge chemical the liquid in the ta	ely. When the tar l solutions P, Q ar ink after 3 minut	ık is ıd R es?	empty, all the respectively. V	e thr Vhat	ree pipes are t is the propor	, 20 minutes and 10 opened. A, B and C rtion of solution R in [DMRC]
	(a) $\frac{5}{11}$	(b) $\frac{6}{11}$	(c)	$\frac{7}{11}$	(d)	$\frac{8}{11}$	
13.	Two pipes A and I	B can fill a cister	n in	$37\frac{1}{2}$ minute	s an	d 45 minute	s respectively. Both
				_			B is turned off after: [SSC]
	(a) 5 minutes	(b) 9 minutes	(c)	10 minutes	(d)	15 minutes	
14.	Three taps A, B an the time, and B an						ly. If A is opened all nk will be full in: [SSC]
	(a) 6 hours	(b) $6\frac{2}{3}$ hours	(c)	$5\frac{1}{2}$ hours	(d)	7 hours	
15 .		3		2			inutes respectively.
_ • •	There is a third p	ipe in the botton ened, then the ci	n of	the cistern to	em	pty it. If all	the three pipes are nuch time the third [SSC]
	(a) 90 minutes	(b) 100 minutes	(c)	110 minutes	(d)	120 minutes	1

5. A tank can be filled by one tap in 20 minutes and by another in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. In how many minutes more

16.	A tank is filled in 5 hours by three pipes A, B and C. Pipe C is twice as fast as B, and B is twice as fast as A. How much time will pipe A alone take to fill the tank? [Bank PO]
	(a) 20 hours (b) 25 hours (c) 35 hours
	(d) Cannot be determined (e) None of these
17.	Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately? [NABARD]
	(a) 1 hour (b) 2 hours (c) 6 hours (d) 8 hours
18.	A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in: [SSC]
	(a) $4\frac{1}{3}$ hours (b) 7 hours (c) 8 hours (d) 14 hours
19.	A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes, and pipe B can empty it in 6 minutes. If both the pipes are opened, how long will it take to empty or fill the tank completely? [Bank PO]
	(a) 6 minutes to empty (b) 6 minutes to fill (c) 9 minutes to empty
	(d) 9 minutes to fill (e) None of these
20.	Pipes A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in: [Bank PO]
	(a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours (c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours
	(a) $1\frac{1}{17}$ hours (b) $2\frac{1}{11}$ hours (c) $3\frac{1}{17}$ hours (d) $4\frac{1}{2}$ hours
21.	Two pipes A and B can fill a tank in 20 minutes and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank? [MAT]
	(a) 12 minutes (b) 15 minutes (c) 25 minutes (d) 50 minutes
22.	Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are opened, in how many hours will the tank be filled? [CBI]
	(a) 2 hours (b) 2.5 hours (c) 3 hours (d) 3.5 hours
23.	A cistern can be filled by a tap in 4 hours, while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the cistern get filled? [Hotel Management]
	(a) 4.5 hours (b) 5 hours (c) 6.5 hours (d) 7.2 hours
24.	Two pipes A and B can fill a cistern in 18 minutes and 9 minutes respectively, while a third pipe C can empty the full tank in 5 minutes. A and B are kept open for 4 minutes in the beginning and then C is also opened. In what time is the cistern emptied?
	(a) 15 minutes (b) 20 minutes (c) 22 minutes (d) 24 minutes
25.	10 buckets of water fill a tank when the capacity of each bucket is 12 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 8 litres?
	(a) 8 (b) 12 (c) 15 (d) 18
	WERS .
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1.	(b) 2. (b) 3. (d) 4. (b) 5. (c) 6. (d) 7. (c) 8. (d) 9. (c) 10. (c)
11.	
91	



${m y}$ ${m S}$ olutions with Necessary Explanation

- 1. Time required to fill the cistern = $\frac{2 \times 3}{3 2}$ = 6 hours.
- 2. Let the faster pipe takes x hours to fill the tank.
 - \therefore The slower pipe takes (x + 5) hours to fill the tank.
 - \therefore The two taps together fill the tank in $\frac{x(x+5)}{x+x+5} = 6$ hours
 - $\therefore x = 10 \text{ or } -3$
 - .. The faster tap fills the tank in 10 hours.
- 3. The tap can fill the tank in $\frac{12 \times 8}{12 8} = 24$ hours
 - \therefore Capacity of tank = 6 litres/min \times (60 \times 24) = 8640 litres
- 4. The two taps together can fill the cistern in $\frac{12 \times 15}{12 + 15} = \frac{20}{3}$ minutes

Including a flow through waste pipe, cistern will be filled in 20 minutes.

- $\therefore \text{ Time taken to empty the cistern} = \frac{20 \times 20/3}{20 20/3} = \frac{20 \times 20/3}{40/3} = 10 \text{ minutes}$
- 5. For the first 5 minutes, part of the tank filled by both the pipes = $\frac{5}{20} + \frac{5}{25} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$
 - \therefore Part of tank to be filled up = $1 \frac{9}{20} = \frac{11}{20}$
 - :. First tap can fill 11/20 of tank in $20 \times \frac{11}{20} = 11$ minutes
- **6.** Part of tanker filled by pipe A in 1 minute = $\frac{1}{60}$

Part of tanker filled by pipe B in 1 minute = $\frac{1}{40}$

Let the time taken to fill the tank be 2x minutes

$$\therefore \qquad \text{Part of tanker filled} = \frac{1}{40} \times x + \left(\frac{1}{60} + \frac{1}{40}\right)x = 1$$

i.e.
$$\frac{x}{40} + \frac{(2+3)}{120}x = 1$$

$$\Rightarrow \frac{x}{40} + \frac{x}{24} = 1$$

$$\Rightarrow \frac{3x + 5x}{120} = 1$$

$$\Rightarrow \qquad x = \frac{1 \times 120}{8} = 15 \text{ minutes}$$

 \therefore Time taken to fill the tank = 2x = 30 minutes

7. Part of tank filled in by first pipe/minute = $\frac{1}{20}$

Part of tank filled in by second pipe/minute = $\frac{1}{24}$

Part of tank filled in by all the two pipes and waste pipe = $\frac{1}{15}$

Let time taken to empty by waste pipe be x minute

$$\therefore \frac{1}{20} + \frac{1}{24} - \frac{1}{x} = \frac{1}{15}$$

or $\frac{1}{x} = \frac{1}{20} + \frac{1}{24} - \frac{1}{15}$

$$=\frac{6+5-8}{120}=\frac{3}{120}=\frac{1}{40}$$

- \therefore x = 40 minutes
- \therefore Capacity of tank = $40 \times 3 = 120$ gallons
- 8. Part of the tank filled in by A and B in 1 minute = $\frac{1}{15} + \frac{1}{20} = \frac{4+3}{60} = \frac{7}{60}$

In the first 4 minutes, part of tank filled = $\frac{4 \times 7}{60} = \frac{28}{60} = \frac{7}{15}$

Remaining part to be filled = $1 - \frac{7}{15} = \frac{8}{15}$

$$\therefore \frac{1}{20} \times x = \frac{8}{15}$$

or
$$x = \frac{20 \times 8}{15} = \frac{4 \times 8}{3}$$

$$= 10 \frac{2}{3} = 10 \text{ min } 40 \text{ sec.}$$

- \therefore Total time taken to fill the tank = x + 4 = 14 minutes 40 seconds
- 9. Part of tank filled during first 2 hours by A, B and C = $\frac{2}{6} = \frac{1}{3}$

Remaining part to be filled = $1 - \frac{1}{3} = \frac{2}{3}$

Part of tank filled by A and B in 7 hours = $\frac{2}{3}$

Part of tank filled by A and B in 1 hour = $\frac{2}{21}$

Part of tank filled by C alone in 1 hour

$$=\frac{1}{6}-\frac{2}{21}=\frac{7-4}{42}=\frac{3}{42}=\frac{1}{14}$$

The tank can be filled by C alone in 14 hours.

- 10. Let the second pipe fills the tank in x hours.
 - .. The first pipe fills the tank in x + 5 hours third pipe fills the tank in x - 4 hours

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{x-4}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{x-4} - \frac{1}{x+5}$$

$$\Rightarrow \frac{1}{x} = \frac{(x+5) - (x-4)}{(x-4)(x+5)}$$

$$\frac{1}{x} = \frac{9}{(x-4)(x+5)}$$
or
$$(x-4)(x+5) = 9x$$

$$\Rightarrow x^2 + x - 9x - 20 = 0$$

$$\Rightarrow x^2 - 8x - 20 = 0$$

$$\Rightarrow (x-10)(x+2) = 0$$

$$\therefore x = 10 \text{ or } -2$$

- \therefore Time required by first pipe = x + 5 = 15 hours
- 11. Let the first pipe fill a tank in x minutes and the second pipe fill a tank in 3x minutes.

$$\therefore \frac{1}{x} + \frac{1}{3x} = \frac{1}{36}$$
or
$$\frac{3+1}{3x} = \frac{1}{36}$$
or
$$x = \frac{36 \times 4}{3} = 48 \text{ minutes}$$

The slower pipe alone can fill the tank in 3x minutes = $3 \times 48 = 144$ minutes.

12. Part of tank filled in one minute by A, B and C

$$= \frac{1}{30} + \frac{1}{20} + \frac{1}{10} = \frac{2+3+6}{60} = \frac{11}{60}$$

P, Q and R are in the ratio $\frac{1}{30} : \frac{1}{20} : \frac{1}{10} = 2 : 3 : 6$

- \therefore Proportion of R in the liquid after 3 minutes = $\frac{6}{2+3+6} = \frac{6}{11}$
- 13. Let pipe B be turned off after x minutes.

$$\therefore \left(\frac{1}{37\frac{1}{2}} + \frac{1}{45}\right)x + \frac{1}{37\frac{1}{2}} \times (30 - x) = 1$$

$$\Rightarrow \left(\frac{2}{75} + \frac{1}{45}\right)x + \frac{2}{75}(30 - x) = 1$$

$$\Rightarrow \qquad \left(\frac{6+5}{225}\right)x + \frac{6}{225}(30-x) = 1$$

$$\Rightarrow \frac{11x + 180 - 6x}{225} = 1$$

$$\Rightarrow \qquad 5x + 180 = 225$$

$$\Rightarrow$$
 5x = 45

$$\Rightarrow$$
 $x = 9 \text{ minutes}$

14. When A and B are opened, part of tank filled in =
$$\left(\frac{1}{12} + \frac{1}{15}\right) = \frac{5+4}{60} = \frac{3}{20}$$

When A and C are opened, part of tank filled in = $\left(\frac{1}{12} + \frac{1}{20}\right) = \frac{5+3}{60} = \frac{2}{15}$

$$\therefore \text{ In first 2 hours, part of tank filled in} = \frac{3}{20} + \frac{2}{15} = \frac{9+8}{60} = \frac{17}{60}$$

In 6 hours, part of tank filled =
$$\frac{17}{60} \times 3 = \frac{51}{60} = \frac{17}{20}$$

Balance to be filled =
$$1 - \frac{17}{20} = \frac{3}{20}$$

It will be filled in 1 hour when A and B are opened

$$\therefore$$
 Total time taken = 6 + 1 = 7 hours

15. Let the time taken by C alone to empty the tank be x minute

Part of tank filled in 1 minute when A, B and C are opened = $\frac{1}{60} + \frac{1}{75} - \frac{1}{x} = \frac{1}{50}$

$$\frac{1}{x} = \frac{1}{60} + \frac{1}{75} - \frac{1}{50}$$
$$= \frac{5+4-6}{300} = \frac{1}{100}$$

$$\therefore$$
 $x = 100 \text{ minutes}$

16. Let time taken by A alone to fill the tank be x hours

time taken by B alone to fill the tank be $\frac{x}{2}$ hours

time taken by C alone to fill the tank be $\frac{x}{4}$ hours

$$\therefore \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/4} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$$

$$\Rightarrow \qquad \frac{7}{x} = \frac{1}{5}$$

$$\Rightarrow$$
 $x = 35 \text{ hours}$

- 17. Let the time taken by A alone to fill the cistern be x hours
 - \therefore The time taken by B alone to fill the cistern = (x + 6) hours

$$\therefore \frac{1}{x} + \frac{1}{(x+6)} = \frac{1}{4}$$

$$\Rightarrow \frac{x+x+6}{x(x+6)} = \frac{1}{4}$$

$$\Rightarrow \qquad (2x+6)4 = x(x+6)$$

$$\Rightarrow \qquad x^2 + 6x = 8x + 24$$

$$\Rightarrow \qquad x^2 - 2x - 24 = 0$$

$$(x-6)(x+4)=0$$

$$\therefore \qquad x = 6 \text{ hours}$$

18. Let the time taken to drain completely be x hours.

$$\therefore \qquad \frac{1}{2} - \frac{1}{x} = \frac{1}{2\frac{1}{3}}$$

$$\Rightarrow \qquad \frac{1}{2} - \frac{1}{r} = \frac{3}{7}$$

$$\Rightarrow \qquad \frac{1}{x} = \frac{1}{2} - \frac{3}{7}$$

$$\Rightarrow \frac{1}{r} = \frac{7-6}{14}$$

$$\Rightarrow \qquad \frac{1}{x} = \frac{1}{14}$$

$$\therefore$$
 $x = 14 \text{ hours}$

19. Part of tank filled by A in one minute = $\frac{1}{10}$

Part of tank emptied by B in one minute = $\frac{1}{6}$

Part of tank emptied by A and B in one minute $=\frac{1}{6}-\frac{1}{10}=\frac{5-3}{30}=\frac{1}{15}$

15 minutes will be taken to empty the tank, when it is full.

- $\therefore \frac{2}{5}$ th of the tank can be emptied in $\frac{2}{5} \times 15 = 6$ minutes
- 20. Pipe A can fill $\frac{1}{5}$ part of tank in 1 hour

Similarly pipe B can fill $\frac{1}{6}$ part of tank in 1 hour

C can empty $\frac{1}{12}$ part of tank in 1 hour

When A, B and C are opened, part of tank filled in 1 hour

$$= \frac{1}{5} + \frac{1}{6} - \frac{1}{12}$$
$$= \frac{12 + 10 - 5}{60} = \frac{17}{60}$$

Time taken to fill the tank = $\frac{1}{17/60} = \frac{60}{17}$ hours = $3\frac{9}{17}$ hours

21. Time taken to fill the tank when A and B are opened

$$= \frac{1}{\frac{1}{20} + \frac{1}{30}} = \frac{1}{\frac{3+2}{60}} = \frac{60}{5} = 12 \text{ minutes}$$

22. Time taken to fill the tank, when A, B and C are opened

$$= \frac{1}{\frac{1}{5} + \frac{1}{10} + \frac{1}{30}} = \frac{5 \times 10 \times 30}{5 \times 10 + 10 \times 30 + 30 \times 5}$$

$$=\frac{30}{1+6+3}=3$$
 hours

23. When both taps are opened, time taken to fill the cistern

$$= \frac{1}{\frac{1}{4} - \frac{1}{9}} = \frac{9 \times 4}{9 - 4} = \frac{36}{5} = 7.2 \text{ hours}$$

24. Part of tank filled in the first minute by A and B

$$= \left(\frac{1}{18} + \frac{1}{9}\right) = \frac{(1+2)}{18} = \frac{1}{6}$$

:. Part of tank filled in first 4 minutes by A and B = $4 \times \frac{1}{6} = \frac{2}{3}$

Part of the tank emptied in 1 minute when A, B and C are opened

$$= \frac{1}{5} - \left(\frac{1}{18} + \frac{1}{9}\right) = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$$

.. Time taken to empty the tank = 30 minutes

$$\therefore \frac{2}{3}$$
 of the tank will be emptied in $\frac{2}{3} \times 30 = 20$ minutes

25. Capacity of tank = $10 \times 12 = 120$ litres

:. Number of buckets needed to fill the tank with 8 litre capacity of bucket = $\frac{10 \times 12}{8} = 15$

i.e. Number of buckets required = 15

Alligation

Alligation is an old and practical method of solving arithmetic problem related to mixtures of ingredients.

Important Formulae

Rule of Alligation =
$$\frac{\text{Amount of Cheaper ingredient}}{\text{Amount of Dearer ingredient}} = \frac{\text{Cost price of Dearer - Mean price}}{\text{Mean price - Cost price of Cheaper}}$$

- 1. A vessel contains x litres of liquid A and y litres is withdrawn and replaced by liquid B and the operation is repeated n times in all. Find the ratio of liquid A left after nth operation to the whole quantity of liquid A initially present in the vessel.
 - Original quantity of liquid A = x litres

Quantity withdrawn first time =
$$y$$
 litres = $\frac{y}{x}$ (x litres)

- \therefore Quantity of liquid A left after 1st operation = $\left(1 \frac{y}{x}\right)$ part
 - Similarly quantity left after 2nd operation = $\left(1 \frac{y}{x}\right)^2$ part
 - Similarly quantity left after *n*th operation = $\left(1 \frac{y}{x}\right)^n$ part
- $\therefore \frac{\text{Quantity of A left after } n \text{th operation}}{\text{Quantity of A initially present}} = \frac{\left(1 \frac{y}{x}\right)^n \times x}{x} = \left(1 \frac{y}{x}\right)^n = \left(\frac{x y}{x}\right)^n$
- 2. If a group of 2 legged and 4 legged creatures gives a head count H and their total number of legs is C, then
 - Number of 4 legged animals = $\frac{C 2H}{2}$
 - Number of 2 legged animals = $\frac{4H C}{2}$

m gram of sugar solution has x% sugar in it. To increase the sugar content in the solution to y%, quantity of sugar needed to be added = $\frac{m(y-x)}{100-y}$

3. If contents of n vessels of sizes $c_1, c_2, ..., c_n$ filled with mixture of liquids A and B in the ratio $a_1:b_1, a_2:b_2, ..., a_n:b_n$ respectively are emptied into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left[\frac{a_1c_1}{a_1 + b_1} + \frac{a_2c_2}{a_2 + b_2} + \dots + \frac{a_nc_n}{(a_n + b_n)}\right]}{\left[\frac{b_1c_1}{a_1 + b_1} + \frac{b_2c_2}{a_2 + b_2} + \dots + \frac{b_nc_n}{a_n + b_n}\right]}$$

4. If the contents of n vessel of equal size filled with mixtures of liquids A and B in the ratio $a_1:b_1, a_2:b_2, \ldots a_n:b_n$, respectively are emptied into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left[\frac{a_1}{a_1 + b_1} + \frac{a_2}{a_2 + b_2} + \dots + \frac{a_n}{a_n + b_n}\right]}{\left[\frac{b_1}{a_1 + b_1} + \frac{b_2}{a_2 + b_2} + \dots + \frac{b_n}{a_n + b_n}\right]}$$

Alligation help us to

(a) find the mean or average value of mixture when two or more ingredients which may be mixed together and the proportion in which they are mixed are given (Alligation medial); and

(b) find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price (Alligation alternate).

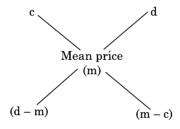
The word *alligation* literally means *linking*. The alligation method is applied to determine the percentage value, ratio, rate, price, speed, etc. and not for absolute values. i.e. whenever per cent, per hour, per kg, etc. are being compared, we can use alligation.

RULE OF ALLIGATION: If the items are mixed in a ratio, then

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean price}}{\text{Mean price} - \text{CP of Cheaper}}$$

CP of cheaper item for unit quantity (c)

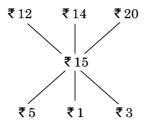
CP of unit quantity of dearer item (d).



 \therefore Cheaper quantity: dearer quantity = (d - m) : (m - c)

EXAMPLE 1 Find the proportion in which three types of sugar at $\stackrel{?}{\underset{?}{?}}$ 14 and $\stackrel{?}{\underset{?}{?}}$ 20 may be mixed so as to obtain a mixture worth $\stackrel{?}{\underset{?}{?}}$ 15 per kg.

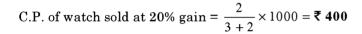
Solution Required proportion = 5:5:(3+1)=5:5:4

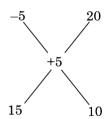


EXAMPLE 2 A person buys two watches for $\ref{1,000}$. He sells one watch at a loss of 5% and the other at a gain of 20% and on the whole he gains $\ref{50}$. Find the cost price of each watch.

Solution Overall percentage profit = $\frac{50}{1000} \times 100 = 5\%$

C.P. of watch sold at 5% less = $\frac{3}{3+2} \times 1000 =$ ₹ 600





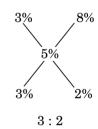
COMBINATION OF DIFFERENT RATES OF INTEREST

EXAMPLE 3 A person has ₹ 5000. He invests a part of it at 3% per annum and the remainder at 8% per annum simple interest. His total income as interest in 3 years is ₹ 750. Find the sum invested at different rates of interest.

Solution Average rate of interest =
$$\frac{750 \times 100}{5000 \times 3} = 5\%$$

Investment at 3% per annum = $\frac{3}{3+2}$ × 5000 = ₹ 3000

Investment at 8% per annum =
$$\frac{2}{3+2}$$
 × 5000 = ₹ 2000



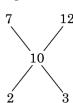
SPEED AND DISTANCE

EXAMPLE 4 A person covers a distance of 100 km in 10 hours, partly by walking at 7 km/h and the rest by running at 12 km/h. Find the distance covered in each part.

Solution Average speed =
$$\frac{100}{10} = 10 \text{ km/h}$$

Ratio of time taken at 7 km/h to 12 km/h = 2:3

$$\therefore$$
 Time taken at 7 km/h = $\frac{2}{2+3} \times 10 = 4$ hours



Distance covered at $7 \text{ km/h} = 7 \times 4 = 28 \text{ km}$ Distance covered at 12 km/h = 100 - 28 = 72 km

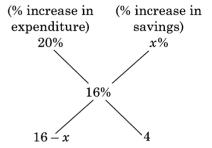
WAGES AND SALARY, EXPENDITURE AND SAVINGS

EXAMPLE 5 The expenditure and savings of an employee are in the ratio 3:1. His income increases by 16%, but at the same time his expenditure also increases by 20%. Find the increase or decrease in his savings.

Solution Expenditure and savings are two constituents of income.

Let x% be the increase in his savings.

$$\frac{16 - x}{4} = \frac{3}{1}$$
$$16 - x = 12$$
$$x = 4\%$$



DIVISION OF SUM INTO PARTS

EXAMPLE 6 ₹ 675 was divided among 75 boys and girls. Each boy gets ₹ 20 whereas a girl gets ₹ 5. Find the number of boys and girls.

Solution Average money/head =
$$\frac{675}{75} = ₹9$$

∴ Number of boys = $\frac{4}{4+11} \times 75 = 20$
Number of girls = $\frac{11}{4+11} \times 75 = 55$
(Boys) (Girls) ₹ 5

COINS

 \Rightarrow

 \Rightarrow

EXAMPLE 7 A sum of ₹ 12 is made up of 30 coins which consist of either 50 paise or 25 paise. How many are there of each kind?

Solution Average value/coin =
$$\frac{12 \times 100}{30}$$
 = 40 paise

$$\therefore \text{ Number of 50-paise coins} = \frac{3}{3+2} \times 30 = 18$$
Number of 25-paise coins = $\frac{2}{3+2} \times 30 = 12$

$$15P \quad 10P$$

$$15: 10 \text{ or } 3: 2$$

REPLACEMENT OF A PART OF MIXTURE BY ONE INGREDIENT

EXAMPLE 8 From a cask of wine containing 25 litres, 5 litres is withdrawn and the cask is refilled with water. The process is repeated a second time and then a third time.

Find the quantity of wine left in the cask and also the ratio of wine to water in the resulting mixture.

Solution Quantity of wine left after 1st withdrawal = $1 - \frac{1}{5} = \frac{4}{5}$ of the cask

Quantity of wine left after 2nd withdrawal = $\left(\frac{4}{5}\right)^2$ of the cask

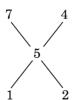
Quantity of wine left after 3rd withdrawal = $\left(\frac{4}{5}\right)^3$ of the cask = $\frac{64}{125} \times 25 = 12\frac{4}{5}$ litres

Ratio of wine to water =
$$\frac{64/125}{1-64/125}$$

= $\frac{64/125}{61/125}$ = $64/61$

EXAMPLE 9 How many kilograms of salt at 4/kg must a person mix with 25 kg of salt at ₹ 7/kg, so that he may on selling the mixture at ₹ 6/kg, gain 20% on the outlay?

Solution Cost price of mixture = $6 \times \frac{100}{120} = ₹ 5/kg$



- \therefore Ratio = 1:2
- ∴ For each kg of salt at ₹ 7 /kg, 2 kg of salt at ₹ 4/kg is used.
- \therefore Required weight of salt = $25 \times 2 = 50 \text{ kg}$

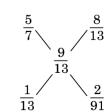
EXAMPLE 10 Milk and water are mixed in vessel A in the proportion 5: 2, and in vessel B in the proportion 8: 5. In what proportion should quantities be taken from two vessels so as to form a mixture in which milk and water will be in the proportion of 9: 4?

Solution In vessel A, quantity of milk = 5/7 of the mixture.

In vessel B, quantity of milk = 8/13 of the mixture.

In the new mixture, quantity of milk = 9/13 of the mixture.

- \therefore Required proportion 1/13:2/91=7:2
- :. 7/9 parts from vessel A and 2/9 parts from vessel B.



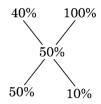
EXAMPLE 11 200 g of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

Solution The existing solution has 40% sugar.

Sugar is to mixed, which is a 100% solution.

Required ratio = 5:1

∴ Required sugar =
$$\frac{200}{5} \times 1 = 40 \text{ g}$$



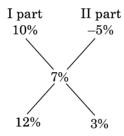


Short cut: Quantity of sugar added

= Weight of sugar solution (Required % value - Present % value)
(100 - Required % value)

i.e.
$$= 200 \frac{(50-40)}{(100-50)} = 40 \text{ g}$$

EXAMPLE 12 A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss? **Solution** Ratio of quantities sold at 10% profit and 5% loss = 12:3=4:1



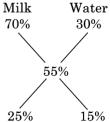
:. Quantity sold at 10% profit = 4/5 (50 kg) = **40 kg** Quantity sold at 5% loss = 50 - 40 = 10 kg

EXAMPLE 13 A vessel of 80 litres is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is emptied by 55%. Find the initial quantity of milk and water.

Solution Required ratio of milk water = 25:15 or 5:3

Quantity of milk = $\frac{80}{5+3} \times 5 = 50$ litres

Quantity of water = $\frac{80}{5+3} \times 3 = 30$ litres



EXAMPLE 14 A container contained 80 litres of milk. From this container 8 litres of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now contained by the container?

Solution The amount of liquid left after n operations, when the container originally contain x units of liquid from which y units is taken out each time is $x \left(\frac{x-y}{x}\right)^n$ units.

$$\therefore \text{ Amount of milk left} = 80 \left(\frac{80 - 8}{80} \right)^3 = 58.32 \text{ kg}.$$

EXAMPLE 15 $\stackrel{?}{\stackrel{?}{\stackrel{?}{$\sim}}}$ 15000 is invested in two parts such that if one part is invested at 6%, and the other at 5%, the total amount in one year as interest from both investments is $\stackrel{?}{\stackrel{?}{\stackrel{?}{$\sim}}}$ 850. How much is invested at 5%.

Solution Total interest at 5% for $\stackrel{?}{\sim} 15000 = \stackrel{?}{\sim} 750$

Balance interest at 1% excess is got from amount x is $\ref{100}$.

$$\therefore \qquad x \times 1 \times \frac{1}{100} = 100$$

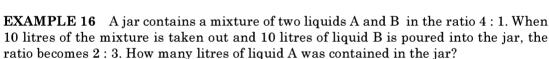
$$\therefore \qquad \qquad x = \mathbf{₹10000}$$

∴ ₹ 10000 is invested at 6% per annum and
 ₹ 5000 is invested at 5% per annum

Applying allegation rule:

Required ratio =
$$2:1$$

∴ Money invested at
$$5\% = \frac{1}{2+1} \times ₹15000 = ₹5000$$



Solution Method I: In original mixture, the % of liquid
$$B = \frac{1}{4+1} \times 100 = 20\%$$

In the resultant mixture, % of liquid B =
$$\frac{3}{2+3} \times 100 = 60\%$$

Replacement is made by liquid B, so that % of B in second mixture = 100% By method of alligation:

- \therefore Ratio of first and second mixtures that should be added is 1:1.
- \therefore Total mixture = 10 + 10 = 20 litres

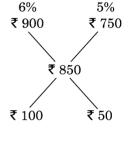
and liquid
$$A = \frac{4}{5} \times 20 = 16$$
 litres

Method II: This method is explained through fraction
Fraction of B in original mixture= 1/5
Fraction of B in second mixture= 1

Fraction of B in resulting mixture = 3/5

Applying method of alligation:

Original mixture and liquid B are added in the same ratio.



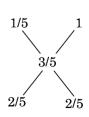
20%

40%

60%

100%

40%



 $10\ litres$ of liquid B is added after taking out $10\ litres$ of mixture from the jar. There should have been $10\ litres$ of mixture left.

 \therefore Quantity of mixture in the jar = 10 + 10 = 20 litres

Quantity of A in the jar =
$$\frac{4}{4+1} \times 20 = 16$$
 litres

Method III: Let the quantity of mixture in the jar be 5x litres.

Then initial proportion of liquids A to B = 4x : x

After taking 10 litres of mixture out, ratio becomes $4x - 10\left(\frac{4}{4+1}\right)$: $x - \left(\frac{1}{4+1}\right)10 + 10$

i.e.
$$(4x-8): x-2+10=2:3$$

i.e.
$$\frac{4x-8}{x+8} = \frac{2}{3} \implies x = 4$$

 \therefore Quantity of A in the mixture = $4x = 4 \times 4 = 16$ litres



Note: Liquid A in original mixture = 4x

Liquid A taken out with 10 litres of mixture = $10 \times \frac{4}{4+1} = 8$ litres

Remaining quantity of A =
$$4x - 10\left(\frac{4}{5}\right) = 12$$
 litres

Liquid B in original mixture = x litres

Liquid B taken out with 10 litres of mixture = $10\left(\frac{1}{5}\right) = 2$ litres

Liquid B added = 10 litres

Total quantity of liquid B = $x - 10\left(\frac{1}{5}\right) + 10$

Ratio of liquid A : B = 2 : 3

Theorem: If x glasses of equal size are filled with a mixture of spirit and water. The ratio of spirit and water in each glass is as follows $a_1 : b_1, a_2 : b_2, ..., a_n : b_n$. If the contents of all the glasses are emptied into a single vessel, then proportion of spirit and water in it is given by

$$\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\cdots+\frac{a_n}{a_n+b_n}\right):\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\cdots+\frac{b_n}{a_n+b_n}\right)$$

EXAMPLE 17 In three vessels each of 10 litres capacity, mixture of milk and water is filled. The ratios of milk and water are 2:1, 3:1 and 3:2 in the three respective vessels. If all the three vessels are emptied into a single vessel, find the proportion of milk and water in the mixture.

Solution Required ratio is $\left(\frac{2}{2+1} + \frac{3}{3+1} + \frac{3}{3+2}\right) : \left(\frac{1}{2+1} + \frac{1}{3+1} + \frac{2}{3+2}\right)$

$$= \left(\frac{2}{3} + \frac{3}{4} + \frac{3}{5}\right) : \left(\frac{1}{3} + \frac{1}{4} + \frac{2}{5}\right) = \frac{40 + 45 + 36}{60} : \frac{15 + 20 + 24}{60} = \frac{121}{60} : \frac{59}{60} = 121 : 59$$



Note: The capacity of vessel may be taken as LCM of (2 + 1), (3 + 1) and (3 + 2) i.e. 60 litres. So without using the theorem, the problem can be solved.

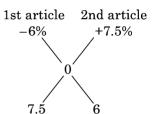
EXAMPLE 18 A person buys two articles for $\ref{1,350}$ and sells one at a loss of 6% and the other at a gain of 7.5% and on the whole he neither gains nor loses. What does each article cost?

Solution Cost of 1st article = $\frac{5}{5+4} \times 1350 =$ ₹ **750**

Cost of 2nd article =
$$\frac{4}{5+4} \times 1350 =$$
₹ 600

 \therefore Required ratio is 7.5: 6 or 5:4

$$= 1.5 \times 5 : 1.5 \times 4 = 5 : 4$$





EXERCISES

1. How much water must be added to a vessel which contains 80 litres of milk at cost price ₹ 3.5/litre so that the cost of milk reduces to ₹ 2/litre?

(a) 20 litres

- (b) 40 litres
- (c) 60 litres
- (d) 70 litres
- (e) None of these
- 2. A sum of ₹ 64 is made up of 80 coins which are either 100 paise or 50 paise coins. How many coins are of 50 paise?

(a) 22

- (b) 32
- (c) 42
- (d) 52
- (e) None of these
- 3. Varun's expenditures and savings are in the ratio of 4:1. His income increases by 20%. If his savings increase by 12%, by how much % should his expenditure increase?

(a) 12%

- (b) 20%
- (c) 22%
- (d) 32%
- (e) None of these
- 4. A dishonest trader professes to sell his article at cost price but he uses 900 g weight instead of 1 kg. What is the percentage gain of the trader in this trade?

(a) 80/9%

- (b) 100/9%
- (c) 110/9%
- (d) 70/9%
- (e) None of these
- 5. A sum of ₹ 390 was divided among 450 boys and girls. Each girl gets 50 paise and each boy gets one rupee. How many girls are there among them?

(a) 100

- (b) 110
- (c) 120
- (d) 130
- (e) None of these
- 6. An amount of ₹70,000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 16000, the sum lent at 6% per annum was ______.

(a) ₹ 10000

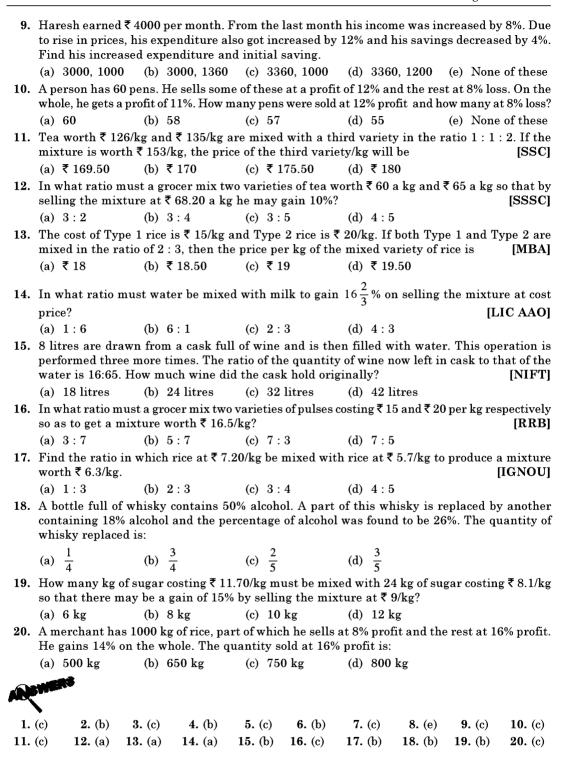
- (b) ₹ 20000
- (c) ₹ 30000
- (d) ₹ 35000
- (e) None of these
- 7. Gold is 19 times as heavy as water and copper is 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?

(a) 5:4

- (b) 4:3
- (c) 3:2
- (d) 2:1
- (e) None of these
- 8. A man has given ₹ 300000 as loan. He lent some of his capital to Ajay at an interest rate of 20% per annum and the rest to Ramesh at an interest rate of 12% per annum. At the end of one year, he got 17% of his capital as interest. How much did he lend to Ajay?

(a) ₹ 120000

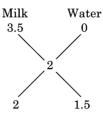
- (b) ₹ 140000
- (c) ₹ 160000
- (d) ₹ 180000
- (e) None of these





Solutions with Necessary Explanation

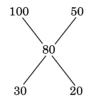
1.



Ratio of milk: water = 2:1.5 or 4:3

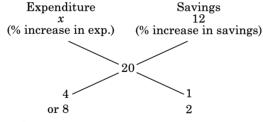
Added water =
$$\frac{80}{4} \times 3 = 60$$
 litres

2. Average value/coin = $\frac{6400}{80}$ = 80 paise

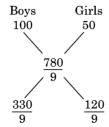


- \therefore 100 paise : 50 paise = 30 : 20 or 3 : 2
- \therefore Number of 50 paise coins = $\frac{80 \times 2}{3+2} = 32$
- 3. x-20=2

or x = 22%



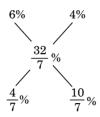
- \therefore % increase in expenditure = 22%
- 4. % gain of the trader = $\frac{100}{900} \times 100 = \frac{100}{9}$ % = $11\frac{1}{9}$ %
- 5. Paise per head = $\frac{39000}{450} = \frac{780}{9}$ paise



 \therefore i.e. required ratio = 330 : 120 or 11 : 4

:. Number of girls among them =
$$\frac{4}{11+4} \times 450 = 120$$

6.

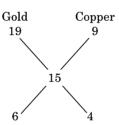


Total simple interest percentage = $\frac{16000 \times 100}{70000 \times 5} = \frac{32}{7}\%$

$$\therefore$$
 Ratio of amounts = $\frac{4}{7}$: $\frac{10}{7}$ = 2:5

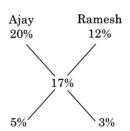
∴ Sum lent at 6% per annum =
$$\frac{2}{2+5} \times ₹70000 = ₹20000$$

7.



Gold: Copper = 6:4 or 3:2

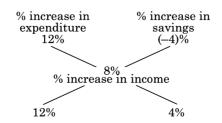
8.



Ratio of amounts that was given to Ajay and Ramesh are in the ratio 5:3

∴ Amount that was given to Ajay =
$$\frac{5}{5+3} \times 300000 = ₹ 187500$$

9.



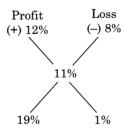
 \therefore Expenditure : Savings = 12 : 4 = 3 : 1

∴ Expenditure =
$$4000 \times \frac{3}{3+1} = ₹3000$$

Savings =
$$4000 - 3000 = ₹ 1000$$

∴ Increased expenditure =
$$3000 \times \left(\frac{112}{100}\right) = ₹3360$$

10.



Ratio of pens sold at 12% profit and 8% loss = 19:1

:. Number of pens sold at 12% profit

$$= \frac{19}{19+1} \times 60 = 57$$

11. Let the price of third variety be ₹ x

First two varieties are mixed in the prepart

First two varieties are mixed in the proportion 1:1

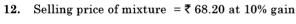
$$\therefore \text{ Average price } = \frac{2126 + 135}{2} = 130.50$$

Mixture is in the proportion 1:1:2 i.e. 2:2 or 1:1

$$\therefore (x-153): 22.50 = 1:1$$
or
$$(x-153) = 22.50$$

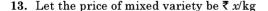
$$\Rightarrow \qquad \qquad x = 7.75.50$$

Hence price of third variety = ₹ 175.50



∴ Cost price of mixture =
$$\left(\frac{100}{110} \times 68.20\right) = ₹62$$

$$\therefore$$
 Required ratio = 3:2

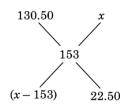


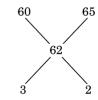
$$\therefore \qquad \frac{20-x}{x-15} = \frac{2}{3}$$

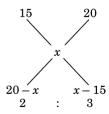
$$\Rightarrow \qquad (20-x)3 = 2(x-15)$$

$$60-3x = 2x-30$$
or
$$5x = 60+30 = 90$$

$$\therefore \qquad x = \text{₹ 18/kg}$$







14. C.P. of 1 litre milk = ₹ 1

Gain % =
$$16\frac{2}{3}$$
% = $\frac{50}{3}$ %

S.P. of 1 litre mixture ₹ 1

C.P. of 1 litre mixture =
$$\frac{300}{350} \times 1 = \frac{6}{7}$$

$$\therefore$$
 Ratio of water and milk = $\frac{1}{7}$: $\frac{6}{7}$ = 1:6



Quantity of wine left in the cask after 4 operations = $x \left(1 - \frac{8}{x}\right)^4$ litres

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

or
$$\left(1 - \frac{8}{x}\right)^4 = \frac{16}{81} = \left(\frac{2}{3}\right)^4$$

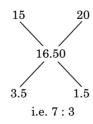
$$1 - \frac{8}{x} = \frac{2}{3}$$

or
$$\frac{8}{x} = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\therefore \qquad \qquad x = 8 \times 3$$

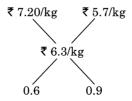
= 24 litres

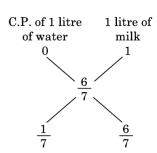
16. Required ratio = 7:3.



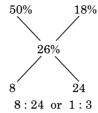
17. Required proportion

$$= (6.3 - 5.7) : (7.2 - 6.3) = 0.6 : 0.9$$
 or $2:3$



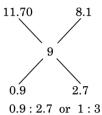


- 18. By rule of alligation, to get mean strength of 26%, given solution of strengths 50% and 18% are to be mixed in the proportion 1:3.
 - \therefore Quantity of whisky replaced = $\frac{3}{4}$



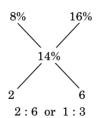
- 19. Required proportion by rule of alligation
 - = 1 part of sugar costing ₹ 11.70/kg and 3 parts of sugar costing ₹ 8.1/kg
 - i.e. Required proportion = 1:3
 - ∴ Quantity of sugar to be mixed with 24 kg of sugar costing ₹ 8.1/kg

$$=\frac{1}{3} \times 24 = 8 \text{ kg}$$



20. Quantity of rice at 8% profit and 16% are in the ratio 1:3.

Quantity of rice sold at 16% profit = $\frac{3}{3+1} \times 1000 = 750 \text{ kg}$



Problems on Trains

Important Formulae

1. Time taken (t) by a train x metres long to cross a stationary person or pole or signal post is equal to the time taken by the train to cover the distance x metres (equal to its length) with its own speed (v)

 $t = \frac{x}{y}$

2. Time taken by a train x metres long to cross a stationary object (like platform, bridge, tunnel, standing train, etc.) y metres long is equal to time taken by the train to cover a total distance (x + y) metres at its own speed (v).

$$t = \frac{x + y}{v}$$

3. (a) A train x metres long running at speed v passes an object of length y metres moving at speed u in the same direction.

Time taken by the train to cross the moving object = $\frac{x+y}{y-u}$

- (b) If the object is a man, signal post or things like this, $t = \frac{x}{y \mu}$
- 4. If the train and object are moving in the opposite direction, i.e. they are approaching towards each other.

Relative speed = v + u

Then time taken by the train to cross the moving object = $\frac{x+y}{y+u}$

Time taken by the train to cross a walking man/moving object = $\frac{x}{y+u}$

5. Two trains A and B start from two points P and Q and move towards each other. After crossing they take a and b hours in reaching Q and P respectively. Then the ratio of their speed is given by

A's speed : B's speed = \sqrt{b} : $\sqrt{a} = \sqrt{b/a}$

- 6. Two trains A and B start simultaneously from stations P and Q and move towards each other at speed S_1 and S_2 km/h respectively. At the point where they meet, one train has covered x km more than the other. Then the distance between P and Q is $\left(\frac{S_1 + S_2}{S_1 S_2}\right)x$ km.
- 7. Two trains A and B are moving in the same direction at speeds S_1 and S_2 km/h. The faster train passes a man sitting in the slower train in t seconds. Then the length of the faster train is $\frac{5}{18}(S_1 \sim S_2)t$ metres.
- 8. Two trains A and B start simultaneously from stations P and Q, x km apart and move towards each other. They meet after t hours. If the difference in speeds of the two trains is 5 km/h. Show that the speeds of the two trains are 1/2(x/t + 5) km/h and 1/2(x/t 5) km/h

i.e. speed of faster train
$$S_1 = \frac{1}{2} \left(\frac{x}{t} + 5 \right) \text{km/h}$$

Slower train
$$S_2 = \frac{1}{2} \left(\frac{x}{t} - 5 \right) \text{km/h}$$

- 9. A train passes a man standing on the platform in t_1 seconds and the platform completely in t_2 seconds. If the length of platform is l, then the length of train is $l\left(\frac{t_1}{t_2-t_1}\right)$ metres.
- 10. If a train covers a certain distance non-stop, its average speed is S_1 km/h, but when it covers the distance with stoppages, its average speed is reduced to S_2 km/h. Then the train stops for $\left(\frac{S_1-S_2}{S_1}\right)$ hour for every hour travelled.
- 11. Two trains of lengths l_1 and l_2 metres run on parallel tracks. When running in the same direction, the faster train passes the slower one in t_1 seconds. But when they are running in the opposite directions at same speeds as earlier, they cross each other completely in t_2 seconds.

Then the speeds of the trains are
$$\left(\frac{l_1+l_2}{2}\right)\left(\frac{t_1+t_2}{t_1t_2}\right)$$
 m/s and $\left(\frac{l_1+l_2}{2}\right)\left(\frac{t_1-t_2}{t_1t_2}\right)$ m/s

12. (a) When two trains are moving in the same direction, time taken to cross each other

$$= \frac{2 \times Product \text{ of two times}}{Difference \text{ of two times}}$$

(b) When the two trains are moving in the opposite direction, time taken to cross each other

$$= \frac{2 \times Product \text{ of two times}}{Sum \text{ of two times}}$$

13. A train overtakes two persons walking in the same direction at x km/h and y km/h respectively and completely passes them in t_1 seconds and t_2 seconds respectively. Then length of train

$$= \frac{\text{Difference of speeds of two man} \times \text{Product of two times}}{\text{Difference of times}}$$

$$= \frac{(x - y) \times t_1 t_2}{t_1 - t_2}$$

Let the speed of train be S km/h and its length be l metres.

$$\therefore \qquad (S-x)t_1 = (S-y)t_2 = l$$

$$\Rightarrow \qquad S(t_1-t_2) = xt_1 - yt_2$$

$$S = \frac{(xt_1 - yt_2)}{(t_1 - t_2)}$$

But

$$l = (S - x)t_1$$

On simplifying the above equation using the expression for S, we get

$$l = \frac{(x - y)t_1t_2}{(t_1 - t_2)}$$

- 14. Two trains of equal length but travelling at different speeds cross a stationery pole in t_1 seconds and t_2 seconds respectively. They will cross each other in time
 - (a) $t = \frac{2t_1t_2}{t_2 t_1}$ when they are travelling in the same direction.
 - (b) $t = \frac{2t_1t_2}{t_2 + t_1}$ when they are travelling in the opposite direction.

EXAMPLE 1 How long does a train 120 metres long running at the rate of 36 km/h take to cross a bridge 144 metres in length?

Solution In crossing the bridge, it must travel its own length plus the length of the bridge.

$$\therefore \qquad \qquad \text{Required time} = \frac{(120 + 144)\text{m}}{36 \text{ km/h}}$$

But

$$1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

$$\therefore \qquad \text{Required time} = \frac{264}{36 \times \frac{5}{18}} = 26.4 \text{ seconds}$$

EXAMPLE 2 Two trains 141 metres and 119 metres in length respectively are running in opposite directions, one at the rate of 30 km and the other at the rate of 42 km an hour. In what time will they be completely clear of each other from the moment they meet?

Solution As the two trains are moving in opposite directions, their relative speed

=
$$42 + 30 = 72$$
 km/h = $72 \times \frac{5}{18} = 20$ m/s

$$\therefore \qquad \text{Required time} = \frac{\text{Total length}}{\text{Relative speed}} = \frac{141 + 119}{20} = 13 \text{ seconds}$$

EXAMPLE 3 A train 120 metres in length travels at 66 km/h. In what time will it pass a man who is walking at 6 km/h

(i) against it, (ii) in the same direction?

Solution

(i) Relative speed =
$$66 + 6 = 72 \text{ km/h} = 72 \times \frac{5}{18} = 20 \text{ m/s}$$

Required time = $\frac{\text{Length of train}}{\text{Relative speed}} = \frac{120}{20} = 6 \text{ seconds}$

(ii) Relative speed =
$$66 - 6 = 60 \text{ km/h} = 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/s}$$

$$\therefore \text{ Required time} = \frac{120}{50/3} = \frac{120 \times 3}{50} = 7.2 \text{ seconds}$$

EXAMPLE 4 Two trains of the same length but with different speeds pass a static pole in 4 seconds and 5 seconds respectively. In what time will they cross each other when they are moving in

- (i) the same direction?
- (ii) the opposite direction?

Solution Let the length of train be l metres.

 \therefore The speeds of trains are l/4 m/s and l/5 m/s.

Total distance to be travelled = 2l metres.

(i) Relative speed =
$$\frac{l}{4} - \frac{l}{5} = \frac{l}{20}$$
 m/s

:. Required time =
$$2l \div \frac{l}{20} = 2l \times \frac{20}{l} = 40 \text{ seconds}$$

(ii) Relative speed =
$$\frac{l}{4} + \frac{l}{5} = \frac{9l}{20}$$
 m/s

$$\therefore$$
 Required time = $2l \div \frac{9l}{20} = 2l \times \frac{20}{9l} = \frac{40}{9} = 4\frac{4}{9}$ seconds



Short cut:

1. Time =
$$\frac{2 \text{ (Product of time taken)}}{\text{Difference of time taken}} = \frac{2(4 \times 5)}{(5-4)} = 40 \text{ seconds}$$

2. Time =
$$\frac{2(\text{Product of time taken})}{(\text{Sum of time taken})} = \frac{2(4 \times 5)}{(4 + 5)} = \frac{40}{9} \text{ seconds} = 4\frac{4}{9} \text{ seconds}$$



Note: If the trains are of different lengths and moving with the same speed:

- 1. When they are moving in the same direction, they can't pass each other.
- 2. When they are moving in the opposite direction, time taken to pass each other

$$= \frac{\text{Total length}}{\text{Speed}} = \frac{4+5}{2} = 4.5 \text{ seconds}$$



Short cut: Required time = Average of two times.

EXAMPLE 5 Two trains of length 100 m and 80 m respectively run on parallel tracks. When running in the same direction the faster train passes the slower one in 18 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in 9 seconds. Find the speed of each train.

Solution Let the speed of trains be S_1 and S_2 m/s.

When they are moving in the same direction, the relative speed

$$S_1 - S_2 = \frac{100 + 80}{18} = 10 \text{ m/s}$$

Similarly, when they are in the opposite direction, the relative speed

$$S_1 + S_2 = \frac{100 + 80}{9} = 20 \text{ m/s}$$

Solving, we get $S_1 = 15$ m/s; $S_2 = 5$ m/s.



:.

Short cut: Speed of faster/slower train = Average length of trains

$$\times \left[\frac{1}{\text{Opposite direction's time}} \pm \frac{1}{\text{Same direction's time}} \right]$$

Speed of faster train =
$$\frac{l_1 + l_2}{2} \left(\frac{t_1 + t_2}{t_1 \times t_2} \right) = \frac{100 + 80}{2} \left(\frac{18 + 9}{18 \times 9} \right) = \frac{90 \times 27}{18 \times 9} = 15 \text{ m/s}$$

Speed of slower train =
$$\frac{100 + 80}{2} \left(\frac{18 - 9}{18 \times 9} \right) = \frac{90 \times 9}{18 \times 9} = 5 \text{ m/s}$$

EXAMPLE 6 A train overtakes two persons who are walking in the same direction as the train is moving, at the rate of 2 km/h and 4 km/h respectively and passes them completely in 9 and 10 seconds respectively. Find the speed and length of the train.

Solution Speeds of men are 2 km/h and 4 km/h

i.e.
$$2 \times \frac{5}{18} = \frac{5}{9}$$
 m/s and $4 \times \frac{5}{18} = \frac{10}{9}$ m/s

Let the speed of train be x m/s

:. Length of train = relative speed × time taken to pass a man

$$= \left(x - \frac{5}{9}\right) \times 9 = \left(x - \frac{10}{9}\right) 10$$

$$x = \frac{100}{9} - \frac{45}{9} = \frac{55}{9} \text{ m/s}$$

$$= \frac{55}{9} \times \frac{18}{5} \text{ km/h} = 22 \text{ km/h}$$
(1)

Length of train =
$$\left(x - \frac{5}{9}\right)9 = \left(\frac{55}{9} - \frac{5}{9}\right)9 = 50 \text{ m}$$



Short cut: Length of train = $\frac{\text{(Difference in speeds of two men)} \times T_1 \times T_2}{T_2 - T_1}$

$$= \left(\frac{10}{9} - \frac{5}{9}\right) \times \frac{9 \times 10}{10 - 9} = \frac{5}{9} \times 9 \times 10 = \mathbf{50} \text{ m}$$

EXAMPLE 7 A train passes a pole in 15 seconds and passes a platform 100 m long in 25 seconds. Find the length of train.

Solution Let the length of train be l metres.

$$\frac{l}{15} = \frac{l+100}{25}$$

or
$$25l = 15l + 1500$$

$$l = \frac{1500}{(25 - 15)} = 150 \text{ m}$$



Short cut: Length of train = (Time to pass a pole × Length of platform)
(Difference in time to cross a pole and platform)

$$=\frac{15\times100}{25-15}=150 \text{ m}$$

EXAMPLE 8 Two trains A and B start from two places P and Q towards Q and P respectively. After passing each other they take 4 hours 48 minutes and 3 hours and 20 minutes to reach Q and P respectively. If the train from P is moving at 45 km/h then find the speed of other train.

Solution Let the speed of train B be x km/h and they meet at M.

 \therefore 45 km/h × (4 hours + 48 minutes)

:.
$$MQ = 45 \times \left[4\frac{4}{5} \right] = 45 \times \frac{24}{5} = 216 \text{ km}$$

Distance MP = $x \times (3 \text{ hours} + 20 \text{ minutes})$

$$= x \times 3\frac{1}{3} = x \times \frac{10}{3} = \frac{10x}{3}$$
 km

.. Time to reach the train from Q to M

$$= \frac{QM}{x} = \frac{PM}{45}$$

or

$$\frac{216}{x} = \frac{10x/3}{45}$$

or
$$10x^{2} = 216 \times 3 \times 45$$
or
$$x^{2} = 108 \times 3 \times 9$$

$$\therefore \qquad x = 9 \times 2 \times 3 = 54 \text{ km/h}$$



Speed of second train = Speed of first train $\times \frac{\sqrt{\text{Time taken by first train after meeting}}}{\sqrt{\text{Time taken by second train after meeting}}}$

$$= 45 \times \frac{\sqrt{4\frac{4}{5}}}{\sqrt{3\frac{1}{3}}} = 45 \times \frac{\sqrt{24/5}}{\sqrt{10/3}} = 45 \times \sqrt{\frac{24}{5} \times \frac{3}{10}} = 45 \times \frac{6}{5} = 54 \text{ km/h}$$

EXAMPLE 9 Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. Both the trains meet 6 hours after they start moving. If the train travelling from P to Q travels 8 km/h faster than the other train, find the speed of the two trains.

Solution Let the speeds of trains be S_1 km/h and S_2 km/h

$$S_1 + S_2 = \frac{162}{6} = 27 \text{ km/h}$$
 (1)

$$S_1 - S_2 = 8 \text{ km/h}$$
 (2)

Solving (1) and (2), we get,

$$S_1 = \frac{27 + 8}{2} = 17.5 \text{ km/h}$$

and

$$S_2 = \frac{27 - 8}{2} = 9.5 \text{ km/h}$$



Distance between the places ± time taken to meet × difference in speeds of trains 2 × time taken to meet

Speeds of the trains =
$$\frac{162+6\times8}{2\times6}$$
 and $\frac{162-6\times8}{2\times6}$

i.e. 17.5 km/h and 9.5 km/h

EXAMPLE 10 The speeds of two trains are in the ratio x: y. They are moving in the opposite direction on parallel tracks. The first train crosses a telegraph pole in a seconds whereas the second train crosses a telegraph pole in b seconds. Find the time taken by the trains to cross each other completely.

Solution Let the speeds of trains be kx and ky metres/second.

 \therefore Length of first train = $kx \times a = kax$ metres

Length of second train = $ky \times b = kby$ metres

Time taken to cross each other =
$$\frac{\text{Sum of lengths}}{\text{Relative speed}} = \frac{kax + kby}{kx + ky} = \frac{ax + by}{x + y} \text{ seconds}$$

EXAMPLE 11 How many seconds will a train 100 metres long running at the rate of 36 km an hour take to pass a telegraph post?

Solution To pass the telegraph post, the train should cover its own length.

:. Speed of train =
$$36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

Required time =
$$\frac{\text{Length of train}}{\text{Speed}} = \frac{100}{10} = 10 \text{ seconds}$$

EXAMPLE 12 How long does a train 180 metres long running at the rate of 36 km/h take to cross a bridge 100 metres in length?

Solution In crossing the bridge, the train must travel its own length plus the length of the bridge.

Speed of train = 36 km/h = 36
$$\times \frac{5}{18}$$
 = 10 m/s

Required time =
$$\frac{180 + 100}{10}$$
 = 28 seconds

EXAMPLE 13 Two trains are moving in the same direction at 50 km/h and 30 km/h. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.

Solution Relative speed = (50 - 30) km/h

$$=20 \times \frac{5}{18} = \frac{50}{9}$$
 m/s

Distance covered in 18 seconds at this speed = $\frac{50}{9} \times 18 = 100 \text{ m}$

:. Length of faster train = 100 m

EXAMPLE 14 A train travelling at 36 km/h took 15 seconds in passing a standing man. Find the length of the train.

Solution Speed of train = 36 km/h =
$$36 \times \frac{5}{18} = 10$$
 m/s

Time taken to pass the man = 15 seconds.

$$\therefore$$
 Length of train = $10 \times 15 = 150 \text{ m}$

(e) None of these

(d) 50 seconds

? EXERCISES

(a) 20 seconds

(b) 30 seconds

	and travels towards Q at 20 km/h speed. Another train starts from Q at 8 am and travels P at a speed of 25 km/h. At what time will they meet?	avels
	(a) 9 am (b) 10 am (c) 10.30 am (d) 11 am (e) None of t	hese
4.	A train 75 metres long overtook a person who was walking at the rate of 6 km/h and pa	
	him in $7\frac{1}{2}$ seconds. Subsequently, it overtook a second person, and passed him in $6\frac{3}{4}$ seconds.	onds.
	At what rate was the second person travelling?	
	(a) 1 km/h (b) 2 km/h (c) 3 km/h (d) 4 km/h (e) None of t	$_{ m hese}$
5.	A train covers a distance between two stations A and B in 45 minutes. If the speed is red by 5 km/h, it will cover the same distance in 48 minutes. What is the distance between two stations A and B (in km)? Also find the speed of the train.	
	(a) 80 km, 60 km/h (b) 90 km, 70 km/h (c) 60 km, 80 km/h	
	(d) 70 km , 70 km/h (e) None of these	
6.	Two trains are running at 40 km/hr and 20 km/hr respectively in the same direction. For train completely passes a man in the slower train in 5 seconds. What is the length of faster train?	
	(a) 23 m (b) $23\frac{2}{9} \text{ m}$ (c) 27 m (d) $27\frac{7}{9} \text{ m}$	
7.	How long does a train 110 metres long running at the speed of 72 km/hr take to crebridge 132 metres in length?	oss a
	(a) 9.8 sec (b) 12.1 sec (c) 12.42 sec (d) 14.3 sec	
8.	A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the leng the train? [Bank	
	(a) 120 metres (b) 180 metres (c) 324 metres (d) Cannot be determined (e) None of these	
9.	A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a teleg post, then the length of the train is [Bank	
	(a) 90 m (b) 100 m (c) 120 m (d) 140 m	
10.	A 300 m long train crosses a platform in 39 seconds, while it crosses a signal po 18 seconds. What is the length of the platform? [Bank	
	(a) 320 m (b) 350 m (c) 650 m	
	(d) Data inadequate (e) None of these	

1. How many seconds will a train 60 m in length, travelling at the rate of 42 km/h, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km/h?

(c) 40 seconds

2. Two trains running at the rate of 45 km and 36 km an hour respectively, on parallel tracks in opposite directions; are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train

(a) 70 m, 100 m (b) 65 m, 95 m (c) 75 m, 105 m (d) 80 m, 110 m (e) None of these 3. Two stations P and Q are 110 km apart on a straight line. One train starts from P at 7 am

observes that he passes the other in 30 seconds. Find the length of trains.

11.		and 0.9 km respe		n/hr and 90 km/hr respective taken by the slower train to o	-
	(a) 36 (l	b) 45	(c) 48	(d) 49	
12.				km/hr. In what time will it page to that in which the train is go	
	(a) 5 sec (1	b) 6 sec	(c) 7 sec	(d) 10 sec	•
13.		5 km/hr and 30 l	km/hr respectivel	opposite directions on paralle y. Find the time taken by th	
	(a) 12 sec (1	b) 24 sec	(c) 48 sec	(d) 60 sec	
14.	_	of each train be		conds respectively to cross a to what time will they cross ea	
	(a) 10 sec (1	b) 12 sec	(c) 15 sec	(d) 20 sec	
15.			-	o cross a man walking with a eed of the train is 63 km/hr?	speed of [SSC]
	(a) 25 (l	b) 30	(c) 40	(d) 45	
16.		_	-	60 km/hr and 40 km/hr respect to they take to cross each other	-
	(a) 9 sec (1	b) 9.6 sec	(c) 10 sec	(d) 10.8 sec	
17.				the same speed. If the length econds, then the speed of each	
	(a) 10 km/hr (l	b) 18 km/hr	(c) 36 km/hr	(d) 72 km/hr	
18.	A train moves past respectively. What is			4 m long in 8 seconds and 20	seconds [SSC]
	(a) 69.5 km/hr	b) 70 km/hr	(c) 79 km/hr	(d) 79.2 km/hr	
19.	What is the length of	of the goods train		s a 250 m long platform in 26 [Ba	seconds. nk PO]
	, ,	b) 240 m	(c) 260 m	(d) 270 m	
20.	140 m long?	_	_	ar. In what time will it pass	a bridge [BSF]
	` '	b) 42 sec	(c) 45 sec	(d) 48 sec	
21.				nes in the same direction at 4 slower train in 36 seconds. Th	
	(a) 50 m	b) 72 m	(c) 80 m	(d) 82 m	
22.				esses another train running in t is the length of the other tra	
	(a) 230 m	b) 240 m	(c) 260 m	(d) 320 m (e) None	-
23.	Two trains, each 10	0 m long, moving	g in opposite direc	etions, cross each other in 8 se ed of the faster train is:	
		b) 45 km/hr	(c) 60 km/hr	(d) 75 km/hr	[]

24.	simultaneously.		the trains reach	other from Patna to Howrah, s their destinations after 9 hours [R]	
	(a) 2:3	(b) 4:3	(c) 6:7	(d) 9:16	
25.		10 seconds. The s	peed of the train is		the AO]
	(a) 45 km/hr	(b) 50 km/hr	(c) 54 km/hr	(d) 55 km/hr	
26.	A train speeds pa is	st a pole in 15 sec	onds and a platfor	m 100 m long in 25 seconds. Its ler [R:	ngth RB]
	(a) 50 m	(b) 150 m	(c) 200 m	(d) Data Inadequate	
27.	A train 800 metre then the length of		at a speed of 78 km	n/hr. If it crosses a tunnel in 1 min [S	ute, SC]
	(a) 130 m	(b) 360 m	(c) 500 m	(d) 540 m	
28.	A train 240 m los 650 m long?	ng passed a pole	in 24 seconds. Ho	w long will it take to pass a platf [R]	orm RBJ
	(a) 65 sec	(b) 89 sec	(c) 100 sec	(d) 150 sec	
29 .	In what time will	a train 100 m lon	g cross an electric j	pole, if its speed be 144 km/hr? [S	SC]
	(a) 2.5 sec	(b) 4.25 sec	(c) 5 sec	(d) 12.5 sec	
30.	A train 280 m lon	ng, running with a	a speed of 63 km/hi	r will pass a tree in [S	SC]
	(a) 15 sec	(b) 16 sec	(c) 18 sec	(d) 20 sec	
31.				standing on the platform in 27 seconds. The ratio of t [Hotel Management	heir
	(a) 1:3	(b) 3:2	(c) 3:4	(d) None of these	
32.		_	· ·	x is 240 m ahead of the engine of a tion. In how much time will the t [IGN0	rain
	(a) 3.6 sec	(b) 18 sec	(c) 36 sec	(d) 72 sec	
33.	-	_		d a man standing on the platform t is the length of the platform?	n in
	(a) 120 m	(b) 240 m	(c) 300 m	(d) None of these	
34.	The length of the in 30 seconds, is	bridge, which a tr	rain 130 metres lon	ng and travelling at 45 km/hr can c [Section Office	
	(a) 200 m	(b) 225 m	() O 1 F		
35.	(a) 200 m	(b) 225 m	(c) 245 m	(d) 250 m	
	Two trains 100 m	netres and 120 me	etres long are runn	ning in the same direction with spe he faster train cross the second?	eeds
	Two trains 100 m	netres and 120 me	etres long are runn	ning in the same direction with spe he faster train cross the second?	
	Two trains 100 m of 72 km/hr and (a) 40 sec A train 100 m lon	netres and 120 me 54 km/hr. In how (b) 42 sec	etres long are runn much time will t (c) 44 sec ne speed of 30 km/l	ing in the same direction with spe he faster train cross the second? [C	BIJ
	Two trains 100 m of 72 km/hr and (a) 40 sec A train 100 m lon	netres and 120 me 54 km/hr. In how (b) 42 sec ng is running at th	etres long are runn much time will t (c) 44 sec ne speed of 30 km/l	ring in the same direction with specific he faster train cross the second? (d) 46 sec	BIJ
36.	Two trains 100 m of 72 km/hr and (a) 40 sec A train 100 m lon man standing nea (a) 10 sec A train is moving	netres and 120 me 54 km/hr. In how (b) 42 sec ng is running at the ar the railway line (b) 12 sec	etres long are runn much time will t (c) 44 sec ne speed of 30 km/l e. (c) 14 sec km/hr. If the length	ting in the same direction with speche faster train cross the second? (d) 46 sec (hr. Find the time taken by it to pa	CBI] .ss a

- 38. Length of a train and that of a platform are equal. If with a speed of 135 km/hr, the train crosses the platform in 40 seconds, then the length of train is
 - (a) 500 m
- (b) 600 m
- (c) 750 m
- (d) 900 m
- **39.** Two trains 250 m and 200 m long are running on parallel track at the rate of 36 km/hr and 45 km/hr respectively. In how much time will they cross each other, if they are running in the same direction?
 - (a) 72 sec
- (b) 144 sec
- (c) 180 sec
- (d) 252 sec
- 40. A train 120 m long moving at a speed of 60 km/hr crosses a train 130 m long coming from opposite direction in 6 seconds. The speed of the second train is
 - (a) 72 km/hr
- (b) 78 km/hr
- (c) 84 km/hr
- (d) 90 km/hr



- 1. (e) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d) 7. (b) 8. (e) 9. (c) 10. (b)
- 11. (c) 12. (b) 13. (b) 14. (b) 15. (b) 16. (d) 17. (c) 18. (d) 19. (d) 20. (a)
- 21. (a) 22. (a) 23. (c) 24. (b) 25. (b) 26. (b) 27. (c) 28. (b) 29. (a) 30. (b)
- 31. (b) 32. (c) 33. (b) 34. (c) 35. (c) 36. (b) 37. (b) 38. (c) 39. (c) 40. (d)



${f S}$ olutions with Necessary Explanation

1. Relative speed = 42 - 30 = 12 km/h

$$= 12 \times \frac{5}{18} = \frac{10}{3}$$
 m/s

Time taken by the train to pass another train = $\frac{\text{Distance travelled}}{\text{Speed}}$ = $\frac{(60 + 84)\text{m}}{10/3\text{ m/s}}$ = $\frac{432}{10} = 43.2 \text{ seconds}$

2. Relative speed of two trains = 45 + 36 = 81 km/h

$$= 81 \times \frac{5}{18} = \frac{45}{2} = 22\frac{1}{2}$$
 m/s

 \therefore Length of both the trains = $\frac{45}{2} \times 8 = 180 \text{ m}$

When two trains are moving in the same directions, relative speed = 45 - 36 = 9 km/h

$$= 9 \times \frac{5}{18} = \frac{5}{2} \text{ m/s}$$

Length of slower train = $\frac{5}{2} \times 30 = 75$ m

 \therefore Length of faster train = 180 - 75 = 105 m



Short cut: Length of slower train = (Relative speed of two train) \times 30

=
$$(45 - 36) \frac{5}{18} \times 30 = 75 \text{ m}$$

Length of faster train = Total length of both train - Length of slower train

$$= (45 + 36) \times \frac{5}{18} \times 8 - 75$$

$$= 180 - 75 = 105 \text{ m}$$

3. The train starts from P and covers a distance of 20 km till 8 am.

Remaining distance = 110 - 20 = 90 km is covered by the trains with relative speed of 20 + 25 = 45 km/h

$$\therefore$$
 They meet after $\frac{90}{45} = 2 \text{ h}$

i.e. at
$$8 + 2 = 10 \text{ h}$$

i.e. at 10 am

- 4. Relative speed of train and first person = $\frac{75}{15/2}$ = 10 m/s = 10 × $\frac{18}{5}$ = 36 km/h
 - \therefore Speed of train = 36 + 6 = 42 km/h

Relative speed of train and second person = $\frac{75}{27/4} = \frac{300}{27} \times \frac{18}{5} = 40 \text{ km/h}$

- \therefore Speed of second person = 42 40 = 2 km/h
- 5. Let the distance between two stations A and B be x km

Let the speed of train be y km/h

$$\therefore \frac{x}{y} = \frac{45}{60} = \frac{3}{4} \Rightarrow x = \frac{3}{4}y \tag{1}$$

When speed is reduced by 5 km/h

$$\frac{x}{y-5} = \frac{48}{60} = \frac{4}{5} \Rightarrow x = \frac{4}{5} (y-5)$$
 (2)

Solving (1) and (2),

$$\frac{3}{4}y = \frac{4}{5}(y - 5)$$

or

$$y\left(\frac{4}{5} - \frac{3}{4}\right) = \frac{4}{5} \times 5 = 4$$

$$\therefore \qquad \qquad y \times \frac{16 - 15}{20} = 4$$

or
∴ Speed of train = 80 km/h

$$y = 4 \times 20 = 80 \text{ km/h}$$

Distance between two stations = $\frac{3}{4} \times 80 = 60 \text{ km}$



Short cut: Speed of train = $\frac{48}{48-45} \times 5 = 80 \text{ km/h}$

Distance = Speed × time =
$$\left[\frac{48}{48 - 45}\right] \times 5 \times \frac{45}{60} = 60 \text{ km}$$

6. Relative speed of faster train w.r.t. slower train = 40 - 20 = 20 km/hr = $20 \times \frac{5}{18}$ m/s = $\frac{50}{9}$ m/s Let length of train be x metres.

$$\therefore \qquad \frac{x}{5} = \frac{50}{9}$$
or
$$x = \frac{250}{9} = 27\frac{7}{9} \text{ m}$$

7. Speed of train = $72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

Let the time taken by train to cross bridge be 't' seconds.

$$\therefore \frac{110 + 132}{20} = t$$
i.e.
$$\frac{242}{20} = t$$
or
$$t = 12.1 \sec t$$

8. Let length of train be x metres.

$$\therefore \qquad x = \left(60 \times \frac{5}{18}\right) \times 9$$
$$= 150 \text{ metres}$$

9. Let length of train be x metres.

Speed of train =
$$\frac{12 \text{ km}}{10 \text{ min}} = \frac{12}{1/6} \text{ km/hr} = 72 \text{ km/hr}$$

= $72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$
 $x = 20 \times 6 = 120 \text{ m}$

10. Let length of platform be x metres.

Speed of train =
$$\frac{300 + x}{39} = \frac{300}{18}$$

i.e. $300 + x = \frac{300 \times 39}{18}$
or $x = 300 \times \frac{39}{18} - 300$
 $= 300 \times \frac{21}{18} = 50 \times 7 = 350 \text{ m}$

11. Let time taken by slower train to cross faster train be t seconds Relative speed = 60 + 90 = 150 km/hr

Total length of train = 1.1 + 0.9 = 2 km.

 $\therefore \quad \text{Time taken to cross each other } t = \frac{\text{Length of train}}{\text{Relative speed}}$

$$= \frac{2 \text{ km}}{150 \text{ km/hr}} = \frac{1}{75} \text{ hour}$$
$$= \frac{60 \times 60}{75} \sec = 12 \times 4 = 48 \sec$$

12. Let the time taken by train to cross a running man be t sec.

Relative speed of train w.r.t. man

=
$$60 + 6 = 66 \text{ km/hr} = 66 \times \frac{5}{18} \text{ m/s}$$

= $\frac{55}{3} \text{ m/s}$

 \therefore Time taken to cross running man, $t = \frac{\text{Length of train}}{\text{Speed}}$

$$=\frac{110}{55/3}=6 \text{ sec}$$

13. Relative speed of train = $45 + 30 = 75 \text{ km/hr} = 75 \times \frac{5}{18} = \frac{125}{6} \text{ m/s}$

Let the time take by slower train to pass the driver of faster train be t seconds.

- $t = \frac{\text{Length of slower train}}{\text{Relative speed of train}}$ $= \frac{500 \text{ m}}{\frac{125}{6} \text{ m/s}} = 24 \text{ sec}$
- **14.** Speed of first train = $\frac{120}{10}$ = 12 m/s

Speed of second train = $\frac{120}{15}$ = 8 m/s

Let t sec be the time in which they cross each other.

- $t = \frac{\text{Total length of trains}}{\text{Relative speed of trains}}$ $= \frac{120 + 120}{(12 + 8)} = \frac{240}{20} = 12 \text{ sec}$
- 15. Let the time taken by the train to cross a walking man be t seconds.

Relative speed of train w.r.t. man = $(63 - 3) = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{m/s} = \frac{50}{3} \text{ m/s}$

Length of train = 500 m

$$t = \frac{500}{50/3} = \frac{500 \times 3}{50} = 30 \text{ sec}$$

16. Relative speed of train, $v_r = 60 + 40 = 100$ km/hr

$$= 100 \times \frac{5}{18} \text{ m/s}$$
$$= \frac{250}{9} \text{ m/s}$$

Length of trains, L = 140 + 160 = 300 m.

Time taken by the trains to cross each other

$$\frac{L}{v_r} = \frac{300}{250/9} = \frac{300 \times 9}{250} = \frac{6 \times 9}{5} = 10.8 \text{ sec}$$

- 17. Let speed of trains be v m/s.
 - \therefore Relative speed of trains $v_r = 2v$ m/s

Time taken to cross each other = 12 sec.

Length of trains = 120 + 120 = 240 m.

$$\frac{L}{v_r} = t$$
or
$$v_r = \frac{L}{t} = \frac{240}{12} = 20 \text{ m/s}$$

$$v = \frac{v_r}{2} = 10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/hr}$$

 $v = \frac{7}{2} = 10 \text{ m/s} = 10 \times \frac{10 \times 10^{-3}}{5} = 36 \text{ km/s}$ 18. Let speed of train be $v \times 10^{-3}$ km/hr. Let length of train be $v \times 10^{-3}$ km/hr.

$$\frac{L}{8} = \frac{L + 264}{20} = v \times \frac{5}{18}$$
i.e.
$$\frac{L}{v \times \frac{5}{18}} = 8 \implies L = v \times \frac{5 \times 8}{18}$$
(1)

similarly
$$\frac{L+264}{20} = v \times \frac{5}{18} \implies L+264 = v \times \frac{5}{18} \times 20$$
 (2)
(2) - (1) $\implies 264 = v \times \frac{5}{18} (20-8) = v \times \frac{5}{18} \times 12$

or $v = 264 \times \frac{18}{60} = 79.2 \text{ km/hr}$

19. Speed of train = $72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

Let length of goods train be x metre.

$$\frac{x + 250}{26} = 20$$
i.e.
$$x + 250 = 20 \times 26 = 520$$

$$\therefore \qquad x = 520 - 250$$

$$= 270 \text{ m}$$

20. Speed of train = 45 km/hr = 45
$$\times \frac{5}{18}$$
 = 12.5 m/s

Length of bridge = 140 m.

Let time taken by train to pass bridge be t seconds.

$$\frac{360 + 140}{t} = 12.5$$
or
$$t = \frac{500}{12.5} = 40 \text{ seconds}$$

- **21.** Let length of each train be l metres
 - \therefore Total length of trains = l + l = 2l.

Relative speed of faster train w.r.t. slower train

$$= 46 - 36 = 10 \text{ km/hr} = 10 \times \frac{5}{18} \text{ m/s} = \frac{25}{9} \text{ m/s}$$

$$\therefore \qquad \text{Relative speed} = \frac{\text{Total length of trains}}{\text{Time taken to pass the train}}$$

i.e.
$$\frac{25}{9} = \frac{2l}{36}$$

or $l = \frac{25 \times 36}{18} = 50 \text{ m}$

22. Let the length of second train be l metres.

Relative speed of train = 120 + 80 = 200 km/hr

$$= 200 \times \frac{5}{18} \text{ m/s} = \frac{500}{9} \text{ m/s}$$

$$\therefore \qquad \frac{270+l}{9} = \frac{500}{9}$$

or
$$270 + l = 500$$

or $l = 500 - 270 = 230 \text{ m}$

23. Let speed of slower train be x/2 and that of faster train be x metres/second.

$$\frac{100 + 100}{8} = x + \frac{x}{2} = \frac{3x}{2}$$
or
$$x = \frac{200}{8} \times \frac{2}{3} = \frac{50}{3} \text{ m/s}$$

$$= \frac{50}{3} \times \frac{18}{5} \text{ km/hr} = 60 \text{ km/hr}$$

24. Ratio of their speeds = inverse ratio of square root of time taken to reach destination

i.e.
$$\frac{v_1}{v_2} = \sqrt{\frac{t_2}{t_1}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

25. Let speed of train be x km/hr

$$\therefore (x-5) \times \frac{5}{18} \,\text{m/s} = \frac{125}{10} = 12.5$$

∴
$$x-5 = \frac{12.5 \times 18}{5} = 45$$

∴ $x = 45 + 5 = 50 \text{ km/hr}$

26. Let length of train be l metres.

$$\frac{l}{15} = \frac{l+100}{25}$$
i.e.
$$\frac{l}{3} = \frac{l+100}{5}$$
or
$$5l = 3l + 300$$
or
$$l = \frac{300}{(5-3)} = 150 \text{ m}$$

27. Let length of tunnel be x metres.

Speed of train =
$$78 \text{ km/hr} = 78 \times \frac{5}{18} \text{ m/s} = \frac{65}{3} \text{ m/s}$$

$$\therefore \frac{800 + x}{60} = \frac{65}{3}$$
or
$$(800 + x) = \frac{60}{3} \times 65$$

$$800 + x = 1300$$
or
$$x = 1300 - 800 = 500 \text{ m}$$

28. Let the time taken by the train to pass a platform be t seconds.

$$\frac{240 + 650}{t} = \frac{240}{24} = 10$$
 or
$$t = \frac{240 + 650}{10} = 89 \text{ sec}$$

29. Let time taken to cross a pole be t seconds.

$$\frac{100}{t} = 144 \times \frac{5}{18} \text{ m/s} = 40$$
or
$$t = \frac{100}{40} = 2.5 \sec$$

30. Time taken by the train to pass a tree

$$= \frac{\text{Length of train}}{\text{Speed of train}} = \frac{280 \text{ m}}{63 \times \frac{5}{18} \text{ m/s}} = \frac{280 \times 18}{63 \times 5} = 8 \times 2 = 16 \text{ seconds}$$

31. Let the speeds of first and second trains be x m/s and y m/s respectively \therefore Length of first train = $x \times 27 = 27x$ metres.

Length of second train = $y \times 17 = 17y$ metres.

Time taken to cross each other = $\frac{27x + 17y}{x + y} = 23$

Simplifying, we get

or
$$27x + 17y = 23x + 23y$$
$$4x = 6y$$
$$\frac{x}{y} = \frac{6}{4} = \frac{3}{2}$$

32. Let the time taken by train to pass jogger be t seconds Relative speed of train w.r.t. jogger = 45 - 9 = 36 km/hr

=
$$36 \times \frac{5}{18}$$
 m/s = 10 m/s

$$t = \frac{\text{Distance travelled by train to pass jogger}}{\text{Relative speed}}$$
= $\frac{240 + 120 \text{ m}}{10 \text{ m/s}} = 36 \text{ sec}$

33. Speed of train = $54 \text{ km/hr} = 54 \times \frac{5}{18} \text{ m/s} = 15 \text{ m/s}.$

Let length of platform be x metres and y metres that of train.

$$\therefore \frac{y}{20} = 15$$
or
$$y = 300 \text{ m}$$

Similarly, speed of train = $\frac{\text{Length of (train + platform)}}{\text{Time to cross platform}}$

i.e.
$$15 = \frac{x + 300}{36}$$

or
$$x = 15 \times 36 - 300 = 540 - 300 = 240$$
 metres.

34. Let length of bridge be x metres.

$$\frac{x+130}{30} = 45 \times \frac{5}{18} = \frac{25}{2}$$
or
$$x+130 = 25 \times 15 = 375$$

$$\therefore x = 375 - 130 = 245 \text{ m}$$

35. Relative speed of faster train w.r.t. slower train

$$= 72 - 54 = 18 \text{ km/hr} = 18 \times \frac{5}{18} \text{ m/s} = 5 \text{ m/s}$$

Time taken by the faster train to cross the slower train = $\frac{(100 + 120) \text{ m}}{5 \text{ m/s}}$ = 44 sec

36. Speed of train = $30 \text{ km/hr} = 30 \times \frac{5}{18} \text{ m/s} = \frac{25}{3} \text{ m/s}$

Time taken by the train to pass standing man = $\frac{\text{Length of train}}{\text{Speed of train}} = \frac{100 \text{ m}}{\frac{25}{3} \text{ m/s}}$

$$=\frac{100\times3}{25}=12 \text{ sec}$$

37. Speed of train = $120 \text{ km/hr} = 120 \times \frac{5}{18} \text{ m/s} = \frac{100}{3} \text{ m/s}$

Time taken by train to cross a platform = $\frac{\text{Length of train} + \text{Length of platform}}{\text{Speed of train}}$

$$= \frac{120 + 130}{\frac{100}{3}} = \frac{250 \times 3}{100} = 7\frac{1}{2} \sec c$$

38. Speed of train = 135 km/hr = $135 \times \frac{5}{18}$ m/s

Let length of train and length of platform be l metres each

Speed of train = $\frac{\text{Length of train} + \text{Length of platform}}{\text{Time taken by train to cross platform}}$

i.e. $\frac{75}{2} = \frac{l+l}{40}$

or $2(2l) = 75 \times 40$

or $l = \frac{75 \times 40}{2 \times 2} = 750 \text{ metres}$

39. Relative speed of faster train w.r.t. slower train = $45 - 36 = 9 \text{ km/hr} = 9 \times \frac{5}{18} \text{ m/s} = 2.5 \text{ m/s}$

Time taken to cross each other = $\frac{Sum \text{ of lengths of train}}{Relative \text{ speed of train}}$

i.e. $t = \frac{250 + 200}{2.5} = \frac{450}{2.5} = 180 \text{ sec}$

40. Speed of first train = $60 \text{ km/hr} = 60 \times \frac{5}{18} \text{ m/s} = \frac{50}{3} \text{ m/s}$

Let the speed of second train be x m/s.

Relative speed of trains = $\left(\frac{50}{3} + x\right)$ m/s

Relative speed of train = $\frac{Distance\ travelled}{Time\ taken\ to\ cross\ train}$

i.e. $\frac{50}{3} + x = \frac{120 + 130}{6} = \frac{250}{6}$

i.e. $\frac{50}{3} + x = \frac{125}{3}$

or $x = \frac{125 - 50}{3} = 25 \text{ m/s}$

i.e. Speed of second train = 25 m/s

= $25 \times \frac{18}{5}$ km/hr = **90 km/hr**

Boats and Streams

Let the speed of a boat or swimmer in still water be Z km/h and the speed of stream or current be S km/h.

- :. Speed of the boat while going downstream = (Z + S) km/h Speed of the boat while going upstream = (Z - S) km/h
- :. Speed of boat in still water = $\frac{1}{2}$ (Downstream speed + Upstream speed)

Speed of the stream =
$$\frac{1}{2}$$
 (Downstream speed – Upstream speed)

Important Formulae

- 1. A man swims downstream d_1 km in t_1 hours and upstream d_2 km in t_2 hours. Then his speed in still water is $\frac{1}{2} \left(\frac{d_1 t_2 + d_2 t_1}{t_1 t_2} \right)$ km/h and speed of current is $\frac{1}{2} \left(\frac{d_1 t_2 d_2 t_1}{t_1 t_2} \right)$ km/h.
- 2. A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream is S km/h, speed of man in still water and speed of stream will be $S\left(\frac{t_2+t_1}{t_2-t_1}\right)$ km/h and $S\left(\frac{t_2-t_1}{t_2+t_1}\right)$ km/h.
- 3. A man can row a boat in still water at Z km/h. In a stream flowing at S km/h, if it takes him t hours to row to a point and come back, then the distance (D) between the two points is

$$\frac{t(Z^2-S^2)}{2Z}$$
km

4. A person can row a boat u_1 km upstream and d_1 km downstream in t_1 hours. Also, he can row u_2 km upstream and d_2 km downstream in t_2 hours.

Then his upstream speed =
$$\frac{u_1d_2 - u_2d_1}{t_1d_2 - t_2d_1}$$
His downstream speed =
$$\frac{u_1d_2 - u_2d_1}{t_2u_1 - t_1u_2}$$

EXAMPLE 1 A man can row upstream at 10 km/h and downstream at 16 km/h. Find the man's rate in still water and the rate of current.

Man's rate in still water = $\frac{1}{2}$ (Upstream speed + Downstream speed) Solution $=\frac{1}{2}(10+16)=13$ km/h Rate of current = $\frac{1}{2}$ (Downstream speed – Upstream speed) $=\frac{1}{2}(16-10)=3$ km/h

EXAMPLE 2 A man swims downstream 30 km and upstream 18 km, taking a time of 3 hours each. What is the velocity of the current?

Solution Downstream speed = $\frac{30}{3}$ = 10 km/h

Upstream speed =
$$\frac{18}{3}$$
 = 6 km/h

Velocity of current = $\frac{1}{2}$ (Downstream speed – Upstream speed)

$$=\frac{1}{2}(10-6)=2$$
 km/h

EXAMPLE 3 A man can row at a speed of 6 km/h in still water. It takes him twice as long as to row up as to row down the river. Find the rate of stream.

Solution Speed of man in still water = 6 km/h

Let the rate of stream = x km/h

Upstream speed = (6 - x) km/h ٠.

Downstream speed = (6 + x) km/h

3x = 6

But
$$(6+x) = 2(6-x)$$
$$\Rightarrow x + 2x = 2 \times 6 - 6$$

$$\Rightarrow \qquad x + 2x = 2 \times 6 - 6$$

$$\therefore$$
 $x = 2 \text{ km/h}$

EXAMPLE 4 A man can row 50 km upstream and 60 km downstream in 15 hours. Also he can row 30 km upstream and 48 km downstream in 12 hours. Find the speed of the man in still water and the speed of the current.

Solution Let upstream speed be u km/hr and downstream speed be v km/hr

$$\therefore \qquad \frac{50}{u} + \frac{60}{v} = 15 \tag{1}$$

$$\frac{30}{u} + \frac{48}{v} = 10\tag{2}$$

Solving (1) and (2),

$$3 \times (1) - 5 \times (2) \Rightarrow$$

$$3 \times \frac{60}{v} - 5 \times \frac{48}{v} = 3 \times 15 - 5 \times 10$$

$$\frac{180 - 240}{v} = 45 - 50$$

$$\frac{-60}{v} = -5$$

$$v = \frac{-60}{-5} = 12 \text{ km/hr}$$

or

From (2),
$$\frac{30}{u} + \frac{48}{12} = 10$$

or

$$\frac{30}{u} = 10 - 4 = 6$$

or

$$u = \frac{30}{6} = 5 \text{ km/hr}$$

Speed of man in still water = $\frac{u+v}{2} = \frac{12+5}{2} = \frac{17}{2} \text{ km/hr}$

Speed of current =
$$\frac{1}{2}(v - u) = \frac{1}{2}(12 - 5)$$

= $\frac{7}{2}$ km/hr

EXAMPLE 5 A man can row upstream at 8 km/hr and downstream at 12 km/hr. Find man's rate in still water and the rate of current.

Solution Rate in still water = $\frac{1}{2}(12+8) = 10 \text{ km/hr}$

Rate of current =
$$\frac{1}{2}(12-8) = 2 \text{ km/hr}$$

EXAMPLE 6 A man can row at 24 km/hr in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.

Solution Let man's upstream speed be x km/hr and downstream speed be 3x km/hr.

Rate in still water = $\frac{1}{2}(x+3x) = 2x \text{ km/hr}$:.

2x = 24 or x = 12 km/hr i.e.

Upstream speed = 12 km/hr and ٠.

Downstream speed = 3x = 36 km/hr

Rate of stream = $\frac{1}{2}(36-12) = 12 \text{ km/hr}$

EXERCISES

- 1. The speed of a boat in still water is 6 km/h and the speed of the stream is 1.5 km/h. A man rows to a place at a distance of 22.5 km and comes back to the starting point. Find the total time taken by him.
 - (a) 4 hours (b) 6 hours (c) 8 hours
- (d) 10 hours (e) None of these
- 2. A person can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of $2\frac{1}{4}$ km/h, find how far he can row in an hour in still water.
 - (b) $11\frac{1}{4}$ km/h (c) $12\frac{1}{4}$ km/h (d) $13\frac{1}{4}$ km/h (a) $10^{\frac{1}{4}} \text{ km/h}$ (e) None of these
- 3. A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of the boat in still water is 9 km/h and the speed of the current is 3 km/h, find the distance between A and B in kilometres.
 - (b) 12 km (a) 10 km (c) 14 km (d) 16 km (e) None of these
- 4. The current of a stream runs at 1 km/h. A motor boat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motor boat in still water in km/h is.
- (b) 6 (c) 8 (a) 4 (d) 12 (e) None of these

5. A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours, Then the velocity of the current is _____.

- (c) 2 km/h (a) 1 km/h(b) 1.5 km/h(d) 2.5 km/h
- 6. If a man's rate with the current is 12 km/h and the rate of current is 1.5 km/h, then the man's rate against the current in km/h is
 - (a) 10 (b) 9 (c) 8 (d) 6 (e) None of these
- 7. A man can row at 5 km/h in still water and the velocity of the current is 1 km/h. It takes him 1 hour to row to a place and back. How far is the place?
- (d) 3 km (a) 1.2 km (b) 1.8 km (c) 2.4 km (e) None of these 8. The speed of a boat in still water is 6 km/h. It takes the boat twice as long to row up as to row
- down the river. Find the rate of the stream.
 - (a) 1 km/hr, 6 km/hr
- (b) 2 km/hr, 9 km/hr
- (c) 3 km/hr, 8 km/hr

- (d) 4 km/hr, 6 km/hr
- (e) None of these

Λ	A 20	. 1	. 1 441 1	i 10 b Al-	
9.		km downstream		tream in 10 hours. Als ad the rate of the curr	
	(a) 1 km/h	(b) 2 km/h	(c) 3 km/h	(d) 4 km/h	(e) None of these
10	` '	` '		in 6 hours and return	` '
10.	in 9 hours. If the s	stream flows at th	e rate of 3 km/h,	, find the speed of the 1	person in still water.
	(a) 12 km/h	(b) 15 km/h	(c) 18 km/h	(d) 20 km/h	(e) None of these
11.				nstream than to trave 10 metre/hour, the sp	
	(a) 2 m/hr	(b) 2.5 m/hr	(c) 3 m/hr	(d) 4 m/hr	
12.		_		er goes 30 km downstre stream (in km/hr) is	eam and comes back [RRB]
	(a) 4	(b) 5	(c) 6	(d) 10	
13.	A man can row at 1 hour to row to a			city of current is 1 km the place?	hr and it takes him [SSC]
	(a) 2.4 km	(b) 2.5 km	(c) 3 km	(d) 3.6 km	
14.	A boat covers a ce	ertain distance do	ownstream in 1	hour, while it comes b	eack in $1\frac{1}{2}$ hours. If
				eed of the boat in still	_
	(a) 12 km/hr		(c) 14 km/hr		(e) None of these
15.		ostream takes 8 h	ours 48 minutes	s to cover a certain dis enstream. What is the	
					[Bank PO]
	speed of the boat (a) 2:1	and speed of the			
	speed of the boat	and speed of the (b)	current respect	ively?	
16.	speed of the boat (a) 2:1 (d) Cannot be de	and speed of the (b) termined (e) l with a speed o	current respect 3:2 None of these f 13 km/hr in s	ively? (c) 8:3 still water. If the spe	[Bank PO]
16.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave	and speed of the (b) termined (e) l with a speed o	current respect 3:2 None of these f 13 km/hr in s	ively? (c) 8:3 still water. If the spe	[Bank PO] ed of the stream is
	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours	(c) 8:3 still water. If the spekm downstream.	[Bank PO] ed of the stream is [RRB]
	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again	ively? (c) 8:3 still water. If the spekm downstream. (d) 5 hours	[Bank PO] ed of the stream is [RRB]
	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again	ively? (c) 8:3 still water. If the spekm downstream. (d) 5 hours	[Bank PO] ed of the stream is [RRB] minutes. The speed
17.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t	ively? (c) $8:3$ still water. If the spekm downstream. (d) 5 hours ast the stream in $11\frac{1}{4}$	[Bank PO] ed of the stream is [RRB] minutes. The speed [LIC AAO]
17.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t	(c) 8:3 still water. If the spekm downstream. (d) 5 hours st the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre	[Bank PO] ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The
17. 18.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi man's speed again (a) 8.5 km/hr A boat running de	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is nst the current is (b) 9 km/hr ownstream cover	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t s (c) 10 km/hr rs a distance of	(c) 8:3 still water. If the spekm downstream. (d) 5 hours st the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre	ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The [MAT] nile for covering the t in still water?
17. 18.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi man's speed again (a) 8.5 km/hr A boat running d same distance up	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is nst the current is (b) 9 km/hr ownstream cover stream, it takes	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t s (c) 10 km/hr rs a distance of 4 hours. What is	(c) 8:3 still water. If the spekm downstream. (d) 5 hours st the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre (d) 12.5 km/hr 16 km in 2 hours, whis the speed of the boars	ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The [MAT] nile for covering the t in still water? [SBI PO]
17. 18. 19.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi man's speed agai (a) 8.5 km/hr A boat running d same distance up (a) 4 km/hr In one hour, a boa	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is nst the current is (b) 9 km/hr ownstream cover stream, it takes (b) 6 km/hr at goes 11 km alo	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t (c) 10 km/hr rs a distance of 4 hours. What is (c) 8 km/hr ng the stream a	(c) 8:3 still water. If the spekm downstream. (d) 5 hours st the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre (d) 12.5 km/hr 16 km in 2 hours, wh	ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The [MAT] nile for covering the t in still water? [SBI PO] tuate stream. The speed of
17. 18. 19.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi man's speed agai: (a) 8.5 km/hr A boat running d same distance up (a) 4 km/hr In one hour, a boat the boat in still w	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is (b) 9 km/hr ownstream cover stream, it takes (b) 6 km/hr at goes 11 km alo rater (in km/hr) is	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t s (c) 10 km/hr rs a distance of 4 hours. What is (c) 8 km/hr ng the stream a s:	(c) 8:3 still water. If the spekm downstream. (d) 5 hours set the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre (d) 12.5 km/hr 16 km in 2 hours, while the speed of the boat (d) Data inadequal 5 km against the speed of the speed of the speed of the speed of the boat	ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The [MAT] nile for covering the t in still water? [SBI PO]
17. 18. 19.	speed of the boat (a) 2:1 (d) Cannot be de A boat can trave 4 km/hr, find the (a) 2 hours A man can row th (in km/hr) of the (a) 2 A man's speed wi man's speed again (a) 8.5 km/hr A boat running d same distance up (a) 4 km/hr In one hour, a boat the boat in still w (a) 3 A man takes twice	and speed of the (b) termined (e) l with a speed o time taken by th (b) 3 hours ree-quarters of a man in still wate (b) 3 th the current is nst the current is (b) 9 km/hr ownstream cover stream, it takes (b) 6 km/hr at goes 11 km alo rater (in km/hr) is (b) 5 e as long to row a	current respect 3:2 None of these f 13 km/hr in s e boat to go 68 l (c) 4 hours kilometre again r is (c) 4 15 km/hr, and t (c) 10 km/hr rs a distance of 4 hours. What is (c) 8 km/hr ng the stream a s: (c) 8 distance agains	ively? (c) 8:3 still water. If the spekm downstream. (d) 5 hours st the stream in 11 $\frac{1}{4}$ (d) 5 the speed of the curre (d) 12.5 km/hr 16 km in 2 hours, when the speed of the board of the	ed of the stream is [RRB] minutes. The speed [LIC AAO] nt is 2.5 km/hr. The [MAT] nile for covering the t in still water? [SBI PO] tate tream. The speed of [SSC] w the same distance

- 22. A boatman goes 2 km against the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will he take to go 5 km in stationary water? [RRB]
 - (a) 40 minutes

(b) 1 hour

(c) 1 hour 15 minutes

- (d) 1 hour 30 minutes
- 23. Rahul can travel 12 miles downstream in a certain river in 6 hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would take only one hour less than the upstream 12 miles. What is the speed of the current in miles/hour? [MAT]
 - (a) $1\frac{1}{3}$ (b) $1\frac{2}{3}$ (c) $2\frac{1}{3}$

- 24. A man can row $7\frac{1}{2}$ km/hr in still water. If in a river running at 1.5 km/hr, it takes him

50 minutes to row to a place and back, how far off is the place?

[RRB]

- (a) 2 km
- (b) 3 km
- (c) 4 km
- (d) 5 km
- 25. There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr, while the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first?
 - (a) cyclist

(b) sailor

(c) both cyclist and sailor

(d) data inadequate



- 1. (c) **2.** (b) **3.** (b) **4.** (b) **5.** (c) **6.** (b) 7. (c) 8. (b) **9.** (c) **10.** (b)
- **12.** (b) 13. (a) 14. (d) **15.** (c) 17. (d) 18. (c) 19. (b) 11. (a) **16.** (c) **20**. (c)
- 21. (b) **22.** (c) 23. (d) **24.** (b) 25. (a)



Solutions with Necessary Explanation

1. Given speed of boat in still water = 6 km/h

Speed of stream = 1.5 km/h

- \therefore Upstream speed of boat = 6 1.5 = 4.5 km/h
- \therefore Time taken for upstream side = 22.5/4.5 = 5 h

Similarly, downstream speed of boat = 6 + 1.5 = 7.5 km/h

- \therefore Time taken for downstream side = 22.5/7.5 = 3 h
- \therefore Total time taken = 5 + 3 = 8 h



Short cut: Total time =
$$\frac{2 \times \text{Distance} \times \text{Speed in still water}}{(\text{Speed in still water})^2 - (\text{Speed of current})^2}$$
$$= \frac{2 \times 22.5 \times 6}{(6)^2 - (1.5)^2} = 8 \text{ hours}$$

2. Let the speed of person in still water be x km/h.

$$\therefore$$
 Downstream speed = $x + 2\frac{1}{4} = \left(x + \frac{9}{4}\right) \text{km/h}$

Upstream speed =
$$\left(x - \frac{9}{4}\right)$$
 km/h

$$\therefore \qquad \text{Distance} = \left(x + \frac{9}{4}\right)6 = \left(x - \frac{9}{4}\right)9$$

$$\Rightarrow \qquad 6x + \frac{27}{2} = 9x - \frac{81}{4}$$

$$\Rightarrow \qquad 9x - 6x = \frac{27}{2} + \frac{81}{4} = \frac{135}{4}$$

$$\Rightarrow \qquad 3x = \frac{135}{4}$$

or
$$x = \frac{45}{4} = 11 \frac{1}{4} \text{ km/h}$$



Speed in still water = Rate of stream $\times \left(\frac{\text{Sum of upstream and downstream time}}{\text{Difference of upstream and downstream time}} \right)$

$$= \frac{9}{4} \times \left(\frac{9+6}{9-6}\right) = \frac{9 \times 15}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4} \text{ km/h}$$

3. Let the distance be x km.

 \therefore Upstream speed = 9 - 3 = 6 km/h

Downstream speed = 9 + 3 = 12 km/h

Total time taken for upstream and downstream journey = $\frac{x}{6} + \frac{x}{12} = 3$ or 3x/12 = 3 or x = 12 km

4. Let the speed of boat in still water be x km/h.

Upstream speed of boat = (x - 1) km/h

Downstream speed of boat = (x + 1) km/h

$$\therefore \text{ Total time taken} = \frac{35}{x-1} + \frac{35}{x+1} = 12 \text{ hours}$$

$$\therefore \frac{35(x+1)+35(x-1)}{(x+1)(x-1)} = 12$$

or
$$\frac{70x}{(x+1)(x-1)} = 12$$

or
$$70x = 12(x+1)(x-1)$$

$$35x = 6(x+1)(x-1)$$

$$x = 6 \text{ or } 1/6$$

i.e 6 km/h or 1/6 km/h.



Short cut:

Distance = Total time $\times \frac{(\text{Speed in still water})^2 - (\text{Speed of current})^2}{2 \times \text{speed in still water}}$

$$35 = 12 \times \frac{(x^2 - 1^2)}{2 \times x}$$

On simplifying, x = 6 or 1/6

i.e. Speed of boat in still water = 6 km/h or 1/6 km/h

 \therefore x > 1, speed of boat = 6 km/h

5. Let the velocity of current be x km/h.

$$\frac{24}{(u-x)} + \frac{36}{(u+x)} = 6\tag{1}$$

$$\frac{36}{(u-x)} + \frac{24}{(u+x)} = 6\frac{1}{2} \tag{2}$$

$$1.5 \times (1) \qquad \Rightarrow \qquad \frac{36}{(u-x)} + \frac{36 \times 1.5}{(u+x)} = 9 \tag{3}$$

(3) - (2)
$$\Rightarrow \frac{1.5 \times 36 - 24}{(u+x)} = 9 - 6\frac{1}{2} = \frac{5}{2}$$

or
$$\frac{30}{u+x} = \frac{5}{2}$$

or
$$u + x = \frac{30 \times 2}{5} = 12 \text{ km/h}$$

$$\therefore (2) \qquad \Rightarrow \qquad \frac{36}{u-x} = 6\frac{1}{2} - \frac{24}{12}$$
$$= 6\frac{1}{2} - 2 = 4\frac{1}{2} = \frac{9}{2}$$

$$\therefore \qquad u - x = \frac{36 \times 2}{9} = 8 \text{ km/h}$$

$$\therefore x = \frac{(u+x) - (u-x)}{2} = \frac{12 - 8}{2} = 2 \text{ km/h}$$

6. Let the man's rate in still water be x km/h.

$$x + 1.5 = 12 \text{ km/h}$$

$$\therefore$$
 $x = 12 - 1.5 = 10.5 \text{ km/h}$

 \therefore The man's rate against the current = 10.5 - 1.5 = 9 km/h



Short cut: The man's rate against current = Man's rate with the current - $2 \times \text{rate of current}$

$$= 12 - 2 \times 1.5 = 9 \text{ km/h}$$

7. Distance = Total time $\times \frac{(\text{Speed in still water})^2 - (\text{Speed of current})^2}{2 \times \text{Speed in still water}}$

$$= 1 \times \frac{(5^2 - 1^2)}{2 \times 5} = 2.4 \text{ km}$$

- 8. Let the man's rate for upstream be x km/h.
 - \therefore The man's rate for downstream = 2x km/h.
 - \therefore Man's rate in still water = $\frac{x + 2x}{2} = \frac{3x}{2}$ km/h.

Here, 3x/2 = 6 or x = 4 km/h

:. Rate of stream = $\frac{1}{2}$ (Downstream rate – Upstream rate) = $\frac{1}{2}(2x - x) = \frac{x}{2} = \frac{4}{2} = 2 \text{ km/h}$



Short cut: Let the rate of stream be x km/h

$$6 + x = 2(6 - x)$$

or
$$3x = 2 \times 6 - 6 = 6$$

$$x = \frac{6}{3} = 2 \text{ km/h}$$

9. Let the speed of man in still water be u km/h and rate of current be v km/h.

$$\therefore \frac{30}{u-v} + \frac{44}{u+v} = 10 \tag{1}$$

$$\frac{40}{u-v} + \frac{55}{u+v} = 13\tag{2}$$

$$4 \times (1) \qquad \Rightarrow \qquad \frac{120}{u - v} + \frac{176}{u + v} = 40$$
 (3)

$$3 \times (2) \qquad \Rightarrow \qquad \frac{120}{u - v} + \frac{165}{u + v} = 39$$
 (4)

Solving (3) and (4), we get,

$$\frac{176 - 165}{u + v} = 40 - 39 = 1$$

$$u + v = \frac{176 - 165}{1} = 11 \text{ km/h}$$

$$∴ (1) ⇒ \frac{30}{u - v} = 10 - \frac{44}{u + v}$$

$$= 10 - \frac{44}{11}$$

$$= 10 - 4 = 6$$

$$u - v = \frac{30}{6} = 5 \text{ km/h}$$

$$u = \frac{11+5}{2} = 8 \text{ km/h}$$

$$v = \frac{11-5}{2} = 3 \text{ km/h}$$



:.

Short cut: (By use of multiple cross – multiplication)

Upstream	Downstream	Time
distance	distance	10
30	44	10
40	55	13
Upstream speed of ma	$\mathbf{n} = \frac{30 \times 55 - 40}{55 \times 10 - 44}$	$\frac{0 \times 44}{4 \times 13} = 5 \text{ km/h}$
Downstream speed of ma	$\mathbf{n} = \frac{30 \times 55 - 40}{30 \times 13 - 40}$	$\frac{0 \times 44}{0 \times 10} = 11 \text{ km/h}$
Speed of ma	$n = \frac{5 + 11}{2} = 8 k$	m/h
	$m = \frac{11-5}{2} = 3 \text{ k}$	

10. The speed of the person in still water = Rate of current $\times \frac{(Sum \ of \ times)}{(Difference \ of \ times)}$

$$= 3 \times \frac{(9+6)}{(9-6)} = 15 \text{ km/h}$$

11. Let speed of stream be x km/hr

 \therefore Downstream speed = (10 + x) miles/hr.

Upstream speed = (10 - x) miles/hr.

$$\therefore \frac{36}{(10-x)} - \frac{36}{(10+x)} = \frac{90}{60}$$

i.e.
$$\frac{\left\{(10+x)-(10-x)\right\}36}{(10+x)(10-x)} = \frac{3}{2}$$

or
$$\frac{72x}{(100 - x^2)} = \frac{3}{2}$$

or
$$144x = 300-3x^2$$

or
$$x^2 + 48x - 100 = 0$$

$$\therefore (x+50) (x-2) = 0$$

$$\Rightarrow x = 2 \text{ miles/hour}$$

12. Let the speed of stream be x km/hr.

 \therefore Downstream speed = (15 + x) km/hr.

Upstream speed = (15 - x) km/hr.

i.e.
$$\frac{30}{(15+x)} + \frac{30}{(15-x)} = 4.5$$
or
$$\frac{30[(15-x) + (15+x)]}{(15+x)(15-x)} = 4.5$$
or
$$\frac{30 \times 30}{15^2 - x^2} = 4.5$$
or
$$900 = 4.5 (15^2 - x^2)$$

or
$$x^{2} = \frac{4.5 \times 15^{2} - 900}{4.5}$$
$$= 15^{2} - 200 = 25$$

13. Let the distance of the place be x km.

or

Speed downstream = 5 + 1 = 6 km/hr Speed upstream = 5 - 1 = 4 km/hr

$$\frac{x}{6} + \frac{x}{4} = 1$$
or
$$x = \frac{1 \times 6 \times 4}{(6 + 4)} = \frac{24}{10} = 2.4 \text{ km}$$

x = 5 km/hr

14. Let speed of boat in still water be x km/hr.

.. Upstream speed =
$$(x - 3)$$
 km/hr
Downstream speed = $(x + 3)$ km/hr

or
$$0.5x = 3 + 4.5 = 7.5$$

or
$$x = \frac{7.5}{0.5} = 15 \text{ km/hr}$$

15. Let speed of boat in still water be x km/hr.

and speed of current = kx km/hr

:. Upstream speed = (x - kx) = (1 - k)x km/hrdownstream speed = (x + kx) = (1 + k) x km/hr

$$\therefore \qquad (1-k)x \times \left(8 + \frac{48}{60}\right) = (1+k)x \times 4$$

or
$$(1-k) 8.8 = (1+k) 4$$

or
$$(1-k) \ 2.2 = (1+k)$$

or
$$(2.2 + 1) k = 2.2 - 1 = 1.2$$

$$\therefore \qquad \qquad k = \frac{1.2}{3.2} = \frac{3}{8}$$

 \therefore Speed of boat: Speed of current = 1: $k = 1: \frac{3}{8}$ or 8:3

16. Downstream speed = 13 + 4 = 17 km/hr

$$\therefore$$
 Time taken by boat to go 68 km downstream = $\frac{68}{17}$ = 4 hours

17. Upstream speed = $\frac{3/4 \text{ km}}{45/4 \times 60 \text{ hr}}$

$$= \frac{3}{4} \times \frac{4 \times 60}{45} = 4 \text{ km/hr}$$

18. Downstream speed = 15 km/hr

Speed of current = .5 km/hr

 \therefore Speed of man in still water = 15 - 2.5 = 12.5 km/hr

Speed against current = 12.5 - 2.5 = 10 km/hr

19. Let speed of boat in still water be x km/hr.

$$\therefore \frac{1}{2} \left(\frac{16}{2} + \frac{16}{4} \right) =$$
Speed of boat in still water

 $x = \frac{1}{2}(8+4)$ i.e.

= 6 km/hr

20. Let speed of boat in still water be x km/hr

$$\therefore \qquad \text{Speed in still water } x = \frac{11+5}{2} = 8 \text{ km/hr}$$

21. Let speed of boat in still water be x km/hr and that of stream be kx km/hr.

$$\frac{S}{(x-kx)} = 2\frac{S}{(x+kx)}$$
or
$$\frac{1}{1-k} = \frac{2}{1+k}$$
or
$$1+k=2(1-k)$$
or
$$k+2k=2-1$$

$$3k=1$$
or
$$k=\frac{1}{2}$$

 \therefore Required ratio = 1: k = 1: $\frac{1}{2} = 3:1$

22. Downstream speed = $\frac{1}{10/60}$ = 6 km/hr

 \mathbf{or}

Upstream speed = $\frac{2}{1}$ = 2 km/hr

Speed of boatman in still water = $\frac{\text{Upstream speed} + \text{Downstream speed}}{2}$ $=\frac{2+6}{2}=4 \text{ km/hr}$

(1)

.. Time taken to go 5 km in stationary water

$$=\frac{5}{4}$$
 = 1.25 hours = 1 hour 15 minutes

23. Let the speed of current be u miles/hour and speed in still water be x miles/hour.

$$\frac{12}{x-u} - \frac{12}{x+u} = 6 \text{ hours}$$

$$\frac{12\{(x+u) - (x-u)\}}{(x+u)(x-u)} = 6 \text{ hours}$$

$$\frac{12 \times 2u}{x^2 - u^2} = 6 \text{ hours}$$

$$\frac{12}{2x-u} - \frac{12}{2x+u} = 1$$

$$\frac{12\{(2x+u)-(2x-u)\}}{(2x+u)(2x-u)}=1$$

$$\frac{12 \times 2u}{4x^2 - u^2} = 1 \tag{2}$$

(1)
$$\Rightarrow \frac{4u}{x^2 - u^2} = 1$$

or $x^2 - u^2 = 4u$
or $x^2 = u^2 + 4u$
(2) $\Rightarrow 4x^2 - u^2 = 24u$
 $4x^2 = u^2 + 24u$

$$x^2 = \frac{u^2 + 24u}{4}$$
 (B)

Equating (A) and (B), we get,

$$u^{2} + 4u = \frac{u^{2} + 24u}{4}$$

$$\Rightarrow \qquad 4u^{2} + 16u = u^{2} + 24u$$

$$\Rightarrow \qquad 3u^{2} = 8u$$
or
$$u = \frac{8}{3} \text{ miles/hour} = 2\frac{2}{3} \text{ miles/hour}$$

24. Let the place be at a distance of x km.

Upstream speed =
$$7\frac{1}{2} - 1\frac{1}{2} = 6 \text{ km/hr}$$

Downstream speed =
$$7\frac{1}{2} + 1\frac{1}{2} = 9 \text{ km/hr}$$

$$\frac{x}{6} + \frac{x}{9} = \frac{50}{60}$$
i.e.
$$\frac{3x + 2x}{18} = \frac{5}{6}$$
or
$$(3x + 2x) \cdot 6 = 5 \times 18$$
or
$$5x = \frac{5 \times 18}{6}$$
or
$$5x = 15$$
or
$$x = 3 \text{ km}$$

$$25. \frac{x}{10 + 4} + \frac{x}{10 - 4} = \frac{x}{14} + \frac{x}{6}$$

$$= \frac{3x + 7x}{42} = \frac{10x}{42}$$

$$= \frac{5x}{21} \text{ hour (Time taken by sailor)}$$

$$\frac{2x}{12} = \frac{x}{6} \text{ hours (Time taken by cyclist)}$$

$$\therefore \frac{5x}{21} > \frac{x}{6}, \text{ Cyclist will return first.}$$



Short cut: Average speed of sailor =
$$\frac{2uv}{u+v}$$

= $\frac{2(10+4)(10-4)}{(10+4)+(10-4)}$
= $\frac{2\times14\times6}{(14+6)}$
= $\frac{2\times14\times6}{20}$ = 8.4 km/hr

(: His speed on both sides are the same)

Average speed of cyclist = 12 km/hr

:. Cyclist will return first.

Elementary Mensuration I (Measurement of Area)

Important Formulae

AREA AND PERIMETER

Triangle

1. (a) Area = 1/2 bh; when b is the base and h is the height.

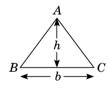
(b)
$$b = \frac{2A}{h}$$
 or $b^2 = 2A \times \frac{b}{h}$ or $b = \sqrt{2A\left(\frac{b}{h}\right)}$

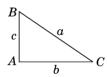
(c)
$$h = \frac{2A}{b} = \sqrt{\frac{2A}{(b/h)}}$$

2.
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
; where $s = \frac{1}{2}(a+b+c)$; $p = a+b+c$

3.
$$a^2 = b^2 + c^2$$
 (Pythagoras theorem)

$$4. \quad A = \frac{1}{2}bc$$





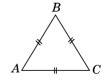
Isosceles triangle

5.
$$A = \frac{1}{4}b\sqrt{(4a^2 - b^2)}$$

Equilateral triangle

$$\angle A = \angle B = \angle C = \frac{180^{\circ}}{3} = 60^{\circ}$$

6.
$$A = \frac{\sqrt{3}}{4}a^2$$



Right-angled isosceles triangle

$$\angle A = 90^{\circ}, \ b = c = \frac{BC}{\sqrt{2}}$$

6. (a)
$$A = \frac{1}{2}b^2 = \frac{1}{4}(hypotenuse)^2$$

Quadrilateral

7.
$$A = \sqrt{4(d_1d_2)^2 - (b^2 + d^2 - a^2 - c^2)^2}$$

 $P = a + b + c + d$

Special Case

Rectangle

8. (a) Area =
$$l \times b$$

 Length = $\frac{A}{b}$ or $b = \frac{A}{l}$

(b) Diagonal
$$AC = BD = \sqrt{l^2 + b^2}$$

(c)
$$P = 2(l + b)$$

Square

9. (a) Area =
$$l^2$$

(b) Diagonal (d) =
$$AC = BD = \sqrt{2}I$$

or $I = \frac{(\text{diagonal})}{\sqrt{2}} = \frac{d}{\sqrt{2}}$

(c)
$$A = l^2 = \frac{d^2}{2}$$
; $P = 4l$

Parallelogram

10. Area = Base × Height
or
$$A = l \times h$$

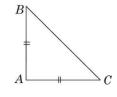
 $P = 2(l + b)$

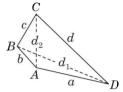
Rhombus

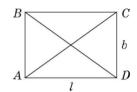
11. (a) Area =
$$\frac{1}{2}$$
 (Product of diagonals)
= $\frac{1}{2} d_1 d_2$

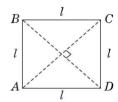
(b) Side
$$l = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

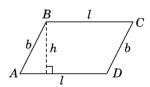
(c)
$$P = 4l$$

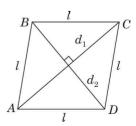










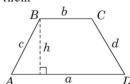


Trapezium

12. Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) × Distance between them

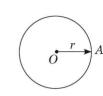
$$A = \frac{1}{2} (a + b) \times h$$

$$P = a + b + c + d$$



Circle

- 13. (a) Area = $\pi r^2 = \frac{\pi}{4}D^2$ where $\pi = \frac{22}{7}$ and D = 2r $D = \sqrt{\frac{4A}{\pi}}$
 - (b) Perimeter $P = 2\pi r = \pi D$ and $D = \frac{P}{\pi}$ $A = \frac{P^2}{4\pi} \implies P = \sqrt{4\pi \times A}$



Arc

14. A part of the circumference of the circle is called arc of the circle.

$$\widehat{ACB} = \frac{\theta}{360^{\circ}} \times 2\pi r = r \, \boldsymbol{\theta}$$



Sector

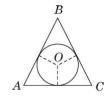
15. Area of sector $AOB = \pi r^2 \times \frac{\theta}{360^{\circ}}$

$$= \frac{1}{2} \times \operatorname{arc} ACB \times r$$



Room

- 16. (a) Area of 4 walls of room = 2h(l+b)
 - (b) Area of floor and 4 walls = 2h(l + b) + lb
 - (c) Area of floor, roof and 4 walls = $2\{h(l+b) + lb\}$
- 17. (a) Polygon: A plane figure bounded by multiple number of sides.
 - (b) Regular polygon: Polygon whose all sides are equal.
- 18. Incircle or Inscribed circle: It is the circle inside the polygon whose all sides are tangent to the circle. For an equilateral triangle of side a, radius of inscribed circle = $\frac{a}{2\sqrt{3}}$



19. Circumcircle: It is the circle whose circumference touches all the vertices of the polygon.

For an equilateral triangle, radius of circumcircle = $\frac{a}{\sqrt{3}}$

(a) Area of a regular polygon = $\frac{1}{2} \times \text{Number of sides}$

× Radius of the inscribed circle



- (b) Area of regular hexagon = $\frac{3\sqrt{3}}{2}$ (side)² = 2.598 (side)²
- (c) Area of a regular octagon = $2(\sqrt{2} + 1) (\text{side})^2 = 4.828 (\text{side})^2$
- (d) For a regular polygon of n equal sides its vertex angle θ is given by $\theta = \left(\frac{n-2}{n}\right)180^{\circ}$
- 20. Cyclic quadrilateral: It is a quadrilateral whose vertices lie on the circumference of the circle.

$$A = \sqrt{s(s-a)(s-b)(s-c)(s-d)}$$
where $s = \frac{a+b+c+d}{2}$

$$\angle A + \angle C = \angle B + \angle D$$



Isosceles triangle

21. (a) Area =
$$A = \frac{b}{4}\sqrt{(4a^2 - b^2)}$$

(b) If the triangle is isosceles right triangle, $A = \frac{1}{2}(\text{side})^2 = \frac{1}{4}(\text{hypotenuse})^2$

Path, verandah, courtyard, etc. around a rectangular or square space

- 22. (a) If the garden is l metres long and b metre broad and a path having a width of w metre is constructed around the garden on its side, area of path = 2w(l + b + 2w) m²
 - (b) If the same path is inside, area of path = $2w [l + b 2w] m^2$
 - (c) If the path outside and inside taken together, total area = 4w[l + b] m²
 - (d) A square field is surrounded by a path w metres wide on its outside. Area of path is A_0 m². What is the area of the field?

Area of field =
$$\left[\frac{A_0 - 4w^2}{4w} \right] \text{ m}^2$$

- (e) If in a parallelogram, two heights h_1 and h_2 and its area are given, then their sides are given by $l=A/h_1;\ b=A/h_2$
- (f) One of the diagonals of a rhombus of side 's' measures d_1 . The area of rhombus is given

by
$$d_1 \sqrt{s^2 - \left(\frac{d_1}{2}\right)^2}$$
.

(g) If d_1 and d_2 are the diagonals of a rhombus, find area and perimeter.

$$s = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

Perimeter P = 4s

$$A = \frac{1}{2} d_1 d_2$$

(h) Area of a regular octagon

$$A = 2\sqrt{2} + 1 (\text{side})^2$$

All the three sides of a triangle is increased by x%. Find the % change in its area.

Solution:
$$S = \frac{a+b+c}{2}$$

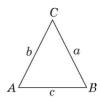
% increase in area = $x \left(2 + \frac{x}{100}\right)$ %

Similarly, when all sides of a square is increased by x%, the % increase in

$$area = x \left(2 + \frac{x}{100}\right)\%$$

The % increase in area of any two dimensional figure when both dimensions

are increased by
$$x\%$$
 is $x\left(2+\frac{x}{100}\right)\%$.



A square is inscribed in a circle of radius r. Then area of inscribed square is $2r^2$. If the circumference and area of a circle are numerically equal, then the diameter is equal to 4 cm

23. Wheel: The number of revolutions of the wheel needed to cover a distance

$$= \frac{\text{Distance to be covered}}{\pi \times \text{Diameter of wheel}}$$

Cuboid

- 24. Total surface area = 2(lb + bh + lh)
- 25. Face diagonals = $\sqrt{l^2 + h^2}$, $\sqrt{b^2 + h^2}$ and $\sqrt{l^2 + b^2}$
- 26. Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$

Cube

- 27. Volume = a^3 cubic units
- 28. Total surface area = $6a^2$ square units

$$\therefore \qquad Volume = \left(\sqrt{\frac{Surface area}{6}}\right)^3$$

- 29. Face diagonal = $\sqrt{2}a$
- 30. Body diagonal = $\sqrt{3}a$

Cylinder

- 31. Volume = $\pi r^2 h$ cubic units
- 32. Area of curved surface = $2\pi rh$ square units
- 33. Area of the base = Area of top = πr^2 square units
- 34. Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$ square units

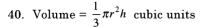
Sphere

- 35. Volume = $\frac{4}{3}\pi r^3$ cubic units
- 36. Surface area = $4\pi r^2$ square units

Hemisphere

- 37. Volume = $\frac{2\pi r^3}{3}$ cubic units
- 38. Area of curved surface = $2\pi r^2$ square units
- 39. Total surface area = $3\pi r^2$ square units

Cone

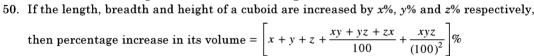


- 41. Slant height $l = \sqrt{r^2 + h^2}$
- 42. Area of base = πr^2 square units
- 43. Area of curved surface = $\pi r l = \pi r \sqrt{r^2 + h^2}$ square units
- 44. Total surface area of cone = $\pi r l + \pi r^2 = \pi r (l + r)$ square units

Frustum of a right circular cone

45.
$$S = \sqrt{H^2 + (R - r)^2}$$

- 46. Volume = $\frac{\pi H}{3} (R^2 + r^2 + Rr)$ cubic unit
- 47. Area of curved or slant surface = $\pi(R+r)$ S square units
- 48. Total surface area of the frustum = $\pi[(R^2 + r^2) + S(R + r)]$ square units
- 49. If the radius of a cylinder is increased by x% and its height increased by y% then volume will be increased by $\left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2y}{(100)^2}\right]\%$

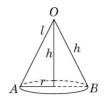


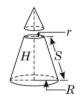
51. If the sides of a cube are increased by x%, then the percentage increase in its surface area,

$$= \left[2x + \frac{x^2}{100}\right]\%$$

- 52. The radius (R) of the largest sphere that can be carved out of a cone of radius r, height h and slant height l is given by $R = \frac{rh}{r+l}$
- 53. Volume flow rate of a liquid through an opening = Cross sectional area of opening × Speed of liquid
- 54. (a) Area of a square = $(\text{side})^2 = \frac{1}{2} (\text{diagonal})^2$
 - (b) Perimeter of a square = $4 \times \text{side}$
- 55. Area of 4 walls of a room = $2(length + breadth) \times height$
- 56. Area of parallelogram = Base \times Height
- 57. Area of a rhombus = $\frac{1}{2}$ × (Product of diagonals)

 When d_1 and d_2 are the two diagonals, then side of rhombus = $\frac{1}{2}\sqrt{d_1^2+d_2^2}$
- 58. (a) Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ (side)²
 - (b) Perimeter of an equilateral triangle = $3 \times \text{side}$





59. Area of an isosceles triangle

$$A = \frac{b}{4} \sqrt{4a^2 - b^2}$$

$$h = \sqrt{a^2 - (b/2)^2} = \frac{1}{2} \sqrt{4a^2 - b^2}$$



60. If a, b, and c are lengths of the sides of a triangle and $s = \frac{1}{2}(a+b+c)$, then

Area of triangle =
$$\sqrt{s(s-a)(s-b)(s-c)}$$

- 61. Area of triangle = $\frac{1}{2}$ Base × Height
- 62. Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides × Perpendicular distance between them) = $\frac{1}{2}(a+b)h$

where a and b are parallel sides and h, perpendicular distance between the sides a and b.

- 63. (a) Circumference of circle = $2\pi r$
 - (b) Area of circle = πr^2
 - (c) arc $AB = \frac{2\pi r\theta}{360^{\circ}}$ where $\angle AOB = \theta$ and r, radius of part of circle forming arc.
 - (d) Area of sector $AOB = \frac{\pi r^2 \theta}{360^{\circ}}$



- (e) Area of sector $AOB = \frac{1}{2} \times \operatorname{arc} AB \times r$
- 64. In a parallelogram

Area =
$$2\sqrt{s(s-a)(s-b)(s-d)}$$

where a and b are two adjacent sides and d is the diagonal connecting the ends of the two sides.

PROBLEMS ON RECTANGLES AND SQUARES

Type I Direct Application of Formula

EXAMPLE 1 Calculate the area of a rectangle 23 metres 7 decimetres long and 14 metres 4 decimetres 8 centimetres wide.

Solution Length = 23.70 m

Breadth = 14.48 m

Area = 23.70×14.48 square metres

= 343.18 square metres

EXAMPLE 2 Find the diagonal of a rectangle whose sides are 12 metres and 5 metres.

Solution Length of diagonal = $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$ metres

Type II Carpeting a Floor

EXAMPLE 3 How many metres of a carpet 75 cm wide will be required to cover the floor of a room which is 20 metres long and 12 metres broad?

Solution Length required =
$$\frac{\text{(Length} \times \text{Breadth) of room}}{\text{Width of carpet}} = \frac{20 \times 12}{0.75} = 320 \text{ m}$$

EXAMPLE 4 What amount will be required in carpeting the floor in the above example if the carpet is available at $\stackrel{?}{\stackrel{?}{\sim}} 30/m$?

Solution



Short cut: Amount required = Rate/m × Required length of carpet

= Rate/m ×
$$\frac{\text{Length of room} \times \text{Breadth of room}}{\text{Width of carpet}}$$

= $30 \times \frac{20 \times 12}{0.75}$ = ₹ 9600

Type III Paving a Courtyard with Tiles

EXAMPLE 5 How many paving stones of size $2.5 \text{ m} \times 2 \text{ m}$ are required to pave a rectangular courtyard 30 m long and 16.5 m wide?

Solution Number of paving stones required = $\frac{\text{Length} \times \text{Breadth of courtyard}}{\text{Length} \times \text{Breadth of each paving stone}}$

$$=\frac{30\times16.5}{2.5\times2}=99$$

EXAMPLE 6 What will be the amount required to pave stones in Example 5, if the paving stone of the given dimension is available at ₹ 10 /piece?

Solution



 $Short\ cut:$

Amount required = Price per stone $\times \frac{\text{Length} \times \text{Breadth of courtyard}}{\text{Length} \times \text{Breadth of each paving stone}}$

$$= 10 \times \frac{30 \times 16.5}{2.5 \times 2} = 799$$

Type IV Paving with Largest Square Tiles

EXAMPLE 7 A hall 39 m 10 cm long and 35 m 70 cm broad is to be paved with square tiles. Find the largest tile so that the tiles exactly fit and also find the number of tiles required.

Solution



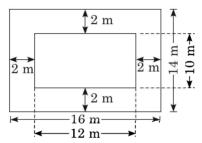
Short cut: Side of largest possible tile = HCF of length and breadth of room

$$= 1.70 \text{ m}$$

Number of tiles required =
$$\frac{\text{Length} \times \text{Breadth of room}}{(\text{HCF of length and breadth of room})^2}$$
$$= \frac{39.10 \times 35.70}{1.00 \times 10^{-3}} = 483$$

Type V Path round a Garden, Verandah round a Room

EXAMPLE 8 A rectangular hall 12 m long and 10 m broad, is surrounded by a verandah 2 metres wide. Find the area of the verandah?



Solution



Short cut:

- (i) When verandah is outside the room, surrounding it,
 - Area of verandah = $2 \times \text{Width of verandah}$

$$\times$$
 [Length + Breadth of room + 2(Width of verandah)]

(ii) When path is within the garden, surrounding by it,

Area of path = $2(Width of path) \times [Length + Breadth of garden - 2(Width of path)]$

- · Verandah is outside the room, formula (i) may be applied.
- \therefore Area of verandah = $2 \times 2 \times [10 + 12 + 2 \times 2] = 4 \times 26 = 104 \text{ m}^2$

Solution Path is within the garden, above formula (ii) may be applied to find the area of path i.e.

area of path =
$$2 \times 2.5 \times [112 + 78 - 2 \times 2.5] = 5 \times 185 = 925 \text{ m}^2$$

 \therefore Cost of construction = Rate/m² × area = $20 \times 925 = ₹ 18500$

I. When area of the path is given, to find the area of garden enclosed (garden is square in shape)

EXAMPLE 10 A path 2 m wide running all round a square garden has an area of 9680 m^2 . Find the area of the part of the garden enclosed by the path.

Solution



Short cut: Area of square garden =
$$\left[\frac{\text{Area of path } - 4 \text{ (width of path)}^2}{4 \times \text{ width of path}}\right]^2$$

:. Area of garden =
$$\left[\frac{9680 - 4(2)^2}{4 \times 2}\right]^2 = \left[\frac{9664}{8}\right]^2 = (1208)^2 = 1459264 \text{ m}^2$$

II. When area of path is given to find the width of path

EXAMPLE 11 A path all around the inside of a rectangular park 37×30 m occupies 570 m. Find the width of the path.

Solution By using formula (ii) in Example 8,

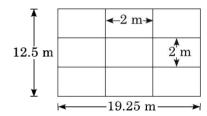
Area of path = $2 \times \text{Width of path} \times [\text{Length} + \text{Breadth of path} - 2 \times \text{Width of path}]$ Let the width of path be x metres.

$$\therefore \qquad \text{Area of path} = 2 \times x \times [37 + 30 - 2 \times x]$$
i.e.
$$570 = 134x - 4x^2$$

$$\Rightarrow \qquad 4x^2 - 134x + 570 = 0$$
Solving we get, $x = 5$ m

III. Paths crossing each other

EXAMPLE 12 An oblong piece of ground measures 19 m 2.5 dm \times 12 m 5 dm. From the centre of each side a path 2 m wide goes across to the centre of the opposite side. What will be the area of the path? Find the cost of paving these paths at the rate of ₹ 13.2/m².



Solution Cost of paving = Area of path \times Rate of paving = $59.5 \times 13.2 = ₹785$



Short cut:

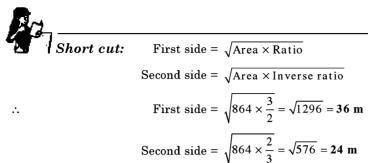
- (a) Area of path = (Width of path) [Length + Breadth of park Width of path]
- (b) Area of park minus path = (Length of park Width of path) \times (Breadth of park Width of path) Here, area of path = $2(19.25 + 12.5 2) = 2 \times 29.75 = 59.5$ m²

$$Cost = Rate \times Area = 13.2 \times 59.5 = ₹785.4$$

Type VI Area and Ratio

EXAMPLE 13 The sides of a rectangular field of 864 m² are in the ratio of 3: 2. Find the sides.

Solution

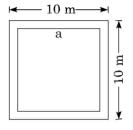


Type VII Miscellaneous Cases (Carpet and Oil Cloth)

EXAMPLE 14 In the centre of a room 10 metres square, there is a square of turkey carpet, and the rest of the floor is covered with oil cloth. The carpet and the oil cloth cost respectively $\stackrel{?}{\underset{?}{|}}$ 15 and $\stackrel{?}{\underset{?}{|}}$ 6.50 per square metre, and the total cost of the carpet and the oil cloth is $\stackrel{?}{\underset{?}{|}}$ 1338.50. Find the width of the oil cloth border.

Solution Area of square room = $10 \times 10 = 100 \text{ m}^2$

Average cost of carpet and oil cloth/m² = $\frac{1338.50}{100}$ = $\frac{1338.50}{100}$



By alligation rule, area of square carpet is $81~\text{m}^2$. Therefore side of carpet is 9~m. Room is of size $10~\text{m} \times 10~\text{m}$. Area of oil cloth = $19~\text{m}^2$

Therefore,

$$10 - 9 = 2 \times Width of border$$

$$\therefore \qquad \text{Width of border} = \frac{1}{2} \text{ metre} = 5 \text{ dm}$$

 \therefore Width of oil cloth = 5 dm

EXAMPLE 15 A square field of 2 km² is to be divided into two equal parts by a fence which coincides with a diagonal. Find the length of the fence.

Solution Area of field = 2 km^2

Diagonal
$$\left[\sqrt{2} \times (\text{side of field})\right] = \sqrt{2} \times \sqrt{2} = 2 \text{ kilometres}$$

EXAMPLE 16 A square field of area 31684 m² is to be enclosed with wire placed at heights 1, 2, 3, 4 metres above the ground. What length of the wire will be required, if its length required for each circuit is 5% greater than the perimeter of the field?

Solution Area of the field = 31684 m^2

$$\therefore \qquad \text{Perimeter} = \sqrt{31684} \times 4 \text{ m} = 178 \times 4$$

$$\therefore$$
 Length of each circuit = $178 \times 4 \times \frac{105}{100}$ m

Here wire is placed at 4 different heights

.. Total length of wire required =
$$178 \times 4 \times \frac{105}{100} \times 4 = 2990.4$$
 metres

PROBLEMS ON TRIANGLES

Type I Direct Application of Formulae

EXAMPLE 17 The base of a triangular field is 880 metres and its height 550 metres. Find the area of the field. Also calculate the changes for supplying water to the field at the rate of $\stackrel{?}{\stackrel{?}{\stackrel{?}{$\sim}}}$ 24.25/ hectometre².

Solution Area of the field =
$$\frac{\text{Base} \times \text{Height}}{2} = \frac{880 \times 550}{2} \,\text{m}^2$$

= $\frac{440 \times 550}{100 \times 100} \,\text{hectometre}^2 = 24.20 \,\text{hectometre}^2$

Cost of supplying water to the whole field =Rate/sq. hectometre \times Area of field

$$= 24.25 \times 24.20 = 3586.85$$

EXAMPLE 18 The base of a triangular field is 3 times its height. If the cost of cultivating the field at ₹ 36.72/hectare is ₹ 495.72, find its base and height.

Area of the field =
$$\frac{495.72}{36.72}$$
 hectares
= $\frac{27}{2}$ hectares

But,

area of field =
$$\frac{1}{2} \times \text{Base} \times \text{Height}$$

= $\frac{1}{2} \times (3 \times \text{Height}) \times \text{Height}$ (: Base = $3 \times \text{Height}$)
= $\frac{3}{2} (\text{Height})^2$

Equating R.H.S. of above two equations,

$$\frac{3}{2}(\text{Height})^2 = \frac{27}{2} \text{ hectares}$$

$$\therefore \qquad (\text{Height})^2 = \frac{27}{3} = 9 \text{ hectares}$$
or
$$\text{Height} = 3 \times 10^2 \text{ m}$$

$$= 300 \text{ m}$$

$$\therefore \text{Page } = 3 \times \text{Height} = 3 \times 300 = 900 \text{ m}$$

 $\therefore \quad \text{Base} = 3 \times \text{Height} = 3 \times 300 = 900 \text{ m}$

EXAMPLE 19 Find the area of a triangle whose sides are 50 metres, 78 metres, 112 metres respectively and also find the perpendicular from the opposite angle on the side 112 metres.

C

Solution Here a = 50 m; b = 78 m; c = 112 m

$$s = \frac{1}{2} (a + b + c)$$

$$= \frac{1}{2} (50 + 78 + 112) = 120 \text{ metres}$$

$$s - a = 120 - 50 = 70 \text{ m};$$

$$s - b = 120 - 78 = 42 \text{ m};$$

$$s - c = 120 - 112 = 8 \text{ m};$$

$$Area = \sqrt{s(s - a)(s - b)(s - c)}$$

$$\therefore \text{ Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{120 \times 70 \times 42 \times 8}$$

$$= \sqrt{120 \times 70 \times 7 \times 6 \times 2 \times 4}$$

$$= \sqrt{7^2 \times 12^2 \times 2^2 \times 10^2}$$

$$= 7 \times 12 \times 2 \times 10 = 1680 \text{ m}^2$$

$$\therefore \qquad \text{Perpendicular} = \frac{\text{Area}}{1/2 \text{ base}} = \frac{1680}{1/2 \times 112} = 30 \text{ m}$$

Type II Faster Methods for Triangle Problems

Solve Examples (20 - 22) by faster methods.

Solution to Example 20: Since Example 17 involves direct application of the formula, a method faster than the one adopted can't be used.

Solution to Example 21: The ratio between base and height in Example 18 is 3:1.

∴ Base =
$$\sqrt{2 \times \text{Area} \times \text{Ratio}}$$

Height = $\sqrt{2 \times \text{Area} \times \text{Inverse ratio}}$

∴ Base = $\sqrt{2 \times \frac{27}{2} \times \frac{3}{1}} = 900 \text{ m}$

Height = $\sqrt{2 \times \text{Area} \times \text{Inverse ratio}}$

$$= \sqrt{2 \times \frac{27}{2} \times \frac{1}{3}} = 300 \text{ m}$$

Solution to Example 22: Since Example 19 involves the direct application of formula, no short cut can be employed.

Problems on Parallelogram, Rhombus and Trapezium

Type I Direct Application of Formulae

EXAMPLE 23 Find the surface area of a metal sheet which is in the form of a parallelogram whose base is 12 cm and height is 7.8 cm.

Solution Surface area = Height \times Base = $7.8 \times 12 = 93.6$ cm²

EXAMPLE 24 Find the area of a rhombus one of whose diagonals measures $8\ cm$ and the other $10\ cm$.

Solution Area = $\frac{1}{2}$ × Product of diagonals = $\frac{1}{2}$ × 8 × 10 = **40 cm²**

EXAMPLE 25 Find the distance between the two parallel sides of a trapezium if the area of trapezium is 250 m², and the two parallel sides are 30 m and 20 m respectively.

Solution Area = $\frac{1}{2}$ × Height × (Sum of parallel sides)

i.e.
$$250 = \frac{1}{2} \times h \times (30 + 20)$$

$$h = \frac{250 \times 2}{30 + 20} = \frac{500}{50} = 10 \text{ m}$$



Short cut: To find the area of a rhombus with one side and one diagonal given.

EXAMPLE 26 Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.

Solution



Short cut: Area of rhombus = Diagonal $\times \sqrt{(\text{Side})^2 - \left(\frac{\text{Diagonal}}{2}\right)^2}$

i.e. Area =
$$24 \times \sqrt{(20)^2 - \left(\frac{24}{2}\right)^2}$$

= $24 \times \sqrt{400 - 144} = 24 \times 16 = 384 \text{ cm}^2$

EXAMPLE 27 The perimeter of a rhombus is 146 cm and one of its diagonal is 55 cm. Find the other diagonal and the area of the rhombus.

Solution

Short cut: Other diagonal =
$$2 \times \sqrt{(\text{Side})^2 - \left(\frac{\text{Diagonal}}{2}\right)^2}$$

Side of rhombus = $\frac{146}{4} = 36.5 \text{ cm}$
Other diagonal = $2 \times \sqrt{(36.5)^2 - \left(\frac{55}{2}\right)^2}$
= $2 \times \sqrt{64 \times 9} = 2 \times 8 \times 3 = 48 \text{ cm}$
 \therefore Area of rhombus = $\frac{1}{2}$ (Product of diagonals) = $\frac{1}{2}$ (48 × 55) = 1320 cm²

PROBLEMS ON REGULAR POLYGONS

A polygon with all sides equal is termed a regular polygon, for example, triangle, quadrilateral, pentagon, hexagon, etc. are polygons.

- 1. Area of a regular polygon = $\frac{1}{2}$ × Number of sides × Radius of the inscribed circle
- 2. Area of a hexagon = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$
- 3. Area of an octagon = $2(\sqrt{2} + 1) (\text{side})^2$

EXAMPLE 28 Find the area of a regular hexagon whose side measures 9 cm.

Solution Area of a regular hexagon =
$$\frac{3\sqrt{3}}{2}a^2$$

Here $a = 9 \text{ cm}$
 \therefore Area = $\frac{3\sqrt{3}}{2}9^2 \text{ cm}^2 = 210.4 \text{ cm}^2 \text{ (approx.)}$

EXAMPLE 29 Find the side of a regular octagonal enclosure whose area is 1 hectare in nearest metre.

Solution Area of a regular octagon =
$$2(1 + \sqrt{2})a^2$$

Given
$$2(1 + \sqrt{2})a^2 = 1 \text{ hectare}$$

$$\therefore \qquad a^2 = \frac{10000}{2(1 + \sqrt{2})} \text{ m}^2 = 2071 \text{ m}^2 \text{ (approx.)}$$
or
$$a = \sqrt{\frac{10000}{2(1 + \sqrt{2})}} = \textbf{46 metres (approx.)}$$

PROBLEMS ON ROOMS AND WALLS

EXAMPLE 30 A room 8 metres long, 6 metres wide and 3 metres high has two windows $1\frac{1}{2}m \times 1m$ and a door $2m \times 1\frac{1}{2}m$. Find the area of the wall and the cost of covering it

with wall paper 50 cm wide at $\stackrel{?}{\underset{?}{?}}$ 2.5/metre. Solution Area of wall = $2(8 + 6)3 = 84 \text{ m}^2$

Area of two windows and a door = $2 \times 1\frac{1}{2} \times 1 + 1 \times 2 \times 1\frac{1}{2} = 6$ m²

 \therefore Area to be covered = 84 - 6 = 78 m²

$$\therefore \text{ Length of paper required} = \frac{78}{50/100} = \frac{7800}{50} = 156 \text{ m}$$

∴ Cost = ₹ 2.5/m × 156 = ₹ 390

LINING A BOX WITH METAL

EXAMPLE 31 A closed box measures externally 90 cm long, 60 cm wide, 45 cm high and is made of wood $2\frac{1}{2}$ cm thick. Find the cost of lining it on the inside with metal at 6 paise/dm².

Solution Internal dimensions are 85 cm, 55 cm and 40 cm, i.e. $8\frac{1}{2}$ dm, $5\frac{1}{2}$ dm and 4 dm.

.. Area of 4 sides =
$$2\left(8\frac{1}{2} + 5\frac{1}{2}\right) \times 4 \text{ dm}^2 = 112 \text{ dm}^2$$

Area of bottom and top = $2 \times 8 \frac{1}{2} \times 5 \frac{1}{2} = \frac{187}{2} \text{ dm}^2$

Total area =
$$112 + \frac{187}{2} = \frac{411}{2} dm^2$$

∴ Cost of lining =
$$6 \times \frac{411}{2}$$
 paise = 1233 paise = ₹ 12.33

PROBLEMS ON CIRCLES

Some formulae which are used to find quick solutions:

(a) Area =
$$\pi$$
(radius)² (b) Radius = $\sqrt{\frac{\text{Area}}{\pi}}$ (c) Diameter = $2\sqrt{\frac{\text{Area}}{\pi}}$

(d) Area =
$$\pi \left(\frac{\text{Diameter}}{2} \right)^2$$
 (e) Perimeter = 2π (radius) (f) Radius = $\left(\frac{\text{Perimeter}}{2\pi} \right)$

(g) Perimeter =
$$\pi$$
(diameter) (h) Diameter = $\left(\frac{\text{Perimeter}}{\pi}\right)$

 $r = \frac{c}{2\pi}$

(i) Arc of a sector =
$$\left(\frac{\theta^{\circ}}{360^{\circ}}\right) \times \text{Circumference}$$

(j) Area of a sector =
$$\left(\frac{\theta^{\circ}}{360^{\circ}}\right) \times \pi \times (\text{radius})^2$$

I. Direct Application of Formulae:

EXAMPLE 32 Find the radius of a circular field whose circumference measures $5\frac{1}{2}$ km.

Solution Circumference (c) = $2\pi r$

:.

Required radius =
$$\frac{\frac{11}{2} \times 1000 \text{ m}}{2\pi}$$

= $\frac{\frac{11}{2} \times 1000 \times 7}{2 \times 22} = \frac{7000}{8} = 875 \text{ metres}$

AREA OF A RING

EXAMPLE 33 The circumference of a circular garden is 1012 m. Find the area outside the garden, a road of 3.5 m width runs round it. Calculate the area of this road and find the cost of gravelling it at the rate of $\sqrt[3]{3/m^2}$.

Solution

Short cut: Area =
$$\frac{\text{(Circumference)}^2}{4\pi} = \frac{(1012)^2}{4 \times \frac{22}{7}} = 81466 \text{ m}^2$$

Area of ring = π [(width of ring) (2 × Inner radius + Width of ring)]

But, inner radius =
$$\sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{81466 \times 7}{22}} = 161 \text{ m}$$

:. Area of ring-shaped road =
$$\pi \times 3.5 \times [3.5 + 2 \times 161] = \frac{22}{7} \times 3.5 \times 325.5 = 3580.5 \text{ m}^2$$

 \therefore Cost of gravelling = $3580.5 \times 3 = ₹ 10741.50$

IDENTICAL CIRCLES PLACED TOGETHER

EXAMPLE 34 An equilateral triangle of side 2 m is placed on the top of a group of 3 circles of radius 1 m each such that the three corners of triangle are at the centres of the

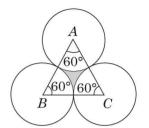
circle: (a) calculate the area common to all the circles and the triangle, (b) calculate the area of the remaining portion of the triangle.

Solution

Short cut: Here, the side of equilateral triangle is double the radius of the circle and therefore all circles touch each other.

 $\therefore \qquad \text{Area of each sector} = \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times \frac{\pi}{3} = \frac{\pi}{6}r^2$

Area of remaining portion = $\frac{\sqrt{3}}{4}a^2 - 3 \times \frac{\pi}{6}r^2 = \frac{\sqrt{3}}{4}(2r)^2 - 3 \times \frac{\pi}{6}r^2$ = $\left[\sqrt{3} - \frac{\pi}{2}\right]r^2 = 0.162 r^2$



- (a) Sum of areas of 3 sectors = $3 \times \frac{\pi}{6} r^2 = \frac{\pi r^2}{2} = \frac{22}{7 \times 2} (1)^2 = 1.57 \text{ m}^2$
- (b) Area of remaining portion = $0.162 r^2 = 0.162 \times (1)^2 = 0.162 \text{ m}^2$

EXERCISES

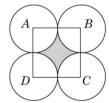
- 1. The radius of a coin is 1 cm. If four of these coins are placed on a table so that the rim of each touches that of the other two, find the area of the unoccupied space between them.
 - (a) 0.25 cm^2

(b) 0.5 cm^2

(c) $1.57 \, \text{cm}^2$

(d) 0.86 cm^2

(e) None of these



- 2. The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?
 - (a) 500
- (b) 1000
- (c) 1500
- (d) 2000
- (e) None of these
- 3. A room is 7 metres long and 5 metres broad, the doors and windows occupy 5 m², and the cost of papering the remaining part of the surface of the walls with paper 75 cm wide, at ₹ 4.20 per piece of 13 m is ₹ 39.20. Find the height of room.
 - (a) 3 m
- (b) 4 m
- (c) 4.5 m
- (d) 5 m
- (e) None of these
- 4. A hall, whose length is 16 m and breadth twice its height takes 168 m of paper 2 m wide for its four walls. Find the area of the floor.
 - (a) 188 m^2
- (b) 196 m^2
- (c) 192 m^2
- (d) 198 m^2
- (e) None of these
- 5. Find the area of a rhombus one of whose diagonals measures 10 cm and the other 12 cm.
 - (a) 120 cm^2
- (b) 60 cm^2
- (c) 50 cm^2
- (d) 40 cm^2
- (e) None of these

[SSC]

- 6. The area of a square field is 69696 cm². Its diagonal will be equal to

- (a) 313.296 cm
- (b) 353.296 cm
- (c) 373.296 cm
- (d) 393.296 cm
- 7. If the ratio of areas of two squares is 225: 256, then the ratio of their perimeters is [SSC]
 - (a) 225:256
- (b) 256:225
- (c) 15:16
- (d) 16:15

8.	A man walked diagonally act by not walking along the edg		imately what was the per cent saved [MBA]
	(a) 20 (b) 24	(c) 30	(d) 33
9.	middle of the park, and rest 2109 m ² , then what is the wi	of the park has been used idth of the road?	o concrete crossroads running in the l as a lawn. If the area of the lawn is [MAT]
	(a) 2.91 m (b) 3 m	(c) 5.82 m	(d) None of these
10.	The length of rectangle is in decreased so as to maintain		per cent would the width have to be $[MAT]$
	(a) $37\frac{1}{2}\%$ (b) 60%	(c) 75%	(d) 120%
11.	1980 cm apart, towards eac	h other in opposite direct	ultaneously from X and Y, which are tions. Both of them make the same et after 10 seconds, the speed of the [MAT]
	(a) 22 cm/sec (b) 44 cm	/sec (c) 66 cm/sec	(d) 132 cm/sec
12.	A man runs round a circular taken by the man to take two		e speed of 12 km/hr. What is the time [MAT]
	(a) 30 minutes (b) 32 min	nutes (c) 34 minutes	(d) None of these
13.	-		oreadth. If the difference between the at is the area of that rectangle?
			[Bank PO]
	(a) 2400 cm^2	(b) 2480 cm^2	(c) 2560 cm^2
	(d) Data inadequate	(e) None of these	
14.	The length of a rectangular p plot at ₹ 26.50/m is ₹ 5300, w		n its breadth. If the cost of fencing the n metres? [Bank PO]
	(a) 40	(b) 50	(c) 120
	(d) Data inadequate	(e) None of these	
15.		sides of the garden setting	able garden. Since he has only 30 m g his house compound wall act as the [RRB]
		=	• •
	(a) $15 \text{ m} \times 6.67 \text{ m}$	(b) 20 m	$n \times 5 \text{ m}$
	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$	(b) 20 m (d) 40 m	1 × 5 m 1 × 2.5 m
16.	 (a) 15 m × 6.67 m (c) 30 m × 3.33 m A rectangle has width α and 	(b) 20 m (d) 40 m I length b. If the width is is the area of the new rect	$n \times 5 \text{ m}$
	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width a and increased by 10%, than what	(b) 20 m (d) 40 m I length b. If the width is is the area of the new rect (c) 110%	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120%
	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width a and increased by 10%, than what	(b) 20 m (d) 40 m (l length b. If the width is is the area of the new rect (c) 110% (c) and one of its diagonal	1×5 m a $\times 2.5$ m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB]
	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width a and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2	(b) 20 m (d) 40 m I length b. If the width is is the area of the new rect (c) 110% Common and one of its diagonal common and	1×5 m a $\times 2.5$ m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB]
17.	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width α and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm^2 (b) 300 cm^2 A circle and a rectangle have 26 cm . What is the area of the	(b) 20 m (d) 40 m (d) 40 m I length b. If the width is is the area of the new rect (c) 110% C6 cm and one of its diago (c) 360 cm ² (c) the same perimeter. The	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm ² sides of the rectangle are 18 cm and [Bank PO]
17.	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width α and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm^2 (b) 300 cm^2 A circle and a rectangle have	(b) 20 m (d) 40 m (d) 40 m I length b. If the width is is the area of the new rect (c) 110% C6 cm and one of its diago (c) 360 cm ² (c) the same perimeter. The	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm ² sides of the rectangle are 18 cm and
17.	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width α and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm^2 (b) 300 cm^2 A circle and a rectangle have 26 cm . What is the area of the	(b) 20 m (d) 40 m (d) 40 m I length b. If the width is is the area of the new rect (c) 110% C6 cm and one of its diago (c) 360 cm ² (e) the same perimeter. The ne circle?	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm ² sides of the rectangle are 18 cm and [Bank PO]
17. 18.	(a) $15 \text{ m} \times 6.67 \text{ m}$ (c) $30 \text{ m} \times 3.33 \text{ m}$ A rectangle has width a and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm^2 (b) 300 cm A circle and a rectangle have 26 cm. What is the area of the (a) 88 cm^2 (d) Cannot be determined	(b) 20 m (d) 40 m (d) 40 m (e) 110% (e) 110% (f) 110% (f) 110% (f) 110% (f) 154 cm² (g) 154 cm² (g) None of these meter is 35 metres, has a 1	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm ² sides of the rectangle are 18 cm and [Bank PO]
17. 18.	 (a) 15 m × 6.67 m (c) 30 m × 3.33 m A rectangle has width a and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm² (b) 300 cm A circle and a rectangle have 26 cm. What is the area of the (a) 88 cm² (d) Cannot be determined A circular ground whose diam 	(b) 20 m (d) 40 m (d) 40 m (e) 110% (e) 110% (f) 110% (f) 110% (f) 110% (f) 154 cm² (g) 154 cm² (g) None of these meter is 35 metres, has a 1	1×5 m 1×2.5 m decreased by 20% and the length is tangle in percentage compared to ab ? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm ² sides of the rectangle are 18 cm and [Bank PO] (c) 1250 cm ²
17. 18.	 (a) 15 m × 6.67 m (c) 30 m × 3.33 m A rectangle has width α and increased by 10%, than what (a) 80% (b) 88% Each side of a rhombus is 2 rhombus is (a) 240 cm² (b) 300 cm A circle and a rectangle have 26 cm. What is the area of the (a) 88 cm² (d) Cannot be determined A circular ground whose dianthe area of the garden in square 	(b) 20 m (d) 40 m (d) 40 m (d) 40 m (e) 110% (e) 110% (f) 26 cm and one of its diago (g) 360 cm ² (g) the same perimeter. The ne circle? (h) 154 cm ² (e) None of these meter is 35 metres, has a 1 tare metres?	a × 5 m a × 2.5 m decreased by 20% and the length is tangle in percentage compared to ab? [RRB] (d) 120% onals is 48 cm long. The area of the [RRB] (d) 480 cm² sides of the rectangle are 18 cm and [Bank PO] (c) 1250 cm² 4 m broad garden around it. What is [SBI PO]

20 .	If the radius of a	circle is increas	sed by 75%, then it	s circumference will increa	ase by [CDS]
	(a) 25%	(b) 50%	(c) 75%	(d) 100%	
21.	If an equilateral is	triangle of area	X and a square of	area Y have the same peri	meter, then Σ [CDS]
	(a) equal to Y		(b) g	reater then Y	
	(c) less than Y		(d) le	ess than or equal to Y	
22 .	An error of 2% in error in the calcu		_	the side of a square. The	percentage o
	(a) 2%	(b) 2.02%	(c) 4%	(d) 4.04%	
23.	to 10 times the or travelling at the	riginal diameter same speed as l	, then the time requeefore, is	s. If the diameter of the circ uired by A to go round the r	
	(a) 20 min	(b) 25 min	(c) 50 min	(d) 100 min	_
24.	triangle, S , the a	rea of the squar	re and C , the area		tes the area o [CDS]
0.5			(c) $T < S < C$		ENT A TEN
25.	The sides of a tri	iangie are 6 cm,	11 cm and 15 cm.	The radius of its incircle is	s [MAT]
	(a) $3\sqrt{2}$ cm	(b) $\frac{4\sqrt{2}}{5}$ cm	(c) $\frac{5\sqrt{2}}{4}$ cm	(d) $6\sqrt{2}$ cm	
26.	A square is inscr circle and the sq		whose radius is 4 c	m. The areas of the portion	n between the
	(a) $(8\pi - 16)$	(b) $(8\pi - 32)$	(c) $(16\pi - 16)$	(d) $(16\pi - 32)$	
27 .		cle is $220~ m cm^2$. T	he area of a squar	e inscribed in this circle w	ill be [CBI]
	` '			(d) 150 cm^2	
28.	A tank is 25 m lo 75 paise/m ² is			cost of plastering its walls	and bottom a
	(a) ₹ 456		(c) ₹ 558		
29.	circle of the triar	ngle is		erimeter, then the radius o	f the inscribed [SSC]
	(a) 1	(b) 1.5	(c) 2	(d) 3	
30.		_	uilateral triangle o		[SSC]
	(a) $22\sqrt{3} \text{ cm}^2$	(b) 231 cm^2	(c) 462 cm^2	(d) 924 cm^2	
31.	The areas of the 14 cm, is	largest circle, t	hat can be drawn	inside a rectangle with sid	les 18 cm and [SSC]
	(a) 49 cm^2		(c) 378 cm^2		
32.				ed by an arc of length 3.5	cm, is [SSC]
	(a) 7.5 cm^2		(c) 8.5 cm^2	(d) 8.75 cm^2	
33.			circle whose perim		[BSRB]
	(a) 154 cm^2	•) 168 cm^2	(c) 308 cm^2	
	(d) Data inadeq		e) None of these		
34.	-	-		nd it. The difference betwe	
			llar path is 132 m.		[SSC]
	(a) 20 m	(b) 21 m	(c) 22 m	(d) 28 cm	

35.	rina the diamete	r of a wheel that i	makes 113 revolut	ions to go 2 km 26 deceme	tres. [SSC]
	(a) $4\frac{4}{13}$ m	(b) $6\frac{4}{11}$ m	(c) $12\frac{4}{11}$ m	(d) $12\frac{8}{11}$ m	
36.	Find the length of graze an area of s		a cow must be te	thered in order that it ma	y be able to [MAT]
	(a) 64 m	(b) 32 m	(c) 56 m	(d) 28 m	
37.	The radius of a w of 11 km will be	heel is 0.25 m. Th	e number of revol	utions it will make to trave	el a distance [RRB]
	(a) 2800	(b) 4000	(c) 5500	(d) 7000	
38.	If the ratio of are	as of two circles is	s 4:9, then the ra	tio of their circumferences	will be [RRB]
	(a) 2:3	(b) 3:2	(c) 4:9	(d) 9:4	
39.	sides are in the r	atio of 6 : 5. The s	maller side of the		angle whose [SSC]
	(a) 25 cm	(b) 30 cm	(c) 36 cm	(d) 60 cm	
40.	Among a square larger area and b	=		circumference 44 cm, which	h figure has [SSC]
	(a) Both have eq	ual area	` ′ -	uare, $33~\mathrm{cm}^2$	
	(c) Circle, 33 cm	2	(d) Squ	uare, $495~\mathrm{cm}^2$	
41.				ken 5% in excess, and the from these measurements	
	(a) 2%	(b) 1%	(c) 0.8%	(d) 0.6%	
42.		lel sides is 24 m.		es 1440 m ² . The perpendicu parallel sides is 5 : 3, the l	
	(a) 45 m	(b) 60 m	(c) 75 m	(d) 120 m	
43.	One of the diagonals is			r diagonal. Its area is 25 cr	n ² . The sum [SSC]
	(a) 10 cm	(b) 12 cm	(c) 15 cm		
44.	The area of a rho the other diagona		The length of one	of its diagonals is 10 cm. T	he length of [SSC]
	(a) 25 cm	(b) 30 cm	(c) 35 cm	(d) 40 cm	
45.				ne same base such that the n, then the altitude of the	
	(a) $10\sqrt{2} \text{ m}$	(b) 100 m	(c) $100\sqrt{2} \text{ m}$	(d) 200 m	
46.	If every side of a old one. K is equa	triangle is double		new triangle is K times the	e area of the [RRB]
	() [5		() 0	(1)	
	(a) $\sqrt{2}$	(b) 2	(c) 3	(d) 4	

(c) 5 cm (d) 6 cm

(a) 3 cm

(b) 4 cm

(a) 40%

(b) 42%

48.	The sides of a triangle are in	the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If t	he perimeter is 52 cm,	then the length
	of the smallest side is	2 3 4		[MAT]
	(a) 9 cm (b) 10 cm	(c) 11 cm	(d) 12 cm	
49.	The areas of a square and re the length of any side of th perimeter of the rectangle.			
	(a) 17 cm (b) 26 cm	(c) 30 cm	(d) 34 cm	
50.	The length of one pair of oppratio of the length and the brarea of the original square?	eadth of the newly forme	ed rectangle becomes 3	
	(a) 25 cm^2 (b) 81 cm	2 (c) 100 cm^2	(d) 225 cm^2 (e)	None of these
51.	If the area of a square increa	ses by 69%, then the sid	le of the square increas	ses by [MAT]
	(a) 13% (b) 30%	(c) 39%	(d) 69%	
52.	A park square in shape has occupied by the road is 1764			
	(a) 576 metres	(b) 600 metres	(c) 640 metres	
	(d) Data inadequate	(e) None of these		
53.	The perimeter of a square is square. If the length of the re		_	the area of the [SSC]
	(a) 24 cm (b) 48 cm	(c) 50 cm	(d) 54 cm	
54.	A 2-metre broad pathway is area of the plot is 96 m^2 . The r			
	(a) ₹ 2400	(b) ₹ 4000	(c) ₹ 4800	
	(d) Data inadequate	(e) None of these		
55.	What will be the cost of garde			
	perimeter of 340 metres at the			ular plot having [Bank PO]
	perimeter of 340 metres at the	ne rate of ₹ 10 per square	e metre?	
56.	perimeter of 340 metres at the (a) ₹ 1700	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these ling 20 cm × 30 cm length	e metre? (c) ₹3440 wise. If a margin of 2 ca	[Bank PO] m is left on each
56.	perimeter of 340 metres at the (a) ₹ 1700 (d) Cannot be determined A typist uses a sheet measure.	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these ling 20 cm × 30 cm lengths	e metre? (c) ₹3440 wise. If a margin of 2 ca	[Bank PO] m is left on each typing is
	perimeter of 340 metres at the (a) ₹ 1700 (d) Cannot be determined A typist uses a sheet measure side and a 3 cm margin on to	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these ling 20 cm × 30 cm length p and bottom, the per ce (c) 64 arallel sides of a trapeziv	e metre? (c) ₹ 3440 wise. If a margin of 2 created the page used for (d) 72 am is 4 cm. The perpendicular the perpe	[Bank PO] m is left on each typing is [MAT] dicular distance
57.	perimeter of 340 metres at the (a) ₹ 1700 (d) Cannot be determined. A typist uses a sheet measure side and a 3 cm margin on to (a) 40 (b) 60. The difference between two pubetween them is 19 cm. If the sides. (a) 29 cm, 25 cm (b) 28 cm.	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these sing 20 cm × 30 cm lengths up and bottom, the per ce (c) 64 arallel sides of a trapezing area of the trapezium is , 24 cm (c) 27cm, 23 cm	e metre? (c) ₹ 3440 wise. If a margin of 2 cont of the page used for (d) 72 Im is 4 cm. The perpendation of 2 cm², find the lengt (d) 26 cm, 22 cm	m is left on each typing is [MAT] dicular distance h of the parallel [RRB]
57.	perimeter of 340 metres at the (a) ₹ 1700 (d) Cannot be determined. A typist uses a sheet measure side and a 3 cm margin on to (a) 40 (b) 60 The difference between two pubetween them is 19 cm. If the sides.	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these sing 20 cm × 30 cm lengths up and bottom, the per ce (c) 64 arallel sides of a trapezity area of the trapezium is , 24 cm (c) 27cm, 23 cm a rectangular plot be ince area be increased?	e metre? (c) ₹ 3440 wise. If a margin of 2 created for the page used for (d) 72 um is 4 cm. The perpendation of 2 cm², find the lengt (d) 26 cm, 22 cm creased by 50% and 20	m is left on each typing is [MAT] dicular distance h of the parallel [RRB]
57.	perimeter of 340 metres at the (a) ₹ 1700 (d) Cannot be determined. A typist uses a sheet measure side and a 3 cm margin on to (a) 40 (b) 60. The difference between two pubetween them is 19 cm. If the sides. (a) 29 cm, 25 cm (b) 28 cm. If the length and breadth of	ne rate of ₹ 10 per square (b) ₹ 3400 (e) None of these sing 20 cm × 30 cm lengths up and bottom, the per ce (c) 64 arallel sides of a trapeziu area of the trapezium is , 24 cm (c) 27cm, 23 cm a rectangular plot be inc	e metre? (c) ₹ 3440 wise. If a margin of 2 created for the page used for (d) 72 um is 4 cm. The perpendation of 2 cm², find the lengt (d) 26 cm, 22 cm creased by 50% and 20	m is left on each typing is [MAT] dicular distance h of the parallel [RRB]

(c) 44% (d) 46%

[MAT]

60.		along one set of si	des and the same i	nt parts, had a perime is 38 cm when folded alo	
	(a) 140 cm^2	(b) 240 cm^2	(c) 560 cm^2	(d) None of these	
61.		time to cross the sield is	same field along it	ally walking at the rate s sides walking at the r	
	(a) 30 m^2	(b) 40 m^2	(c) 50 m^2	(d) 60 m^2	
62.	floor is increased then the area is o	by 21 m ² . If the led decreased by 5 m	ength is increased ² . The perimeter o		
	(a) 30 m	(b) 32 m	` '	(d) 40 m	
63.	The area of a tria triangle is	angle is 216 cm² a	and its sides are in	the ratio 3 : 4 : 5. The	perimeter of the [SSC]
	(a) 6 cm	(b) 12 cm	(c) 36 cm	· /	
64.	their correspondi	ing altitudes will	be	of their areas is $a:b$. T	hen the ratio of [SSC]
	(a) $ax:by$	(b) $\frac{a}{x}:\frac{b}{y}$	(c) $ay:bx$	(d) $\frac{x}{a}:\frac{b}{y}$	
65.			n of a square enclo e is bent into the fe	ses an area of 484 cm ² .` orm of a circle?	What will be the [SSC]
	(a) 462 cm^2	(b) 539 cm^2	(c) 616 cm^2	(d) 693 cm^2	
66.	The number of re 176 m, is	evolutions of a w	heel of diameter 4	0 cm makes in travelli	ng a distance of [SSC]
	(a) 140	(b) 150	(c) 160	(d) 166	
67.	wire is bent into	a semi-circular s	hape, the area of t	area of the square is 8. he semi-circle will be	1 cm ² . When the [SSC]
	(a) 22 cm^2	(b) 44 cm^2	(c) 77 cm^2	(d) 154 cm^2	
68.	of area 784 cm^2 .		maximum diamet ce of each plate is	er are cut off from a squ	are paper sheet [SSC]
	(a) 22 cm	(b) 44 cm	(c) 66 cm	(d) 88 cm	
69.	If the perimeter of	of an isosceles rig	th triangle is (6+	$3\sqrt{2}$)m, then the area of	of the triangle is [MAT]
	(a) 4.5 m^2	(b) 5.4 m^2	(c) 9 m^2	(d) 81 m^2	
70.	The length of a re The length of the		5 m more than its	breadth. The area of th	e hall is 750 m ² . [SSC]
	` '	` '	(c) 25 m	` '	
71.	The area of a rec breadth of the re-		. If the length is 1	5% more than the brea	dth, what is the [Bank PO]
	(a) 15 metres	(b)	26 metres	(c) 34.5 metre	es
	(d) Cannot be de	etermined (e)	None of these		
72.	_	-		cm, 76 cm and 80 cm r n of the areas of these	
	(a) 31 cm	(b) 62 cm	(c) 124 cm	(d) 961 cm	local

- 73. The number of marble slabs of size $20 \text{ cm} \times 30 \text{ cm}$ required to pave the floor of a square room of side 3 metres, is
 - (a) 100
- (b) 150
- (c) 225
- (d) 250
- 74. The difference of the areas of two squares drawn on two line segments of different lengths is 32 cm^2 . Find the length of the greater line segment if one is longer than the other by 2 cm.

[SSC]

- (a) 7 cm
- (b) 9 cm
- (c) 11 cm
- (d) 16 cm
- 75. If the circumference and the area of a circle are numerically equal, then the diameter is equal to [SSC]
 - (a) $\pi/2$
- (b) 2π
- (c) 2
- (d) 4



- 1. (d) 2. (b) 3. (b) 4. (c) 5. (b) 6. (c) 7. (c) 8. (c) 9. (b) 10. (a)
- 11. (c) 12. (b) 13. (c) 14. (e) 15. (b) 16. (b) 17. (d) 18. (e) 19. (a) 20. (c)
- 21. (c) 22. (d) 23. (c) 24. (c) 25. (c) 26. (d) 27. (c) 28. (c) 29. (c) 30. (c)
- **31.** (b) **32.** (d) **33.** (e) **34.** (b) **35.** (b) **36.** (c) **37.** (d) **38.** (a) **39.** (d) **40.** (c)
- 41. (c) 42. (c) 43. (c) 44. (b) 45. (d) 46. (d) 47. (c) 48. (d) 49. (d) 50. (c)
- **51.** (b) **52.** (b) **53.** (b) **54.** (d) **55.** (c) **56.** (b) **57.** (c) **58.** (e) **59.** (c) **60.** (a)
- 61. (d) 62. (d) 63. (d) 64. (c) 65. (c) 66. (a) 67. (c) 68. (b) 69. (a) 70. (d)
- 71. (e) 72. (c) 73. (b) 74. (b) 75. (d)



${f S}$ olutions with Necessary Explanation

1. Area of $\square ABCD = 2 \times 2 = 4 \text{ cm}^2$

Area of 4 sectors =
$$4 \times \frac{\pi}{4}r^2 = \pi r^2 = \pi (1)^2 = \pi \text{ cm}^2$$

- \therefore Unoccupied space = $4 3.14 = 0.86 \text{ cm}^2$
- 2. Distance to be travelled = 11 km = 11000 m

Radius of wheel =
$$1\frac{3}{4}$$
 m

 \therefore Circumference of the wheel = $2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{7}{4} = 11 \text{ m}$$

- .. In travelling 11000 m, the number of revolution required = 11000/11 = 1000
- 3. Total cost of papering = ₹ 39.20

Length of paper required =
$$\frac{13 \times 39.20}{4.20} = \frac{364}{3}$$
 m

Area of paper =
$$\frac{364}{3} \times \frac{75}{100} = 91 \text{ m}^2$$

Area of walls = 91 + 5 (Opening) = 96 m^2

491

.. Area of wall =
$$2 \times (7 + 5) \times \text{Height} = 96 \text{ m}^2$$

i.e. $24 \times \text{height} = 96 \text{ m}^2$

$$\therefore \qquad \text{height} = 96/24 = 4 \text{ m}$$

Let height = h metres

$$\therefore$$
 Breadth = $2h$ metres

Area of walls =
$$2(l + b) \times h = 2(16 + 2h) \times h \text{ m}^2$$

$$\therefore$$
 Area of paper = 168 \times 2 = 336 m²

$$\therefore$$
 2(16 + 2h)h = 336 m²

$$(8+h)h = 84$$

4.

Solving
$$h = 6 \text{ m}$$

$$\therefore$$
 Breadth $b = 2h = 12 \text{ m}$

Area of floor = $l \times b = 16 \times 12 = 192 \text{ m}^2$

5. Area of rhombus =
$$\frac{1}{2}$$
 (product of diagonals) = $\frac{1}{2} \times (10 \times 12) = 60 \text{ cm}^2$

6.
$$a^2 = 69696 \text{ cm}^2$$
; diagonal = $a\sqrt{2} = \sqrt{a^2} \times \sqrt{2}$
= $\sqrt{69696} \sqrt{2} = 264\sqrt{2} \text{ cm} = 373.296$

7.
$$\frac{A_1}{A_2} = \frac{a_1^2}{a_2^2} = \frac{225}{256} = \frac{15^2}{16^2}$$

$$\therefore \text{ Ratio of perimeters} = \frac{4a_1}{4a_2} = \frac{a_1}{a_2} = \frac{15}{16}$$

8. Distance through sides
$$= a + a = 2a$$

Distance along diagonal = $a\sqrt{2}$

$$\therefore \qquad \text{Per cent saved} = \frac{2a - a\sqrt{2}}{2a} \times 100$$

$$= \frac{2 - \sqrt{2}}{2} \times 100$$

$$= \frac{0.586}{2} \times 100 = 29.3\% \approx 30\%$$

9. Let width of road be x metres.

:.

Area of road =
$$60 \times 40 - 2109$$

= $2400 - 2109 = 291 \text{ m}^2$
Length of road = $60 + (40 - x)$ metres.

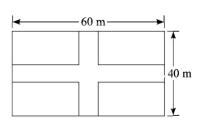
Area of road =
$$(100 - x) x = 291$$

i.e.
$$x^2 - 100x + 291 = 0$$

$$(x-3)(x-97)=0$$

$$\therefore$$
 $x = 3 \text{ or } 97 \text{ metres}.$

$$\therefore$$
 Width of road = 3 metres



- 10. Let the percentage by which width have to be decreased be x.
 - :. Resultant effect due to increase and decrease

$$=60-x-\frac{60x}{100}=0$$

i.e.
$$60 - x - \frac{3}{5}x = 0$$

i.e.
$$60 - \frac{8}{5}x = 0$$

$$\Rightarrow \qquad x = \frac{60 \times 5}{8} = 37.5\%$$

- 11. Let speed of smaller wheel be x cm/sec.
 - :. Distance travelled in 10 sec

$$= x \times 100 = 10 x \text{ cm}$$

- \therefore Speed of larger wheel = 2x cm/sec
- :. Distance travelled in 10 sec

$$= 2x \times 10 = 1980 - 10x$$

$$30x = 1980$$

or
$$x = \frac{1980}{30} = 66 \text{ cm/sec}$$

- 12. Circumference of field = $2\pi r = 2\pi \times 50 = 100 \pi$ metres
 - \therefore For 20 rounds, distance to be covered = $100\pi \times 20$

 $=2000\pi$ metres

Οy

diameter

1980 cm

diameter

.. Time taken for 20 rounds =
$$\frac{\text{Distance}}{\text{Speed}}$$

= $\frac{2000\pi \,\text{m}}{12 \times \frac{5}{18} \,\text{m/s}} = \frac{2000 \times 18}{60} \pi \,\text{sec}$
= $600\pi \,\text{sec}$
= $\frac{600\pi}{60} \,\text{min} = 10 \,\pi = 31.4 \,\simeq \,32 \,\text{minutes}$

- 13. Let breadth of rectangle be x centimetres
 - \therefore Length of rectangle = 1.6 x centimetres

Given
$$1.6x - x = 24 \text{ cm}$$

or
$$0.6 x = 24$$

or
$$x = \frac{24}{0.6} = 40 \text{ cm}$$

- \therefore Area of rectangle = $64 \times 40 = 2560 \text{ cm}^2$
- 14. Let length of plot be x metres
 - \therefore Breadth of plot = (x 20) metres

$$\therefore$$
 Perimeter of plot = $2[x + (x - 20)] = (4x - 40)$ metres

.. Cost of fencing at $\angle 26.5$ /m = $(4x - 40) \times 26.5 = 5300$

26 cm

26 cm

(1)

(2)

i.e.,
$$4x - 40 = \frac{5300}{26.5} = 200$$

$$\therefore \qquad x = \frac{200 + 40}{4} = 60 \text{ metres}$$

15.
$$l + 2b = 30$$
.

From the options, 20 + 2(5) = 30.

 \therefore Dimensions are 20 m \times 5 m.

16. Area of rectangle = ab.

Area of new rectangle =
$$(1.1a) (0.8b) = 0.88 ab$$

= 88% of ab

17. From $\triangle ABC$.

$$OA = OC = \frac{48}{2} = 24 \text{ cm}.$$

From ΔOAB ,

$$(OB)^2 = (AB)^2 - (OA)^2$$

= $(26)^2 - (24)^2$
= $(26 + 24) (26 - 24)$
= $50 \times 2 = 100$

:.

$$OB = 10 cm$$

 \Rightarrow

$$DB = 2 \times OB = 2 \times 10 = 20 \text{ cm}$$

Area =
$$\frac{1}{2}$$
 × AC × DB = $\frac{1}{2}$ × 48 × 20 = **480 cm²**

18. Perimeter of circle = $2(18 + 26) = 88 \text{ cm} = 2\pi r$

or
$$r = \frac{44}{\pi} \, \mathrm{cm}$$

Area of circle =
$$\pi r^2 = 2\pi r \times \frac{r}{2} = 88 \times \frac{44}{2\pi} = \frac{88 \times 22}{\frac{22}{7}} = 88 \times 7 = 616 \text{ cm}^2$$

19. Area of garden =
$$\pi [(17.5 + 1.4)^2 - (17.5)^2]$$

$$= \pi \left[18.9^2 - 17.5^2 \right]$$

$$=\pi \left[(18.9 + 17.5)(18.9 - 17.5) \right] = \pi \times 36.4 \times 1.4$$

=
$$\frac{22}{7}$$
 × 36.4×1.4 = 22×5.2×1.4 = 114.4×1.4 = **160.16** m²

20. Circumference ∝ radius

... On increasing radius by 75%, its circumference will also increase by 75%.

21.
$$X = \frac{\sqrt{3}}{4}a^2$$
 or $a^2 = \frac{4X}{\sqrt{3}}$

Perimeter of triangle = $3a = 3 \times (4x)^{1/2} \times 3^{-1/4}$

i.e.
$$3a = 6x^{1/2} 3^{-1/4}$$

$$Y = b^2 \quad \text{or} \quad b = Y^{1/2}$$

Perimeter of square =
$$4b = 4Y^{1/2}$$

Equating (1) and (2), since perimeters are the same.

$$6X^{1/2} 3^{-1/4} = 4Y^{1/2}$$

i.e.
$$2 \times 3^{1-1/4} \cdot X^{1/2} = 2^2 \cdot Y^{1/2}$$

 $3^{3/4} \cdot X^{1/2} = 2Y^{1/2}$

Squaring both sides,

$$(3^{3/4})^2 \cdot (X^{1/2})^2 = (2Y^{1/2})^2$$

 $3^{3/2} \cdot X = 4Y$

$$\therefore X = \frac{4}{3^{3/2}} Y.$$

X < Y

i.e.

22.
$$a^2 \left[\frac{(1.02)^2 - (1.0)^2}{a^2} \right] \times 100 = 4.04\%$$

Required percentage of error = 4.04%

- 23. Circumference ∞ Diameter
 - .. On increasing diameter to 10 times, circumference will increase to 10 times its original length.

Speed of A = 8 rounds/40 min = 1 round/5 min.

: Distance to be travelled increased to 10 times.

New speed of A = $\frac{1 \text{ round}}{5 \times 10 \text{ min}} = 1 \text{ round/50 min}$

24. Equating perimeters of equilateral triangle, square and circle, we have

$$3a = 4b = 2\pi r \tag{1}$$

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

i.e.
$$T = \frac{\sqrt{3}}{4}a^2 \tag{2}$$

Similarly
$$S = b^2$$
 (3)

Similarly
$$C = \pi r^2$$
 (4)

From (1), 3a = 4b

or $b = \frac{3}{4}a$

From (3),
$$S = \left(\frac{3}{4}a\right)^2 = \frac{9}{16}a^2 \tag{5}$$

From (1), $3\alpha = 2\pi r$ or $r = \frac{3a}{2\pi}$

From (4),
$$C = \pi \times \left(\frac{3a}{2\pi}\right)^2 = \frac{9}{4} \frac{a^2}{\pi}$$
 (6)

Comparing (2), (5) and (6),

$$T = \frac{\sqrt{3}}{4}a^2$$
; $S = \frac{9}{16}a^2$; $C = \frac{9}{4\pi}a^2$

25. Let r be radius of incircle

$$FO = OE = OD = Y$$

Given

$$a = 11 \text{ cm}; b = 6 \text{ cm}; c = 15 \text{ cm}$$

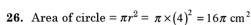
$$S = \frac{a+b+c}{2} = \frac{11+6+15}{2} = 16 \text{ cm}$$

Area =
$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$=\sqrt{16\times(16-11)(16-6)(16-15)}$$

$$= \sqrt{16 \times 5 \times 10 \times 1} = 20\sqrt{2} \text{ cm}^2$$

$$\therefore \quad \text{Radius of incircle} = \frac{\Delta}{S} = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4} \text{ cm}.$$



Diagonal of inscribed square = Radius of circle = 4 cm

$$\therefore \qquad \text{Side of square} = \frac{4 \times 2}{\sqrt{2}} = 4\sqrt{2} \text{ cm}$$

$$\therefore \qquad \text{Area of square} = \left(4\sqrt{2}\right)^2 = 32 \text{ cm}^2$$

$$\therefore$$
 Required area = $(16\pi - 32)$ cm²/m

$$\pi r^2 = \frac{22}{7} \times r^2 = 220 \text{ cm}^2$$

$$r = \sqrt{\frac{220}{22} \times 7} = \sqrt{70} \text{ cm}.$$

Diagonal of square inscribed = Diameter of circle

$$= 2\sqrt{70}$$
 cm

∴ Side of square inscribed =
$$\frac{2\sqrt{70}}{\sqrt{2}} = \sqrt{140}$$
 cm

$$= 2\sqrt{35} \text{ cm}$$

$$\therefore \qquad \text{Area of square} = \left(2\sqrt{35}\right)^2$$

$$=4\times35$$

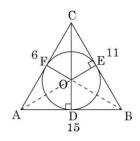
$$= 140 \text{ cm}^2$$

28. Total surface area of tank =
$$[2(25 + 12) \times 6] + 25 \times 12$$

$$= 444 + 300 = 744 \text{ m}^2$$

 \therefore Cost of plastering at 75 paise/m² = 744 × 0.75

$$=\frac{2232}{4}=₹558$$



29. Radius =
$$\frac{\text{Area}}{\text{Semi-perimeter}} = \frac{\text{Area}}{\text{Area/2}}$$

= $\text{Area} \times \frac{2}{\text{Area}} = 2$

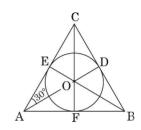
30.
$$AB = BC = AC = 42 \text{ cm};$$

OA =
$$\frac{AE}{\cos 30^{\circ}} = \frac{AC}{2 \cos 30^{\circ}} = \frac{42}{2 \times \frac{\sqrt{3}}{2}} = 14\sqrt{3} \text{ cm}$$

:. OE = OA sin 30° =
$$14\sqrt{3} \times \frac{1}{2} = 7\sqrt{3}$$
 cm

Area of incircle =
$$\pi r^2 = \pi (OE)^2$$

= $\pi \left(7\sqrt{3}\right)^2$
= $49 \times 3 \times \frac{22}{7}$
= 462 cm^2



31. Required area =
$$\pi \times \left(\frac{14}{2}\right)^2 = 7^2 \times \frac{22}{7} = 154 \text{ cm}^2$$

32. Given
$$R = 5$$
 cm; $S = 3.5$ cm

$$\theta = \frac{S}{R} = \frac{3.5}{5} = 0.7 \text{ rad}$$

Area =
$$\frac{1}{2}R^2 \theta = \frac{1}{2} \times 5^2 \times 0.7 = 8.75 \text{ cm}^2$$

33. Given
$$\pi r + 2r = 36 \text{ cm}$$

i.e.
$$(\pi + 2)r = 36$$
 cm

$$\therefore \qquad r = \frac{36}{\pi + 2} = \frac{36}{\frac{22}{7} + 2} = \frac{36 \times 7}{(22 + 14)} = 7 \text{ cm}$$

Area of semi-circle = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77 \text{ cm}^2$

$$2\pi R - 2\pi r = 2\pi (R - r) = 2\pi \times x = 132 \text{ m}$$

$$\therefore \qquad x = \frac{132}{2\pi} = \frac{132}{2 \times \frac{22}{7}} = \frac{132 \times 7}{2 \times 22} = 21 \,\mathrm{m}$$

35. Let diameter of wheel be
$$x$$
 metres

$$\therefore$$
 Circumference of wheel = $\pi \times x$ metres

$$\therefore$$
 In 113 revolution, distance covered = 113 $\times \pi \times x$

∴
$$113 \times \pi \times x = 2 \text{ km } 26 \text{ decametres}$$

= 2000 m + 260 m = 2260 m

$$\therefore \qquad x = \frac{2260}{113 \times \frac{22}{7}} = \frac{2260 \times 7}{113 \times 22} = \frac{10 \times 7}{11} = 6\frac{4}{11} \text{ m}$$

36. Let the length of rope be l metres

$$\therefore \qquad \qquad \pi l^2 = 9856$$

$$\Rightarrow l^2 = \frac{9856}{\pi} = \frac{9856}{\frac{22}{7}} = \frac{9856 \times 7}{22} = 448 \times 7 = 64 \times 7^2$$

$$l = 8 \times 7 = 56 \text{ metres}$$

37. r = 0.25 m;

Let number of revolutions made be N.

$$\therefore N \times 2\pi \times r = 11 \text{ km}$$

i.e.
$$N \times 2 \times \frac{22}{7} \times 0.25 = 11 \times 1000 \text{ m}.$$

$$N = \frac{11000 \times 7}{2 \times 22 \times 0.25} = 7000$$

38. Area of circle \propto (Radius)²

$$\frac{a_1}{a_2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9} \text{ or } \frac{r_1}{r_2} = \frac{2}{3}.$$

Perimeter ∝ Radius

$$\therefore \frac{p_1}{p_2} = \frac{r_1}{r_2} = \frac{2}{3} \quad \text{or} \quad 2:3$$

39. Circumference of circle = $2\pi \times r = 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264$ cm

Let smaller side of rectangle be x.

$$\therefore \qquad \text{Larger side} = \frac{6}{5}x$$

$$\therefore$$
 Perimeter of rectangle = $2\left(x + \frac{6}{5}x\right) = 264$

i.e.
$$2 \times \frac{11x}{5} = 264$$

$$\therefore \qquad x = \frac{264 \times 5}{22} = 12 \times 5 = 60 \text{ cm}$$

40. Perimeter of square = 44 cm

:. Its area =
$$(11)^2 = 121 \text{ cm}^2$$

Perimeter of circle = $2\pi r = 44$ cm

i.e.
$$2 \times \frac{22}{7} \times r = 44$$
or
$$r = 7 \text{ cr}$$

:. Its area =
$$(\pi r)^2 = \pi \times (7)^2 = \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$$

Area of circle is larger by 33 cm².

41. Let l and b be the length and breadth of rectangle.

Then $l(1.05) \times b(0.96) = 1.0080 \ lb$.

:. Error per cent in area calculated

$$= \frac{1.0080 \, lb - lb}{lb} \times 100 = 0.8 \,\%$$

42. Area of trapezium ABCD = $\frac{1}{2}(5x + 3x) \times 24 = 96x \text{ m}^2$

$$\therefore$$
 96 $x = 1440$

$$x = \frac{1440}{96} = 15 \,\text{m}$$

- \therefore Larger side = $5x = 5 \times 15 = 75$ m
- 43. Let the diagonals be d_1 and d_2 .

$$d_1 = 2d_2;$$

Area =
$$\frac{1}{2}d_1d_2 = \frac{1}{2}(2d_2)(d_2) = 25 \text{ cm}^2$$

$$d_2^2 = 25 \text{ cm}^2$$

$$\Rightarrow$$
 $d_2 = 5 \text{ cm}$

∴
$$d_1 = 2d_2 = 10$$
 cm

- \therefore Sum of diagonals = 10 + 5 = 15 cm
- **44.** Area = $\frac{1}{2}d_1d_2 = \frac{1}{2} \times 10 \times d_2 = 150$

$$d_2 = \frac{150}{5} = 30 \text{ cm}$$

45. Area of triangle =
$$\frac{1}{2}bh_1$$
 (1)

Area of parallelogram =
$$bh_2$$
 (2)

Equating (1) and (2),

$$\frac{1}{2}bh_1 = bh_2 h_1 = 2h_2$$
 (3)

3x

В

24 m

or Given

$$h_2 = 100 \text{ m}.$$

From (3), we have

$$h_1 = 2 \times 100 = 200 \text{ m}$$

46. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)} = \Delta$

If every side is doubled, new area will be

$$\sqrt{s(s-a)(s-b)(s-c)\times 2\times 2\times 2\times 2}=4\Delta$$

$$K = 4$$
.

47. Perimeter 2s = a + b + c = 30 cm;

Area =
$$\sqrt{s(s-a)(s-b)(s-c)}$$
 = 30

i.e.
$$\sqrt{15(15-13)(15-b)(15-c)} = 30$$

$$(15 - b) (15 - c) = 30$$

 \therefore Smallest side = 5 cm.

499

Let sides be 6x, 4x and 3x centimetres

$$\therefore \qquad \text{Perimeter} = 6x + 4x + 3x = 52 \text{ cm}$$

or
$$x = \frac{52}{13} = 4$$

 \therefore Smallest side = $3x = 3 \times 4 = 12$ cm

49. Let length of side of square be a centimetre.

$$\therefore$$
 Length of rectangle = $(a + 5)$ cm

Breadth of rectangle = (a - 3) cm

Given
$$a^{2} = (a + 5) (a - 3)$$

$$\Rightarrow \qquad a^{2} + 2a - 15 = a^{2}$$

$$\Rightarrow \qquad 2a - 15 = 0$$

$$\Rightarrow \qquad a = \frac{15}{2}$$

 $\therefore \text{ Perimeter of rectangle} = 2 \left[(a+5) + (a-3) \right]$

=
$$2[2\alpha + 2] = 4\alpha + 4 = 4 \times \frac{15}{2} + 4 = 34$$
 cm

50. Let the side of square be x centimetres.

Length of rectangle = (x + 5) centimetres

Now,
$$\frac{x+5}{x} = \frac{3}{2}$$
or
$$3x = 2(x+5)$$

$$\Rightarrow x = 10 \text{ cm.}$$

 \therefore Area of original square = $x^2 = 10^2 = 100 \text{ cm}^2$

51. Let the side of square be increased by x%

$$x + x + \frac{x^2}{100} = 69$$
i.e.
$$2x + \frac{x^2}{100} = 69$$
or
$$x^2 + 200 \ x = 6900$$

$$x^2 + 200x - 6900 = 0$$
or
$$(x + 230) \ (x - 30) = 0$$

$$x = 30\%$$

52. Let the side along the outer edge of road be x.

:. Side along the inner edge of road = x - 6Area occupied by the road = $x^2 - (x - 6)^2 = 1764$

i.e.
$$[x + (x - 6)][x - (x - 6)] = 1764$$

i.e. $(2x - 6)6 = 1764$
or $2x - 6 = 294$

$$\therefore \qquad x = \frac{294 + 6}{2} = 150$$

:. Perimeter along the outer edge of road

$$= 4 \times 150 = 600 \text{ metres}$$

53. Perimeter of square = 48 cm

$$\therefore \qquad \text{Side of square} = \frac{48}{4} = 12 \text{ cm}$$

Area of square = $(12)^2 = 144 \text{ cm}^2$

Area of rectangle = $144 - 4 = 140 \text{ cm}^2$

Given length of rectangle = 14 cm

$$\therefore \qquad \text{Breadth of rectangle} = \frac{\text{Area}}{\text{Length}} = \frac{140}{14} = 10 \text{ cm}$$

- :. Perimeter of rectangle = 2(l + b) = 2(14 + 10) = 48 cm
- **54.** Let l and b be the length and breadth of plot.

:.
$$lb = 96 \text{ m}^2$$

Area of path = $(l-4)(b-4) - lb = 16 - 4(l+b)$

- \therefore l and b are not known, value of area of path can't be determine.
- .. Data is inadequate.
- 55. Perimeter = 340 cm; Width of garden = 1 m
 - .. Perimeter of outer edge of garden = 348 m

$$\therefore \qquad \text{Average perimeter} = \frac{340 + 348}{2} = 344 \text{ m}$$

.. Cost of gardening at
$$\overline{<} 10/\text{m}^2 = (344 \times 1) \times 10$$

= $\overline{<} 3440$

- **56.** Total area = $20 \times 30 = 600 \text{ cm}^2$
 - $\therefore \qquad \text{Area left for margin} = (30 \text{ cm} \times 2 \text{ cm}) \times 2 + (20 \text{ cm} \times 3 \text{ cm}) \times 2 = 240 \text{ cm}^2$

∴ Per cent of page used =
$$\frac{(600 - 240)}{600} \times 100$$

= $\frac{360}{600} \times 100$
= 60%

57. Let l_1 and l_2 be the parallel sides of trapezium.

$$l_1 = l_2 + 4; \quad h = 19 \text{ cm};$$

$$\therefore \qquad \text{Area} = \frac{h}{2}(l_1 + l_2) = \frac{19}{2}(l_2 + 4 + l_2) = 475 \text{ cm}^2$$
or
$$2l_2 + 4 = \frac{475 \times 2}{19} = 25 \times 2 = 50$$

:.
$$l_2 = \frac{50-4}{2} = 23 \text{ cm}$$

:.
$$l_1 = l_2 + 4 = 23 + 4 = 27$$
 cm

(2)

58. Increase in area by percentage =
$$50 + 20 + \frac{50 \times 20}{100}$$

= $70 + 10 = 80\%$

$$\therefore \qquad \text{Total increase} = \frac{80}{100} = \frac{4}{5} \text{ times}$$

59. Percentage increase in area of rectangle

$$= 20 + 20 + \frac{20 \times 20}{100} = 44\%$$

60. Let l and b be the length and breadth of paper.

When folded along breadth,

$$2\left(\frac{l}{2} + b\right) = 34$$
 or $l + 2b = 34$ (1)

When folded along length.

$$2\left(l + \frac{b}{2}\right) = 38$$
 or $2l + b = 38$ (2)

Solving (1) and (2), we get,

 \mathbf{or}

ving (1) and (2), we get,

$$2 \times (2) - (1) \Rightarrow 3l = 76 - 34 = 42$$

 $l = 14$ cm.

$$\therefore \qquad (2) \Rightarrow b = 38 - 2l$$

$$= 38 - 28 = 10$$
 cm

$$\therefore \qquad \text{Area of paper} = 14 \times 10 = 140 \text{ cm}^2$$

61. Length of diagonal = $\frac{52 \text{ m}}{60 \text{ sec}} \times 15 \text{ sec}$

$$= 13 \text{ m}$$

Let length of semi perimeter be x.

$$x = \frac{68 \text{ m}}{60 \text{ sec}} \times 15 \text{ sec}$$

$$= 17 \text{ m}$$

$$\therefore \qquad \text{Length} = 12 \text{ m and width} = 5 \text{ m}$$
Area of field = $12 \times 5 = 60 \text{ m}^2$

62. Let length and breadth of rectangle be l and b.

Solving (1) and (2), we get

$$l = 12 \text{ m}; b = 8 \text{ m};$$

:. Perimeter of floor = 2(l + b) = 2(12 + 8) = 40 m

63. Area of triangle = Δ

$$S = \frac{3x + 4x + 5x}{2} = 6x$$

$$S - 3x = 3x; \quad S - 4x = 2x; \quad S - 5x = x;$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6x \times 3x \times 2x \times x} = 6x^{2}$$

$$6x^{2} = 216$$

$$x^{2} = 36$$

$$x = 6 \text{ cm}$$
f triangle = $3x + 4x + 5x = 12x$

Perimeter of triangle = 3x + 4x + 5x = 12x= $12 \times 6 = 72$ cm

64.
$$\frac{b_1}{b_2} = \frac{x}{y}$$
; $\frac{\Delta_1}{\Delta_2} = \frac{a}{b}$

i.e., or

 \mathbf{or}

$$\therefore \qquad \Delta_1 = \frac{1}{2} b_1 h_1 \tag{1}$$

i.e.
$$\Delta_2 = \frac{1}{2}b_2 h_2$$
 (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{\Delta_1}{\Delta_2} = \frac{b_1 h_1}{b_2 h_2} = \frac{b_1}{b_2} \times \frac{h_1}{h_2}$$

i.e.
$$\frac{a}{b} = \frac{x}{y} \times \frac{h_1}{h_2}$$

$$\frac{h_1}{h_2} = \frac{ay}{bx} = ay : bx$$

65. Area of square = 484 cm^2

$$\therefore \qquad \text{Perimeter of square} = 4 \times \sqrt{484} = 4 \times 22 = 88 \text{ cm}$$

When it is bent into a circle,

$$\pi d = 88 \text{ cm}$$

or
$$\frac{22}{7}d = 88$$

or $d = \frac{88 \times 7}{22} = 28$ cm

$$\therefore \qquad \text{Area of circle} = \frac{\pi d^2}{4} = \frac{22}{7} \times \left(\frac{28}{2}\right)^2$$
$$= 22 \times 2 \times 14$$
$$= 616 \text{ cm}^2$$

66. Circumference of wheel having diameter 40 cm

$$= \pi d = \frac{22}{7} \times 40 = \frac{880}{7} \text{ cm}$$

Number of revolution required to travel 176 m

$$=\frac{176 \,\mathrm{m}}{\pi d}=\frac{176 \times 100}{\frac{880}{7}}=\frac{176 \times 100 \times 7}{880}=140$$

67. Area of square = 81 cm^2

$$\therefore$$
 Length of wire = $4 \times \sqrt{81} = 36$ cm

Periphery of semi-circle = $2r + \pi r = (\pi + 2)r = 36$

$$r = \frac{36}{(\pi+2)} = \frac{36}{\left(\frac{22}{7} + 2\right)} = \frac{36 \times 7}{(22+14)} = 7 \text{ cm}$$

$$\therefore \qquad \text{Area of semi-circle} = \frac{\pi r^2}{2} = \frac{\pi \times 7^2}{2}$$
$$= \frac{22}{7} \times \frac{7^2}{2} = 77 \text{ cm}^2$$

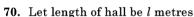
68. Side of square sheet = $\sqrt{784}$ = 28 cm

Maximum diameter of a circular plate = 14 cm

:. Circumference of each plate =
$$\pi d = \frac{22}{7} \times 14 = 44$$
 cm

69. Side of isosceles triangle =
$$\frac{6+3\sqrt{2}}{(1+1+\sqrt{2})} = 3 \text{ m}$$

$$\therefore \qquad \text{Area of triangle} = \frac{1}{2}bh = \frac{1}{2} \times 3 \times 3 = 4.5 \text{ m}^2$$



$$\therefore$$
 breadth of hall = $(l-5)$ metres.

Area of hall =
$$l(l-5) = 750$$

$$\Rightarrow \qquad l^2 - 5l - 750 = 0$$

i.e.
$$(l-30)(l+25)=0$$

$$\therefore$$
 $l = 30 \text{ metres}$

71. Let breadth of rectangle be b metres

Length of rectangle =
$$1.15b$$
 metres.

Area of rectangle =
$$1.15b \times b = 460$$

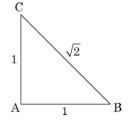
i.e.
$$1.15 b^2 = 460$$

or
$$b^2 = \frac{460}{1.15} = 400$$

$$b = 20 \text{ metres}$$

72. Sum of areas of squares =
$$\left(\frac{24}{4}\right)^2 + \left(\frac{32}{4}\right)^2 + \left(\frac{40}{4}\right)^2 + \left(\frac{76}{4}\right)^2 + \left(\frac{80}{4}\right)^2$$

= $6^2 + 8^2 + 10^2 + 19^2 + 20^2$



$$= 36 + 64 + 100 + 361 + 400$$

= 961

- \therefore Side of large square = $\sqrt{961}$ = 31 cm
- \therefore Perimeter of large square = $4 \times 31 = 124$ cm
- **73.** Area of room = $3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2$

Area of 1 marble slab = $20 \text{ cm} \times 30 \text{ cm} = 600 \text{ cm}^2$

- $\therefore \text{ Number of slabs required to pave the floor} = \frac{9 \text{ m}^2}{600 \text{ cm}^2} = \frac{9 \times 100 \times 100}{600} = 150$
- 74. Let length of greater line segment be x centimetres
 - \therefore Length of smaller line segment = (x-2) cm

Area of square formed by first line segment = $(x)^2$

Area of square formed by second line segment = $(x-2)^2$

Given
$$(x)^2 - (x-2)^2 = 32$$

 $\Rightarrow (x+x-2)2 = 32$
 $2x-2 = 16$
or $x = \frac{16+2}{2} = 9$

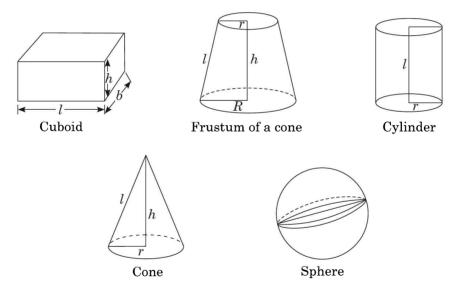
- :. Length of greater line segment = 9 cm
- **75.** $2\pi r = \pi r^2$

or
$$r=2$$

 \therefore Diameter = 4

Elementary Mensuration II (Measurement of Volume and Surface Areas)

An object which occupies space has three dimensions: length, breadth and depth. Such an object is called a *solid*. Some commonly known solids are shown in the following figures:



lmportant Formulae

CUBOID

Let length = l; breadth = b and height = h units

- 1. Volume of cuboid = $l \times b \times h$ cubic units = $\sqrt{A_1 \times A_2 \times A_3}$ where A_1 = Area of base or top; A_2 = Area of side face; A_3 = Area of other side face.
- 2. Area of whole surface of cuboid = 2(lb + bh + lh) sq. units.

3. Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.

CUBE

Let the edge of cube be a units. Then

- 4. Volume of cube = a^3 cubic units.
- 5. Whole surface area of cube = $6a^2$ sq. units
- 6. Diagonal of cube = $\sqrt{3} a$ units.

CYLINDER

Let the radius of the base of cylinder be r units and height be h units. Then

- 7. Volume of cylinder = $\pi r^2 h$ cubic units.
- 8. Curved surface area of cylinder = $2\pi rh$ sq. units.
- 9. Total surface area of cylinder = $(2\pi rh + 2\pi r^2) = 2\pi r(h + r)$ sq. units.

SPHERE

Let the radius of sphere be r units. Then

- 10. Volume of sphere = $4/3\pi r^3$ cubic units.
- 11. Surface area of sphere = $4\pi r^2$ sq. units.
- 12. Volume of a hemisphere = $2/3\pi r^3$ cubic units.
- 13. Curved surface area of hemisphere = $2\pi r^2$ sq. units.
- 14. Whole surface area of hemisphere = $3\pi r^2$ sq. units.

RIGHT CIRCULAR CONE

Let r be the radius of base, h, the height and l the slant height of a cone. Then

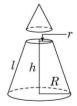
- 15. Slant height $l = \sqrt{h^2 + r^2}$.
- 16. Volume of cone = $1/3 \pi r^2 h$ cubic units.
- 17. Curved surface area of the cone = $\pi r l$ sq. units = $(\pi r \sqrt{r^2 + h^2})$ sq. units.
- 18. Total surface area of cone = $\pi rl + \pi r^2 = \pi r(l + r)$ sq. units.

FRUSTUM OF A RIGHT CIRCULAR CONE

If a cone is cut by a plane parallel to the base so as to divide the cone into two parts as shown in the figure, the lower part is called the frustum of a cone.

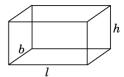
Let the radius of the base of frustum = R, radius of top = r, height = h and slant height = l units.

- 19. Slant height $l = \sqrt{h^2 + (R r)^2}$ units.
- 20. Curved surface area = $\pi(R + r)l$ sq. units.
- 21. Total surface area = $\pi\{(R+r)l + r^2 + R^2\}$ sq. units.
- 22. Volume = $\frac{\pi h}{3} (r^2 + R^2 + rR)$ cubic units.



RIGHT PARALLELOPIPED

It is a type of cuboid in which the shape of a side face is rectangular whereas the shape of the base or the top face is a parallelogram (neither a rectangle nor a square).



- 23. Surface area (of side faces) = 2h(b + l) sq. units.
- 24. Surface area (of the base or the top face) = $2\sqrt{s(s-a)(s-b)(s-d)}$ sq. units
- 25. Total surface area = $2h(b+l)+4\sqrt{s(s-a)(s-b)(s-d)}$ sq. units
- 26. Volume = Base area \times height.

PROBLEMS ON CUBES AND CUBOIDS

Type I Direct Application of Formulae

EXAMPLE 1 Find the volume and the surface area of a slab of stone measuring 4 metres in length, 2 metres in width and 1/4 metre in thickness.

Solution Volume = $4 \times 2 \times 1/4 = 2$ cubic metres

Surface area =
$$2(lb + bh + lh)$$

= $2(4 \times 2 + 2 \times 1/4 + 4 \times 1/4)$
= $2(8 + 1/2 + 1) = 19$ sq. units

EXAMPLE 2 A brick measures 20 cm \times 10 cm \times 7 $\frac{1}{2}$ cm. How many bricks will be

required for a wall 25 m long, 2 m high and $\frac{3}{4}$ m thick?

Solution Volume of wall = $25 \times 2 \times \frac{3}{4} = \frac{75}{2}$ cubic metres

Volume of 1 brick =
$$\frac{20}{100} \times \frac{10}{100} \times \frac{15}{200} = \frac{3}{2000}$$
 cubic metres

:. Required number of bricks =
$$\frac{75}{2} \div \frac{3}{2000} = \frac{75}{2} \times \frac{2000}{3} = 25000$$

EXAMPLE 3 Find the volume of a cuboid whose areas of base and two adjacent faces are 180 cm², 96 cm² and 120 cm² respectively.

Solution Volume of a cuboid = $\sqrt{\text{Area of base} \times \text{Area of one face} \times \text{Area of other face}}$ = $\sqrt{180 \times 96 \times 120} \text{ cm}^3 = 1440 \text{ cm}^3$

Type II Some Quicker Methods

EXAMPLE 4 A closed wooden box measures externally 9 cm long, 7 cm broad, 6 cm high. If the thickness of the wood is half a centimetre, find (i) the capacity of the box and (ii) the weight supposing that 1m^3 of wood weighs 0.9 g.

$$Solution \quad \text{Capacity} = (\text{External length} - 2 \times \text{thickness}) \\ \times (\text{External breadth} - 2 \times \text{thickness}) \\ \times (\text{External height} - 2 \times \text{thickness}) \\ \times (\text{External height} - 2 \times \text{thickness}) \\ \text{Volume of material} = \text{External volume} - \text{Capacity} \\ \text{Here,} \quad \text{Capacity} = (9 - 2 \times 0.5) \ (7 - 2 \times 0.5) \ (6 - 2 \times 0.5) \\ = 8 \times 6 \times 5 = \textbf{240 cm}^3 \\ \text{Volume of wood} = \text{External volume} - \text{Capacity} \\ = 9 \times 7 \times 6 - 240 = 378 - 240 = \textbf{138 cm}^3 \\ \text{Weight of wood} = \text{Volume of wood} \times \text{Density of wood} \\ = 138 \times 0.9 = \textbf{124.2 g}$$

EXAMPLE 5 The surface area of a cube is $30\frac{3}{8}$ sq. metres. Find its volume.

Solution Volume of cube =
$$\left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$$

Here, Surface area = $\frac{243}{8}$ m²
 \therefore Volume of cube = $\left(\sqrt{\frac{243/8}{6}}\right)^3$
= $\left(\sqrt{\frac{243}{48}}\right)^3 = \left(\sqrt{\frac{81}{16}}\right)^3$
= $\left(\frac{9}{4}\right)^3 = \frac{729}{64} = 11\frac{25}{64}$ cu.m

RAINFALL IN A GIVEN AREA AND SIMILAR PROBLEMS

EXAMPLE 6 The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rainfall there on a hectare of land, taking the weight of water to be 1 metric tonne for 1 cubic metre.

Solution



Short cut: Volume of water = Height of water × Base area

Given water level = 43 cm; Area of land = $1 \text{ hectare} = 10^4 \text{ m}^2$

: Volume of water =
$$\frac{43}{100} \times 10^4 \text{ m}^3 = 4300 \text{ m}^3$$

: Weight of water = $4300 \times 1 = 4300$ metric tonnes.

EXAMPLE 7 A rectangular tank is 50 metres long and 29 m deep. If 1000 m³ of water be drawn off the tank, the level of the water in the tank goes down by 2 metres. How many cubic metres of water can the tank hold?

Solution



Short cut: Base area =
$$\frac{\text{Volume of water}}{\text{Level of water}} = \frac{1000}{2} = 500 \text{ m}^2$$

Total volume = Depth × Base area =
$$29 \times 500 = 14500 \text{ m}^3$$

CONVERSION OF A SQUARE BAR INTO CUBES

EXAMPLE 8 A cubic metre of copper weighing 9000 kg is rolled into a square bar 9 m long. An exact cube is cut off from the bar. How much does it weigh?

Solution Here, the given volume of copper is rolled into a square bar of given length. Then an exact cube is cut off from this square bar.

Given volume = $1m^3$

i.e.,
$$1 \text{ m}^3 = \text{Area of square base} \times \text{length}$$

$$\Rightarrow$$
 Area of square base \times length = 1 m³

$$\Rightarrow \qquad \text{Area of square base} = \frac{1}{\text{length}} = \frac{1}{9} \text{m}^2$$

$$\therefore \qquad \text{Side of square base} = \sqrt{\frac{1}{9}} = \frac{1}{3} \text{m}$$

$$\therefore \qquad \text{Volume of cut off cube} = (\text{side})^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27} \text{m}^3$$

Weight of cube =
$$\frac{1}{27} \times 9000 = 333.3 \text{ kg}$$



Short cut: Volume of cube cut off =
$$\left(\sqrt{\frac{\text{Volume of original solid}}{\text{Length of solid}}}\right)^3$$

$$\therefore \qquad \text{Volume} = \left(\sqrt{\frac{1}{9}}\right)^3 = \frac{1}{27} \,\text{m}^3$$

:. Weight of cube =
$$\frac{1}{27} \times 9000 = 333.3 \text{ kg}$$

OBJECT CHANGING ITS SHAPE

EXAMPLE 9 A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

Solution The underlying concept for these type of questions is *total volume of a solid* remains the same even when its shape changes.

∴ Old Volume = New Volume

⇒
$$1\text{m}^3 = 60000 \times \text{Thickness}$$

⇒ Thickness = $\frac{1}{60000} = 0.0017 \text{ cm}$

EXAMPLE 10 Three cubes of metals whose edges are 3, 4 and 5 cm respectively are melted and formed into a single cube. If there is no loss of metal in the process, find the side of the new cube.

Solution



Short cut: When many cubes integrate into one cube, the side of the new cube is

given by side = $\sqrt{\text{Sum of cubes of sides of all the cubes}}$

Here, side =
$$\sqrt{3^3 + 4^3 + 5^3} = \sqrt{27 + 64 + 125} = \sqrt{216} = 6$$
 cm

EXAMPLE 11 A cube of sides 3 cm is melted and similar cubes of sides 1 cm each are formed. How many such cubes are possible?

Solution Quicker method

Number of cubes possible =
$$\left(\frac{\text{Original length of cube}}{\text{New length of cube}}\right)^3$$

i.e. Number of cubes = $\left(\frac{3}{1}\right)^3 = 27$

PROBLEMS ON CYLINDERS

Type I Direct Application of Formula

EXAMPLE 12 Find the volume of a cylinder which has a height of 14 metres and a base of radius 3 metres. Find the curved surface of the cylinder.

Solution Volume =
$$\frac{22}{7} \times (3)^2 \times 14 = 396 \text{ m}^3$$

Curved surface area = Circumference × Height

=
$$2 \times \frac{22}{7} \times 3 \times 14 = 264 \text{ m}^2$$

Type II Indirect Method

EXAMPLE 13 A hollow cylindrical tube opens at both ends is made of iron 2 cm thick. If the external diameter is 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

Solution



:.

Short cut: Volume of metal = π [(Outer radius)² – (Inner radius)²] × Height

Here outer radius = 50/2 = 25 cm

Inner radius = 25 - 2 = 23 cm

Height = 140 cm

Volume =
$$\pi[25^2 - 23^2] \times 140$$

$$= \frac{22}{7} [25^2 - 23^2] \times 140 \qquad \left[\because \pi = \frac{22}{7} \right]$$

$$=440 (25 + 23) (25 - 23)$$

$$= 440 \times 48 \times 2 = 42240 \text{ m}^3$$



Note: The above problem can also be solved by using the equation

Volume = $\pi \times$ Height \times (2 \times Outer radius – Thickness) \times (Thickness)

(when the outer radius is given)

Volume = $\pi \times$ Height \times (2 \times Inner radius + Thickness) \times (Thickness)

(when the inner radius is given)

Here using the first relation,

Volume =
$$\frac{22}{7} \times 140 \times (2 \times 25 - 2)$$
 (2) = $\frac{22}{7} \times 140 \times 48 \times 2 = 42240 \text{ m}^3$

EXAMPLE 14 Find the weight of a lead pipe 3.5 cm long if the external diameter is 2.4 cm, the thickness of the lead is 2 mm and 1 cm³ of lead weighs 11.4 gm.

Solution Weight of lead pipe = Unit weight of pipe × Volume of pipe

= 11.4 g/cc
$$\times \frac{22}{7} \times 3.5 (2 \times 2.4 - 0.2) (0.2) \text{ cm}^3$$

= 11.4 $\times \frac{22}{7} \times 3.5 \times 4.6 \times 0.2$
= 11.4 $\times 22 \times 0.5 \times 4.6 \times 0.2 = 115.368 \text{ g}$

ROLLING A SQUARE INTO A CYLINDER

EXAMPLE 15 A rectangular sheet with dimensions $22 \text{ m} \times 10 \text{ m}$ is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

Solution Volume =
$$\frac{\text{Height} \times (\text{Other side of the sheet})^2}{4\pi}$$
Here, Volume =
$$\frac{10 \times (22)^2}{4 \times \frac{22}{7}} = 385 \text{ m}^3$$

PROBLEM ON CONES

EXAMPLE 16 Find what length of canvas 2 metres in width is required to make a conical tent 8 metres in diameter and 5.6 metres in slant height. Also find the cost of the canvas at the rate of $\stackrel{?}{\underset{\sim}{\sim}}$ 3.20/m.

Solution Curved surface area = $\pi r l$

$$= \frac{22}{7} \times \left(\frac{1}{2} \times 8\right) \times 5.6$$

$$= 22 \times 4 \times 0.8 = 70.4 \text{ m}^2$$

 \therefore Length of the canvas = $70.4 \div 2 = 35.2$ metres

Cost of the canvas = ₹ $35.2 \times 3.2 = ₹ 112.64$

EXAMPLE 17 The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its

- (i) Curved surface area, (ii) Total surface area (iii) Volume, and
- (iv) The cost of colouring its total surface at the rate of 14 paise/m².

Solution

(i) Curved surface area =
$$\pi r l = \frac{22}{7} \times \frac{14}{2} \times 12 =$$
7 264 m²

(ii) Total surface area =
$$\pi r(r+l) = \frac{22}{7} \times \frac{14}{2} \left(\frac{14}{2} + 12 \right) = 22 \times 19 = 418 \text{ m}^2$$

(iii) Volume =
$$\frac{1}{3} \pi r^2 h$$

Here,
$$h = \sqrt{l^2 - r^2} = \sqrt{12^2 - 7^2} = 9.75 \text{ m}$$

:. Volume =
$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 9.75 = 22 \times 7 \times 3.25 = 500.5 \text{ m}^3$$

(iv) Required cost = ₹418 ×
$$\frac{14}{100}$$
 = ₹ 58.52

EXAMPLE 18 A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm and a height of 5 cm. Find the area of its whole surface and volume.

Solution R = 5 cm, r = 3 cm, and h = 5 cm

$$l = \sqrt{h^2 + (R - h)^2} = \sqrt{5^2 + (5 - 3)^2} = \sqrt{29} = 5.385 \text{ cm}$$

 \therefore Whole surface area of the frustum = $\pi(R^2 + r^2 + Rl + rl)$

=
$$\frac{22}{7}$$
 (5² + 3² + 5 × 5.385 + 3 × 5.385) = **242.25** cm²

$$Volume = \frac{\pi h}{3} (R^2 + r^2 + Rr)$$

$$= \frac{22}{7} \times \frac{5}{3} (5^2 + 3^2 + 5 \times 3)$$
$$= \frac{22}{7} \times \frac{5}{3} \times 49 = 256.67 \text{ cm}^3$$

PROBLEMS ON SPHERES

Type I Direct Application of Formula

EXAMPLE 19 Find the volume and the surface area of a sphere of diameter 42 cm.

Solution Radius = 42/2 = 21 cm

:. Volume =
$$\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 38808 \text{ cm}^3$$

Surface area =
$$4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 = 5544 \text{ cm}^2$$

EXAMPLE 20 Find the volume, curved surface area and the total surface area of a hemisphere of radius 21 cm.

Solution Volume =
$$\frac{2}{3}\pi r^3$$

= $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 19404 \text{ cm}^3$

Curved surface area =
$$2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = 2772 \text{ cm}^2$$

Total surface area =
$$3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 = 4158 \text{ cm}^2$$

Type II Indirect Method

Spheres Changing Shapes

The basic principle in solving questions of this type where solid in the shape of any object (sphere, cone etc.) changes into some other shape, the volume remains unchanged.

EXAMPLE 21 A metallic sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.

Solution When a sphere is converted into a cylinder, the length of wire is given by

Length of cylinder =
$$\frac{4 \times (\text{Radius of sphere})^3}{3 \times (\text{Radius of cylinder})^2}$$

i.e.,
$$l = \frac{4 \times (18/2)^3}{3 \times (4/20)^2} = \frac{4 \times 9^3 \times 20^2}{3 \times 4^2}$$

$$= \frac{4 \times 9^3 \times 10^2}{3 \times 2^2} = 9^2 \times 3 \times 100 = 24300 \text{ cm}$$



Note:

1. Radius of sphere =
$$\sqrt{\frac{3}{4}}$$
 [(Length of cylinder) × (Radius of cylinder)²]

2. Radius of cylinder =
$$\sqrt{\frac{4 \times (\text{Radius of sphere})^2}{3 \times (\text{Length of cylinder})}}$$

EXAMPLE 22 A copper sphere of 36 cm diameter is drawn into a cylindrical wire of length 2.43 m. What is the radius of wire?

Solution Radius of wire =
$$\sqrt{\frac{4 \times (\text{Radius of sphere})^3}{3 \times \text{Length of wire}}}$$

= $\sqrt{\frac{4 \times 36^3}{3 \times 243}} = \sqrt{\frac{4 \times 4^3 \times 9^3}{9^3}} = 4^2 = 16 \text{ cm}$

EXAMPLE 23 A cylinder of radius 2 cm and height 15 cm is melted and the same mass is used to create a sphere. What will be the radius of the sphere?

Solution Radius of sphere =
$$\sqrt{\frac{3}{4}}$$
 (Length of cylinder) × (Radius of cylinder)²
= $\sqrt{\frac{3}{4} \times 15 \times 2^2} = \sqrt{45} = 3\sqrt{5}$ cm

EXAMPLE 24 How many bullets can be made out of a lead cylinder 28 cm high and 6 cm radius, each bullet being 1.5 cm in diameter?

Solution Number of bullets =
$$\frac{\text{Volume of cylinder}}{\text{Volume of 1 bullet}}$$

$$= \frac{\pi \times 6 \times 6 \times 28}{\frac{4}{3}\pi \times (0.75)^3} = \frac{6 \times 6 \times 28 \times 3}{4 \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}}$$

$$= 2 \times 2 \times 28 \times 4 \times 4 = 1792$$

EXAMPLE 25 Find the number of lead balls of diameter 1 cm each that can be made from a sphere of diameter 16 cm.

Solution Number of balls =
$$\frac{\text{Volume of big sphere}}{\text{Volume of 1 small sphere}}$$

$$= \frac{\frac{4}{3}\pi \times \frac{16}{2} \times \frac{16}{2} \times \frac{16}{2}}{\frac{4}{3}\pi \times 0.5 \times 0.5 \times 0.5}$$

$$= \frac{8 \times 8 \times 8}{0.5 \times 0.5 \times 0.5} = 4096$$



Short cut: Number of balls =
$$\left(\frac{\text{Bigger radius}}{\text{Smaller radius}}\right)^3 = \left(\frac{8}{0.5}\right)^3 = 16^3 = 4096$$

TYPICAL CASES

Problems Involving Ratios

I. Two spheres

- (i) $(Ratio of radii)^2 = Ratio of surface areas$
- (ii) $(Ratio of radii)^3 = Ratio of volume$
- (iii) (Ratio of surface areas)³ = (Ratio of volume)²

II. Two cylinders

A. When the volumes are equal

- (i) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$
- (ii) Ratio of curved surface areas = Inverse ratio of radii
- (iii) Ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$

B. When the radii are equal

- (i) Ratio of volumes = Ratio of heights
- (ii) Ratio of curved surface areas = Ratio of heights
- (iii) Ratio of volumes = Ratio of curved surface areas

C. When the heights are equal

- (i) Ratio of volumes = $(Ratio of radii)^2$
- (ii) Ratio of curved surface areas = Ratio of radii
- (iii) Ratio of volumes = $(Ratio of curved surface areas)^2$

D. When the curved surface areas are equal

- (i) Ratio of volumes = Ratio of radii
- (ii) Ratio of volumes = Inverse ratio of heights
- (iii) Ratio of radii = Inverse ratio of heights

III. Two cubes

- (i) Ratio of volumes = $(Ratio of sides)^3$
- (ii) Ratio of surface areas = $(Ratio of sides)^2$
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

IV. Two cones

A. When the volumes are equal

- (i) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$
- B. When the radii are equal
 - (i) Ratio of volumes = Ratio of heights
- C. When heights are equal
 - (i) Ratio of volumes = $(Ratio of radii)^2$
- D. When curved surface areas are equal
 - (i) Ratio of radii = Inverse ratio of slant heights

EXAMPLE 26 The curved surface areas of two spheres are in the ratio 1:4. Find the ratio of their volumes.

Solution We have (ratio of surface areas) $^3 = (Ratio of volumes)^2$

$$\therefore$$
 (1:4)³ = (Ratio of volumes)²

i.e.
$$1:64 = (\text{ratio of volumes})^2$$

$$\therefore \qquad \text{Ratio of volumes} = \sqrt{1:64} = 1:8$$



Short cut: Ratio of radii =
$$\sqrt{\frac{1}{4}} = \frac{1}{2}$$

Ratio of volumes =
$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 1:8$$

EXAMPLE 27 The radii of two spheres are in the ratio of 1:2. What is the ratio of their surface areas?

Solution We have ratio of surface areas =
$$(\text{Ratio of radii})^2 = \left(\frac{1}{2}\right)^2 = 1:4$$

EXAMPLE 28 Two cylinders of equal volume have their heights in the ratio of 1:2. What will be the ratio of their radii?

Solution We have the ratio of radii =
$$\sqrt{\text{Inverse ratio of heights}} = \sqrt{2:1} = \sqrt{2}:1$$

EXAMPLE 29 If the heights of two cones are in the ratio 1:4 and their diameters are in the ratio 4:5, what is the ratio of their volumes?

Solution Volume of cone = $\frac{1}{3}\pi(\text{radius})^2 \times \text{height}$

$$\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi \left(\text{Radius of first cone}\right)^2 \times \left(\text{Height}\right)}{\frac{1}{3}\pi \left(\text{Radius of second cone}\right)^2 \times \left(\text{Height}\right)}$$

$$= \left(\frac{\text{Radius of first cone}}{\text{Radius of second cone}}\right)^2 \left(\frac{\text{Height of 1st cone}}{\text{Height of 2nd cone}}\right)$$
Ratio of volumes = (ratio of radii)² (ratio of heights)
$$= (4:5)^2 (1:4)$$

$$= \frac{16}{25} \times \frac{1}{4} = \frac{4}{25} = 4:25$$

V. For a cone and also for a cylinder

- (i) Ratio of volumes = $(Ratio of radii)^2 \times (Ratio of heights)$
- (ii) Ratio of heights = $(Inverse ratio of radii)^2 \times (Ratio of volumes)$
- (iii) Ratio of radii = $\sqrt{\text{(Ratio of volumes)} \times \text{(Inverse ratio of heights)}}$

VI. For a cylinder

- (i) Ratio of curved surface areas = (Ratio of radii) × (Ratio of heights)
- (ii) Ratio of heights = (Ratio of curved surface areas) × (Inverse ratio of radii)
- (iii) Ratio of radii = (Ratio of curved surface areas) × (Inverse ratio of heights)

VII. For a cone

- (i) Ratio of curved surface areas = (ratio of radii) × (ratio of slant heights)
- (ii) Ratio of slant heights = (ratio of curved surface areas) × (inverse ratio of radii)
- (iii) Ratio of radii = (ratio of curved surface areas) × (inverse ratio of slant heights)

EXAMPLE 30 If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new volume and the previous volume?

Solution Let the initial radius and height of the cylinder be r cm and h cm respectively

Then

:.

$$V_1 = \pi r^2 h$$

$$V_2 = \pi (2r)^2 \ h/2 = 2\pi r^2 h$$

$$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = \mathbf{2}:\mathbf{1}$$

EXAMPLE 31 A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all round it to a width of 7 m to form a circular embankment. Find the height of this embankment.

Solution Volume of earth dug out =
$$\pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8$$

= $\frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$
Area of embankment = $\pi[(5.6 + 7)^2 - (5.6)^2]$
= $\frac{22}{7} \times (5.6 + 7 + 5.6) (5.6 + 7 - 5.6)$
= $\frac{22}{7} \times 18.2 \times 7 = 400.4 \text{ m}^2$
Height of embankment = $\frac{\text{Volume of earth}}{\text{Area of embankment}} = \frac{788.48}{400.4} = 1.97 \text{ m}$

EXAMPLE 32 A right-angled triangle having base 6.3 m and height equal to 10 cm is turned around the height. Find the volume of the cone thus formed. Also find the surface area.

Solution Height of cone thus formed = Height of triangle

Radius = Base of the triangle

Slant height = Hypotenuse of the triangle

Surface area = π (radius) × (slant height)

$$= \frac{22}{7} (6.3) (\sqrt{6.3^2 + 10^2})$$

$$= 22 \times 0.9 \sqrt{139.69}$$

$$= 22 \times 0.9 \times 11.82 = 234.04 \text{ m}^2$$

Theorem: For Cuboid:

If the length, breadth and height of a cuboid are increased by x%, y% and z% respectively, then

its volume is increased by
$$\left[x+y+z+\frac{xy+yz+xz}{100}+\frac{xyz}{(100)^2}\right]\%$$

For Cube:

If the side of a cube is increased by x%, then its volume will be increased by

$$\left[3x + \frac{3x^2}{100} + \frac{x^3}{100^2}\right]\% \quad \text{or} \quad \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$$

For Sphere:

If the radius of a sphere is changed by x% then its volume will be changed by $\left[3x + \frac{3x^2}{100} + \frac{x^3}{100}\right]$ %

For Cylinder:

- I. If the height is changed by x% and radius remains the same, then volume will be changed by x%.
- II. If radius is changed by x% and the height remains the same, the volume will be changed by

$$\left[2x + \frac{x^2}{100}\right]\% \quad \text{or} \quad \left[\left(1 + \frac{x}{100}\right)^2 - 1\right] \times 100\%$$

III. If the radius is changed by x% and the height is changed by y% then volume will be changed by

$$\left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2y}{100}\right]\%$$

IV. If height and radius both change by x%, then volume will change by $\left[3x + \frac{3x}{100} + \frac{x^3}{100^2}\right]\%$

EXAMPLE 33 Each edge of a cube is increased by 50%. What is the percentage increase in its volume? Also find the percentage increase in its surface area?

Solution Percentage increase in volume

$$= \left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\%$$

$$= \left[\left(1 + \frac{50}{100} \right)^3 - 1 \right] \times 100\%$$

$$= \left[(1.5)^3 - 1 \right] \times 100\%$$

$$= \left[\frac{6.75 - 2}{2} \right] \times 100\% = \frac{4.75}{2} \times 100\% = 237.5\%$$
antaga in area = \[2x \ x^2 \]

Percentage increase in area = $\left[2x + \frac{x^2}{100}\right]$

=
$$(2 \times 50) + \frac{(50 \times 50)}{100} = 125\%$$

EXAMPLE 34 Each of the radius and the height of a cone is increased by 20%. Find the percentage increase in volume.

EXAMPLE 35 The radius of a sphere is increased by 5%. Find the percentage increase in its surface area.

Solution Required percentage value =
$$2x + \frac{x^2}{100} = 2 \times 5 + \frac{5^2}{100} = 10.25$$

i.e. surface area of sphere is increased by 10.25%.

EXAMPLE 36 Each edge of a cube is decreased by 50%. Find the percentage decrease in its surface area and volume.

Solution For a two dimensional figure, percentage increase or decrease in area is given

$$by \left[2x + \frac{x^2}{100} \right] \%$$

i.e. required percentage =
$$2(-50) + \frac{(-50)^2}{100} = -100 + 25 = -75\%$$

i.e. area decreases by 75%

Percentage decrease in volume

$$= \left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100 = \left[\left(1 - \frac{50}{100} \right)^3 - 1 \right] \times 100$$
$$= \left[(0.5)^3 - 1 \right] \times 100 = -0.875 \times 100 = -87.5$$

i.e. volume decreases by 87.5%

EXAMPLE 37 Find the ratio of volumes of a cube to that of a sphere which will fit inside the cube.

Solution Let side of the cube be a units and diameter of the sphere be a units

$$\frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{a^3}{\frac{4}{3}\pi \left(\frac{a}{2}\right)^3}$$

$$= \frac{1}{\frac{4}{3}\pi \times \frac{1}{8}} = \frac{6}{\pi} = 6 : \pi$$

EXAMPLE 38 A cube of maximum volume (each corner touching the surface from inside) is cut from a sphere. Find the ratio of the volume of the cube and the sphere.

Solution Let the radius of the sphere be r cm and side of the cube be x cm. Then diagonal of cube = diameter of sphere

or
$$\sqrt{3} x = 2r \implies x = \frac{2r}{\sqrt{3}}$$
Ratio of volume = $\frac{\text{Volume of cube}}{\text{Volume of sphere}}$

$$= \frac{x^3}{\frac{4}{3}\pi r^3} = \frac{(2r/\sqrt{3})^3}{\frac{4}{3}\pi r^3} = \frac{8/(3\sqrt{3}) \times r^3}{\frac{4}{3}\pi \times r^3}$$

$$= \frac{8}{\sqrt{3}} \div 4\pi = \frac{2}{\pi\sqrt{3}}$$

$$\therefore \text{Ratio = 2 : } \sqrt{3}\pi$$

EXAMPLE 39 The volumes of two cubes are in the ratio of 8: 125. Find the ratio of their edges and surface areas.

Solution We have,

٠.

Ratio of sides = $(Ratio of volumes)^{1/3}$

Ratio of surface areas = $(Ratio of volumes)^{2/3}$

Ratio of sides = $(8:125)^{1/3} = 2:5$

Ratio of surface areas = $(8:125)^{2/3} = 4:25$

EXAMPLE 40 A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6:5. Find the smaller side of the rectangle.

Solution Length of wire = Circumference of the circle

$$= 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

Perimeter of rectangle = 264 cm

Length of rectangle =
$$\frac{6}{2(6+5)} \times 264 = 72$$
 cm

Breadth of rectangle =
$$\frac{5}{2(6+5)} \times 264 = 60$$
 cm

EXAMPLE 41 A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 cc, what is the approximate volume of the cone?

Solution Edge of the cube = $\sqrt[3]{343} = 7$ cm

Radius of cone = 7/2 = 3.5 cm

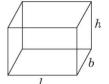
Height of cone = 7 cm

 $\therefore \qquad \text{Volume of cone} = \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 7 = \frac{11 \times 49}{6} \approx 90 \text{ cm}^3$$

Important Formulae

- 1. Right parallelopiped
 - (a) Surface area (of side faces) = 2h(l + b)
 - (b) Surface area (of base or top face) = $2\sqrt{s(s-a)(s-b)(s-d)}$ sq. units.



- (c) Total surface area = $2h(l+b) + 4\sqrt{s(s-a)(s-b)(s-d)}$ sq. units.
- (d) Volume = Base area \times Height.
- 2. Cylinder

Let the radius of base of cylinder be r units and its height be h units. Then

- (a) Volume of the cylinder = $(\pi r^2 h)$ cubic units.
- (b) Curved surface area of the cylinder = $(2\pi rh)$ sq. units.
- (c) Total surface area of the cylinder = $(2\pi rh + 2\pi r^2)$ sq. units.
- 3. Hollow cylinder: Let the outer radius of cylinder be R units and inner radius of cylinder be r units. Let the height of cylinder be h units and thickness be (R-r)=t units.
 - (a) Volume of cylinder = $\pi h(R^2 r^2)$ cubic units.

$$=\pi h(R+r)t$$
 cubic units.

- (b) Outer curved surface area of cylinder = πRh square units.
- (c) Inner curved surface area = πrh square units.
- (d) Total curved surface area of cylinder = $\pi h(R + r)$ square units
- (e) Total surface area of cylinder $= \pi h(R+r) + 2\pi(R^2 r^2)$ sq. units $= \pi(R+r) \{h + 2(R-r)\}$ sq. units $= \pi(R+r) \{h + 2t\}$ sq. units.

PROBLEMS ON CYLINDERS

Type I Direct Application of Formulae

EXAMPLE 42 Find the volume of a cylinder which has a height of 14 metres and a base of radius 4 metres. Also find the curved surface area of the cylinder.

Solution

Volume =
$$\pi r^2 h \text{ m}^3$$

= $\frac{22}{7} \times 4 \times 4 \times 14 = 704 \text{ m}^3$

Curved surface area = Circumference × Height

$$= (2\pi r) \times h \text{ m}^2$$

=
$$2 \times \frac{22}{7} \times 4 \times 14 = 352 \text{ m}^2$$

EXAMPLE 43 A hollow cylindrical tube opens at both ends is made of iron 2 cm thick. If the external diameter is 50 cm and the length of the tube is 105 cm, find the volume of iron in it.

Solution Volume of metal = $\pi h(R^2 - r^2)$

External radius $R = 50 \div 2 = 25$ cm

 \therefore Inner radius, r = R - t

$$= 25 - 2 = 23$$
 cm

:. Volume = $\frac{22}{7} \times 105 \times (25^2 - 23^2)$ = $330 \times 96 = 31680 \text{ m}^3$



Short cut: Volume = $\pi \times \text{Height} \times (2 \times \text{Outer radius} - \text{Thickness}) \times (\text{Thickness})$

(if outer radius is known)

Volume = $\pi \times \text{Height} \times (2 \times \text{Inner radius} + \text{Thickness}) \times (\text{Thickness})$

(if inner radius is given)

Here.

Volume =
$$\frac{22}{7} \times 105 \times (2 \times 25 - 2) (2)$$

= $\frac{22}{7} \times 105 \times 48 \times 2 = 31680 \text{ cu. m}$

Rolling a Square into a Cylinder

EXAMPLE 44 A rectangular sheet with dimensions $22 \text{ m} \times 12 \text{ m}$ is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

Solution



Short cut: In this case, use the rule,

Volume =
$$\frac{\text{Height} \times (\text{Other side of the sheet})^2}{4\pi}$$

:. Here, Volume =
$$\frac{12 \times (22)^2}{4 \times \frac{22}{7}} = 462 \text{ m}^3$$
.

Cones

EXAMPLE 45 Find the length of canvas 2 metres in width required to make a conical tent 8 metres in diameter and 5.6 metres in slant height. Find out the cost of the canvas at the rate of 3.20/metre.

Solution Curved surface area = πrl

$$= \frac{22}{7} \times \frac{1}{2} \times 8 \times 5.6$$

$$= 22 \times 4 \times 0.8 = 70.4 \text{ m}^2$$
Length of canvas = $\frac{\text{Curved surface area}}{\text{Width of canvas}} = \frac{70.4}{2} = 35.2 \text{ m}$

Cost of canvas at ₹ 3.2/m = $35.2 \times 3.2 = ₹ 112.64$

EXAMPLE 46 The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its

- (i) Curved surface area (ii) Total surface area (iii) Volume and,
- (iv) Cost of colouring its total surface at the rate of ₹ 1.2/m²

Solution

(i) Curved surface area =
$$\pi r l = \frac{22}{7} \times \frac{14}{2} \times 12$$

= $22 \times 12 = 264 \text{ m}^2$

(ii) Total surface area =
$$\pi r(r+l) = \frac{22}{7} \times \frac{14}{2} \left(\frac{14}{2} + 12\right)$$

= $\frac{22}{7} \times \frac{14}{2} \times 19 = 22 \times 19 = 418 \text{ m}^2$

(iii) Volume =
$$\frac{1}{3} \pi r^2 h$$

But $h = \sqrt{l^2 - r^2}$
= $\sqrt{(12)^2 - (7)^2} = 9.75$ metres
Volume = $\frac{1}{3} \times \frac{22}{7} \times 7^2 \times 9.75 = 500.5$ m³

(iv) Cost of colouring = $418 \times 1.2 = ₹ 501.6$

EXAMPLE 47 Frustum of a cone has a base diameter of 10 cm, and top of 6 cm diameter and a height of 5 cm. Find the area of its whole surface and volume.

Solution R = 5 cm; r = 3 cm; h = 5 cm

$$l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{5^2 + (5 - 3)^2} = \sqrt{29} = 5.385 \text{ cm}$$

.. Whole surface area of the frustum

$$= \pi[(R^2 + r^2 + (R + r)l]$$

$$= \frac{22}{7}[5^2 + 3^2 + (5 + 3)5.385] = 242.25 \text{ m}^2$$
Volume = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

$$= \frac{22}{7} \times \frac{5}{3}(5^2 + 3^2 + 5 \times 3) = 256.67 \text{ m}^3$$

Important Formulae

I. Two Spheres

- (i) $(Ratio of radii)^2 = Ratio of surface areas$
- (ii) $(Ratio of radii)^3 = Ratio of volume$
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

II. Two Cubes

- (i) Ratio of volumes = $(Ratio of sides)^3$
- (ii) Ratio of surface areas = $(Ratio of sides)^2$
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

III. Two cylinders

- (a) When the volumes are equal
 - (i) Ratio of radii = \(\sqrt{Inverse ratio of heights} \)
 - (ii) Ratio of curved surface areas = Inverse ratio of radii
 - (iii) Ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$
- (b) When the radii are equal
 - (i) Ratio of volumes = Ratio of heights
 - (ii) Ratio of curved surface areas = Ratio of heights
 - (iii) Ratio of volume = Ratio of curved surface areas
- (c) When the heights are equal
 - (i) Ratio of volumes = $(Ratio of radii)^2$
 - (ii) Ratio of curved surface areas = Ratio of radii
 - (iii) Ratio of volumes = (Ratio of curved surface areas)²
- (d) When curved surface areas are equal
 - (i) Ratio of volumes = Ratio of radii
 - (ii) Ratio of radii = Inverse ratio of heights
 - (iii) Ratio of volumes = Inverse ratio of heights

IV. Two Cones

- (a) When the volumes are equal Formula (i) of cylinders holds
- (b) When radii are equal Formula (i) of cylinders holds
- (c) When the heights are equal Formula (i) of cylinders holds
- (d) When the curved surface areas are equal Formula (iii) of cylinders holds with a change
 - i.e. Ratio of radii = Inverse ratio of slant heights

(e) None of these

(e) None of these

(e) None of these

(c) $3:2:1, \sqrt{2}:\sqrt{2}:1$

(d) 100 cm

(d) 36:25

(d) 223.83 m^2

1. A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in

2. The volumes of two cubes are in the ratio of 8: 125. Then find the ratio of their edges and

3. A cylinder, a hemisphere and a cone stand on the same base and have the same heights. Find the ratio of their volumes and also the ratio of the areas of their curved surface. (b) $1:2:3, 1:\sqrt{2}:\sqrt{2}$

4. A right angled triangle having base 6.3 m and height equal to 10 cm is turned around the

(c) 213.83 m^2

(c) 80 cm

(c) 4:25

(e) None of these

height. Find the volume of the cone thus formed. Also find the surface area.

the ratio 6:5. Find the smaller side of the rectangle. (b) 60 cm

(b) 9:25

(b) 233.83 m^2

EXERCISES

(a) 40 cm

surface areas. (a) 16:25

(a) $1:1:1, 1:1:\sqrt{2}$ (d) $2:3:1, \sqrt{2}:1:\sqrt{2}$

(a) 253.83 m^2

5.	If the heights of the is the ratio of the		ne ratio 1 : 5 and th	neir diameters are in the ra	ıtio 5 : 6, what
	(a) 1:6	(b) 1:36	(c) 5:6	(d) 5:36 (e) I	None of these
6.				ts surface area is 88 cm ² .	
	(a) 24 cm^3	(b) 48 cm^3	(c) 64 cm^3	(d) 120 cm^3	
7.			e is 20 cm. Its vol		[SSC]
	_		(c) 1000 cm^3		
8.				he ratio of their edges is	[SSC]
	(a) 1:3		(c) 3:1	_	
9.	. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off t				drawn off the
	(a) $10\frac{1}{2}$ cm	(b) $11\frac{3}{7}$ cm	(c) $12\frac{6}{7}$ cm	(d) 14 cm	
10.		_		eight 15 cm and base dian	neter 16 cm is [SSC]
	(a) $60\pi \text{ cm}^2$	(b) $68\pi \text{ cm}^2$	(c) $120 \pi \text{ cm}^2$	(d) $136\pi \text{ cm}^2$	-
11.	A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl at transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is				f the diameter
	(a) $66\frac{2}{3}\%$	(b)	$78\frac{1}{2}\%$	(c) 100%	
12.			ternal diameters 4 height of the con-	cm and 8 cm respectively is	is melted into [RRB]
	(a) 12 cm	(b) 14 cm	(c) 15 cm	(d) 18 cm	
13.	The volumes of t	wo spheres are in	the ratio of 64:2	27. The ratio of their surfa	ce areas is [RRB]
	(a) 1:2	(b) 2:3	(c) 9:16	(d) 16:9	

(a) 8 cm

(b) 9 cm

14.	The slant height of a curved surface	a right circular con-	e is 10 m, and i	its height is 8 m. F	ind the area of its [RRB]	
		$40\pi \mathrm{m}^2$ (c)	$60\pi \mathrm{\ m}^2$	(d) $80\pi \mathrm{m}^2$		
15.	How many cubes of 1			, ,	[RRB]	
				(d) 10000		
16.	A cylindrical vessel o	of radius 4 cm cont	ains water. A s	olid sphere of radiv	us 3 cm is lowered	
	into the water until i	it is completely im	nersed. The wa	iter level in the ves	ssel will rise by	
					[MBA]	
	(a) $\frac{2}{9}$ cm (b)	$\frac{4}{2}$ cm (a)	9 - cm	(d) 9 cm		
	(a) 9 cm (b)	$\frac{1}{9}$ cm (c)	$\frac{-4}{4}$ cm	$\frac{1}{2}$ cm		
17.	The volume of the la				e of edge 7 cm is [MAT]	
	(a) 13.6 cm^3 (b)) 89.8 cm^3 (c)	$121~\mathrm{cm}^3$	(d) 147.68 cm^3		
18.	Three cubes with sid				be whose diagonal	
	is $12\sqrt{3}$ cm. The side	es of the cubes are			[MAT]	
	(a) 3 cm, 4 cm, 5 cm	l	(b) 6 cm	, 8 cm, 10 cm		
	(c) 9 cm, 12 cm, 15 cm	cm	(d) None	e of these		
19.	The length of an edge	e of a hollow cube o	pened at one fa	ace is $\sqrt{3}$ metres. V	What is the length	
	of the largest pole th				[MAT]	
	(a) $\sqrt{3}$ m (b)) 3 m (c)	$3\sqrt{3}$ m	(d) $\frac{3}{\sqrt{2}}$ m		
				VS	-	
20.	A metallic sheet is corners, a square is corners.					
	volume of the box (in		e an open box. I	if the length of the	[MAT]	
			6420	(d) 8960	. ,	
21.	The number of bricks	s, each measuring 2	$5 \text{ cm} \times 12.5 \text{ cm}$	\times 7.5 cm, required	to construct a wall	
	6 m long, 5 m high an	nd 0.5 m thick, whil	e the mortar occ	cupies 5% of the vol		
	(a) 2040 (b)) 5740 (a)	6060	(J) 0100	[MBA]	
99	(a) 3040 (b) A solid metallic sphere			(d) 8120	one with diameter	
22.	of the base as 12 cm.			a and recast into a c	[CBI]	
		_		(d) 6 cm		
23.	If the measured value	,		` '	rect to one decimal	
	place) made in calcul				[CBI]	
	(a) 2.1 (b)) 3.2 (c)	4.6	(d) 5.4		
24.	Consider the volume	es of the following:				
	1. A parallelopiped			height 4 cm.		
	2. A cube of each side					
		3. A cylinder of radius 3 cm and length 3 cm.4. A sphere of radius 3 cm.				
		The volumes of these in the decreasing order is [Civil Services]				
			-	(d) 4, 3, 2, 1	- "•	
25 .	The radii of two cone	es are in the ratio 2	: 1, their volur	mes are equal. Find	d the ratio of their	
	heights.				[CBI]	
			2:1	(d) 4:1		
26.	The radius and heigh what is its slant heigh		r cone are in th	e ratio 3 : 4. If its v	volume is $96\pi \text{ cm}^3$, [CBI]	

(c) 10 cm

(d) 12 cm

[CBI]

(d) 336

27. 66 cm³ of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be

28. X and Y are two cylinders of the same height. The base of X has diameter which is equal to

(c) 168

(a) 84

(a) 5600

(b) 6000

(b) 90

	half the diameter of the base of Y. If the	height of X is doubled, the volume of X becomes		
	(a) equal to the volume of Y	(b) double the volume of Y		
	(c) half the volume of Y	(d) greater than the volume of Y		
29.		dth, and the length of the wall is seven times its		
	height. If volume of the wall be 16128 m	3 , its width is [CBI]		
		5 m (d) 6 m		
30.	The radius of cylinder is half its height a how many litres of milk can it contain?	nd area of the inner part is 616 cm ² . Approximately [SBI PO]		
	(a) 1.4 (b) 1.5 (c)	()		
31.	The capacity of a cylindrical tank is 246.4 of the base?	litres. If the height is 4 metres, what is the diameter $[Bank\ PO]$		
	1.	14 m (d) 28 m (e) None of these		
32.	The cost of paint in ₹ 36/kg. If 1 kg of poutside of a cube having 2.5 m side?	paint covers 1.5 m ² , how much will it cost to paint [Bank PO]		
	(a) ₹ 700 (b) ₹ 784 (c)	₹ 884 (d) ₹ 900 (e) None of these		
33.	A hall is 15 m long and 12 m broad. If the equal to the sum of areas of the four wall	the sum of the areas of the floor and the ceiling is lls, the volume of the hall is [LIC AAO]		
		1200 (d) 1800		
34.	A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden so sphere of radius 9 cm. The percentage of the wood wasted is [SS]			
	(a) 25% (b) $25\pi\%$ (c)	50% (d) 75%		
35 .	If three metallic spheres of radii 6 cm, 8 c diameter of the new sphere will be:	m and 10 cm are melted to form a single sphere, the [DMRC]		
	(a) 12 cm (b) 24 cm (c)	30 cm (d) 36 cm		
36.	A large cube is formed from the material obtained by melting three smaller cubes of sides 3 cm, 4 cm and 5 cm respectively. What is the ratio of the total surface areas of the smaller cubes and the large cube? [MAT]			
		25:18 (d) 27:20		
37.	A rectangular water tank is $80 \text{ m} \times 40 \text{ m}$. Water flows into it through a pipe 40 cm^2 at the opening at a speed of 10 km/hr . By how much, the water level will rise in the tank in half an hour? [MBA]			
	(a) $\frac{3}{2}$ cm (b) $\frac{4}{9}$ cm (c)	$\frac{5}{8}$ cm (d) None of these		
38.	The volume of the greatest sphere that or radius 1 cm and height 5 cm is	can be cut off from a cylindrical log of wood of base [CBI]		
	(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c)	5π (d) $\frac{20}{3}\pi$		
39.	A cylinder with base radius of 8 cm and cm. The radius of the cone will be:	height of 2 cm is melted to form a cone of height 6 $[RRB]$		
	(a) 4 cm (b) 5 cm (c)	6 cm (d) 8 cm		
40.	How many bricks, each measuring 25 cm $8 \text{ m} \times 6 \text{ m} \times 22.5 \text{ cm}$?	m m imes 11.25~cm imes 6~cm, will be needed to build a wall [BSF]		

(c) 6400

(d) 7200





Solutions with Necessary Explanation

Length of wire = Perimeter of rectangle
 Circumference of circle

i.e.
$$2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264$$
 cm

.. Perimeter of rectangle = 264

$$\therefore \qquad \text{Length} = \frac{6}{11 \times 2} \times 264 = 72 \text{ cm}$$

Breadth =
$$\frac{5}{11 \times 2} \times 264 = 60$$
 cm

2. Ratio of volumes of cubes = $(Ratio of sides)^3$

i.e. $8:125=(2)^3:(5)^3$

 \therefore Edges are in the ratio 2:5

Ratio of surface areas = $(Ratio of sides)^2 = (2)^2 : (5)^2 = 4 : 25$

3. Let the heights be h units

Let the radii be r units

Let the volumes be respectively v_1 , v_2 and v_3 .

Let curved surface areas be respectively CSA₁, CSA₂ and CSA₃.

 $=2h:2r:\sqrt{2}r$

$$v_{1}: v_{2}: v_{3} = \pi r^{2}h: \frac{2}{3}\pi r^{3}: \frac{1}{3}\pi r^{2}h$$

$$= h: \frac{2}{3}r: \frac{h}{3}$$
or
$$3h: 2r: h \text{ or } 3: 2: 1$$

$$CSA_{1}: CSA_{2}: CSA_{3} = 2\pi rh = 2\pi r^{2}: \pi rl$$

$$= 2h: 2r: l$$

$$= 2h: 2r: \sqrt{r^{2} + h^{2}}$$

$$= 2h: 2r: \sqrt{2r^{2}}$$

$$(\because r = h)$$

or
$$2:2:\sqrt{2}$$

or
$$\sqrt{2}:\sqrt{2}:1$$

Surface area of cone = πrl

But
$$l = \sqrt{r^2 + h^2} = \sqrt{(6.3)^2 + 10^2} = 11.81 \text{ m}$$

:. Surface area =
$$\frac{22}{7} \times 6.3 \times 11.81 = 233.83 \text{ m}^2$$

- 5. Ratio of volumes of two cones = $\frac{r_1^2 h_1}{r_2^2 h_2}$ = $\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$ = $\left(\frac{5}{6}\right)^2 \times \left(\frac{1}{5}\right) = 5:36$
- **6.** Surface area = $2 \times l \times b + 2(l+b) \times h = 88 \text{ cm}^2$

Let
$$l = 3x$$
; $b = 2x$; $h = x$;

$$\therefore \quad \text{Surface area} = 2 \times 3x \times 2x + 2(3x + 2x)x = 88 \text{ cm}^2$$

i.e.
$$12x^2 + 10x^2 = 88$$

$$x^2 = \frac{88}{22} = 4$$

$$\therefore$$
 $x=2$

.. Volume of cuboid =
$$3x \times 2x \times x = 6x^3$$

= $6(2)^3 = 48 \text{ cm}^3$

7. Given $4a = 20 \text{ cm} \implies a = 5 \text{ cm}$;

:. Volume =
$$a^3 = 5^3 = 125 \text{ cm}^3$$

8.
$$\frac{a_1^3}{a_2^3} = \frac{27}{1} \implies \frac{a_1}{a_2} = \frac{3}{1}$$

9. Water drawn =
$$\pi \times \left(\frac{35}{2}\right)^2 \times h = 11000 \text{ cm}^3$$

or
$$h = \frac{11000 \times 4}{\pi \times 35^2} = \frac{11000 \times 4}{\frac{22}{7} \times 35^2}$$

= $\frac{11000 \times 4}{22 \times 35 \times 5} = \frac{80}{7} = 11\frac{3}{7}$ cm

10. Given h = 15 cm; $r = \frac{16}{2} = 8$ cm

$$l = \sqrt{h^2 + r^2} = \sqrt{15^2 + 8^2} = 17 \text{ cm}$$

Curved surface area = $\pi rl = \pi \times 8 \times 17 = 136 \pi \text{ cm}^2$

11. $\frac{2}{3}\pi r^3$ = Volume of hemispherical bowl.

Let h be the height of cylinder.

$$\therefore$$
 $r = 1.5 h$

Volume of cylinder =
$$\pi r^2 h = \pi r^2 \times \frac{r}{1.5} = \frac{\pi r^3}{1.5} = \frac{2\pi r^3}{3}$$

∴ Both volumes are equal, required percentage = 100%

12. Volume of sphere =
$$\frac{4}{3}\pi \left[\left(\frac{8}{2} \right)^3 - \left(\frac{4}{2} \right)^3 \right]$$

= $\frac{4}{3}\pi (4^3 - 2^3)$
= $\frac{4}{3}\pi \times 56 \text{ cm}^3$

Diameter of cone = 2r = 8 cm.

$$r = 4 \text{ cm}$$

$$\therefore \qquad \text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4)^2 \times h = \frac{4}{3}\pi \times 56$$

[: Volume of cone = Volume of sphere]

i.e.
$$16h = 4 \times 56$$

$$h = \frac{4 \times 56}{16}$$

= 14 cm

13.
$$\frac{V_1}{V_2} = \frac{64}{27} = \frac{r_1^3}{r_2^3} \implies \frac{r_1}{r_2} = \frac{4}{3}$$

Surface area = $4\pi r^2$

$$\therefore$$
 area $\propto r^2$

$$\therefore \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{16}{9}$$

14. l = 10 m; h = 8 m;

Curved surface area of cone = $\pi r l$

$$r^2 = \sqrt{l^2 - h^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6 \text{ cm}$$

$$\therefore \qquad \text{C.S.A} = \pi \times 6 \times 10 = 60 \,\pi \,\text{m}^2$$

15. Volume of cube of 1 m side $V_1 = 1^3 = 1 \text{ m}^3$

Volume of cube of 10 cm side $V_2 = 10^3 = 1000 \text{ cm}^3 = 0.001 \text{ m}^3$

$$\therefore$$
 Required number of cubes = $\frac{V_1}{V_2} = \frac{1}{0.001} = 1000$

16. Volume of sphere of 3 cm radius
$$V_1 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$$

Sectional area of cylinder, $A_1 = \pi r^2 = \pi \times (4)^2 = 16 \pi \text{ cm}^2$

:. Height through which water water will rise

$$= \frac{V_1}{A_1} = \frac{36 \pi \text{ cm}^3}{16 \pi \text{ cm}^2} = \frac{9}{4} \text{ cm}$$

17.
$$d = 7$$
 cm; $h = 7$ cm

Volume of cone =
$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{\pi d^2 h}{4}$$

= $\frac{1}{3} \times \frac{\pi (7^2) \times 7}{4}$
= $\frac{1}{3} \times \frac{22}{7} \times \frac{7^3}{4}$
= $\frac{11 \times 7^2}{6}$
= $\frac{539}{6} = 89.83 \approx 89.8 \text{ cm}^3$

18. Total volumes of cubes =
$$(3x)^3 + (4x)^3 + (5x)^3$$

= $(27 + 64 + 125)x^3$
= 216 x^3

$$\therefore$$
 Side of single cube = $\sqrt{216x^3} = 6x$

$$\therefore$$
 Diagonal of cube = $6x\sqrt{3} = 12\sqrt{3}$

$$\therefore \qquad \qquad x = \frac{12\sqrt{3}}{6\sqrt{3}} = \mathbf{2}$$

:. Sides are 6 cm, 8 cm and 10 cm.

19. Required length =
$$\sqrt{3} \times \sqrt{3} = 3$$
 m

20. Size of sheet is
$$48 \text{ m} \times 36 \text{ m}$$

After cutting squares of 8 m size from its corners,

Size of sheet =
$$(48 - 16) \times (36 - 16)$$

= $32 \text{ m} \times 20 \text{ m}$

$$\therefore \qquad \text{Volume of box} = 32 \times 20 \times 8 = 5120 \text{ m}^3$$

21. Volume of wall =
$$6 \text{ m} \times 5 \text{ m} \times 0.5 \text{ m} = 15 \text{ m}^3$$

Volume of 1 brick = $25 \times 12.5 \times 7.5 = 2343.75 \text{ cm}^3$

Required number of bricks =
$$\frac{95\% \text{ of } 15 \text{ m}^3}{9375/4}$$

= $\frac{0.95 \times 15 \times 10^6 \text{ cm}^3}{2343.75 \text{ cm}^3}$

$$= \frac{95 \times 15 \times 10^4}{9375/4}$$

$$= \frac{95 \times 15 \times 10^4 \times 4}{9375}$$

$$= \frac{19 \times 3 \times 10^4 \times 4}{375}$$

$$= \frac{19 \times 4 \times 10^4}{125} = 19 \times 4 \times 8 \times 10 = 6080$$

22. Volume of spherical ball =
$$\frac{4}{3}\pi \left(\frac{6}{2}\right)^3 = 36\pi \text{ cm}^3$$
 (1)

Volume of cone =
$$\frac{1}{3}\pi(6)^2 \times h = 12\pi h$$
 (2)

Equating (1) and (2), we get

$$12\pi h = 36\pi$$

$$\Rightarrow$$

$$h = \frac{36 \pi}{12 \pi} = 3 \text{ cm}$$

23. % error =
$$\frac{r_c^3 \sim r_e^3}{r_c^3} \times 100$$

= $\frac{r_c^3 \sim (1.015)^3 r_c^3}{r_c^3} \times 100$
= $\frac{1.04567 \sim 1}{1} \times 100 = 4.567\% \simeq 4.6\%$

24. Required order is 4, 3, 2, 1.

25.
$$\frac{r_1}{r_2} = \frac{2}{1}$$
; $\frac{V_1}{V_2} = 1$

i.e.
$$V_1 = \frac{1}{3}\pi r_1^2 h_1; \quad V_2 = \frac{1}{3}\pi r_2^2 h_2$$
 i.e.
$$\frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

$$\therefore \qquad \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2} = \frac{1^2}{2^2} = \frac{1}{4}$$

26. Let radius = 3x and height = 4x;

Volume of cone =
$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3x)^2 \times 4x = 96\pi$$

i.e.
$$\frac{1}{3}\pi \times 9x^2 \times 4x = 96\pi$$

i.e.
$$12x^3 = 96$$

$$\Rightarrow x^3 = \frac{96}{12} = 8$$

$$\therefore x = 2$$

$$\therefore h = 4x = 8 \text{ cm}; r = 3x = 6 \text{ cm}.$$

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = 10 \text{ cm}$$

27. Required length =
$$\frac{66 \text{ cm}^3}{\frac{\pi}{4}(0.1)^2} = \frac{66 \times 4}{\pi \times 0.01}$$

= $\frac{66 \times 4 \times 100}{\frac{22}{\pi}} = 3 \times 4 \times 7 \times 100 = 8400 \text{ cm} = 84 \text{ m}$

28.
$$h_1 = h_2$$
; $d_1 = \frac{1}{2}d_2$;

Volume of cylinder
$$X V_1 = \pi d_1^2 h_1$$
 (1)

Volume of cylinder
$$YV_2 = \pi d_2^2 h_2$$
 (2)

$$V_1 = \pi d_1^2 h_1 = \pi d_1^2 h \tag{A}$$

$$V_2 = \pi d_2^2 h_2 = \pi (2d_1)^2 \times h$$

$$= 4 \pi d_1^2 h$$

$$= 4V_1$$
(B)

If height of X is doubled, volume of X will become double

Volume of X = Half the volume of Y

29.
$$h = 6b$$
; $l = 7h$

Given

:.

Volume of wall =
$$lbh = (7h) \left(\frac{h}{6}\right) \times h = \frac{7}{6}h^3$$

$$\frac{7}{6}h^3 = 16128$$

⇒
$$h^3 = \frac{16128 \times 6}{7}$$

= 2304 × 6 = 13824

h = 24

.. Width
$$b = \frac{h}{6} = \frac{24}{6} = 4 \text{ m}$$

30.
$$r = \frac{h}{2}$$
;

$$A = 2\pi r h = 616 \text{ cm}^2$$

$$2\pi r (2r) = 616$$

$$r^2 = \frac{616}{4\pi} = \frac{154}{\frac{22}{7}} = \frac{154 \times 7}{22} = 49$$

$$r = 7 \text{ cm}$$
Volume of cylinder = $\pi r^2 h = \pi (7)^2 (2 \times 7)$

$$= \frac{22}{7} \times 7^3 \times 2$$

$$= 44 \times 49$$

$$= 2156 \text{ cm}^3$$

Approximate volume in litres = $\frac{2156}{1000}$ = 2.156 \approx 2.2 litre

31.
$$V = 246.4$$
 litre; $h = 4$ m; $d = ?$

$$V = \frac{\pi d^2}{4} \times h$$

$$\therefore d = \sqrt{\frac{4V}{\pi h}} = \sqrt{\left(4 \times \frac{246.4 \text{ m}^3}{1000}\right) / \frac{22}{7} \times 4 \text{ m}}$$

$$= \sqrt{\frac{7 \times 246.4}{22000}} \text{ m}$$

$$= \sqrt{\frac{7 \times 246.4 \times 1000}{22}} \text{ cm}$$

$$= \sqrt{7 \times 11.2 \times 1000} \text{ cm} = \sqrt{100 \times 784}$$

$$= 10 \times 28 = 280 \text{ cm} = 2.8$$

$$= 10 \times 28 = 280 \text{ cm} = 2.8 \text{ m}$$

32. Required cost =
$$\frac{6(2.5)^2 \times 1}{1.5} \times 36 = 25 \times 36 = ₹900$$

33.
$$2 \times 15 \times 12 = 2(15 + 12) \times h$$

$$h = \frac{2 \times 15 \times 12}{2 \times 27}$$
$$= \frac{20}{3} \text{ m}$$

.. Volume of hall =
$$15 \times 12 \times \frac{20}{3} = 1200 \text{ m}^3$$

34. Volume of wooden solid sphere = $\frac{4}{3}\pi(9)^3 = \frac{4}{3} \times \pi \times 9^3 = 972 \pi \text{ cm}^3$

Volume of cone =
$$\frac{1}{3}\pi \left(\frac{18}{2}\right)^2 \times 9 = 243\pi \text{ cm}^3$$

$$\therefore \qquad \text{Required percentage} = \frac{972 \pi - 243 \pi}{972 \pi} \times 100 = 75 \%$$

35. Required diameter =
$$\sqrt[3]{6^3 + 8^3 + 10^3}$$

= $\sqrt[3]{216 + 512 + 1000}$
= $\sqrt[3]{1728} = 12$ cm

36. Side of large cube =
$$\sqrt[3]{3^3 + 4^3 + 5^3} = 6$$
 cm

Total surface area = $6 a^2$

Total surface area for smaller cubes =
$$6(3^2 + 4^2 + 5^2)$$

= $6(9 + 16 + 25)$

$$= 300 \text{ cm}^2$$

T.S.A of large cube = $6 \times 6^2 = 216 \text{ cm}^2$

Required ratio =
$$\frac{300}{216} = \frac{25}{18}$$

37. Speed of water =
$$10 \text{ km/hr} = 10 \times \frac{5}{18} \text{ m/s} = \frac{25}{9} \text{ m/s}$$

Let h be the height through which water will rise in half an hour.

$$\therefore \qquad \text{Quantity of inflow} = AV$$

$$= \frac{40}{10000} \text{ m}^2 \times \frac{25}{9} \text{ m/s}$$
$$= \frac{1}{90} \text{ m}^3/\text{s}$$

.. Quantity of inflow in 30 minutes

$$=\frac{1}{90}$$
 m³/s × 30 × 60 sec = 20 m³

$$\therefore \qquad \text{Required height} = \frac{20}{80 \times 40} \text{m} = \frac{20 \times 100}{80 \times 40} \text{cm} = \frac{5}{8} \text{cm}$$

38. Volume of greatest sphere =
$$\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ cm}^3$$

39. Volume of cylinder =
$$\pi(8)^2 \times 2 = 128\pi \text{ cm}^3$$

Let radius of cone be r cm.

$$\therefore \qquad \text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 \times (6) \text{ cm}^3 = 2\pi r^2 \text{ cm}^3$$

Equating both volumes, we get

$$128 \pi = 2\pi r^2$$

$$r^2 = \frac{128\,\pi}{2\,\pi} = 64$$

$$r = \sqrt{64} = 8 \text{ cm}$$

40. Volume of wall = $8 \text{ m} \times 6 \text{ m} \times 22.5 \text{ cm}$

$$= 48 \times 0.225 \text{ m}^3$$

Volume of brick = $25 \text{ cm} \times 11.25 \text{ cm} \times 6 \text{ cm}$

$$\therefore \quad \text{Required number of bricks} = \frac{8 \times 6 \times 22.5 \times 10^4}{25 \times 11.25 \times 6} = \frac{8 \times 2 \times 10^4}{25}$$
$$= 8 \times 2 \times 4 \times 10^2 = 6400$$

Problems on Clock

A clock is a device which measures and shows time. In this chapter some facts related to a clock are included based on which you have to work out problems.

Important Facts

- 1. In every hour the two hands coincide once.
- 2. Each hour space equals $360^{\circ}/12 = 30^{\circ}$.
- 3. Each minute space equals $360^{\circ}/60 = 6^{\circ}$.
- 4. When the two hands are at right angles (90°), they are 90/6 = 15 minutes apart. This occurs twice in every hour.
- 5. When two hands are in the opposite direction, they are 180/6 = 30 minutes apart. This occurs only once in every hour.
- 6. When the hands are coincident, they are obviously zero degrees and zero minutes apart.
- 7. The two hands are in the same straight line when they are either coincident or opposite to each other.
- 8. The hands coincide 11 times in every 12 hours, because there is a common position 12 o'clock between 11 and 1 o'clock. Hence they coincide only 22 times a day.
- 9. The hands of a clock are at right angles twice in every hour but in 12 hours they are at right angles only 22 times. It is so because there are two positions common in every 12 hours. i.e. 3 o'clock and 9 o'clock.
- 10. Two hands of a clock will be together between h and (h + 1) o' clock at (60/11) h minutes past h o'clock.

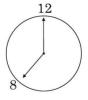
EXAMPLE 1 Find the time between 8 o'clock and 9 o'clock, when the two hands of a clock are in the same straight line.

Solution Two hands of a clock are in the same straight line in two cases.

(i) When the two hands are in exactly opposite direction.

This equals $180^{\circ}/6^{\circ} = 30$ minutes apart.

At 8 o'clock, minute hand is at $8 \times 5 = 40$ minutes spaces behind the hour hand.



Therefore, the minute hand will have to gain (40-30) = 10 minutes space over the hour hand.

- \therefore Gain of 55 minutes space = 60 minutes
- :. Gain of 10 minutes space = $\frac{60}{55} \times 10 = \frac{120}{11}$ minutes
- .. The minute hand will be exactly opposite direction to the hour hand at $10\frac{10}{11}$ minutes past 8 o'clock.
- (ii) When the two hands coincide (i.e. 0 minutes apart) In this case the minute hand will have to gain $8 \times 5 = 40$ minute space over the hour hand.
 - \therefore Gain of 55 minutes space = 60 minutes

Gain of 40 minutes space =
$$\frac{60}{55} \times 40 = \frac{480}{11} = 43\frac{7}{11}$$
 minutes

 \therefore Two hands will coincide at 43 $\frac{7}{11}$ minutes past 8 o'clock.

Between h and (h + 1) o'clock, the minute hand is 5h minute space behind the hour hand.

Case (a) The minute hand is m minutes behind the hour hand.

The minute hand must gain (5h - m) minute space over the hour hand.

Case (b) The minute hand is m minutes ahead of the hour hand.

The minute hand must gain (5h + m) minute space over the hour hand. Gain of 55 minutes space = 60 minutes.

Gain of $(5h \pm m)$ minute space = 60/55 $(5h \pm m)$ minutes

=
$$12/11 (5h \pm m)$$
 minutes past h o'clock

When minute hand is behind the hour hand, angle between hour hand and minute

(M) M

hand is given by angle = $30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees, where H represents hours and M represents minutes.

- (iii) When the minute hand is ahead of the hour hand, the angle between the two hands at M minutes past H hour is given by angle = $30\left(\frac{M}{5} H\right) \frac{M}{2}$ degrees.
- (iv) When the minute hand overtakes the hour hand of a clock at intervals of m, gain or loss/day by the clock = $\left(\frac{720}{11} m\right) \left(\frac{60 \times 24}{m}\right)$

EXAMPLE 2 At what time between 3 o'clock and 4 o'clock will the hands of a clock be together?

Solution At 3 o'clock, hour hand is at 3 and the minute hand is at 12. i.e., they are 15 minutes apart. Minute hand must gain 15 minutes over the hour hand so that both of them will be together.

But 55 minutes are gained by minute hand in 60 minute.

- \therefore 15 minutes will be gained in $\left(\frac{60}{55} \times 15\right)$ min = $16\frac{4}{11}$ min
- \therefore The hands will coincide at $16\frac{4}{11}$ minutes past 3.

EXAMPLE 3 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Solution Usually minute hand gains 55 minutes over the hour hand in 60 minutes. To be together, minute hand must gain 60 minutes over the hour hand.

- \therefore 60 minutes are gained in $\left(\frac{60}{55} \times 60\right)$ min = $65\frac{5}{11}$ minutes
- : Both of them are together after 65 minutes,

Gain in 65 min =
$$\left(65 \frac{5}{11} - 65\right) = \frac{5}{11}$$
 minutes

- :. Gain in 24 hours = $\frac{5}{11} \times \frac{60 \times 24}{65} = 10 \frac{10}{43}$ minutes
- .. Clock gain $10\frac{10}{43}$ minutes in 24 hours.

EXAMPLE 4 A clock in set at 9 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 2 p.m. on the following day?

Solution Time interval between today 9 a.m. and tomorrow 2 p.m = 29 hours 24 hours 10 minute of the clock = 24 hours of correct clock

- \therefore $\frac{145}{6}$ hours of this clock = 24 hours of the correct clock
- ∴ 29 hours of this clock = $\left(24 \times \frac{6}{145} \times 29\right)$ hours of correct clock

$$= \left(24 \times \frac{6}{145} \times 29\right) \text{ hours of correct clock}$$

= 28 hours 48 min after 9 a.m. = 32 hours 43 min.

i.e. time is 48 minute past 1 p.m.

EXAMPLE 5 At what time between 5 o'clock and 6 o'clock will the hands of a clock be at right angle.

Solution At 5 o'clock minute hand will be 25 minutes space behind the hour hand.

When the two hands are at right angles, they are 15 minutes apart.

Therefore they can be at right angles in the following two cases:

 $\textbf{Case I.} \quad \text{When minute hand is 15 minutes behind the hour hand.}$

Here minute hand will have to gain (25-15) = 10 minutes space.

 \therefore 10 minutes space will be gained by it in $\frac{60}{55} \times 10 = 10 \frac{10}{11}$ minutes.

 \therefore They are at right angles at $10\frac{10}{11}$ minutes past 5.

Case II. When minute hand is 15 minutes space a head of the hour hand:

Here minute hand will have to gain (25 + 15) = 40 minutes space.

- ∴ 40 minutes space will be gained in $\left(\frac{60}{55} \times 40\right) = 43 \frac{7}{11}$ minutes.
- \therefore They are at right angles at $43\frac{7}{11}$ minutes past 5.

EXAMPLE 6 Find the angle between the hour hand and the minute hand of a clock when the time is 4.35.

Solution Angle traced by the hour hand in 12 hours = 360°.

 \therefore Angle traced by it in 4 hours 35 min $\left(\frac{55}{12} \text{ hours}\right)$

$$=\left(\frac{55}{12}\times\frac{360}{12}\right)^{\circ}=137\frac{1}{2}^{\circ}$$

Angle traced by minute hand in 60 min = 360°

Angle traced by it in 35 minutes = $\left(\frac{360}{60} \times 35\right)^{\circ} = 210^{\circ}$

 $\therefore \text{ Required angle} = \left(210 - 137 \frac{1}{2}\right)^{\circ} = 72 \frac{1}{2}^{\circ}$

EXAMPLE 7 At what time between 9 o'clock and 10 o'clock will the hand of a clock be in the same straight line but pointing in opposite direction.

Solution At 9 o'clock, the hour hand is at 9 and the minute hand is at 12, i.e., two hands are 15 minutes space apart.

To be in same straight line but pointing in the opposite direction, they will be at 30 minutes space apart.

- \therefore Minute hand will have to gain (30-15)=15 minutes space over the hour hand. 55 minutes space is gained in 60 minutes.
- \therefore 15 minutes space will be gained in $\left(\frac{60}{55} \times 15\right)$ min = $\frac{180}{11}$ = 16 $\frac{4}{11}$ minutes
- .. The hands will be in the same straight line, but pointing in opposite direction at $16\frac{4}{11}$ minutes past 9.

? EXERCISES

Study the following questions given below carefully and answer most appropriately from the given alternatives.

clock was 15 minutes past 7. What is the time lag between the two times?

1. The time shown by a clock was 2 o'clock. After a certain time the mirror image shown by the

(a) 5 h 15 min. (b) 4 h 45 min. (c) 3 h 15 min. (d) 2 h 45 min. (e) None of these

2.		-		ery hour for the nu he clock from 8 hou	imber same as that of urs to 12 hours?
	(a) 45	(b) 50	(c) 54	(d) 60	(e) None of these
3.	The time taken be complete its strik	•	ı two single strike	is 2 seconds. Wha	t is the time taken to
	(a) 20 seconds	(b) 18 seconds	(c) 22 seconds	(d) 16 seconds	(e) None of these
4.				hind him 2 seconds an will catch the t	after thief started at hief.
	(a) 1 second	(b) 2 seconds	(c) 3 seconds	(d) 4 seconds	(e) None of these
5.	Between 1 hour together?	and 12 hours hov	v many times the	hour needle and r	ninute needle will be
	(a) 11 times	(b) 12 times	(c) 13 times	(d) 10 times	(e) None of these
6.	fast at 2 p.m. on	the following Mor	2 minutes slow at a nday. When was it		nd is 4 min 48 seconds [RRB]
	(a) 2 p.m. on Tu	-			
	(b) 2 p.m. on We	-			
	(c) 3 p.m. on Th	v			
	(d) Inmon H'mi	d a			
_	(d) 1 p.m. on Fri	-			
7.	At what angle th	-	are inclined at 1	5 minutes past 5?	[LIC AAO]
7.	At what angle th	e hands of a clock	are inclined at 1: (c) $67\frac{1}{2}^{\circ}$		[LIC AAO]
	At what angle the (a) $58\frac{1}{2}^{\circ}$	e hands of a clock (b) 64°	(c) $67\frac{1}{2}$ °	(d) $72\frac{1}{2}^{\circ}$	[LIC AAO] lles will coincide each [RRB]
	At what angle the (a) $58\frac{1}{2}^{\circ}$ At what time (in other?	e hands of a clock (b) 64° minutes) between	(c) $67\frac{1}{2}$ °	(d) $72\frac{1}{2}^{\circ}$ clock, both the need	lles will coincide each
8.	At what angle the (a) $58\frac{1}{2}^{\circ}$ At what time (in other? (a) $5\frac{1}{11}''$	e hands of a clock (b) 64° minutes) between (b) 12 4/11" ween 7 o'clock and	(c) $67\frac{1}{2}^{\circ}$ a 3 o'clock and 4 o'c (c) $13\frac{4}{11}''$	(d) $72\frac{1}{2}^{\circ}$ clock, both the need (d) $16\frac{4}{11}''$	lles will coincide each
8.	At what angle the (a) $58\frac{1}{2}$ ° At what time (in other? (a) $5\frac{1}{11}$ " At what time bet	e hands of a clock (b) 64° minutes) between (b) $12\frac{4}{11}''$ ween 7 o'clock and ther?	(c) $67\frac{1}{2}^{\circ}$ a 3 o'clock and 4 o'c (c) $13\frac{4}{11}''$	(d) $72\frac{1}{2}^{\circ}$ clock, both the need (d) $16\frac{4}{11}''$	lles will coincide each [RRB]
8.	At what angle the (a) $58\frac{1}{2}^{\circ}$ At what time (in other? (a) $5\frac{1}{11}''$ At what time bet line but, not toge	e hands of a clock (b) 64° minutes) between (b) 12 4/11" ween 7 o'clock and ther?	(c) $67\frac{1}{2}^{\circ}$ a 3 o'clock and 4 o'c (c) $13\frac{4}{11}''$	(d) $72\frac{1}{2}^{\circ}$ clock, both the need (d) $16\frac{4}{11}''$	lles will coincide each [RRB]
8.	At what angle the (a) $58\frac{1}{2}$ ° At what time (in other? (a) $5\frac{1}{11}$ " At what time bettline but, not toge (a) 5 minute pass	e hands of a clock (b) 64° minutes) between (b) 12 4/11" ween 7 o'clock and ther? st 7 past 7	(c) $67\frac{1}{2}^{\circ}$ a 3 o'clock and 4 o'c (c) $13\frac{4}{11}''$	(d) $72\frac{1}{2}^{\circ}$ clock, both the need (d) $16\frac{4}{11}''$	lles will coincide each [RRB]

- 10. How many times in a day, are the hands of a clock in straight line but opposite in direction? [RRB]
 - (a) 20
- (b) 22
- (c) 24
- (d) 48
- 11. How many times in a day, are the hands of a clock at right angle?

[IAS]

- (a) 22
- (b) 24
- (c) 44
- (d) 48
- 12. The reflex angle between the hands of a clock at 10.25 is

[SCRA]

- (a) 180°
- (b) $192\frac{1}{2}$ ° (c) 195°
- (d) $197\frac{1}{2}$ °
- 13. An accurate clock shows 8 o'clock in the morning. Through how may degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
 - (a) 144°
- (b) 150°
- (c) 168°
- (d) 180°
- 14. The angle between the minute hand and the hour hand of a clock when the time is 10.30 is
 - (a) 120°
- (b) 135°
- (c) 55°
- (d) 60°
- 15. The angle between the hour hand and the minute hand of a clock when the time is 4.55 is:
 - (a) $177\frac{1}{2}$ ° (b) $182\frac{1}{2}$ °
- (c) 180°
- (d) 175°

7. (c)



- **1.** (d) **2.** (c)
- **3.** (b)
- **4.** (b)
- **5.** (a)
- **6.** (b)
- 8. (d)
- **9.** (d) **10.** (a)

11. (c) **12.** (d) **13.** (d) **14.** (b) 15. (b)



Solutions with Necessary Explanation

1. Initial time = 2 o'clock.

Mirror image = 7 h 15 min.

- \therefore The time then will be 4 hours 45 minutes (i.e. 12-7 hours 15 minutes)
- \therefore Time lag = 4 h 45 min 2h = 2 h 45 min.
- 2. For all half an hour once
 - .. Total strikes for half hour = 4

For hour from 8 to 12, number of hourly strikes

$$= 8 + 9 + 10 + 11 + 12 = 50$$

- Total strikes = 4 + 50 = 54
- 3. Time taken for completing single strike = 2 seconds

Time taken to complete its strike at 10 o'clock = 9×2 seconds = 18 seconds.

4. Relative speed between policeman and thief = 10 - 5 = 5 m/s

The policeman lags behind thief by 2 seconds at the start.

Time taken by policeman to catch the thief = $\frac{Distance covered by thief in 2 seconds}{Relative speed of policeman}$

$$= \frac{5 \text{ m/s} \times 2 \text{ sec}}{5 \text{ m/s}} = 2 \text{ second s}$$

- 5. In every hour, the hour needle and minute needle will be together once. So, in 11 hours, 11 times and once for 12 o'clock.
 - .: In total 12 times.
 - : between 1 hour and 12 hours, number of coincidence = 11 times.
- 6. Time interval between Monday 12 noon to next Monday 2 p.m.

$$\therefore$$
 Watch gains $\left(2+4\frac{4}{5}\right)$ minutes in 170 hours

i.e. $\frac{34}{5}$ minutes gained in 170 hours.

$$\therefore$$
 Watch gained 2 minutes in $\left(\frac{170}{34/5} \times 2\right)$ hours

$$=\frac{170\times2\times5}{34}$$

$$=50 \text{ hours}$$

 \therefore Watch is correct 2 days 2 hours after 12 noon of Monday

i.e. at 2 p.m. on Wednesday

7. Angle traced by hour hand in 5 hours 15 minutes $\left(\frac{21}{4} \text{ hours}\right)$

$$=\left(\frac{360}{12}\times\frac{21}{4}\right)^{\circ}=157\frac{1}{2}^{\circ}$$

Angle traced by minute hand in 15 minutes

$$=\left(\frac{360}{60}\times15\right)^{\circ}=90^{\circ}$$

$$\therefore$$
 Required angle = $157\frac{1}{2} - 90 = 67\frac{1}{2}^{\circ}$

8. At 3 o'clock, minute hand is at 15 minutes space apart from the hour hand.

In order to coincide, it should gain 15 minutes space

55 minutes space is gained in 60 minutes

 \therefore 15 minutes space is gained in $\left(\frac{60}{55} \times 15\right)$ minutes

$$=\frac{180}{11}=16\frac{4}{11}$$
 minutes

Both hands will coincide at $16\frac{4}{11}$ minutes past 3

9. At 7 o'clock minute hand is at 25 minute space apart.

In order that both hands to be in straight line, minute hand should gain 5 minute space. 55 minute space is gained in 60 minutes

 \therefore 5 minutes spaces is gained in $\left(\frac{60}{55} \times 5\right)$ minutes = $5\frac{5}{11}$ minutes

.. Required time = $5\frac{5}{11}$ minutes past 7.

- 10. The hands of a clock point in opposite directions in the same straight line 11 times in every 12 hours. So in a day, hands point in the opposite direction 22 times.
- 11. In 12 hours, they are at right angles 22 times. In 24 hours, they are at right angles 44 times.
- 12. Angle traced by hour hand in 10.25 hours or $\frac{125}{12}$ hours

$$= \left(\frac{360}{12} \times \frac{125}{12}\right)^{\circ} = 312 \frac{1}{2}^{\circ}$$

Angle traced by minute hand in 25 minutes

$$= \left(\frac{360}{60} \times 25\right)^{\circ} = 150^{\circ}$$

:. Reflex angle =
$$360^{\circ} - \left(312\frac{1}{2} - 150\right)^{\circ}$$

= $360^{\circ} - 162\frac{1}{2}^{\circ}$
= $197\frac{1}{2}^{\circ}$

- 13. Time inerval between 8 a.m. and 2 p.m. = 6 hours Hours hand will move through 30° in every hours. In 6 hours, movement of hour hand = $6 \times 30 = 180^{\circ}$
- 14. Angle traced by hour hand in $10\frac{1}{2}$ hours or $\frac{21}{2}$ hours

$$= \left(\frac{360}{12} \times \frac{21}{2}\right)^{\circ} = 315^{\circ}$$

Angle traced by minute hand in 30 minutes

$$= \left(\frac{360}{60} \times 30\right)^{\circ} = 180^{\circ}$$

- \therefore Required angle = $(315 180)^{\circ} = 135^{\circ}$
- 15. Angle traced by hour hand in 4 hours 55 min $\left(\frac{59}{12} \text{ hours}\right)$

$$= \left(\frac{360}{12} \times \frac{59}{12}\right)^{\circ} = \frac{295}{2} = 147 \frac{1}{2}^{\circ}$$

Angle traced by minute hand in 55 minutes

$$= \left(\frac{360}{60} \times 55\right)^{\circ} = 330^{\circ}$$

$$\therefore \qquad \text{Required angle} = \left(330 - 147 \frac{1}{2}\right)^{\circ} = 182 \frac{1}{2}^{\circ}$$

Problems on Calendar

ODD DAYS: The number of days more than the complete number of weeks in a given period are called odd days.

ORDINARY YEAR: The year with 365 days is called an ordinary year. It has one odd day.

LEAP YEAR: The year with 366 days is called a leap year. Every year divisible by 4, which is not a centenary year, is a leap year. Every fourth centenary year is a leap year.

Examples:

- (a) Years such as 1984, 1996, 2008, etc. are leap years.
- (b) Years such as 400, 800, 1600, etc. are leap years.
- (c) Years such as 600, 900, 1400, 1900, 2100, etc. are not leap years.

COUNTING OF ODD DAYS

1 ordinary year = 365 days = 52 weeks + 1 day

Ordinary year has 1 odd day.

1 leap year = 366 days = 52 weeks + 2 days

Leap year has 2 odd days.

100 years = 76 ordinary years + 24 leap years

 $= (76 \times 1) + (24 \times 2) \text{ odd days}$

= 76 + 48 = 124 odd days

= 17 weeks + 5 days

= 5 odd days.

 \therefore Number of odd days in 100 years = 5 odd days.

Important Facts

- 1. An ordinary year contains 365 days, i.e. 52 weeks + 1 odd day.
- 2. A leap year contains 366 days, i.e. 52 weeks + 2 odd days.

A year divisible by 4 and a century year divisible by 400 will be a leap year.

- 3. 100 years contain 24 leap years and 76 ordinary years.
 - :. 100 years contain 5 odd days.
- 4. 200 years contain $(5 \times 2) = 10$ days, i.e. 3 odd days.
- 5. 300 years contain $(5 \times 3) = 15$ days, i.e. 1 odd day.
- 6. 400 years is a leap year and hence it will contain $(5 \times 4 + 1) = 21$ days, i.e. 0 odd day.

For finding the day of the week on a particular date when no reference day is given.

- (i) Count the net number of odd days for the period upto the given date, and
- (ii) Write Sunday for 0 odd day, Monday for 1 odd day, Tuesday for 2 odd days and so on.

If the reference day is given; then

- (a) Find the net number of odd days for the period between the reference day and the given date.
- (b) The day of the week on the particular date is equal to number of odd days ahead or behind the reference day depending on whether the reference day falls before or after the given date.

For the calendars of two different years to be the same, they must satisfy the following two conditions simultaneously.

- (i) Both the years must be of the same type, i.e. both must be ordinary years or both must be leap years. If one is ordinary year and the other a leap year, their calendars can never match.
- (ii) 1st January of both the years must fall on the same day of the week, i.e. the net number of odd days for the period between these two years should be zero.



Notes:

While counting the number of odd days, always check

- (i) Whether the interval has leap years and if yes,
- (ii) Whether February of the leap year falls in that interval or not.

Also remember that

- (i) The last day of a century year can only be either Sunday, Monday, Wednesday or Friday. It can't be Tuesday, Thursday or Saturday.
- (ii) Any date in March is the same day of the week as the corresponding date in November of that year. Another pair showing similar corresponding feature is April–July.

EXAMPLE 1 What was the day of the week on 13th September 1986?

Solution 13th September 1986 = 1985 years + Period from 1.1.86 to 13.9.86

Number of odd days in 1600 years = 0

Number of odd days in 300 years = 1

85 years = 21 leap years + 64 ordinary years

 $= (21 \times 2 + 64 \times 1)$ odd days

= 106 odd days

```
= (15 \text{ weeks} + 1 \text{ day})
                  = 1 odd dav.
\therefore 1985 years = (0 + 1 + 1) odd days = 2 odd days.
Period from 1.1.86 to 13.9.86 = 31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 13
                                      =256 \,\mathrm{days}
                                      = (36 \text{ weeks} + 4 \text{ days})
                                      = 4 odd days
```

- Total number of odd days = 2 + 4 = 6 odd days.
- Required day is Saturday.

EXAMPLE 2 What was the day of the week on 26th January 1950?

Solution 26th January 1950 = 1949 years +26 days.

Odd days in 1600 years = 0Odd days in 300 years = 1

50 years = 12 leap years + 38 ordinary years

 $= (12 \times 2 + 38 \times 1)$ odd davs

= 62 odd days

= (8 weeks + 6 days)

= 6 odd days.

1.1.1950 to 26.1.1950 = 26 days = 3 weeks + 5 odd days.

 \therefore Total number of odd days = 0 + 1 + 6 + 5

= 12 odd days

= (1 week + 5 days)

= 5 odd days

.. Required day is Friday.

EXAMPLE 3 January 1, 2013 was Tuesday. What day of the weeks lies on Jan. 1, 2014? **Solution** Year 2013 is an ordinary year and therefore it is having 1 odd day. 1st day of the vear 2013 is Tuesday.

:. 1st day of the year 2014 will be Wednesday

(b) Tuesday

(: Year 2013 is having 1 odd day)

(d) Wednesday (e) None of these

(e) None of these

Hence Jan 1, 2014 lies on Wednesday.



EXERCISES

(a) Sunday

(a) Monday

- 1. The birthday of Arjun falls between 20th and 23rd of March. Anil remembers that birthday of Arjun falls between 21st and 24th of March. So the birthday of Arjun will be on
 - (a) 21st March (b) 22nd March (c) 23rd March (d) 24th March (e) None of these

(c) Wednesday (d) Thursday

- 2. The 11th October 1986 was a Saturday. What was the day on 30th January, 1948?
 - (b) Monday (c) Tuesday
- 3. If 12th December 1985 was Thursday, what will be the day on 10th January 2100?

Problems on Calendar 4. 11th October 1986 was Saturday. When will be the same day and same month fall on Saturday? (a) in 1990 (b) in 1995 (c) in 2026 (d) in 2086 (e) None of these 5. 10th January 2105 will be a Wednesday. What was the day on 31st December 1704? (a) Sunday (b) Monday (c) Tuesday (d) Thursday (e) None of these **6.** How many days are there in m weeks m days? (a) $7m^2$ (b) 8*m* (c) 14m(d) 7 7. Which of the following is not a leap year? (d) 2400 (a) 800 (b) 1600 (c) 600 8. The last day of a century cannot be: (a) Monday (b) Tuesday (c) Wednesday (d) Friday 9. The calendar of the year 2010 will be the same for the year: (a) 2012 (b) 2014 (c) 2018 (d) 2021 10. 27th March 2005 was on Monday. What was the day of the week on 27th March 2002? (b) Tuesday (c) Thursday (d) Saturday (a) Sunday 11. What was the day of the week on 30th January 1948? (a) Wednesday (b) Thursday (c) Friday (d) Saturday 12. What was the day of the week on 13th September 2001? (a) Wednesday (b) Thursday (c) Friday (d) Saturday 13. Today is Wednesday. After 96 days, it will be (a) Friday (b) Saturday (c) Sunday (d) Monday 14. 26th January 2013 was Saturday. What was the day of the week on 26th January 2012? (a) Monday (b) Thursday (c) Friday (d) Sunday 15. January 1, 2006 was Sunday. What was the day of the week on January 1, 2012? (a) Sunday (b) Tuesday (c) Thursday (d) Saturday 1. (b) **2.** (e) **4.** (e) **5.** (a) **3.** (a) **6.** (b) 7. (c) 8. (b) **9.** (d) **10.** (c) 11. (c) **12.** (b) **13.** (d) **14.** (b) **15.** (a)



Solutions with Necessary Explanation

1. Birthday of Arjun – either 21st or 22nd March.

As Anil remembers, birthday falls either on 22nd or 23rd of March.

As both the above statements are true, the birthday of Arjun is on 22nd March.

2. 11th October 1986 - Saturday

11th October 1986 to 30th January 1948

30th January 1948 - 30th January 1986 + 30th January 1986 - 11th October 1986

= 38 years + 1 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 11 days

= 38 years + 254 days

 $= 2 + (9 \times 5) + 1 + 254$ odd days

= 48 + 255 = 302 odd days

302 % 7 = 2 odd days.

:. 30th January 1948 will be 1 day back from Saturday i.e. Friday.

3. 12th December 1985 - Monday

10th January 2100 - ?

12th December 1985 - 10th January 2100

- = 12th Dec 1985 12th Dec 2099 + 12th Dec 2099 10th Jan 2100
- = 114 years + 29 days
- $= 2 + 1 + 6 + 3 \times 5 + 29$ odd days
- = 53 odd days.

53 % 7 = 4 odd days

i.e. Jan 2100 will be on Monday. Here answer is (a).

- 4. The same day same month and same week will be observed after 400 years. Here answer is None of these.
- 5. 10th Jan 2105 Wednesday

31st Dec 1704 - ?

31st Dec 1704 - 31st Dec 2104 + 31st Dec 2104 - 10th Jan 2105

= 400 years + 10 days = 10 odd days

10 % 7 = 3 odd days. So required day = Wednesday $-3 = \mathbf{Sunday}$.

- 6. m weeks m day = $7 \times m + m = 8m$ days
- 7. A century year divisible by 400 is a leap year.
 - ∴ 600 is not a leap year
- 8. 100 years contain 5 odd days.
 - : Last day of 1st century is Friday

200 years contain $5 \times 2 = 10$ days = 3 odd days.

:. Last day of 2nd century is Wednesday.

300 years contains $5 \times 3 = 15$ days = 1 odd day.

:. Last day of 3rd century is Monday.

400 years contain 0 odd days

- :. Last day of 4th century year is Sunday
- :. Last day of a century year cannot be Tuesday

Thursday or Saturday.

9. Number of odd days in 2010 = 1

Number of odd days in 2011 = 1

Number of odd days in 2012 = 2

Number of odd days in 2013 = 1

Number of odd days in 2014 = 1

Number of odd days in 2015 = 1

Number of odd days in 2016 = 2

Number of odd days in 2017 = 1

Number of odd days in 2018 = 1

Number of odd days in 2019 = 1

Number of odd days in 2020 = 2

Total number = 14

Sum of odd days = 14 days = 0 odd day.

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:. Calendar for 2021 will be the same as for the year 2010.

10 27th March 2002 to 27th March 2005 = 3 years

$$= 2$$
 ordinary year $+ 1$ leap year $= 2 + 2 = 4$ odd days.

1.15

:. Day on which 27th March 2003 falls

$$= Monday - 4 = Thursday$$

11. Odd days in 1600 years = 0

Odd days in 300 years = 1

47 years = 11 leap years + 36 ordinary years

$$= (11 \times 2 + 36 \times 1)$$
 odd days

$$= 58 \text{ days} = (8 \text{ weeks} + 2 \text{ days})$$

= 2 odd days

Period from 1.1.48 to 30.1.48 = 30 days 2 odd days.

 \therefore Total number of odd days = 0 + 1 + 2 + 2 = 5 odd days

Hence 30th January 1948 was on Friday.

12. 2000 years = 0 odd days.

Period from 1.1.2001 to 13.9.2001 = 31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 13 = 256 days.

$$= (36 \text{ weeks} + 4 \text{ days})$$

- : 13th September 2001 was Thursday.
- 13. 96 days = 13 weeks + 5 days

- \therefore Week after 96 days = Wednesday + 5 = **Monday**
- 14. 2012 was a leap year and so it is having 2 odd days.

26th January 2013 was Saturday

- \therefore 26th January 2012 = Saturday -2 = Thursday
- 15. 1st January 2006 was Sunday
 - 1.1.2006 to 1.1.2012 = 6 years = 1 leap year + 5 ordinary years

$$= (1 \times 2 + 5 \times 1)$$
 odd days

$$= 7 \text{ days}$$

$$= 0$$
 odd day.

∴ 1st January 2012 was Sunday.

Time and Distances

The speed of a body is the rate at which it is moving.

$$Speed = \frac{Distance}{Time}$$

$$Average speed = \frac{Total \ distance \ covered}{Total \ time \ taken}$$

Speed is measured in km/hr or m/s.

$$1 \text{ km/hr} = \frac{5}{18} \text{m/s}$$
$$1 \text{ m/s} = \frac{18}{5} \text{km/hr}$$

or

BODIES MOVING IN THE SAME DIRECTION

Relative speed = Difference in speed

Distance between moving objects = Relative speed \times Time of travel

BODIES MOVING IN THE OPPOSITE DIRECTION

- (i) The relative speed of one w.r.t. the other is the sum of their speeds.
- (ii) Increase or decrease in distance between them is the product of their relative speed and time.
- (iii) The distance between two bodies moving towards each other will get reduced at the rate of their relative speed (i.e. sum of their speeds). The time of their meeting (or crossing) is given by

$$Meeting time = \frac{Initial distance between the two bodies}{Sum of their speeds}$$

AVERAGE SPEED: If a certain distance d_1 is covered at speed S_1 and distance d_2 at S_2 , their average speed for the whole journey is given by

Average speed =
$$\frac{S_1 S_2 (d_1 + d_2)}{S_2 d_1 + S_1 d_2}$$

When $d_1 = d_2$,

Average speed =
$$\frac{2S_1S_2}{S_1 + S_2}$$

If a person covers 1/nth part of the total journey at speed S_1 and the remaining at speed S_2 , then his average speed for the whole journey is given by

Average speed =
$$\frac{nS_1S_2}{(n-1)S_1 + S_2}$$

If a person covers m/nth part of the total journey at speed S_1 and the remaining journey at speed S_2 , then his average speed for the total journey is given by

Average speed =
$$\frac{nS_1S_2}{(n-m)S_1 + mS_2}$$

When a person covers a certain distance D km at speed $S_1 \text{ km/h}$ he is late by l_1 hours. If he covers it at $S_2 \text{ km/h}$, he will be early by l_2 hours. Then

$$D = \frac{S_1 S_2}{S_2 - S_1} (l_1 - l_2) \,\mathrm{km}$$

If a runner improves his timing by time t by running at n times his original speed, his usual time

$$= \frac{t}{1 - 1/n} = \frac{\text{Improvement in time}}{1 - \left(\frac{\text{Old speed}}{\text{New speed}}\right)}$$

Two persons A and B start their journey at the same time in opposite directions from two points and after passing each other they complete their remaining journey arriving at the two points in a and b hours respectively. Then the ratio of their speeds is

$$\frac{S_a}{S_b} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

Two persons cover a certain distance D at S_1 and S_2 km/h respectively. If the difference in time taken by the two is t hours, then the distance covered is given by

$$D = \frac{S_1 S_2}{S_1 - S_2} \times t \text{ km} = \frac{\text{Product of two speeds}}{\text{Difference of two speeds}} \times \text{Difference in time taken}$$

A train starts from P at time t_1 and reaches Q at time t_2 . Another train starts from Q at t_3 (later than t_1) and reaches P at t_4 . The two trains will meet on their way at

Meeting time = Second train's starting time

+
$$\frac{(1\text{st arrival} - 2\text{nd's starting time})}{\text{Sum of time taken by both}} \times (\text{Time taken by the second})$$

First train's starting time + $\frac{(2nd's \ arrival - 1st \ starting \ time)}{Sum \ of \ time \ taken \ by \ both} \times (Time \ taken \ by \ first)$

EXAMPLE 1 A and B walk around a circle of circumference 1800 m at the speed of 15 m/minutes and 60 m/minutes respectively. If both start simultaneously from the same point and walk in the same direction, when will they be together again at the starting point for the first time?

Solution Circumference of circle = 1800 m

Speed of A and B are 150m/min. and 60 m/min.

A gains (150 - 60) = 90 m in 1 min.

:. A gains 1800 m in 1800/90 = 20 min.

So A and B will be together again for the first time after 20 min.

Time taken by A to complete 1 full circle = 1800/150 = 12 min.

Time taken by B to complete 1 full circle = 1800/60 = 30 min.

Thus, A will be at the starting point after every 12 minutes and B after every 30 minutes. So they will be together at the starting point after every 60 minutes (i.e. LCM of 12 and 30 is 60).

Hence, A and B will be again at the starting point for the first time after 60 minutes.

Theorem: If a person covers a certain distance at a speed of x km/h and he covers the same distance at a speed of y km/h, then the average speed of the person for the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/h.

If S_1 km/h and S_2 km/h are two different speeds of a person and the difference between arrival times is given as t hours then the distance covered by the person is given by

$$\left(\frac{S_1 \times S_2}{S_1 - S_2} \times t\right) \text{km}.$$

If S_1 km/h and S_2 km/h are speeds of a person for his onward and return journey respectively and t hours is the total time taken by him for his whole journey, then the distance travelled

by the person is given by $\left(t \times \frac{S_1 \times S_2}{S_1 + S_2}\right) \text{km}$.

MEETING POINT: If two trains travel at speeds of S_1 km/h and S_2 km/h and time lag at start between the two trains is t hours, then the meeting point's distance is given by

$$\left(\frac{S_1 \times S_2 \times t}{S_1 - S_2}\right) \mathbf{km}.$$

Theorem: If two persons start at the same time in opposite directions from two points and after passing each other they complete their journeys in a and b hours respectively, then speed of the first person: speed of the second person = \sqrt{b} : \sqrt{a} .

If two persons travel the same distance at speeds of S_1 km/h and S_2 km/h respectively and one of them takes t hours more than the other to complete his journey, then the distance

covered by them is given by $\frac{S_1 \times S_2}{S_1 \sim S_2} \times t \text{ km}$.

Time of rest per hour =
$$\frac{\text{Difference of speed}}{\text{Speed without stoppage}}$$

A person travels a certain distance by two modes of conveyance. The whole journey took t hours, then the distance travelled by the person in one side

= Total time
$$\times \frac{\text{Multiplication of speeds}}{\text{Sum of speeds}}$$

EXAMPLE 2 A person covers a certain distance at a speed of 60 km/h and returns to the starting point at a speed of 40 km/h. Find the average speed of the person for the whole journey.

Solution Average speed =
$$\frac{2 \times 60 \times 40}{60 + 40} = 48 \text{ km/h}$$

EXAMPLE 3 P covers a certain distance at a speed of 25 km/h and he reached his destination 5 minutes late. With a speed of 30 km/h, he reaches his destination 10 minutes earlier. Find the distance between the starting point and destination.

Solution Let the required distance be x km.

∴ Time taken to cover
$$x \text{ km}$$
 at 25 km/h speed = $\frac{x}{25}$ hours

Similarly time taken to cover x km at 30 km/h speed = $\frac{x}{30}$ hours

Difference between the time taken = 5 + 10 = 15 min = 1/4 hour

$$\therefore \qquad \frac{x}{25} - \frac{x}{30} = \frac{1}{4}$$

or
$$6x - 5x = \frac{150}{4}$$

i.e.
$$x = \frac{150}{4} \, \mathrm{km}$$



Short cut: Required distance =
$$\frac{25 \times 30}{30 - 25}$$
 km/h × $\left(\frac{1}{4}\text{h}\right)$ = $150 \times \frac{1}{4} = \frac{150}{4}$ km

EXAMPLE 4 A bus travels for 7 hours, the first half at 30 km/h and the second half at 40 km/h. Find the distance

Solution Let the distance be x km

Time taken during the first half of total distance =
$$\frac{(x/2)}{30} = \frac{x}{60}$$
 hour

Time taken during the second half of total distance =
$$\frac{(x/2)}{40} = \frac{x}{80}$$
 hour

 \therefore Time taken for the whole journey = $\frac{x}{60} + \frac{x}{80} = 7$ hours

i.e.
$$\frac{4x + 3x}{240} = 7 \text{ hours}$$

$$\therefore \qquad \qquad x = 240 \text{ km}$$



Short cut: Required distance =
$$\frac{2 \times 7 \times 30 \times 40}{30 + 40} = 240 \text{ km}$$



Note: Here half of the journey is meant by half of the total distance covered and not the half of time taken for the whole journey.

EXAMPLE 5 Two persons X and Y walk from a place P at 5 km/h and 6 km/h respectively. How much distance will they be apart after 4 hours:

- (i) If they walk in the same direction?
- (ii) If they walk in opposite directions?

Solution

- (i) When they walk in the same direction, the relative speed of Y with respect to X is 1 km/h.
 - :. After 4 hours they will be 4×1 km/h = 4 km apart.
- (ii) When they walk in opposite directions, the relative speed of Y with respect to X is 6 + 5 = 11 km/h.
 - \therefore After 4 hours they will be $4 \times 11 = 44$ km apart.

EXAMPLE 6 A thief runs at a speed of 10 m/s. A policeman runs behind him at a speed of 12.5 m/s after 10 seconds the thief has started running. After how many metres, will the policeman catch the thief?

Solution During the first 10 seconds, the thief has covered 10 m/s \times 10 = 100 m.

The time required by the policeman to cover 100 m is given by $\frac{100}{(12.5-10)}$ = 40 seconds

:. The policeman will catch the thief at a distance of 12.5 m/s \times 40 = 500 metres



Short cut: Required distance = $\frac{S_1 \times S_2 \times \text{Difference in time }(t)}{S_1 \sim S_2}$

Required distance =
$$\frac{10 \times 12.5 \times 10}{2.5}$$
 = 500 metres

EXAMPLE 7 Two persons cover the same distance at speeds of 25 km/h and 30 km/h respectively. Find the distance travelled if one person takes 25 minutes more than the other.

Solution Let the distance be x km.

 \therefore Time taken by the first person = x/25 hour

Time taken by the second person = x/30 hour

$$\frac{x}{25} - \frac{x}{30} = \frac{25}{60}$$
i.e.
$$\frac{6x - 5x}{150} = \frac{25}{60}$$
or
$$x = \frac{25 \times 150}{60} = 62.5 \text{ km}$$



Short cut: Required distance =
$$\frac{S_1 \times S_2}{S_1 \sim S_2} \times (t_1 \sim t_2)$$

= $\frac{25 \times 30}{30 \sim 25} \times \frac{25}{60}$
= $\frac{25 \times 30 \times 5}{60} = \frac{125}{2} = 62.5 \text{ km}$

EXAMPLE 8 A person travels a certain distance at a speed of 60 km/h and with stoppages he travels the same distance at a speed of 40 km/h. How many minutes/hour does he stop?

Solution Let the total distance be x km

Time taken at 60 km/h speed = x/60 hour

Time taken at 40 km/h speed = x/40 hour

$$\therefore \qquad \text{Time taken to rest} = \frac{x}{40} - \frac{x}{60} = \frac{3x - 2x}{120} = \frac{x}{120} \text{ hour}$$

:. Rest/hour =
$$\frac{x}{120} \div \frac{x}{40} = \frac{x}{120} \times \frac{40}{x} = \frac{1}{3}h = 20 \text{ min}$$



Short cut: Time of rest/hour =
$$\frac{S_1 \sim S_2}{\text{Speed without stoppage}}$$

$$=\frac{60-40}{60}=\frac{1}{3}$$
 hour = 20 minutes

EXAMPLE 9 A bus running at a speed of 30 km/h leaves Trivandrum at 10 am and another bus running at a speed of 40 km/h leaves the same place at 3 pm in the same direction. How many kilometres from Trivandrum will they be together?

Solution The first bus has advanced 30×5 km and the second bus gains (40 - 30) or 10 km/h.

- .. The second bus will gain 30×5 km in $\frac{30 \times 5}{10}$ or 15 h
- \therefore Required distance from Trivandrum = 15 hours \times 40 km/h = 600 km



Short cut: The meeting point's distance from starting point

$$= \frac{S_1 \times S_2 \times \text{Difference in time}}{\text{Difference in speed}}$$

where S_1 and S_2 are the speeds of the first and second buses respectively.

:. Required distance =
$$\frac{30 \times 40 \times (3 \text{ pm} \sim 10 \text{ am})}{(40 \sim 30)} = \frac{30 \times 40 \times 5}{10} = 600 \text{ km}$$

Theorem: When two persons A and B are running such that the ratio of their speeds is

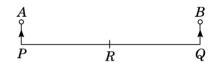
$$a:b$$
, then Distance travelled by $A=2\times Distance$ between two points $\left(\frac{a}{a+b}\right)$

Distance travelled by B = 2 × Distance between two points
$$\left(\frac{b}{a+b}\right)$$

EXAMPLE 10 When two persons A and B walk from P to Q, which are at a distance of 21 km, at 3 km/h and 4 km/h respectively, B reaches Q, returns immediately and meets A at R. Find the distance from P to R.

Solution When A reaches R, B also reaches R after returning from Q.

 \therefore Distance travelled by A = PRDistance travelled by B = PQ + QR



- \therefore Distance travelled by A and B together = $2 \times PQ = 2 \times 21 = 42$ km i.e., as the ratio of speeds is 3:4
- $\therefore PR$, distance travelled by $A = \left(\frac{3}{3+4}\right) \times 42 = 18 \text{ km}$



Short cut: Distance travelled by A = 2 × Distance between two points × $\left(\frac{a}{a+b}\right)$

$$= 2 \times 21 \times \frac{3}{(3+4)} = 18 \text{ km}$$

Theorem: If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journey in a and b hours respectively, then the ratio of speeds from A to B is $\sqrt{b}:\sqrt{a}$.

EXAMPLE 11 A person starts by a car from Trivandrum to Kollam, and at the same time another person starts from Kollam by a car to Trivandrum. After passing each other they complete their journey in 2 hours and 1/2 hour respectively. At what rate does the second person drives the car if the first car runs at a speed of 40 km/h?

Solution

Ratio of speeds = \sqrt{b} : \sqrt{a}

$$=\sqrt{1/2}:\sqrt{2}=\sqrt{1/4}=\frac{1}{2}$$

 \therefore The second person's speed = First person's speed \times 2

$$= 40 \times 2 = 80 \text{ km/h}$$

Theorem: If two persons cover the same distance at two different speeds a and b km/h and if t minutes is the difference in time taken by the two persons to cover the distance, then the distance covered is given by $\frac{a \times b}{a \sim b} \times \frac{t}{60}$ km.

EXAMPLE 12 Two buses travels to a place at speeds of 45 km/h and 60 km/h respectively. If the second bus takes 5 hours less than the first for completing the journey, find the total distance covered by the bus during the journey.

Solution As the second bus takes 5 hours less, distance covered = $\frac{45 \times 60}{45 \sim 60} \times 5 = 900 \text{ km}$.

> Theorem:

Speed and time taken are inversely proportional.

$$S_1 T_1 = S_2 T_2 = S_3 T_3 = \cdots$$

where $S_1, S_2, S_3, ...$ are the speeds and $T_1, T_2, T_3...$ are the time taken to travel the same distance.

EXAMPLE 13 A person covers a distance in 40 minutes if he runs at a speed of 45 km/h on an average. Find the speed at which he must run to reduce the time of journey to 30 minutes.

Solution Here $45 \times 40 = S_2 \times 30$

$$S_2 = \frac{45 \times 40}{30} = 60 \text{ km/h}$$



EXERCISES

- 1. How many km/h does a man walk who passes through a street 600 m long in 5 minutes?
 - (a) 9/5 km/h
- (b) 18/5 km/h
- (c) 27/5 km/h
- (d) 36/5 km/h
- (e) None of these
- 2. A man takes 6 hours 30 minutes in walking to a certain place and riding back. He would have gained 2 hours 10 minutes by riding both ways. How long would he take to walk both ways?
 - (a) 480 minutes (b) 520 minutes (c) 560 minutes (d) 600 minutes (e) None of these
- 3. Two persons begin together to write out a booklet containing 8190 lines. The first person starts with the first line, writing at the rate of 200 lines an hour; and the second person starts with the last line, then writes 8189th line and so on, proceeding backward at the rate of 150 lines an hour. At what time will they meet?
 - (a) 22.4 hours
- (b) 23.4 hours
- (c) 24.4 hours
- (d) 25.4 hours
- (e) None of these
- 4. A motor car does a journey in 10 hours, the first half at 21 km/h, and the rest at 24 km/h. Find the distance.
 - (a) 220 km
- (b) 224 km
- (c) 228 km
- (d) 232 km
- (e) None of these

(a) 3599

by a car in 4 hours if the speed of train is 45 km/h? (b) 3600

	(a) 3 km	(b) 4 km	(c) 5 km	(d) 6 km	(e) None of these
7.	the rates of 20 km	n/h and 25 km/h re	espectively. When	-	wards each other at may have travelled ons.
	(a) 680 km	(b) 700 km	(c) 710 km	(d) 720 km	(e) None of these
8.				peed of 25 km/h an ainutes. What dista	d walked back at a nce did he travel?
	(a) 29 km	(b) 25 km	(c) 20 km	(d) 14 km	(e) None of these
9.	to catch the thief that of policeman policeman?	he also started ru	nning. Assume thar will the thief h	ne speed of the thie las run before he i	e policeman started f to be 10 km/h and s overtaken by the
	(a) 1 km	(b) 2 km	(c) 3 km	(d) 4 km	(e) None of these
10.	distance from Na	gercoil will it mee one-third faster tl	t a train which lea han the former do	aves Nagercoil for	l at 8 AM At what Trivandrum at 8.20 rom Trivandrum to
	(a) 26 km	(b) 32 km	(c) 46 km	(d) 48 km	(e) None of these
11.	50 km/hr and 40 k more than the sec	cm/hr respectively cond. The distance	. By the time they between P and Q	met the first train h is:	other at a speed of as travelled 100 km [SSC]
	(a) 500 km	(b) 630 km	(c) 660 km	(d) 900 km	
12.				hr. The theft is disc vill he overtake the	overed at 3 p.m. and thief? [RRB]
	(a) 4.30 p.m.	(b) 4.45 p.m.	(c) 5 p.m.	(d) 5.15 p.m.	
13.	.				peed been increased That is the distance [Bank PO]
	(a) 420 km(d) Cannot be de	` '	480 km None of these	(c) 640 km	
14.		less. If he had mov	-		en 40 minutes more. [SSC]
	(a) 35	(b) $36\frac{2}{3}$	(c) $37\frac{1}{2}$	(d) 40	
15.	one-third distance 60 km/hr. The ave	e with a speed of 2 erage speed of the	20 km/hr, and the car for the whole j	last one-third dista ourney is:	10 km/hr, the next ance with a speed of [IAS]
	(a) 18 km/hr	(b) 24 km/hr	(c) 30 km/hr	(d) 36 km/hr	
16.					t a speed of 2 km/hr. nd school is: [SSC]

5. Supposing that telegraph poles on a rail road are 50 metres apart, how many will be passed

6. A person has to reach a certain place and certain time and he finds that he will be 15 minutes late, if he walk at 4 km/h, and 10 minutes earlier, if he walk at 6 km/h. How far has he to walk?

(d) 3602

(e) None of these

(c) 3601

17.	A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour as: [RRB]
	(a) 35.55 km/hr (b) 36 km/hr (c) 71.11 km/hr (d) 71 km/hr
18.	A farmer travelled a distance of $61~\rm km$ in 9 hours. He travelled partly on foot at $4~\rm km/hr$ and partly on bicycle at 9 km/hr. The distance travelled on foot is: [UPSC]
	(a) 14 km (b) 15 km (c) 16 km (d) 17 km
19.	A person has to cover a distance of $6\mathrm{km}$ in $45\mathrm{minutes}$. If he covers one-half of the distance in two-third of the total time, to cover the remaining distance in the remaining time, his speed (in km/hr) must be: [SSC]
	(a) 6 (b) 8 (c) 12 (d) 15
20.	A boy rides his bicycle 10 km at an average speed of 12 km/hr and again travels 12 km at an average speed of 10 km/hr. His average speed for the entire trip is approximately: [SSC] (a) 10.4 km/hr (b) 10.8 km/hr (c) 11 km/hr (d) 12.2 km/hr
21.	A person travels equal distances with speeds of 3 km/hr, 4 km/hr and 5 km/hr and takes a
	total time of 47 minutes. The total distance (in km) is: [RRB]
99	
22.	The distance between two cities A and B is 330 km. A train starts from A at 8 a.m. and travels towards B at 60 km/hr. Another trains starts from B at 9 a.m. and travels towards A at 75 km/hr. At what time do they meet? [LIC AAO]
	(a) 10 a.m (b) 10.30 a.m (c) 11 a.m. (d) 11.30 a.m.
23.	In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip as reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is: [MAT]
	(a) 1 hour (b) 2 hours (c) 3 hours (d) 4 hours
24.	In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is [MAT]
	(a) 5 kmph (b) 6 kmph (c) 6.25 kmph (d) 7.5 kmph
25.	A train, when moves at an average speed of 40 km/hr , reaches its destination on time. When its average speed becomes 35 km/hr , then it reaches its destination 15 minutes late . Find the length of journey. [Bank PO]
	(a) 30 km (b) 40 km (c) 70 km (d) 80 km (e) None of these
26.	A walks around a circular field at the rate of one round/hour, while B runs around it at the rate of six rounds/hour. They start in the same direction from the same point at 7.30 a.m. They shall first cross each other at: [IAS]
	(a) 7.42 a.m (b) 7.48 a.m (c) 8.10 a.m (d) 8.30 a.m
27.	Two cars P and Q start at the same time from A and B which are 120 km apart. If the two cars travel in opposite directions, they meet after one hour and if they travel in same direction (from A towards B), then P meets Q after 6 hours. What is the speed of car P? [SBI PO]
	(a) 60 km/hr (b) 70 km/hr (c) 120 km/hr (d) Data inadequate (e) None of these
96	A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the
20.	policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after six minutes? [SSC]
	(a) 100 m (b) 150 m (c) 190 m (d) 200 m
29.	If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is: $[RRB]$
	(a) 50 km (b) 56 km (c) 70 km (d) 80 km

less in that time. Find his speed.

(b) 5 km/hr

(a) 4 km/hr

30.	30. A and B walk around a circular track. They start at 8 a.m. f directions. A and B walk at a speed of 2 rounds per hour a How many times shall they cross each other before 9.30 a	nd 3 rounds per hour respectively.
) 8
31.	31. It takes 8 hours for a 600 km journey, if 120 km is done by 20 minutes more, if 200 km is done by train and the rest be train to that of the car is:	r train and the rest by car. It takes by car. The ratio of the speed of the [MBA]
) 4:3
32.	32. A person travels from P to Q at a speed of 40 km/hr and a 50%. What is his average speed for both the trips?	returns by increasing his speed by [MBA]
	(a) 36 km/hr (b) 45 km/hr (c) 48 km/hr (d)) 50 km/hr
33.	33. Starting from his house, a student walks at a speed of 2 minutes late. Next day he increases his speed by 1 km/hr early. How far is the school from his house?	
	(a) 1 km (b) 1.5 km (c) 1.75 km (d)) 2 km
34.	34. A car travelling with 5/7 of its actual speed covers 42 km Find the actual speed of the car.	in 1 hour 40 minutes 48 seconds. [SSC]
	(a) $17\frac{6}{7}$ km/hr (b) 25 km/hr (c) 30 km/hr (d)) 35 km/hr
35.	35. A man travels 600 km by train at 80 km/hr, 800 km by ship at 400 km/hr and 100 km by car at 50 km/hr. What is distance?	the average speed for the entire [SSC]
	(a) 60 km/hr (b) $60 \frac{5}{123} \text{ km/hr}$ (c) 62 km/hr (d)	$65\frac{5}{123} \text{ km/hr}$
36.	36. A is faster than B. A and B each walk 24 km. The sum of the of time taken by them is 14 hours. Then A's speed is equal	<u>.</u>
	(a) 3 km/hr (b) 4 km/hr (c) 5 km/hr (d)) 7 km/hr
37.	37. Walking $\frac{6}{7}$ th of his usual speed, a man is 12 minutes lat	e. The usual time taken by him to
	cover that distance is:	[RRB]
	(a) 1 hour (b) 1 hour 12 min	
	(c) 1 hour 15 min (d) 1 hour 20 min	
38.	38. Robert is travelling on his cycle and has calculated to reac 10 km/hr; he will reach there at 12 noon if he travels at travel to reach A at 1 p.m.?	= = = = = = = = = = = = = = = = = = = =
	(a) 8 km/hr (b) 11 km/hr (c) 12 km/hr (d)) 14 km/hr
39.	39. A train can travel 50% faster than a car. Both start from point B 75 km away from A at the same time. On the way, minutes while stopping at the stations. The speed of the	however, the train lost about 12.5
	(a) 100 km/hr (b) 110 km/hr (c) 120 km/hr (d)) 130 km/hr
40.	40. The jogging track in a sports complex is 726 metres in cistart from the same point and walk in opposite direct respectively. They will meet for the first time in:	ion at 4.5 km/hr and 9.75 km/hr [MAT]
	(a) 4.9 min (b) 5.28 min (c) 5.5 min (d)) 6 min
41.	41. A man can reach a certain place in 30 hours. If he reduces	his speed by $\frac{1}{15}$ th, he goes 10 km

(c) $5\frac{1}{2}$ km/hr (d) 6 km/hr

[SSC]

42.	30 minutes more than B to destination is:	reach the destination. Th	are in the ratio of 3:4. A takes e time taken by A to reach the [SSC]
	(a) 1 hour (b) $1\frac{1}{2}$ hou	urs (c) 2 hours (d)	$2\frac{1}{2}$ hours
43.	Anna left for city A from city B	3 at 5.20 a.m. She travelled a ed was reduced to 60 km/hr.	at the speed of 80 km/hr for 2 hour If the distance between two cities [Bank PO]
	(a) 9.20 a.m.	(b) 9.25 a.m.	(c) 9.35 a.m.
	(d) 10.05 a.m	(e) None of these	
44.			, stopping for 3 minutes after every n from the starting point? [MAT]
	(c) 6 hours 27 min	(d) 6 hours 30 min	
45 .	A car is running at a speed of	108 km/hr. What distance w	vill it cover in 15 second? [RBI]
	(a) 45 metres(d) Cannot be determined	(b) 55 metres(e) None of these	(c) 450 metres
46.	A person crosses a 600 m long	street in 5 minutes. What i	s his speed in km/hr? [RRB]
	(a) 3.6 (b) 7.2	(c) 8.4 (d)	10
47.	How long will a boy take to rur 9 km/hr?	n round a square field of side	35 metres, if he runs at the rate of [SSC]
	(a) 50 sec (b) 52 sec	(c) 54 sec (d)	56 sec
48.	One of the two buses complete	es a journey of 300 km in $7\frac{1}{2}$	hours and the other a journey of
	450 km in 9 hours. The ratio o		[RRB]
			8:9
	(a) 2:3 (b) 3:4	(c) 4:5 (d)	0.0
49.	()	eed of 70 km/hr with its sp	peed increasing every 2 hours by [Bank PO]
49.	A motor car starts with a sp 10 km/hr. In how many hours	eed of 70 km/hr with its sp	peed increasing every 2 hours by
49.	A motor car starts with a sp 10 km/hr. In how many hours	eed of 70 km/hr with its sp will it cover 345 km? (b) 4 hours 5 min	peed increasing every 2 hours by [Bank PO]
	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined	eed of 70 km/hr with its specifications will it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed	peed increasing every 2 hours by [Bank PO]
	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined A train covers a distance of 10 lets.	eed of 70 km/hr with its specifications will it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time
	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of	eed of 70 km/hr with its sp will it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed distance will be:	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time
50. 51.	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of (a) 10 min (c) 13 min	eed of 70 km/hr with its sp will it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed distance will be: (b) 11 min 20 sec (d) 13 min 20 sec	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time
50. 51.	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 1/4 hours (d) Cannot be determined A train covers a distance of 10 1 taken by it to cover the same of a) 10 min (c) 13 min A man walking at the rate of 5	eed of 70 km/hr with its sp will it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed distance will be: (b) 11 min 20 sec (d) 13 min 20 sec km/hr crosses a bridge in 15	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time [SSC] minutes. The length of the bridge
50. 51.	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of 10 min (c) 13 min A man walking at the rate of 5 (in metres) is: (a) 600 (b) 750	eed of 70 km/hr with its special it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed distance will be: (b) 11 min 20 sec (d) 13 min 20 sec km/hr crosses a bridge in 15 (c) 1000 (d) 0 metres in 1 minute, where	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time [SSC] minutes. The length of the bridge [SSC]
50. 51.	A motor car starts with a sp 10 km/hr. In how many hours (a) 2 \frac{1}{4} hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of (a) 10 min (c) 13 min A man walking at the rate of 5 (in metres) is: (a) 600 (b) 750 A truck covers a distance of 55	eed of 70 km/hr with its special it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its special istance will be: (b) 11 min 20 sec (d) 13 min 20 sec km/hr crosses a bridge in 15 (c) 1000 0 metres in 1 minute, where their speeds is:	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time [SSC] minutes. The length of the bridge [SSC] 1250 as a bus covers a distance of 33 km
50.51.52.	A motor car starts with a sp 10 km/hr. In how many hours (a) $2\frac{1}{4}$ hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of a 10 min (c) 13 min A man walking at the rate of 5 (in metres) is: (a) 600 (b) 750 A truck covers a distance of 55 in 45 minutes. The ratio of the (a) 3:4 (b) 4:3	eed of 70 km/hr with its special it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its speed distance will be: (b) 11 min 20 sec (d) 13 min 20 sec km/hr crosses a bridge in 15 (c) 1000 0 metres in 1 minute, where the speeds is: (c) 3:5 (d) 2 km after every hour. If the ald distance travelled in 12 hours will it in the second	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time [SSC] minutes. The length of the bridge [SSC] 1250 as a bus covers a distance of 33 km [SSC] 50:3 e distance travelled in the first one
50.51.52.	A motor car starts with a sp 10 km/hr. In how many hours (a) $2\frac{1}{4}$ hours (d) Cannot be determined A train covers a distance of 10 taken by it to cover the same of (a) 10 min (c) 13 min A man walking at the rate of 5 (in metres) is: (a) 600 (b) 750 A truck covers a distance of 55 in 45 minutes. The ratio of the (a) 3:4 (b) 4:3 The speed of a car increases by	eed of 70 km/hr with its special it cover 345 km? (b) 4 hours 5 min (e) None of these km in 12 minutes. If its special istance will be: (b) 11 min 20 sec (d) 13 min 20 sec km/hr crosses a bridge in 15 (c) 1000 0 metres in 1 minute, where the sir speeds is: (c) 3:5 (d) 2 km after every hour. If the	peed increasing every 2 hours by [Bank PO] (c) $4\frac{1}{2}$ hours d in decreased by 5 km/hr, the time [SSC] minutes. The length of the bridge [SSC] 1250 as a bus covers a distance of 33 km [SSC] 50:3 e distance travelled in the first one

54. An athlete runs 200 metres race in 24 seconds. His speed is:

[SSC]

- (a) 20 km/hr
- (b) 24 km/hr
- (c) 28.5 km/hr
- (d) 30 km/hr
- 55. A man travelled from the village to the post office at the rate of 25 km/hr and walked back at the rate of 4 km/hr. If the whole journey took 5 hours 48 minutes, find the distance of the post office from the village. [SSC]
 - (a) 15 km
- (b) 18 km
- (c) 20 km
- (d) 22 km



- 1. (d) 2. (b) 3. (b) **4.** (b)
 - **5.** (c) **6.** (c)
- 7. (d)
- 8. (c) 9. (a)
- **10.** (b)

- 11. (d) **12.** (c)
- **13.** (b) **14.** (d)
- 15. (a)
- 18. (c)
 - **19.** (c) **20**. (b)

- 21. (b) **22.** (c)
- 24. (a) **23.** (a)
- **16.** (c) **25.** (c) **26.** (a)
- 17. (c) **27**. (b)
- 28. (a) **29**. (a)
 - **30.** (c)

- **31.** (c) **32.** (c) **33.** (c)
- **34.** (d)
- **35.** (d) **36.** (b)
- **37.** (b)
- 38. (c) **39.** (c)
- **40**. (b)

- **41.** (b) **42.** (c)
- **43.** (e)
- **44.** (a)
- **46.** (b)
- **47.** (d)
- **49.** (c) **50**. (d)

- **51.** (d)
- **52.** (a)
- **53.** (c)
- **54.** (d)
- **45.** (c) **55.** (c)
- **48.** (c)

${f S}$ olutions with Necessary Explanation

- Speed in m/s = $\frac{600}{5 \times 60}$ = 2 1.
 - $\therefore \text{ Speed in km/h} = 2 \times \frac{18}{5} = 7\frac{1}{5} \text{ km/h}$
- 2. Walking + Riding = 6 hours 30 minutes

2 Riding = 6 hours 30 min - 2 hours 10 min.

(2)

(1)

Solving (1) and (2),

 $2 \times (1) - (2) \Rightarrow 2$ walking = 13 hours - 4 hours 20 min.

= 8 hours 40 minutes.

 $= (8 \times 60 + 40)$ minutes

= 520 minutes



Short cut: One way riding saves 2 hours 10 minutes

- ⇒ One way walking takes 2 hours 10 minutes more than one way riding.
- .. Both way walking takes 6 hours 30 min + 2 hours 10 min = 8 hours 40 minutes
 - $= (8 \times 60 + 40) \text{ minutes} = 520 \text{ minutes}$
- **3.** Their relative speed = 200 + 150 = 350 lines/h

Time at which they meet = 8190/350 hour = 117/5 hour = 23.4 hour

4. Let the distance be x km.

Total time taken =
$$\frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10$$
 hours

or
$$\frac{x(24+21)}{2\times 21\times 24} = 10$$
 hours

$$\therefore x = \frac{10 \times 2 \times 21 \times 24}{45} = 2 \times 2 \times 7 \times 8 = 224 \text{ km}$$

- 5. Total distance covered by the train in 4 hours = $45 \times 4 = 180$ km Telegraph poles are 50 m apart.
 - :. Number of poles in 180 km will be $\frac{180000}{50} + 1 = 3601$
- 6. Required distance (Distance he has to walk) = $\frac{Product \text{ of two speeds}}{Difference \text{ of two speeds}} \times Difference \text{ in time}$

$$= \frac{4 \times 6}{6 \sim 4} \times \left(\frac{15 + 10}{60}\right) = \frac{24}{2} \times \frac{25}{60} = 5 \text{ km}$$

7.



Short cut: Distance between two bus stations

= Distance more travelled by one bus
$$\times$$
 $\frac{Sum \text{ of speeds}}{Difference \text{ of speeds}}$

=
$$80 \frac{(25 + 20)}{(25 \sim 20)} = 80 \times \frac{45}{5} = 720 \text{ km}$$

8. Total distance = Total time $\left(\frac{\text{Product of speeds}}{\text{Sum of speeds}}\right)$

$$= 5 \frac{48}{60} \left(\frac{25 \times 4}{25 + 4} \right) = 5 \frac{4}{5} \left(\frac{100}{29} \right) = \frac{29}{5} \times \frac{100}{29} = 20 \text{ km}$$

- 9. Relative speed of the policeman w.r.t. thief = 12 10 = 2 km/h
 - \therefore Time taken by the policeman to catch the thief = $\frac{200/1000}{2} = \frac{1}{10}$ hour
 - \therefore The distance thief has run before he was caught by the policeman = $10 \times \frac{1}{10} = 1$ km
- 10. Speed of first train, $S_1 = 10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/h}$

Speed of second train,
$$S_2 = 36 \times 1\frac{1}{3} = 48 \text{ km/h}$$

Total distance between the two places = 68 km

Difference between first and second times of departure $T_1 - T_2 = -20$ minutes = $-\frac{1}{3}$ hour

Distance of meeting point from Nagercoil = $68 - 36\left(\frac{1}{3}\right) = 56$

$$84 \times t = 56$$

$$48t = \frac{56}{84} \times 48 = \frac{8}{12} \times 48 = 32 \text{ km}$$

Distance of meeting point from Nagercoil =
$$48 \times \left[\frac{68 - 36\left(\frac{1}{3}\right)}{36 + 48} \right]$$

$$= 48 \left[\frac{68 - 12}{36 + 48} \right] = 32 \text{ km}$$

11. Distance between P and Q = Distance more travelled by one train
$$\times \frac{\text{Sum of speeds}}{\text{Difference of speeds}}$$

=
$$100 \text{ km} \times \frac{50 + 40}{50 - 40}$$

= $100 \times \frac{90}{10}$
= 900 km

12. Relative speed of owner w.r.t. thief = 75 - 60 = 15 km/hr.

The distance covered by thief in 30 minutes = $60 \times \frac{1}{2} = 30 \text{ km}$

- \therefore Owner will overtake thief in $\frac{30}{15} = 2$ hours
- :. He will overtake thief at 5 p.m.



Short cut: Required time =
$$\frac{S_2 \times \text{Difference in time }(t)}{S_1 \times S_2}$$

where
$$S_1 = \text{Speed of man}$$

$$S_2 =$$
Speed of thief

$$t =$$
Time lag between both

Here
$$t = 3.00 - 2.30 = 30$$
 minutes = $\frac{1}{2}$ hour; $S_1 = 75$ km/hr; $S_2 = 60$ km/hr

$$\therefore \text{ Required time} = \frac{60 \times \frac{1}{2}}{15} = 2 \text{ hours}.$$

13. Let the distance covered be S km.

14. Let distance be S km and usual speed be u km/hr

$$\frac{S}{u} - \frac{S}{u+3} = \frac{40}{60} = \frac{2}{3}$$
or
$$\frac{3S}{u(u+3)} = \frac{2}{3}$$

or
$$9S = 2u(u+3)$$
 (1)

Similarly by
$$\frac{S}{u-2} - \frac{S}{u} = \frac{40}{60} = \frac{2}{3}$$

or
$$\frac{2S}{u(u-2)} = \frac{2}{3}$$

$$3S = u(u-2) \tag{2}$$

Dividing (1) by (2), we get

$$3=\frac{2(u+3)}{u-2}$$

$$\Rightarrow$$
 $u = 12 \text{ km/hr}$

and, therefore, from (2),

$$S = \frac{(12)(12-2)}{3}$$

$$= 4 \times 10 = 40 \text{ km/hr}$$

15. Let the total distance travelled be S km and average speed be x km/hr.

$$\frac{S/3}{10} + \frac{S/3}{20} + \frac{S/3}{60} = \frac{S}{x}$$
or
$$\frac{1}{30} + \frac{1}{60} + \frac{1}{180} = \frac{1}{x}$$
or
$$x = \frac{1}{6+3+1} = 18 \text{ km/hr}$$

16. Let the distance to his school be
$$S \text{ km}$$
.

Average speed =
$$\frac{2 \times 3 \times 2}{3+2}$$
 = 2.4 km/hr

Time taken for whole journey = 5 hours.

$$2S = \text{Average speed} \times \text{Time}$$

$$= 2.4 \times 5 = 12 \text{ km}$$
 or
$$S = 6 \text{ km}$$

17. Average speed =
$$\frac{2 \times 64 \times 80}{64 + 80} = \frac{2 \times 64 \times 80}{144}$$

= $\frac{1 \times 64 \times 10}{9} = \frac{640}{9} = 71.11 \text{ km/hr}$

18. Let distance travelled by foot be x km, and therefore distance by bicycle = (61 - x) km.

$$\frac{61 - x}{9} + \frac{x}{4} = 9$$
or
$$(61 - x) 4 + 9x = 9 \times 36$$
or
$$5x = 9 \times 36 - 4 \times 61$$

$$= 4[81 - 61]$$

$$= 80$$
or
$$x = \frac{80}{5} = 16 \text{ km}$$

19. Required speed =
$$\frac{6 \text{ km}}{\frac{45}{60} \text{ hr}}$$
 = 8 km/hr

Half of journey (speed) =
$$\frac{3}{\frac{30}{60}}$$
 = 6 km/hr

Let speed for next half be x km/hr

$$\therefore \qquad \text{Average speed} = \frac{2 \times 6 \times x}{6 + x} = 8$$
i.e.
$$\frac{12x}{6 + x} = 8$$
or
$$12x = 8 \times 6 + 8x$$
or
$$4x = 8 \times 6$$
or
$$x = 12 \text{ km/hr}$$

20. Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Total distance travelled = 10 + 12 = 22 km

Total time taken =
$$\left(\frac{10}{12} + \frac{12}{10}\right) = \frac{50 + 72}{60} = \frac{122}{60}$$

= $\frac{61}{30}$ hours

Average speed =
$$\frac{22}{\frac{61}{30}} = \frac{22 \times 30}{61} = 10.8 \text{ km/hr}$$

21. Let total distance be S km

٠:.

If u_1 , u_2 , u_3 are their speeds for 3 equal distances and t minutes is the total time taken, then

Total distance travelled =
$$\frac{u_1 u_2 u_3 \times t}{20(u_1 u_2 + u_2 u_3 + u_3 u_1)}$$
$$= \frac{3 \times 4 \times 5 \times 47}{20(3 \times 4 + 4 \times 5 + 5 \times 3)} = \frac{3 \times 47}{(12 + 20 + 15)}$$
$$= \frac{3 \times 47}{47} = 3 \text{ km}$$

22. Distance travelled by train from A from 8 a.m. to 9 a.m. = $60 \text{ km/hr} \times 1 \text{ hr} = 60 \text{ km}$ Distance between two trains at 9 a.m. = 330 - 60 = 270 kmRelative speed of second trains w.r.t first = 75 - (-60) = 135 km/hr

∴ Time taken by them to meet =
$$\frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{270}{135}$$

At 11 a.m. both the trains will meet.

23. Let the duration of flight be *t* hours.

$$\frac{600}{t} - \frac{600}{t + \frac{1}{2}} = 200$$

$$\frac{600}{t} - \frac{2 \times 600}{2t + 1} = 200$$

$$(2t + 1) 600 - t \times 1200 = 200t (2t + 1)$$
i.e.
$$3(2t + 1) - 6t = t(2t + 1)$$

$$\Rightarrow 6t + 3 - 6t = 2t^2 + t$$

$$\Rightarrow 2t^2 + t - 3 = 0$$

$$\therefore t = 1 \text{ hour}$$

Hence answer is (a).

24. Let Abhay's speed be x km/hr

$$\therefore$$
 Time taken to travel 30 km by Abhay = $\frac{30}{x}$

$$\therefore$$
 Time taken to travel 30 km by Sameer = $\left(\frac{30}{x} - 2\right)$

$$\therefore \frac{30}{2x} = \left(\frac{30}{x} - 2\right) - 1$$

i.e.
$$\frac{30}{2x} = \frac{30}{x} - 3$$

or
$$\frac{30}{x} - \frac{30}{2x} = 3$$

$$10\left[\frac{1}{x} - \frac{1}{2x}\right] = 1$$

or
$$10\frac{(2-1)}{2r} = 1$$

or
$$x = \frac{10}{2} = 5 \text{ km/hr}$$

25. Let the length of journey be x km.

$$\frac{x}{35} = \frac{x}{40} + \frac{15}{60}$$

$$\frac{x}{35} - \frac{x}{40} = \frac{1}{4}$$

$$\frac{8x - 7x}{7 \times 8 \times 5} = \frac{1}{4}$$

$$x = \frac{7 \times 8 \times 5}{4} = 70 \text{ km}$$

 \mathbf{or}

26. Relative speed of B w.r.t A = (6 - 1) rounds/hour

- :. Time taken to complete one round = 12 minutes
- .. They will meet for the first time after 12 minutes i.e. at 7.42 a.m.
- 27. Relative speed of P w.r.t Q = 120 km/hr

$$i.e u+v=120 (1)$$

Relative speed of P w.r.t. Q, when they are in same direction

$$= u - v = \frac{120}{6} = 20 \tag{2}$$

Adding (1) and (2), we get

$$2u = 140$$

$$u = 70$$

 \mathbf{or}

 \therefore Speed of P, u = 70 km/hr

28. Relative speed of policeman w.r.t thief = 11 - 10 = 1 km/hr

Distance already covered by thief = 200 m

After 6 minutes, distance covered by policeman

=
$$1 \text{ km/hr} \times \frac{6}{60} \text{ hr}$$

= $\frac{1}{10} \text{ km} = 100 \text{ m}$

- \therefore Distance between policeman and thief = 200 100 = 100 m
- **29.** Let the actual distance travelled be x km.

$$\therefore \frac{x}{10} = \frac{x+20}{14}$$

 \Rightarrow x = 50 km

30. Relative speed of B w.r.t. A = (3 + 2) rounds/hour = 5 rounds/hour

They will cross each other 5 times in every hour and 2 times in half an hour. In total, they will cross each other 7 times upto 9.30 a.m.

31. Let speed of train be x km/hr and that of car be kx km/hr.

$$\therefore \qquad \frac{120}{x} + \frac{480}{kx} = 8 \tag{1}$$

$$\frac{200}{x} + \frac{400}{kx} = 8\frac{1}{3} \tag{2}$$

Simplifying (1),
$$\Rightarrow$$
 120k + 480 = 8kx (A)

Simplifying (2),
$$\Rightarrow 200k + 400 = \frac{25}{3}kx$$
 (B)

Dividing (A) by (B),

$$\frac{120k + 480}{200k + 400} = \frac{8kx \times 3}{25kx}$$

i.e.
$$\frac{120(k+4)}{200(k+2)} = \frac{24}{25}$$

$$\frac{5(k+4)}{8(k+2)} = 1$$

$$\Rightarrow$$

$$\frac{k+4}{k+2} = \frac{8}{5}$$

Cross multiplying, we get,

$$8k + 16 = 5k + 20$$
$$8k - 5k = 20 - 16$$
$$3k = 4$$
$$k = \frac{4}{3}$$

 \therefore Speed of train: Speed of car = x: kx = 1: k

$$\therefore$$
 Required ratio = 1: $k = 1$: $\frac{4}{3}$ or 3:4

32. Average speed =
$$\frac{2(40)(60)}{(40+60)} = 2 \times 4 \times 6 = 48$$
 km/hr

33. Let the distance to school be x km.

$$\frac{x}{2.5} - \frac{x}{3.5} = \frac{6+6}{60} = \frac{1}{5}$$

i.e.

$$\frac{x(7-5)}{7\times2.5}=\frac{1}{5}$$

 \mathbf{or}

$$x = \frac{7 \times 2.5}{5 \times 2} = 1.75 \text{ km}$$

34.
$$\frac{42 \text{ km}}{1 \text{ hr } 40 \text{ min } 48 \text{ sec}} = \frac{5}{7} \text{ of actual speed}$$

$$\therefore \text{ Actual speed} = \frac{7}{5} \times \frac{42}{1 \text{ hr } 40 \text{ min } 48 \text{ sec}}$$

$$= \frac{7 \times 42}{5 \text{ hr } 200 \text{ min } + 4 \text{ min}}$$

$$= \frac{7 \times 42}{8 \text{ hr } 24 \text{ min}} = \frac{7 \times 42 \text{ km}}{8.4 \text{ hr}}$$

$$= 35 \text{ km/hr}$$

35. Total distance = 600 + 800 + 500 + 100 = 2000 km

Total time taken =
$$\frac{600}{80} + \frac{800}{40} + \frac{500}{400} + \frac{100}{50}$$

= $\frac{15}{2} + 20 + \frac{5}{4} + 2$
= **30.75 hours**

:. Average speed =
$$\frac{2000}{30.75} = \frac{8000}{123}$$

$$=65\frac{5}{123} \, \text{km/hr}$$

- **36.** Sum of speeds = 7 km/hr
 - \therefore Speed of A is greater than $\frac{7}{2}$ km/hr.

If speed of A = 4 km/hr, that of B = 3 km/hr such that their sum = 7 km/hr Sum of times = 14 hours is also satisfied with the above conditions.

- \therefore Speed of A = 4 km/hr.
- **37.** New speed = $\frac{6}{7}$ of usual speed

New time = $\frac{7}{6}$ of usual time

- $\therefore \qquad \text{Usual time} = 6 \times \frac{1}{5} \text{ hour } = 1 \text{ hour } 12 \text{ min}$
- **38.** Relative speed = 15 10 = 5 km/hr

Difference in time = 2 hours.

 \therefore Relative distance = $5 \times 2 = 10 \text{ km}$

To reach at 1 p.m.

Let distance travelled be x km

$$\therefore \frac{x}{10} - \frac{x}{15} = 2$$

$$\Rightarrow x(3-2) = 2 \times 30$$
or
$$x = 60 \text{ km}$$

At 10 km/hr speed, time taken to travel = $\frac{60}{10}$ = 6 hours

6 hours before 2 p.m. = 8 a.m.

- \therefore Required speed to reach at 1 a.m. = $\frac{60}{5}$ = 12 km/hr
- **39.** Let speed of car be x km/hr and that of train be 1.5x km/hr Distance between A and B = 75 km

$$\therefore \frac{75}{x} = \frac{75}{1.5x} + \frac{12.5}{60} \text{ hours}$$

i.e.
$$\frac{75}{x} - \frac{75}{1.5x} = \frac{5}{24}$$

$$\Rightarrow \qquad \frac{75(1.5-1)}{1.5x} = \frac{5}{24}$$

$$\Rightarrow \qquad x = \frac{75 \times 0.5 \times 24}{1.5 \times 5} = 5 \times 24 = 120 \text{ km/hr}$$

40. Relative speed of Deepak w.r.t. his wife

$$= 4.5 + 3.75$$

$$= 8.25 \text{ km/hr}$$

∴ Time taken to rotate periphery of sports complex =
$$\frac{726 \text{ m}}{8.25 \text{ km/hr}} = \frac{0.726 \text{ km}}{8.25 \text{ km/hr}}$$

$$= \frac{0.726}{8.25} \text{ hr}$$

$$= \frac{0.726}{8.25} \times 60 \text{ minutes}$$

= 5.28 minutes

41. Let the speed of person be x km/hr

$$x \times 30 = \frac{14}{15}x \times 30 + 10$$

$$\therefore \qquad 30\left(x - \frac{14}{15}x\right) = 10$$
or
$$\frac{30x}{15} = 10$$
or
$$x = \frac{10 \times 15}{30} = 5 \text{ km/hr}$$

42. Let time taken by A be x hours

$$\therefore$$
 Time taken by B = $\left(x - \frac{1}{2}\right)$ hours

Distance covered by
$$A = 3k \times x$$
 (1)

Distance covered by B =
$$4k \times \left(x - \frac{1}{2}\right)$$
 (2)

Equating (1) and (2), we get

$$3k \times x = 4k \times \left(x - \frac{1}{2}\right)$$
$$3x = 4\left(x - \frac{1}{2}\right)$$
$$4x - 3x = 4 \times \frac{1}{2} = 2$$

 \mathbf{or}

x = 2 hours

or

i.e. Time taken by A = 2 hours

43. Let time taken by Anna at speed of 60 km/hr be x hours.

$$\therefore 80 \times 2\frac{1}{4} + 60 \times x = 350$$

$$\therefore x = \frac{350 - 180}{60}$$

$$= \frac{170}{60} = \frac{17}{6} \text{ hours} = 2 \text{ hours } 50 \text{ minutes}$$

.. Total time of travel = 2 hours 15 minutes + 2 hours 50 minutes = 5 hours 5 minutes

So Anna reached city A at 10.25 a.m.

Hence answer is (e).

44. Time of travel for a distance of 600 km at a speed of 100 km/hr = $\frac{600}{100}$ = 6 hours

Time of halt at the rate of 3 min/75 km

$$= 3 \times 7 = 21$$
 minutes

- .: Total time of travel = 6 hour 21 min
- 45. Speed of car = $108 \text{ km/hr} = 108 \times \frac{5}{18} \text{ m/s} = 30 \text{ m/s}$
 - \therefore Distance covered = $30 \times 15 = 450 \text{ m}$
- **46.** Speed of person = $\frac{600 \text{ m}}{5 \text{ minutes}} = \frac{600 \text{ m}}{5 \times 60 \text{ sec}} = 2 \text{ m/s}$
 - \therefore Speed of person (in km/hr) = $2 \times \frac{18}{5} = 7.2$ km/hr
- 47. Time taken by boy to run round square field

$$= \frac{\text{Perimeter}}{\text{Speed}} = \frac{4 \times 35 \text{ m}}{9 \text{ km/hr}} = \frac{4 \times 35 \text{ m}}{9 \times \frac{5}{18} \text{ m/s}}$$
$$= \frac{4 \times 35 \times 2}{5} \text{ sec} = 56 \text{ seconds}$$

- **48.** Ratio of their speeds = $\frac{300/7.5}{450/9} = \frac{40}{50} = \frac{4}{5}$ or **4:5**
- **49.** In first 2 hours, distance covered = $70 \times 2 = 140 \text{ km}$ In the second 2 hours, distance covered = $80 \times 2 = 160 \text{ km}$ Remaining distance to be covered = 345 - (140 + 160) = 45 km
 - .. Time taken by motor car to travel 45 km at a speed of 90 km/hr = $\frac{45}{90} = \frac{1}{2}$ hour
 - \therefore Required time = $4\frac{1}{2}$ hours
- **50.** Speed of train = $10 \text{ km}/12 \text{ min} = \frac{10}{\frac{12}{60}} = 50 \text{ km/hr}$

If speed is decreased by 5 km/hr, new speed = 45 km/hr

.. Time taken to cover same distance

$$= \frac{10}{45} \text{ hours} = \frac{10 \times 60}{45} \text{ minutes}$$
$$= \frac{120}{9} = \frac{40}{3} \text{ minutes}$$

= 13 minutes 20 seconds

51. Let length of bridge be L metres.

$$L = \frac{5 \times 5 \times 60 \times 15}{18}$$

$$= 5 \times 5 \times 10 \times 5 = 1250 \text{ m}$$

52. Speed of truck =
$$\frac{550 \text{ m}}{60 \text{ sec}} = \frac{55}{6} \text{ m/s}$$

Speed of bus =
$$\frac{33 \text{ km}}{45 \text{ min}} = \frac{33}{\frac{45}{60}} \text{ km/hr} \times \frac{5}{18} \text{ m/s}$$

= $\left(\frac{33 \times 60}{45}\right) \times \frac{5}{18} \text{ m/s}$
= $\left(11 \times 4 \times \frac{5}{18}\right) \text{ m/s} = \frac{110}{9} \text{ m/s}$

$$\therefore \text{ Ratio of their speed} = \frac{55/6}{110/9} = \frac{55}{6} \times \frac{9}{110}$$
$$= \frac{3}{4} \text{ or } 3:4$$

53. Total distance travelled is 12 hours

$$= 35 + 37 + 39 + \dots$$
 up to 12 terms

$$=\frac{n}{2}\left\{2a+(n-1)d\right\}$$

$$= \frac{12}{2} \left\{ 2 \times 35 + (12 - 1)2 \right\}$$

$$= 6 \{70 + 22\} = 552 \text{ km}$$

54. Speed of athlete =
$$\frac{200 \text{ m}}{24 \text{ sec}} = \frac{25}{3} \text{m/s} = \frac{25}{3} \times \frac{18}{5} \text{km/hr} = 30 \text{ km/hr}$$

55. Average speed =
$$\frac{2 \times 25 \times 4}{(25+4)} = \frac{200}{29} \text{ km/hr}$$

Distance travelled = Average speed \times Time

$$=\frac{200}{29}\times 5\frac{4}{5}$$

$$=\frac{200}{29}\times\frac{29}{5}=40 \text{ km}$$

:. Distance of post office to village =
$$\frac{40}{2}$$
 = 20 km

Heights and Distances

ANGLES OF ELEVATION AND DEPRESSION: The line of sight or line of vision is a straight line from our eye to the object we are viewing. In the adjoining figure *RP* is the line of sight.

The angle of elevation is the angle through which our eyes move above from the horizontal. Here it is $\angle QRP = \theta$

The angle of depression is the angle through which our eyes move below the horizontal line.

Here it is $\angle RPQ = \alpha$

Generalised cases:

From $\triangle ABC$,

$$\tan \theta_2 = \frac{h_2}{d}$$

$$d = \frac{h_2}{\tan \theta_2} = h_2 \cot \theta_2 \tag{1}$$

 \mathbf{or}

Similarly from ΔBCD ,

$$\tan \theta_1 = \frac{h_1}{d}$$

$$d = \frac{h_1}{\tan \theta_1} = h_1 \cot \theta_1$$
 (2)

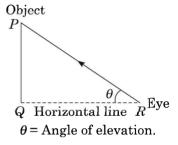
or

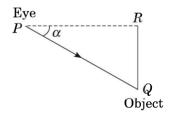
From (1) and (2),

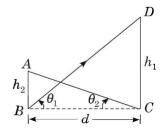
$$h_1 \cot \theta_1 = h_2 \cot \theta_2$$

 \therefore Height difference $h_1 \sim h_2 = d(\tan \theta_1 \sim \tan \theta_2)$

$$d = \frac{h_1 \sim h_2}{(\tan \theta_1 \sim \tan \theta_2)}$$





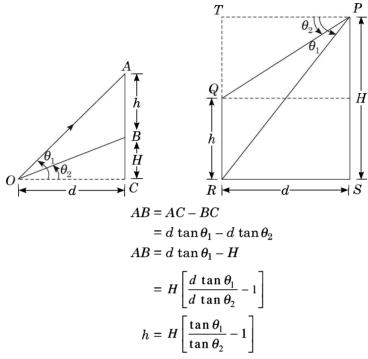


From $\triangle OCB$,

$$BC = d \tan \theta_2 \tag{1}$$

From $\triangle OAC$,

$$AC = d \tan \theta_1 \tag{2}$$



From ΔPRT and ΔPQT

:.

•:•

:.

$$h = d(\tan \theta_2 - \tan \theta_1)$$

 $H = d \tan \theta_2$

$$h = H \left[1 - \frac{\tan \theta_1}{\tan \theta_2} \right]$$

$$d = \frac{h}{\tan \theta_2 \sim \tan \theta_1}$$

If A and B are two positions of a car or a ship or any other vehicle or any person and the distance AB = d is covered in time t, the speed of the vehicle or ship is given by

$$\begin{array}{c|c}
h \\
\hline
A & B & C \\
\hline
\downarrow - d \rightarrow \downarrow
\end{array}$$

$$S = \frac{d}{t} = \frac{h}{t} (\cot \theta_1 - \cot \theta_2)$$

A tower of height h stands vertical on the ground. Its angle of elevation from two points, distance d apart and lying along its mutually \perp^r directions on the ground are θ_1 and θ_2 .

Then
$$h = \frac{d}{\sqrt{\cot^2 \theta_1 + \cot^2 \theta_2}}$$

:.

EXAMPLE 1 300 m from the foot of a cliff on the level ground, the angle of elevation of the top of a cliff is 30°. Find the height of this cliff.

Solution Let the height of cliff be x metres. From ΔPQR ,

$$\tan 30^{\circ} = \frac{PQ}{QR} = \frac{x}{300} = \frac{1}{\sqrt{3}}$$
$$x = \frac{300}{\sqrt{3}} = 100\sqrt{3} = 173.2 \text{ m}$$

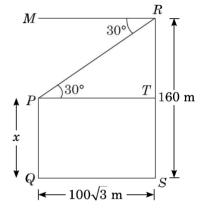
EXAMPLE 2 The horizontal distance between two towers is $100\sqrt{3}$ metres. The angle of depression of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 160 m, find height of the first tower.

Solution Let RS be the second tower 160 m high. Let PQ be the first tower of height x metres $MR \mid\mid PT$ and $\angle MRP = \angle RPT = 30^{\circ}$ \therefore From $\triangle PRT$,

$$\tan 30^{\circ} = \frac{RT}{PT} = \frac{1}{\sqrt{3}} = \frac{RT}{100\sqrt{3}}$$

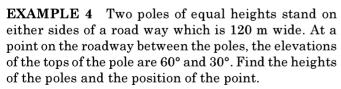
 $\therefore RT = 100 \text{ m}$

$$\therefore PQ = RS - RT = 160 - 100 = 60 \text{ m}$$



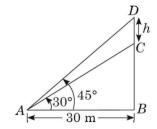
EXAMPLE 3 Angles of elevation of top and bottom of a flag kept on a flag post at 30 metres distance are 45° and 30° respectively. What is the height of the flag?

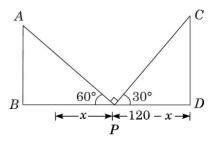
Solution
$$\tan 45^\circ = \frac{DB}{AB}$$
 or $DB = 30 \text{ m}$ $\tan 30^\circ = \frac{CB}{AB}$ or $CB = \frac{30}{\sqrt{3}} \text{ m}$ Height of flag $CD = 30 - \frac{30}{\sqrt{3}} = 30 - 10\sqrt{3}$ $= 30 - 17.32 = 12.68 \text{ m}$



Solution Let AB and CD be the two poles of equal length l.

Let the point P be at a distance x metre from B such that BP = x and PD = 120 - x.





From $\triangle ABP$.

$$\tan 60^\circ = \frac{l}{x} \qquad \text{or} \qquad x = \frac{l}{\tan 60^\circ} \tag{1}$$

From ΔPCD ,

$$\tan 30^{\circ} = \frac{l}{120 - x}$$
 or $120 - x = \frac{l}{\tan 30^{\circ}}$ (2)

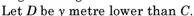
(1) + (2)
$$\Rightarrow$$
 120 = $\frac{l}{\tan 60^{\circ}} + \frac{l}{\tan 30^{\circ}}$
 $120 \times \tan 60^{\circ} \tan 60^{\circ}$

$$l = \frac{120 \times \tan 60^{\circ} \tan 30^{\circ}}{\tan 30^{\circ} + \tan 60^{\circ}}$$
$$= \frac{120 \times \sqrt{3} \times 1/\sqrt{3}}{(1/\sqrt{3} + \sqrt{3})} = \frac{120}{4/\sqrt{3}} = 30\sqrt{3} = 52 \text{ m}$$

$$x = \frac{l}{\tan 60^{\circ}} = \frac{30\sqrt{3}}{\sqrt{3}} = 30 \text{ m}$$

EXAMPLE 5 An aeroplane when 3000 m high passes vertically above another at an instant when the angles of elevation at the same observing point are 60° and 45° respectively. How many metres lower is one than the other?

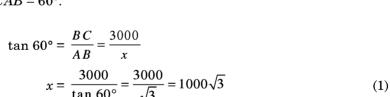
Solution Let C and D be the position of two aeroplanes, C being at a height of 3000 m from B.



Let A be the point of observation.

Now $\angle DAB = 45^{\circ}$, $\angle CAB = 60^{\circ}$.

Let AB = x metres.



or

From $\triangle ABD$,

From $\triangle ABC$,

$$\tan 45^{\circ} = \frac{BD}{AB} = \frac{3000 - y}{x} = 1$$

$$x = 3000 - y \tag{2}$$

Equating (1) and (2),

$$3000 - y = 1000\sqrt{3}$$

$$y = 3000 - 1000\sqrt{3}$$
$$= 3000 - 1732 = 1268 \text{ m}$$

 \therefore Height difference between the two aeroplanes = 1268 m

EXAMPLE 6 The shadow of a vertical pole is $\sqrt{3}$ of its height. Find the angle of elevation.

Solution Let BC be the height of pole = h.

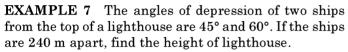
Length of shadow = $AB = h\sqrt{3}$

From $\triangle ABC$,

$$\tan \theta = \frac{BC}{AB} = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = 30$$

 \therefore Angle of elevation = 30°



Solution Let AQ = x m

$$\therefore \qquad QB = (240 - x) \text{ m}$$

From $\triangle APQ$

$$\tan 60^\circ = \frac{y}{x} = \sqrt{3}$$
$$y = x\sqrt{3}$$

 \Rightarrow

From ΔBPQ

$$\tan 45^{\circ} = \frac{y}{240 - x} = 1$$

$$y = 240 - x \tag{2}$$

 \Rightarrow

 \Rightarrow

From (1) and (2), we get

$$\frac{y}{\sqrt{3}} = 240 - y$$
or
$$y\left(1 + \frac{1}{\sqrt{3}}\right) = 240$$

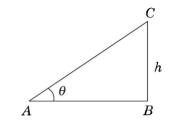
$$\Rightarrow \qquad y = \frac{240\sqrt{3}}{(1 + \sqrt{3})} \mathbf{m}$$

EXAMPLE 8 From the top of a cliff 200 m high the angles of depression of two boats which are due south of observer are 60° and 30°. Find the distance between the two boats.

Solution Let PQ = 200 m, height of cliff. From $\triangle APQ$,

$$\tan 30^{\circ} = \frac{PQ}{AQ} = \frac{1}{\sqrt{3}} = \frac{200}{AQ}$$

$$AQ = 200\sqrt{3} \text{ m}$$



45°

 $\rightarrow 4$ -240-x

459

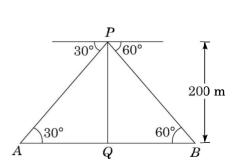
(1)

(1)

y

60°

60°



Q

R

(1)

From ΔBPQ

$$\tan 60^{\circ} = \frac{PQ}{BQ} = \frac{200}{BQ} = \sqrt{3}$$

$$BQ = \frac{200}{\sqrt{3}}$$
(2)

Distance between the two boats = $AQ + BQ = 200\sqrt{3} + \frac{200}{\sqrt{3}} = 460 \text{ m}$ (approx.)

EXAMPLE 9 A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angles of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β . Find the height of the tower.

Solution Let PQ be height of flagstaff = h and QR, height of tower = H. From ΔQRS ,

$$\tan \alpha = \frac{QR}{RS} = \frac{H}{RS}$$

$$RS = \frac{H}{\tan \alpha}$$
(1)

or

 \Rightarrow

From ΔPRS ,

$$\tan \beta = \frac{PQ + QR}{RS} = \frac{(h + H)}{H} \tan \alpha$$

or

$$H \tan \beta = (h + H) \tan \alpha \tag{2}$$

on rearranging, we get

$$H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

EXAMPLE 10 From the top and bottom of a building of height h, the angles of elevation of the top of a tower are α and β respectively. Find the height of the tower.

Solution Let DC be the height of building. Let AB be the height of tower = H. From ΔADE ,

$$\tan \alpha = \frac{AE}{DE} = \frac{H - h}{DE}$$

or

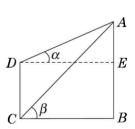
$$DE = \frac{H - h}{\tan \alpha}$$

From $\triangle ABC$,

$$\tan \beta = \frac{AB}{BC} = \frac{H}{DE} = \frac{H}{(H - h)/\tan a}$$
$$= \frac{H \tan \alpha}{(H - h)}$$



$$H = \frac{h \tan \beta}{\tan \beta - \tan \alpha}$$



the height of the building in metres?

Find their heights in metres.

(b) $12\sqrt{2}$

(b) $26\sqrt{3}$

(b) $13/\sqrt{3}$

6.5 m from the house. Find the length of the ladder in metres.

? EXERCISES

(a) 18

(a) $20\sqrt{3}$

(a) 3.25

	Find the height in	-	-	aeroplane are observ	ved to be 30° and 60°.
	(a) $\sqrt{3}/2$	_	-	(d) $2\sqrt{3}/12$	(a) None of those
5.	From the top of a	cliff, 60 m high	, the angle of depre	ession of the top and height of the tower.	
	(a) 40 m	(b) 50 m		-	(e) None of these
6.	-	_	of an unfinished to	ower at a point 120 n t its angle of elevation	n from its base is 45°.
	(a) 90 m	(b) 92 m	(c) 97 m	(d) 87.84 m	(e) None of these
7.		chor to be 45° a	and 30° respective	ea-level, observes the ly. Find the distance	-
	(a) 140 m	(b) 150 m	(c) 156 m	(d) 146.4 m	(e) None of these
8.	makes an angle of	depression of seconds, the	45° with the man's angle of depressio	speeding away from s eye when at a distan n becomes 30°. Wha still water?	nce of 60 metres from
	(a) 32 km/hr	(b) 36 km/hr	(c) 38 km/hr	(d) 40 km/hr	(e) 42 km/hr
9.	-	_		evation of the top of the foot of the tower	
	(a) 149 m	(b) 156 m	(c) 173 m	(d) 200 m	
10.	coming directly to	wards it. If it t	takes 12 minutes f	observes a car movin for the angle of depre he observation tower	ssion to change from
	(a) 14 min 35 sec	(b) 15 min 49	9 sec (c) 16 min 23	3 sec (d) 18 min 5 se	c
11.				tower, which makes	

1. A tower is 30 m high. An observer from the top of the tower makes an angle of depression 60° at the base of the building and angle of depression of 45° at the top of the building, what is

2. Two towers of equal height stand on either side of a wide road which is 100 m wide. At a point on the road between the pillar, the elevations of the tops of the pillars are 60° and 30°.

3. The angle of elevation of a ladder leaning against a house is 60° and the foot of the ladder is

4. From an aeroplane vertically over a straight horizontal road, the angles of depression of two

(d) 15

(d) $22\sqrt{3}$

(e) None of these

(e) None of these

(e) None of these

(c) $10\sqrt{3}$

(c) $30\sqrt{3}$

(c) 13

and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P? [Bank PO]

(a) $4\sqrt{3}$ units

(b) 8 units

(c) 12 units

- (d) Data inadequate
- (e) None of these
- 12. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is [RRB]
 - (a) 30°
- (b) 45°
- (c) 60°
- (d) 90°
- 13. The angle of elevation of a ladder leaning against a wall is 45°, and the foot of the ladder is 4.242 m away from the wall. The length of ladder is
 - (a) 8.4 m
- (b) 7.2 m
- (c) 6 m
- (d) 5.4 m
- 14. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he views the tower 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.
 - (a) 12 m
- (b) 15 m
- (c) 18 m
- (d) 21 m
- 15. Two towers are situated at the opposite sides of bank of a river. One tower is 60 m high. From the top of this tower, angles of depression of top and bottom of the other tower are 30° and 60° respectively. Find the width of the river and the height of the other tower (in metres).

 - (a) $20,40\sqrt{3}$ (b) $40\sqrt{3},20$ (c) $20\sqrt{3},40$ (d) $40,20\sqrt{3}$



- 1. (e)
- **2.** (e)
- **3.** (c)
 - **4.** (a) **5.** (a)
- **6.** (d)
- 8. (a)
- **9.** (c)
- **10.** (c)

- 11. (d)
- **12.** (a)

- 7. (d)

- **13.** (c)
- **14.** (c)
- 15. (c)

Solutions with Necessary Explanation

1. Let the height of building AB be 'H' metres

From $\triangle APQ$,

$$\tan 60^{\circ} = \frac{QP}{AP} = \frac{30}{AP} = \sqrt{3}$$

or

$$AP = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

From ΔBRQ .

$$\tan 45^\circ = \frac{QR}{RR} = \frac{QR}{AP} = 1$$
 (: $BR = AP$)

:.

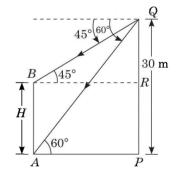
$$QR = AP = 10\sqrt{3} \text{ m}$$

- Height of building H = RP = QP QR
 - $=30-10\sqrt{3}=(10\sqrt{3}-10)\sqrt{3}=7.32\times1.732=12.68 \text{ m}$

Hence answer is (e).

2. Let AB and PQ be the two towers and let its height be h metres each. Let the point on the road be at a distance x metres from B.

From
$$\triangle ABT$$
, $\tan 60^{\circ} = \frac{AB}{BT} = \frac{h}{x}$



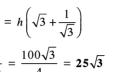
or
$$x = \frac{h}{\tan 60^{\circ}} = \frac{h}{\sqrt{3}}$$
 (1)

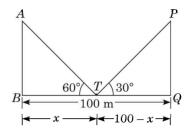
From
$$\Delta PQT$$
, $\tan 30^{\circ} = \frac{PQ}{QT} = \frac{h}{100 - x}$

or
$$100 - x = \frac{h}{\tan 30} = \sqrt{3} h$$
 (2)

(1) + (2)
$$\Rightarrow$$
 100 = $\frac{h}{\sqrt{3}} + \sqrt{3} h = h \left(\sqrt{3} + \frac{1}{\sqrt{3}} \right)$

or
$$h = \frac{100}{\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)} = \frac{100\sqrt{3}}{4} = 25\sqrt{3}$$

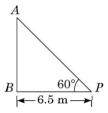




Hence answer is (e).

3. Let length of ladder AP be 'l' metres

From
$$\triangle ABP$$
, $\cos 60^\circ = \frac{BP}{AP} = \frac{6.5}{l}$
or $l = \frac{6.5}{1/2} = 13 \text{ m}$ (: $\cos 60 = 1/2$)



4. Let 'H' be the elevation of aeroplane above the road in miles.

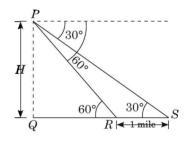
From
$$\Delta PQR$$
, $\tan 60^{\circ} = \frac{H}{QR}$
or $QR = \frac{H}{\tan 60^{\circ}} = \frac{H}{\sqrt{3}}$ (1)
From ΔPQS , $\tan 30^{\circ} = \frac{H}{QS}$

$$\circ = \frac{H}{QS}$$

$$= \frac{H}{QR + RS} = \frac{H}{QR + 1}$$

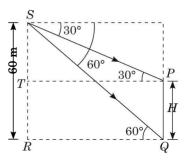
or
$$QR + 1 = \frac{H}{\tan 30^{\circ}} = \sqrt{3} H$$
 (2)

$$(2) - (1) \Rightarrow \qquad 1 = H\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2H}{\sqrt{3}}$$
or
$$H = \frac{\sqrt{3}}{2} \mathbf{m}$$



5. Let H be height of tower PQ.

From
$$\Delta QRS$$
, $\tan 60^{\circ} = \frac{60}{QR}$
or $QR = \frac{60}{\tan 60} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ (1)
From ΔPTS , $\tan 30^{\circ} = \frac{ST}{PT} = \frac{60 - H}{QR} = \frac{60 - H}{20\sqrt{3}}$
for $60 - H = 20\sqrt{3} \times \tan 30$



or
$$60 - H = 20$$

or $H = 60 - 20 = 40$ m.

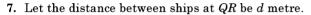
6. From
$$\triangle PQR$$
, $\tan 45^{\circ} = \frac{PQ}{RQ} = 1$
or $PQ = RQ = 120 \text{ m}$ (1)

Let P' be the extended point of tower such that elevation to point P' is 60° .

From
$$\Delta P'QR$$
, tan $60^{\circ} = \frac{P'Q}{RQ} = \frac{P'P + PQ}{RQ} = \sqrt{3}$

or
$$P'P + PQ = RQ\sqrt{3}$$

$$P'P = RQ\sqrt{3} - PQ = 120\sqrt{3} - 120 = 87.84 \text{ m}$$



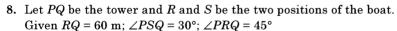
From
$$\triangle PQS$$
, $\tan 45^\circ = \frac{PS}{SQ} = 1$
or $SQ = PS = 200 \text{ m}$ (1)
From $\triangle PRS$, $\tan 30^\circ = \frac{PS}{RS} = \frac{1}{\sqrt{3}}$

or
$$\frac{PS}{SQ + QR} = \frac{1}{\sqrt{3}}$$

or
$$SQ + QR = PS\sqrt{3} = 200\sqrt{3}$$

$$\therefore \qquad QR = 200\sqrt{3} - 200$$

i.e. distance between two ships = 346.4 - 200 = 146.4 m



From
$$\Delta PQR$$
, tan $45^{\circ} = \frac{PQ}{RO} = 1$

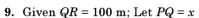
i.e.
$$PQ = RQ = 60 \text{ m}$$
 (1)

From
$$\triangle PQS$$
, $\tan 30^\circ = \frac{PQ}{QS} = \frac{60}{QR + RS} = \frac{60}{60 + RS} = \frac{1}{\sqrt{3}}$ (2)

(2)
$$\Rightarrow$$
 $60 + RS = 60\sqrt{3}$
 \Rightarrow $RS = 60\sqrt{3} - 60 = 60(\sqrt{3} - 1) \text{ m}$

.: Required speed =
$$\frac{RS}{\text{Time}} = \frac{60 (\sqrt{3} - 1)}{5} \text{ m/s} = 12 \times 0.73 \text{ m/s}$$

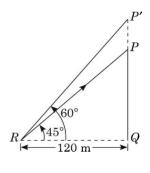
= $12 \times 0.73 \times \frac{18}{5} \text{ km/h} = 31.5 \text{ km/hr} \approx 32 \text{ km/hr}$



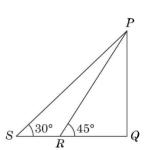
From
$$\triangle PQR$$
, $\tan 30^{\circ} = \frac{RQ}{PQ} = \frac{100}{x} = \frac{1}{\sqrt{3}}$

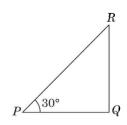
∴
$$x = 100\sqrt{3} \text{ m}$$

= 173.2 m = 173 m



45°





10. Let the time taken be t seconds.

Let PQ be the tower and S and R be two positions of the car.

Now $\angle PSQ = 30^{\circ}$; $\angle PRQ = 45^{\circ}$;

Let SR = x and RQ = y

Let height of tower PQ = H

From $\triangle PQR$, $\tan 45^\circ = \frac{PQ}{RQ} = 1$

i.e.

$$PQ = RQ$$

$$H = v$$

(2)

From $\triangle PQS$, tan $30^{\circ} = \frac{PQ}{SQ} = \frac{1}{\sqrt{3}}$

i.e.

$$PQ\sqrt{3} = SQ$$

i.e.

$$H\sqrt{3} = x + y$$

$$H\sqrt{3} = x + y$$
 $(\because SQ = SR + RQ)$

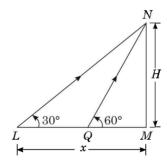
Solving (1) and (2)

$$(2) - (1) \Rightarrow (x + y) - y = H\sqrt{3} - H$$
$$\Rightarrow x = H(\sqrt{3} - 1)$$

- \therefore Distance covered in 12 minutes = $H(\sqrt{3}-1)$
- :. Time taken to reach foot of tower after this point

$$= \frac{12}{H(\sqrt{3}-1)} \times H = \frac{12}{\sqrt{3}-1} \min = 16 \min 23 \sec.$$

11. Since no sides of triangle LMN or QMN are given, it is not possible to find the result.



12. Let QR represents tree and PQ represents its shadow as shown in the figure.

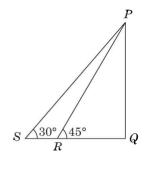
It is given that $PQ = \sqrt{3} QR$

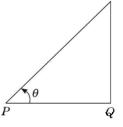
$$\therefore \qquad \tan \theta = \frac{RQ}{PQ} = \frac{RQ}{\sqrt{3} QR} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^{\circ}$$

13. In figure, ladder is represented by AC.

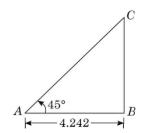
$$\therefore \qquad \cos 45^{\circ} = \frac{AB}{AC} = \frac{4.242}{AC}$$





60 m

$$AC = \frac{4.242}{\cos 45^{\circ}} = \frac{4.242}{1/\sqrt{2}}$$
$$= \frac{3\sqrt{2}}{1/\sqrt{2}} = 3 \times 2 = 6 \text{ m}$$



14. Let the tree be represented by *BC*.

AD is the distance between positions of man.

$$DB = ?$$

Let H be the height of tree and DB = x

$$\therefore \tan 30^\circ = \frac{H}{AB} = \frac{H}{36 + x}$$

or
$$H = (36 + x) \frac{1}{\sqrt{3}}$$

Similarly

$$\tan 60^\circ = \frac{H}{DB} = \frac{H}{x} = \sqrt{3}$$

or

$$H = x\sqrt{3} \tag{2}$$

(1)

A

Equating (1) and (2), we get

$$\frac{36+x}{\sqrt{3}} = x\sqrt{3}$$

$$\mathbf{or}$$

$$36 + x = 3x$$

$$\mathbf{or}$$

$$x = \frac{36}{(3-1)} = 18 \text{ m}$$

15. Let AB and CD be the two towers.

$$CD = 60 \text{ m}$$
; $\angle ECA = 30^{\circ}$; $\angle ECB = 60^{\circ}$

From
$$\Delta BDC$$
,

$$\cos 30^{\circ} = \frac{CD}{RC}$$

or

$$BC = \frac{CD}{\cos 30^{\circ}} = \frac{60}{\sqrt{3}/2}$$

$$=\frac{120}{\sqrt{3}}=40\sqrt{3} \text{ m}$$

From ΔBCE ,

$$EC = BC \cos 60^{\circ}$$

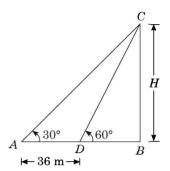
= $40\sqrt{3} \times \frac{1}{2} = 20\sqrt{3} \text{ m}$

$$\therefore \qquad \text{Width of river} = 20 \sqrt{3} \text{ m}$$

Height of tower =
$$AB = BE - AE$$

$$= CD - AE$$
$$= 60 - EC \tan 30^{\circ}$$

=
$$60 - 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 60 - 20 = 40 \text{ m}$$



Trigonometry

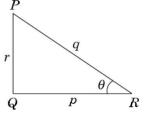
PYTHAGORUS THEOREM: In the right-angled triangle PQR,

$$PR^2 = PQ^2 + QR^2$$

or

$$q^2 = r^2 + p^2$$

i.e. Square of hypotenuse = Sum of square of base and square of altitude.



TRIGONOMETRIC RATIOS: Ratios of sides of right-angled triangles with respect to its angles are called *trigonometric ratios*.

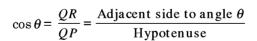
Given $\angle PQR = \theta$

Then the ratio PR/QP is called sine of angle θ .

$$\sin\theta = \frac{PR}{OP}$$

$$\sin \theta = \frac{\text{Opposite side to angle } \theta}{\text{Hypotenuse}}$$





$$\tan \theta = \frac{PR}{QR} = \frac{\text{Opposite side to angle } \theta}{\text{Adjacent side to angle } \theta}$$

Other three trigonometric ratios are cosecant, secant and cotangent of angle θ .

$$\csc\theta = \frac{1}{\sin\theta}; \quad \sec\theta = \frac{1}{\cos\theta}; \quad \cot\theta = \frac{1}{\tan\theta}$$

If one of the six ratios is known, the other five ratios can be calculated.

TRIGONOMETRIC RATIO FOR $\theta = 30^{\circ}$

From ΔPSR .

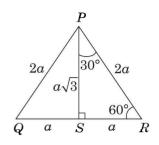
From ΔPSR ,

:.

$$\sin 30^{\circ} = \frac{SR}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^{\circ} = \frac{PS}{PR} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^{\circ} = \frac{SR}{PS} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$



$$\cos 30^{\circ} = \frac{1}{\sin 30^{\circ}} = \frac{2a}{a} = 2$$

$$\sec 30^{\circ} = \frac{1}{\cos 30^{\circ}} = \frac{2a}{a\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\cot 30^{\circ} = \frac{1}{\tan 30^{\circ}} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

 $\cot 30^{\circ} = \frac{1}{\tan 30^{\circ}} = \frac{a\sqrt{3}}{a} = \sqrt{10^{\circ}}$ Trigonometric ratio for $\theta = 60^{\circ}$.

$$\sin 60^{\circ} = \frac{PS}{RR} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^{\circ} = \frac{SR}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{PS}{SR} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

$$\csc 60^{\circ} = \frac{1}{\sin 60^{\circ}} = \frac{2a}{a\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\sec 60^{\circ} = \frac{1}{\cos 60^{\circ}} = \frac{2a}{a} = 2$$

$$\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Trigonometric ratio for $\theta = 45^{\circ}$.

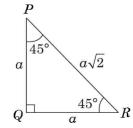
In the right-angled triangle PQR, right angled at Q and $\angle P = \angle R = 45^{\circ}$.

$$\therefore \qquad \qquad QP = QR$$

Let it be a units.

Then by Pythagorus theorem, $PR^2 = PQ^2 + QR^2 = a^2 + a^2 = 2a^2$

$$\therefore PR = a\sqrt{2}$$



$$\sin 45^\circ = \frac{PQ}{PR} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{QR}{PR} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{PQ}{QR} = \frac{a}{a} = 1$$

$$\therefore \qquad \csc 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

$$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{\tan 45^\circ} = 1$$

Trigonometric Ratio of Different Angles

x =	= 0/4	1/4	2/4	3/4	4/4	
fn θ	0°	30°	45°	60°	90°	
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$-\sqrt{x}$
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	← Write values of sine in reverse order
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∝	← sin/cos
cosec	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1	← 1/sin
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞	← 1/cos or cosec written in reverse order
cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	← 1/tan or tan written in reverse order

Trigonometric Ratio of Different Angles

	0	1	2	3	4
Angle Function	0°	30°	45°	60°	90°
sin	$\sqrt{0/4}$	$\sqrt{1/4}$	$\sqrt{2/4}$	$\sqrt{3/4}$	$\sqrt{4/4}$
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

ANGLES OF ELEVATION AND DEPRESSION

Suppose we are viewing an object. The line of sight or the line of vision is a straight line from our eye to the object we are viewing.

If the object is at a higher level from our eyes, the line of sight is above the horizontal line at the level of our eyes. Angle so subtended by the line of sight with the horizontal is called *Angle of Elevation* of the object.

If the line of sight is below the horizontal, angle so subtended by the line of sight with the horizontal is called *Angle of Depression* of the object.

EXAMPLE 1 A person is standing at a certain distance from a tower. When the top of the tower is viewed, the angle of elevation is 60°. If the height of the tower is 30 m, what is the distance of the person from the tower?

Solution
$$\tan 60^\circ = \frac{AB}{BC}$$

or $\frac{AB}{BC} = \sqrt{3}$
or $\frac{30}{BC} = \sqrt{3}$
or $BC = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

EXAMPLE 2 The angles of elevation of top and bottom of a flag at a distance of 30 m are 45° and 30° respectively. What is the height of the flag AB?

Solution
$$\tan 45^\circ = \frac{AC}{DC}$$

or $AC = DC \tan 45^\circ$
 $= 30 \tan 45^\circ$
 $= 30 \text{ m}$
 $\tan 30^\circ = \frac{BC}{DC}$
or $BC = DC \tan 30^\circ$
or $BC = 30 \tan 30^\circ$
 $= 30 \times \frac{1}{\sqrt{3}} = 10\sqrt{3} \text{ m}$
∴ Height of flag $AB = 30 - 10\sqrt{3}$

EXAMPLE 3 Two poles of equal heights stand on either side of a roadway which is 120 m wide. At a point on the roadway between the poles, the elevations of the top of the pole are 60° and 30°. Find the heights of the poles and the position of the point.

Solution Let PQ and RS be two poles and x m be the height of poles.

=30-17.32==12.68 m

Let
$$QT = y \text{ m}$$

$$TS = (120 - y) \text{ m}$$

 \Rightarrow

 \Rightarrow

In ΔPQT ,

$$\tan 60^{\circ} = \frac{PQ}{QT} = \frac{x}{y}$$

$$x = y\sqrt{3}$$
(1)

In ΔRST ,

$$\tan 30^\circ = \frac{RS}{TS} = \frac{x}{120 - y}$$

$$120 - y = x\sqrt{3}$$

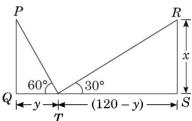
Solving (1) and (2), we get,

$$y\sqrt{3} \times \sqrt{3} = 120 - y$$

$$\Rightarrow \qquad 4y = 120$$
or
$$y = 30 \text{ m}$$

$$\therefore (1) \Rightarrow \qquad x = y\sqrt{3}$$

$$\Rightarrow \qquad x = 30\sqrt{3} = 52 \text{ m}$$



(2)

EXAMPLE 4 The angles of depression of two ships from the top of a lighthouse are 45° and 30°. If the ships are 100 m apart, find the height of lighthouse.

Solution Let AB, the height of lighthouse be x m.

$$MN \mid\mid PQ$$

$$\angle MAP = \angle APB = 30^{\circ}.$$

$$\angle NAQ = AQB = 45^{\circ}$$
Let
$$PB = y \text{ m}$$

$$BQ = (100 - y) \text{ m}$$
In $\triangle ABP$,
$$\tan 30^{\circ} = \frac{AB}{BP} = \frac{1}{\sqrt{3}} = \frac{x}{y}$$
or
$$y = x\sqrt{3}$$

$$\tan 45^{\circ} = \frac{AB}{BQ} = 1$$
or
$$x = (100 - y)$$
(2)

Solving (1) and (2), we get,

or

Solving (1) and (2), we get,

$$y = (100 - y)\sqrt{3} \quad \text{or} \quad y(1 + \sqrt{3}) = 100\sqrt{3}$$

$$\therefore \qquad y = \frac{100\sqrt{3}}{1 + \sqrt{3}} \text{ m}$$

$$\therefore \qquad x = \frac{y}{\sqrt{3}} = \frac{100}{1 + \sqrt{3}} \text{ m}$$
i.e. Height of lighthouse = $\frac{100}{1 + \sqrt{3}} \text{ m}$.

EXAMPLE 5 A tower is $200\sqrt{3}$ m high. Find the angle of elevation of its top from a point 200 m away from its roots.

Solution

$$\tan \theta = \frac{PQ}{QR}$$

$$= \frac{200\sqrt{3}}{200} = \sqrt{3}$$

$$Q = \tan^{-1}(\sqrt{3}) = 60^{\circ}$$

 $200\sqrt{3}$



::

Note: More applications on Trigonometry are included in the Chapter Heights and Distances.



EXERCISES

1. Q	is 14 km	south and	16 km	west of P .	Find the	distance	of Q	from F
------	----------	-----------	-------	---------------	----------	----------	------	--------

- (a) 15.6 km
- (b) 18.8 km
- (c) 21.2 km
- (d) 24.4 km
- (e) 25.8 km
- 2. The angle of elevation of the top of an unfinished tower at a point 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point be 60°?
 - (a) 90 m
- (b) 92 m
- (c) 97 m
- (d) 87.84 m
- (e) None of these
- 3. The shadow of a tower standing on a level plane found to be 60 m longer when the angle of the sun is 30° than when it is 45°. Find the height of the tower when it is 45°.

- (a) $60(\sqrt{3}+1)$ (b) $30(\sqrt{3}+1)$ (c) $\frac{60}{\sqrt{3}+1}$ (d) $30(\sqrt{3}-1)$ (e) None of these
- 4. From an aeroplane vertically, over a straight horizontal road, the angles of depression of two consecutive milestones on the opposite sides of the aeroplane are observed to be 30° and 60°. Find the height in miles of the aeroplane above the road.
 - (a) $\sqrt{3}/2$
- (b) $\sqrt{3}/4$
- (c) $\sqrt{3}/8$
- (d) $2\sqrt{3}/12$
- (e) None of these
- 5. An observer on the top of a cliff, 200 m above the sea level, observes the angle of depression of two ships at anchor to be 45° and 30° respectively. Find the distance between the ships, if the line joining them stretches to the base of the cliff.
 - (a) 40 m
- (b) 150 m
- (c) 156 m
- (d) 146.4 m
- (e) None of these
- 6. A man observes angle of elevation of the top of a tower having a height of 100 m as 30°. After walking certain distance towards the tower, he observes the angle of elevation as 45°. What is the distance through which man walked towards the tower?
- (b) 73.2 m
- (c) 72.3 m
- (d) 82.4 m
- 7. To an observer at the ground, angle of elevation of top of a tower is 30°. If the height of the tower is 100 m, find the distance between top of tower and point of observation (in metres)
- (b) 150
- (c) 200
- (d) 250
- 8. The angle of elevation of top of a tower is 30°. On walking 100 m towards the tower, the angle of elevation of top of a tower becomes 60°. Find the height of tower.
 - (a) 50 m
- (b) $50\sqrt{3}$ m
- (c) 100 m
- (d) $100\sqrt{3}$ m

- 9. A ladder having a length of 16 m is leaning against a wall at an angle of 60° with the ground. Find the distance between the wall and foot of ladder.
 - (a) 12 m
- (b) 10 m
- (d) 6 m
- 10. Two towers are situated at a certain distance apart. A person observed the angle of depression of top and bottom of second tower from the top of first tower as 30° and 60° respectively. If the height of first tower is 60 m, find the height of second tower and distance between the two towers.
 - (a) $40\sqrt{3}$ m. 20 m

(b) $40 \text{ m}, 20\sqrt{3} \text{ m}$

(c) 30 m, $20\sqrt{3}$ m

(d) 20 m, $30\sqrt{3}$ m



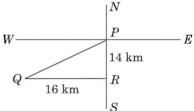
- **2.** (d)
- **3.** (b)
- **4.** (b)
- **5.** (d)
- **6.** (b)
- 7. (c)
- 8. (b)
- **9.** (c)
- **10.** (b)



Solutions with Necessary Explanation

1.
$$PQ = \sqrt{PR^2 + QR^2}$$

 $= \sqrt{14^2 + 16^2}$
 $= \sqrt{196 + 256}$
 $= \sqrt{452} = 21.2 \text{ km}$



2. PQ is the unfinished tower.

$$QR = 120 \text{ m}$$

 \therefore From $\triangle PQR$,

$$\tan 45^\circ = \frac{PQ}{QR} = 1$$

$$PQ = QR = 120 \text{ m}$$

From ΔSQR ,

$$\tan 60^\circ = \frac{SQ}{QR} = \frac{SP + PQ}{QR}$$

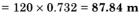
$$QR \tan 60^\circ = SP + PQ$$

 \mathbf{or}

٠.

$$120\sqrt{3} = SP + 120$$

$$SP = 120(\sqrt{3} - 1)$$

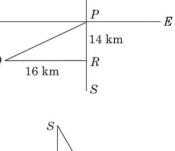


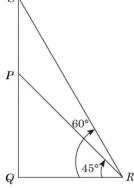
3. Height of tower = x m.

From
$$\Delta PQS$$
,

$$\tan 45^\circ = \frac{PQ}{SQ} = 1$$

$$PQ = QS$$





i.e.

$$x = QS$$

From ΔPQR ,

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{x}{QS + 60}$$

i.e.

$$\frac{1}{\sqrt{3}} = \frac{x}{x+60}$$

Rearranging,

$$x\sqrt{3} = x + 60$$

 \mathbf{or}

$$x(\sqrt{3}-1)=60$$

$$x = \frac{60}{(\sqrt{3} - 1)} = \frac{60}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$
$$= \frac{60(\sqrt{3} + 1)}{(3 - 1)} = 30(\sqrt{3} + 1)$$

4. Given
$$QS + SR = 1$$
 mile.

$$PS = ?$$

From ΔPQS ,

$$\tan 30^{\circ} = \frac{PS}{QS}$$

 \mathbf{or}

$$QS = \frac{PS}{\tan 30^{\circ}} = \sqrt{3} PS$$

From ΔPSR ,

$$\tan 60^{\circ} = \frac{PS}{SR}$$

or

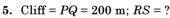
$$SR = \frac{PS}{\tan 60^{\circ}} = \frac{PS}{\sqrt{3}}$$

$$\therefore (1) \Rightarrow \sqrt{3} PS + \frac{PS}{\sqrt{3}} = 1$$

$$\Rightarrow (3+1)PS = \sqrt{3}$$

 \mathbf{or}

$$PS = \frac{\sqrt{3}}{4}$$



From ΔPQS ,

$$\tan 45^\circ = \frac{PQ}{QS} = 1$$

:.

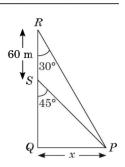
$$QS = 200 \text{ m}$$

From ΔPQR ,

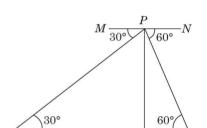
$$\tan 30^\circ = \frac{PQ}{QR} = \frac{1}{\sqrt{3}}$$

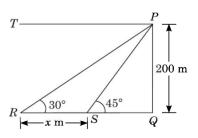
$$QR = PO \sqrt{3} = 200\sqrt{3}$$

$$RS = QR - QS = 200\sqrt{3} - 200 = 146.4 \text{ m}$$









6. Let the distance through which man walked be x metres.

From
$$\Delta DBC$$
,

$$\tan 45^{\circ} = \frac{100}{DB} = 1$$
 $DB = 100 \text{ m}$

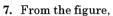
From $\triangle ABC$,

or

$$\tan 30^{\circ} = \frac{100}{AB} = \frac{100}{x + DB}$$
$$= \frac{100}{x + 100} = \frac{1}{\sqrt{3}}$$

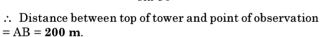
or
$$\frac{x+100}{100} = \sqrt{3}$$

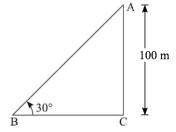
 $x = 100\sqrt{3} - 100$ or = 173.2 - 100= 73.2 m



$$\sin 30^{\circ} = \frac{AC}{AB} = \frac{100}{AB}$$

:. AB =
$$\frac{100}{\sin 30^{\circ}}$$
 = 200 m





100 m

8. Let H be the height of tower

From $\triangle ADB$,

$$\tan 60^{\circ} = \frac{AB}{DB} = \frac{H}{DB}$$

 \mathbf{or}

$$H = DB \tan 60^{\circ}$$

$$H = DB\sqrt{3}$$

 \mathbf{or}

$$DB = \frac{H}{\sqrt{3}}$$

(1)

From $\triangle ABC$,

$$\tan 30^{\circ} = \frac{AB}{BC} = \frac{H}{CD + DB} = \frac{H}{100 + DB}$$

$$\frac{H}{(100 + DB)} = \frac{1}{\sqrt{3}}$$

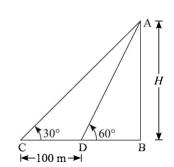
$$H = \frac{100 + DB}{\sqrt{3}}$$

or

 \mathbf{or}

 \mathbf{or}

$$DB = H\sqrt{3} - 100$$



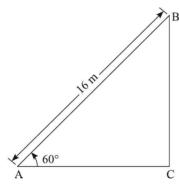
(2)

Equating (1) and (2), we get

$$\frac{H}{\sqrt{3}} = H\sqrt{3} - 100$$
or
$$H\sqrt{3} - \frac{H}{\sqrt{3}} = 100$$
or
$$\frac{2H}{\sqrt{3}} = 100$$
or
$$H = \frac{100 \times \sqrt{3}}{2} = 50\sqrt{3} \text{ m}$$

9. Distance between foot of ladder and wall is given by

$$AC = AB \cos 60^{\circ} = 16 \cos 60^{\circ} = 16 \frac{1}{2} = 8 \text{ m}$$



10. Let AB and CD be the two towers. Hight of first tower = AB = 60 m.

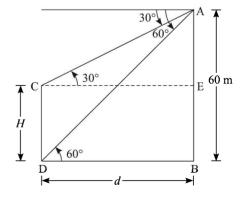
Let H metres be the height of second towr and d metres be the distance between the towers AB and CD.

From AABD,

:.

tan
$$60^{\circ} = \frac{60}{d} = \sqrt{3}$$

or $d = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$
From $\triangle ACE$,
tan $30^{\circ} = \frac{AE}{CE} = \frac{AB - H}{d}$
 $= \frac{60 - H}{20\sqrt{3}}$
i.e. $\frac{1}{\sqrt{3}} = \frac{60 - H}{20\sqrt{3}}$
or $60 - H = \frac{20\sqrt{3}}{\sqrt{3}} = 20$
 $\therefore H = 60 - 20 = 40 \text{ m}$



Odd Man Out and Series

A series is a sequence of numbers obtained by a pre-defined rule applied to the previous number.

In the given series one of the terms will not fit in the series, which is termed here $Odd\ Man\ Out$

In other words, 'Odd Man Out' can be defined as a number which does not belong to the set of the given numbers in which all others are alike in a particular manner.

EXAMPLE 1 Find the odd one out from the given series.

Solution All numbers except 27 are perfect squares.

EXAMPLE 2 Find the odd one out from the given series.

Solution All numbers except 9 are prime numbers.

SERIES: A set of numbers or sequence of numbers formed by applying a particular pre-defined rule to a number in succession is termed series. For example,

- (i) 2, 4, 6, 8, 10, ..., i.e. the numbers are got added by 2 in each step.
- (ii) 2, 6, 18, 54, ..., i.e. the numbers in the sequence are obtained by multiplying each term with 3 in succession, i.e. $2, 2 \times 3 = 6, 6 \times 3 = 18, 18 \times 3 = 54$.

Directions: In each of the following question one series is given. You have to fill in the space marked with a character choosing the correct alternative, in which it follows the same pattern as the first one.

EXAMPLE 3 4 14 36 114 460
$$2$$
 a b c d e

Find the value of *e*.

Solution The first series is $\times 1 + 10, \times 2 + 8, \times 3 + 6, \times 4 + 4...$

$$a = 2 \times 1 + 10 = 12$$

$$b = 12 \times 2 + 8 = 32$$

$$c = 32 \times 3 + 6 = 102$$

$$d = 102 \times 4 + 4 = 412$$

$$e = 412 \times 5 + 2 = 2062$$

EXAMPLE 4 108 52 24 10 3
$$64$$
 a b c d e

What is the value of c?

Solution
$$a = (64 - 4) \div 2 = 30$$

 $b = (30 - 4) \div 2 = 13$
 $c = (13 - 4) \div 2 = 4.5$

What should replace c?

Solution First series is
$$\div 3 \times 2$$

$$a = 1080 \div 3 \times 2 = 720$$

$$b = 720 \div 3 \times 2 = 480$$

$$c = 480 \div 3 \times 2 = 320$$



EXERCISES

Directions (1-25): In each of the following questions, a number sequence is given in which except one of the terms in the sequence are alike. You have to find out the number which is odd in the sequence.

- **1.** 4, 16, 25, 36, 64, 144
 - (a) 16
- (b) 25
- (c) 36
- (d) 64

- 2. 8, 27, 64, 125, 216
 - (a) 27
- (b) 64
- (c) 125
- (d) 216

- **3.** 17, 35, 43, 53, 62, 80
 - (a) 35
- (b) 43
- (c) 53
- (d) 62

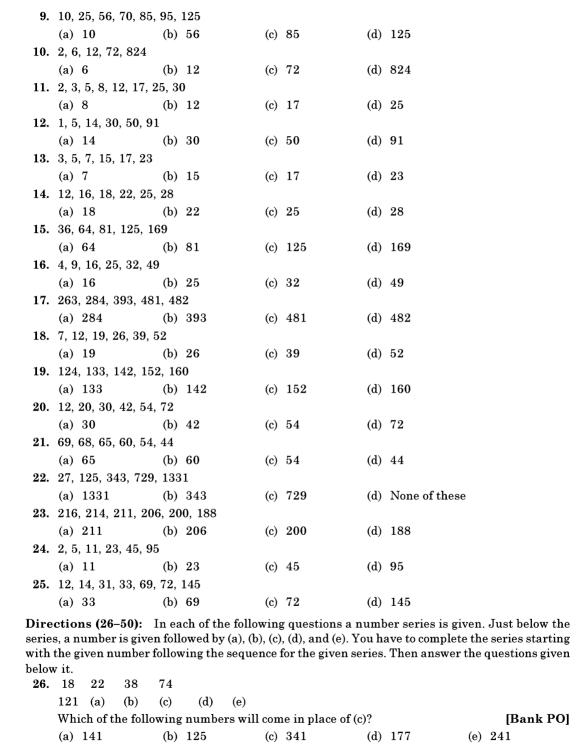
- **4.** 24, 18, 222, 82, 421
 - (a) 18
- (b) 222
- (c) 82
- (d) 421

- **5.** 6, 15, 21, 26, 33, 39
 - (a) 15
- (b) 21
- (c) 26
- (d) 33

- **6.** 14, 49, 63, 72, 77, 91
 - (a) 49
- (b) 63
- (c) 72
- (d) 77

- **7.** 8, 27, 125, 212, 343
 - (a) 8
- (b) 27
- (c) 125
- (d) 212

- **8.** 4, 9, 25, 49, 64, 121
 - (a) 9
- (b) 25
- (c) 49
- (d) 64



	2 Whi	(a)	(b)	(c)	(d)	(e)						
	Whi	ah af +l										
		cm or u	ne follo	owing	numbe	rs will	con	ie in place	of (d)?			
	(a)	12		(b)	230		(c)	3	(d)	51	(e)	1205
28.	4	2	2	3								
	12	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	numbe	rs will	com	ie in place	of (e)?			
	(a)	45		(b)	6		(c)	9	(d)	18	(e)	None of these
29.	264	136	72	40								
	488	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	numbe	rs will	con	ie in place	of (a)?			
	(a)	128		(b)	248		(c)	38	(d)	23	(e)	68
30.	2	17	121	729								
	5	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	numbe	rs will	con	ie in place	of (b)?			
	(a)	289		(b)	41		(c)	17393	(d)	1448	(e)	5796
31.	11	15	38	126								
	7	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne folle	owing	will co	me in j	place	e of (c)?				[Bank PO]
	(a)	102		(b)	30		(c)	2140	(d)	80	(e)	424
32 .	2	3	8	27								
	5	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	will co	me in j	place	e of (e)?				
	(a)	184		(b)	6		(c)	925	(d)	45	(e)	14
33.	2	3	9	40.5								
	4	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	will co	me in 1	place	e of (b)?				
	(a)	486		(b)	81		(c)	3645	(d)	18	(e)	6
34.	12	28	64	140								
	37	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	will co	me in j	place	e of (e)?				
	(a)	1412		(b)	164		(c)	696	(d)	78	(e)	340
35.	5	12	60	340								
	7	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	will co	me in 1	place	e of (d)?				
	(a)	172		(b)	5044		(c)	1012	(d)	20164	(e)	28
36.	1	9	65	393								
	2	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	numbe	rs wou	ıld c	ome in the	place of	(c)?		[BOB PO]
	(a)	490		(b)	853		(c)	731	(d)	729	(e)	None of these $$
37.	8	8	12	24								
	36	(a)	(b)	(c)	(d)	(e)						
	Whi	ch of th	ne follo	owing	numbe	rs wou	ıld c	ome in the	e place of	(e)?		
	(a)	810		(b)	36		(c)	54	(d)	108	(e)	None of these

38.	424	208	100	46								
	888	(a)	(b)	(c)	(d)	(e)						
	Wha	t num	ber wo	ould c	ome in	the pla	ace o	f (b)?				
	(a) 2	20		(b)	440		(c)	216	(d)	56	(e)	None of these
39.	4	5	9.75	23.5								
	7	(a)	(b)	(c)	(d)	(e)						
	Wha	t num	ber wo	ould c	ome in	the pla	ace o	f (d)?				
	(a) 3	32.5		(b)	271.5		(c)	8	(d)	14.25	(e)	None of these
40.	5	294	69	238								
	13	(a)	(b)	(c)	(d)	(e)						
	Whic	h of t	he folle	owing	numbe	ers wo	ald c	ome in t	he place of	(e)?		
	(a) 2	246		(b)	206		(c)	125	(d)	302	(e)	None of these
41.	15	16	25	50								
	189	(a)	(b)	(c)	(d)	(e)						
	Whic	h of t	he foll	owing	memb	ers wil	l con	ne in pla	ce of (e)?			[Bank PO]
	(a) 3				273			394		426	(e)	None of these
42 .	6	3.5	4.5	8.25								
	40	(a)	(b)	(c)	(d)	(e)						
	Whic	h of tl	he follo	owing	numbe	ers wil	l com	ie in pla	ce of (c)?			
	(a) 2				21.5			33.75		69.5	(e)	None of these
43.	9	10	22	69								
	5	(a)	(b)	(c)	(d)	(e)						
	Whic	h of t	he foll	owing	numbe	ers wil	l con	ie in pla	ce of (b)?			
	(a) :			(b)				14		45	(e)	None of these
44.	2	10	27	60								
	5	(a)	(b)	(c)	(d)	(e)						
	Whic	h of t	he foll	owing	numbe	ers wil	l con	ie in pla	ce of (b)?			
	(a) 3			(b)				34		38	(e)	None of these
45 .	5	149	49	113								
	146	(a)	(b)	(c)	(d)	(e)						
	Whic	h of t	he foll	wing	numbe	ers wil	l con	ie in pla	ce of (d)?			
	(a) 2	290		(b)	234		(c)	254	(d)	218	(e)	None of these
46 .	6	3.0	4.5	2.25								
	40	(a)	(b)	(c)	(d)	(e)						
	Whic	h of tl	he follo	owing	numbe	ers wil	l com	e in pla	ce of (c)?			
	(a) 2	20.5		(b)	21.5		(c)	33.75	(d)	69.5	(e)	15
47.	5	9	26	90								
	13	(a)	(b)	(c)	(d)	(e)						
	Whic	h of tl	he folle	owing	numbe	ers wil	l com	ie in pla	ce of (e)?			
	(a) 2			_	2292			1716		3432	(e)	None of these
48.	4	9	25	103					, ,		. ,	
	3	(a)	(b)	(c)	(d)	(e)						
	Whic						ll cor	ne in pla	ace of (c)?			
	(a) 3			(b)				91		79	(e)	None of these

2 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (a)?

(a) 4 (b) 6 **50.** 1260 628 312 154

788 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (d)?

(a) 194

(b) 45.5

(c) 48

(c) 2

(d) 72.5

(d) 3

(e) None of these

(e) None of these

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1. (b)	2. (b)	3. (b)	4. (c)	5. (c)	6. (c)	7. (d)	8. (d)	9 . (b)	10 . (d)
I • (0)	-• (~)	U • (D)	1. (C)	0. (0)	U • (C)	•• (0.)	O. (u)	0. (0)	10. (u)



Solutions with Necessary Explanation

- 1. All numbers except 25 are perfect squares of even numbers.
- 2. All numbers in the sequence are perfect cubes except 64 which is both perfect square and cube.
- 3. All number except 43 are having digit-sum equal to 8.

i.e.
$$3 + 5 = 5 + 3 = 6 + 2 = 8$$
.

4. All numbers except 82 are having product of digit equal to 8,

i.e.
$$2 \times 4 = 1 \times 8 = 2 \times 2 \times 2 = 4 \times 2 \times 1 = 8$$
.

- 5. All numbers except 26 are multiplies of three.
- **6.** All numbers except 72 are multiplies of 7.
- 7. Except 212, all numbers are cubes of prime numbers.
- 8. Except 64, all numbers are squares of prime numbers.
- 9. Except 56, all number are multiples of 5.
 - \therefore 56 is wrong.
- **10.** $2 \times 6 = 12$; $6 \times 12 = 72$; $12 \times 72 = 864$
 - \therefore 824 is wrong.
- 11. The series is +1, +2, +3, +5, +6, +7.
 - \therefore 17 + 6 = 23 \neq 25
 - ∴ 25 is wrong
- 12. $+2^2$, $+3^2$, $+4^2$, $+5^2$, $+6^2$ is the series.
 - $30 + 5^2 = 55 \neq 50$
 - \therefore 50 is wrong
- 13. Except 15 all number are prime.
- 14. Except 25, all number are even.

15. The sequence is 6^2 , 8^2 , 9^2 , 11^2 , 13^2 .

All numbers except 125 are perfect squares.

- ∴ 125 is wrong
- 16. Except 32, all numbers in the sequence are perfect squares.
 - ∴ 32 is wrong
- 17. Second digit = First digit \times Third digit

Except 481, all other terms in the sequence follow the above condition.

- ∴ 481 is wrong
- 18. The sequence is +5, +7, +9, +11, +13.
 - $\therefore 19 + 9 = 28 \neq 26$
 - ∴ 26 is wrong
- 19. The sequence is an A.P. with common difference 9.
 - $\therefore 142 + 9 = 151 \neq 152$
 - ∴ 152 is wrong
- **20.** The sequence is +8, +10, +12, +14, +16.

$$\therefore$$
 12 + 8 = 20; 20 + 10 = 30; 30 + 12 = 42;
42 + 14 = 56 \neq 54; 56 + 16 = 72

- ∴ 54 is wrong
- **21.** The sequenc is -1, -3, -5, -7, -9.

$$\therefore$$
 69 - 1 = 68; 68 - 3 = 65; 65 - 5 = 60; 60 - 7 = 53 \neq 54; 53 - 9 = 44

- ∴ 54 is wrong
- 22. All terms in the given sequence are perfect cubes.
 - .. None of these numbers are wrong.
- 23. Subtract consecutive prime numbers successively.

i.e.
$$-2, -3, -5, -7, -11$$

$$\therefore$$
 216 - 2 = 214; 214 - 3 = 211; 211 - 5 = 206; 206 - 7 = 199 \neq 200; 199 - 11 = 188

- \therefore 200 is wrong.
- **24.** The sequence is $\times 2 + 1$, $\times 2 + 1$...

$$\therefore 2 \times 2 + 1 = 5$$
; $5 \times 2 + 1 = 11$; $11 \times 2 + 1 = 23$; $23 \times 2 + 1 = 47 \neq 45$; $47 \times 2 + 1 = 95$

- ∴ 45 is wrong
- **25.** The sequence is $\times 1 + 2$, $\times 2 + 3$, alternatively.

$$\therefore 12 \times 1 + 2 = 14; 14 \times 2 + 3 = 31; \\ 31 \times 1 + 2 = 33; 33 \times 2 + 3 = 69;$$

$$69 \times 1 + 2 = 71 \neq 72$$
; $71 \times 2 + 3 = 145$

- \therefore 72 is wrong
- **26.** The series is $+2^2$, $+4^2$, $+6^2$, ...
 - \therefore required number = $121 + 2^2 + 4^2 + 6^2 = 177$
- **27.** The series is $\times 2 1$, $\times 3 + 3$, $\times 4 3$, $\times 5 + 5$, ...
 - \therefore required number = 230
- **28.** The series is \times 0.5, \times 1, \times 1.5, \times 2 ...
 - \therefore required number = $12 \times 0.5 \times 1 \times 1.5 \times 2.0 \times 2.5 = 45$

- **29.** The series is $\div 2 + 4$, $\div 2 + 4$...
 - \therefore required number = $488 \div 2 + 4 = 244 + 4 = 248$
- **30.** The series is $\times 8 + 1, \times 7 + 2, \times 6 + 3 \dots$
 - \therefore required number = $(5 \times 8 + 1) \times 7 + 2 = 289$
- **31.** The series is $\times 1 + 4$, $\times 2 + 8$, $\times 3 + 12$, ...
 - :. required number = $[(7 \times 1 + 4) \times 2 + 8] \times 3 + 12 = 102$
- **32.** The series is $\times 1 + 1$, $\times 2 + 2$, $\times 3 + 3$, ...
 - : required number = $((((5 \times 1 + 1) \times 2 + 2) \times 3 + 3) \times 4 + 4) \times 5 + 5 = 925$
- 33. The series is $\times 1.5, \times 3, \times 4.5, \dots$
 - \therefore required number = $4 \times 1.5 \times 3 = 18$
- **34.** The series is $\times 2 + 4$, $\times 2 + 8$, $\times 2 + 12$, ...
 - \therefore required number = $((((37 \times 2 + 4) \times 2 + 8) \times 2 + 12) \times 2 + 16) \times 2 + 20 = 1412$
- **35.** The series is $\times 8 28$, $\times 7 24$, $\times 6 20$, ...
 - \therefore required number = $(((7 \times 8 28) \times 7 24) \times 6 20) \times 5 16 = 5044$
- **36.** The series is $\times 8 + 1, \times 7 + 2, \times 6 + 3, \dots$
 - \therefore required number = $((2 \times 8 + 1) \times 7 + 2) \times 6 + 3 = 729$
- 37. The series is $\times 1, \times 1.5, \times 2, \dots$
 - \therefore required number = $36 \times 1 \times 1.5 \times 2 \times 2.5 \times 3 = 810$
- **38.** The series is $\div 2 4$.
 - \therefore required number = $(888 \div 2 4) \div 2 4 = 216$
- **39.** The series is $\times 1 + 1$, $\times 1.5 + 2.25$, $\times 2 + 4$, $\times 2.5 + 6.25$, $\times 3 + 9$, ...
 - \therefore required number = $(((7 \times 1 + 1) \times 1.5 + 2.25) \times 2 + 4) \times 2.5 + 6.25 = 87.5$
- **40.** The series is $+(17)^2$, $-(15)^2$, $+(13)^2$, $-(11)^2$...
 - \therefore required number = $13 + 17^2 15^2 + 13^2 11^2 + 9^2 = 206$
- **41.** The series is $+1^2$, $+3^2$, $+5^2$, $+7^2$, ...
 - \therefore required number = $189 + 1^2 + 3^2 + 5^2 + 7^2 + 9^2 = 354$
- **42.** The series is \times 0.5 + 0.5, \times 1 + 1, \times 1.5 + 1.5, \times 2 + 2, ...
 - \therefore required number = $((40 \times 0.5 + 0.5) \times 1 + 1) \times 1.5 + 1.5 = 33.75$
- **43.** The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3, \dots$
 - \therefore required number = $(5 \times 1 + 1) \times 2 + 2 = 14$
- **44.** The series is $\times 2 + 6$, $\times 2 + 7$, $\times 2 + 6$, ...
 - \therefore required number = $(5 \times 2 + 6) \times 2 + 7 = 39$
- **45.** The series is $+(12)^2$, $-(10)^2$, $+(8)^2$, $-(6)^2$, ...
 - \therefore required number = $146 + 12^2 10^2 + 8^2 6^2 = 218$
- 46. The series is $\div 2, \times 1.5, \dots$
 - \therefore required number = $((40 \div 2) \times 1.5) \div 2 = 15$
- **47.** Ans. 2860. The series is $\times 1 + 4$, $\times 2 + 8$, $\times 3 + 12$, ...
 - \therefore required number = ((((13 × 1 + 4) × 2 + 8) × 3 + 12) × 4 + 16) × 5 + 20 = 2860
- **48.** The series is $\times 2 + 1$, $\times 3 2$, $\times 4 + 3$, $\times 5 4$, ...
 - \therefore required number = $((3 \times 2 + 1) \times 3 2) \times 4 + 3 = 79$
- **49.** The series is $\times 2 2$, $\times 3 + 2$, $\times 4 2$, $\times 6 + 2$, ...
 - \therefore required number = $2 \times 2 2 = 2$
- **50.** The series is $\div 2 2$ in each step.
 - \therefore required number = $(((788 \div 2 2) \div 2 2) \div 2 2) \div 2 2 = 45.5$

Data Sufficiency

In this type of questions, you are required to judge whether the data given is sufficient to answer the question or not. Usually a question will be followed by two or three statements. These may contain data or some other related information by which the question can possibly be solved.

ANALYSIS: Questions in this chapter will cover topics such as percentage, time and work, time and distance, etc. We have already learnt how to solve this type of questions. These questions are asked in a different pattern.

Steps to Solve these Questions: A question of this pattern should be solved by following a systematic approach as follows.

Two statement data sufficiency

Directions: This type of questions consists of a question followed by two statements labelled (A) and (B). You have to decide if these statements are sufficient to answer the question. Give the answer as

- (a) If statement (A) alone is sufficient to answer the question but statement (B) alone is not sufficient to answer the question.
- (b) If statement (B) alone is sufficient to answer the question but statement (A) alone is not sufficient to answer the question.
- (c) If statement (A) and statement (B) taken together are sufficient to answer the question.
- (d) If statement (A) alone is sufficient and statement (B) alone is also sufficient to answer the question.
- (e) If statement (A) and statement (B) even taken together are not sufficient to answer the question.



Note: If a question involves two unknown, then two distinct equations are required for it. If this condition is fulfilled then (c) is the answer, otherwise (e) is the answer.

EXAMPLE 1 What is the volume of a rectangular box R?

- (A) The total surface area of R is 12 m²
- (B) The height of R is 50 cm

Solution Volume of rectangular box = Length \times Breadth \times Height

The two pieces of information are not sufficient to answer the question.

.. The answer is (e).

EXAMPLE 2 What is the value of x + y?

- (A) x 2y = 5
- (B) $x^2 25 = 4xy 4y^2$.

Solution This question involves two unknowns, but the two equations are not distinct. The second equation can be rearranged to $x^2 - 4xy + 4y^2 = 25$ which gets reduced to $(x-2y)^2 = 5^2$ or x-2y=5, which is same as (A).

.. The answer is (e).

EXAMPLE 3 What is the value of (x - y)?

- $(A) \quad x y = y x$
- (B) $x y = x^2 y^2$

Solution The expression (A) involves two unknowns, but the first equation is sufficient.

i.e.
$$(x - y) = (y - x) = -(x - y)$$

 $\Rightarrow 2(x - y) = 0 \text{ or } (x - y) = 0$

The second expression or expression (B) implies

or
$$(x-y) = (x^2 - y^2)$$
$$(x-y) = (x+y)(x-y)$$
$$\Rightarrow (x-y)(x+y-1) = 0$$
$$\Rightarrow (x-y) = 0 \text{ or } (x+y-1) = 0$$

As we don't know the accurate value of x and y, this expression can't be used or expression (B) is not sufficient.

Hence (a) is the answer.

Questions Based on Inequalities

EXAMPLE 4 If a and b are integers, i.e. (a + b) an odd number?

- (A) 8 < a < 11
- (B) 7 < b < 10

Solution Statement (A) implies a may be 9 or 10. Statement (B) implies b may be 8 or 9. Hence a + b may be either odd or even. Thus even with the help of both statements we can't answer the question. Hence the answer is (e).

EXAMPLE 5 Are two triangles congruent?

- (A) They are both equilateral triangles.
- (B) They both have equal bases and equal heights.

Solution Equilateral triangles with same bases are congruent by the side-side postulate. Hence the answer is (c).

EXAMPLE 6 What are the dimensions of a rectangle?

- (A) The perimeter of the rectangle is 14.
- (B) The diagonal of the rectangle is 5.

Solution Statement (A) implies 2(l+b) = 14 or l+b=7.

Statement (B) implies $l^2 + b^2 = 5^2$

Solving above two equations, we get x and y.

Hence answer is (c).

EXAMPLE 7 How long will it take two pipes P and Q to empty or fill a tank that is 3/4 full?

- (A) Pipe P can fill the tank in 12 minutes.
- (B) Pipe Q can empty it in 8 minutes.

Solution The information contained in (A) and (B) together is needed to find the answer. Hence the answer is (c).

EXAMPLE 8 Is x^2 an integer?

- (A) x is a negative whole number.
- (B) $4x^2$ is an integer.

Solution Statement (A) gives an affirmative answer.

Statement (B) may give an integer or a fraction. Hence (a) is the answer.

EXAMPLE 9 What percentage marks did a boy get in a test of 4 subjects?

- (A) He got 90 in English and 84 in Maths.
- (B) He got 75 in Hindi and 76 in Sanskrit.

Solution From statements (A) and (B), we get total marks of the student. But nothing is mentioned about the total marks for all the four individual subjects. So we can't answer the question. Hence the answer is (e).

EXAMPLE 10 How many letters can be typed by two typists in one day?

- (A) A working day consists of 6 hours.
- (B) Four typists can type 600 letters in 3 days.

Solution Only statement (B) is sufficient to answer the question. The length of working day is not relevant in this solution. Hence the answer is (b).



EXERCISES

Q. (1-10) Each of the questions below consists of a question followed by two statements numbered I and II. You have to decide whether the data provided in the statements are sufficient to answer the question. Read the statements and give answer.

- (a) If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
- (b) If the data in statement II alone is sufficient to answer the questions, while the data in statement I alone is not sufficient to answer the question.
- (c) If the data either in statement I alone or in statement II alone is sufficient to answer the question.

- (d) If the data even in both statements I and II together are not sufficient to answer the question.
- (e) If the data in both statements I and II together are necessary to answer the question.
- 1. The average of 8 results is 20. Find the last result, i.e. 8th result.
 - I. The average of the first two is $15\frac{1}{2}$ and that of the next three is $21\frac{1}{3}$.
 - II. The sixth result is less than the seventh by 4.
- 2. A tank has two taps. Tap A drains out at the rate of 5 litres per minute and tap B at the rate of 6 litres per minute. If the tank is full, in how many hours will it be emptied if both the taps are opened simultaneously?
 - I. The diameters of tap A and B are 2 cm and 2.5 cm respectively.
 - II. Capacity of the tank is given.
- 3. If x and y are positive integers, what is the unitary digit of the number x?
 - I. x is divisible by 2, 3 and 8.
 - II. x^{2n} ends in 6 but x^{2n+1} ends in 4.
- 4. Suresh was 29 years old when his son Anil was born. What is the present age of Anil?
 - I. The sum of their ages now is 73 years.
 - II. The difference of their ages now is known.
- 5. A piece of cloth was washed. After washing, it was found to have lost 20% of its length and 10% of its breadth. Find the original length of the cloth.
 - I. The absolute decrease in its area is 1400 cm².
 - II. The perimeter of the original cloth was 300 cm.
- 6. The simple interest of a certain sum for 10 years is ₹ 900. Find the sum.
 - I. The sum gets trebled in 5 years.
 - II. The sum at simple interest amounts to ₹ 405 in 2 years and ₹ 675 in 5 years.
- 7. Is a two-digit positive integer k divisible by 18?
 - I. When k is divided by 3, the remainder is 2.
 - II. When k is multiplied by 3, we get the result in three digits.
- 8. In a bag, there are coins of 50 paise, 25 paise and 20 paise. Find the number of 25 paise coins
 - I. There are 8 rupees in the form of 20 paise coins.
 - II. There are ₹ 73 in the bag in all.
- 9. A goat is placed for grassing inside a triangular field and is tethered to one of the vertices of the field by a rope. What would be the area over which the goat can graze inside the field?
 - I. The rope is 7 metres long.
 - II. The field is equilateral triangle of side 10 metres.
- 10. Three men A, B and C go walking round a circle 1 km in circumference at different rates of speed. If they all start together and walk in the same direction, when will they first be together again?
 - I. Walking speeds of A, B and C are 80, 125 and 120 metres per minute respectively.
 - II. A, B and C complete the circle in $12\frac{1}{2}$ minutes, 8 minutes and $8\frac{1}{3}$ minutes respectively.
- 11. What is the two-digit number?

[SBI PO]

- I. The difference between the two digits is 9.
- II. The sum of the digits is equal to the difference between the two digits.

12. What is the difference between the digits of a two-digit number?

[Bank PO]

- I. The sum of the digits of that number is 8.
- II. One-fifth of that number is 15 less than half of 44.
- 13. What is the two-digit number whose first digit is a and the second digit is b? The number is greater than 9. [MAT]
 - I. The number is a multiple of 51.
 - II. The sum of the digits a and b is b.
- 14. Divya is twice as old as Shruti. What is the difference in their ages?

[Bank PO]

- I. Five years hence, the ratio of their ages would be 9:5.
- II. Ten years back, the ratio of their ages was 3:1.
- 15. What is Reena's present age?

[Bank PO]

- I. Reena's present age is five times her son's present age.
- II. Reena's age two years hence will be three times her daughter's age at that time.
- 16. Average age of employees working in a department is 30 years. In the next year, ten workers will retire. What will be the average age in the next year? [IMT]
 - I. Retirement age is 60 years.
 - II. There are 50 employees in the department.
- 17. What is Sonia's present age?

[Bank PO]

- I. Sonia's present age is five times Deepak's present age.
- II. Five years ago her age was twenty five times Deepak's age at that time.
- 18. What is the speed of the boat in still water?
 - I. The speed downstream of the boat is thrice the speed upstream.
 - II. The sum of the speeds of the boat, upstream and downstream is 12 km/hr.
- 19. What is the speed of the boat in still water.

[Bank PO]

- I. It takes 2 hours to cover the distance between A and B downstream.
- II. It takes 4 hours to cover the distance between A and B upstream.
- 20. A boat takes a total time of three hours to travel downstream from P to Q and upstream back from Q to P. What is the speed of the boat in still water? [SBI PO]
 - I. The speed of the current is 1 km/hr.
 - II. The distance between P and Q is 4 km.
- 21. What is the speed of the boat in still water?

[Bank PO]

- I. The boat covers a distance of 48 km in 6 hours while running upstream.
- II. The boat covers the same distance in 4 hours while running downstream
- 22. A train crosses another train running in the opposite direction in x seconds. What is the speed of the train? [SBI PO]
 - I. Both the trains have the same length and are running in the same speed.
 - II. One train crosses a pole in 5 seconds.
- 23. What is the speed of a running train which takes 9 seconds to cross a signal post?

[Bank PO]

- I. The length of the train is 90 metres.
- II. The train takes 27 seconds to cross a platform of 180 metres.
- 24. A train crosses a pole in 10 seconds. What is the length of train?

[Bank PO]

- The train crosses another train running in opposite direction with a speed of 80 km/hr in 22 seconds.
- II. The speed of the train is 108 km/hr.

- **25.** A train crosses a signal post in *x* seconds. What is the length of train?
- [NABARD]

- I. The train crosses a platform of 100 metres in y seconds.
- II. The train is running at the speed of 80 km/hr.
- 26. What is the speed of the train?

[Bank PO]

- I. 280 metres long train crosses a signal pole in 18 seconds.
- II. 280 metres long train crosses a platform in 45 seconds.
- 27. What was the length of a running train crossing another 180 metre long train running in the opposite direction? [Bank PO]
 - I. The relative speed of the two trains was 150 km/hr.
 - II. The trains look 9 seconds to cross each other.
- 28. What is the speed of a train whose length is 210 metres?

[Bank PO]

- The train crosses another train of 300 metres length running in opposite direction in 10 seconds.
- II. The train crosses another train running in the same direction at the speed of 60 km/hr in 30 seconds.
- 29. What is the length of a running train?

[SBI PO]

- I. The train crosses a man in 9 seconds.
- II. The train crosses a 240 metre long platform in 24 seconds.
- 30. Towns A, B and C are on a straight line. Town C is between A and B. The distance from A to B is 100 km. How far is A from C?
 [MBA]
 - I. The distance from A to B is 25% more than the distance from C to B.
 - II. The distance from A to C is $\frac{1}{4}$ of the distance from C to B.
- **31.** What is the usual speed of the train?

[MBA]

- The speed of the train is increased by 25 km/hr to reach the destination 150 km away in time.
- II. The train is late by 30 minutes
- 32. Two cars pass each other in opposite direction. How long would they take to be 500 km apart?
 [MAT]
 - I. The sum of their speeds is 135 km/hr.
 - II. The difference of their speeds in 25 km/hr.
- **33.** What is the percentage profit earned by selling the product?

[Bank PO]

- I. The profit earned was ₹ 50.
- II. Had it been sold for ₹310, the profit would have been ₹70.
- 34. A man mixes two types of rices (X and Y) and sells the mixture at the rate of ₹ 17/kg. Find his profit percentage. [MBA]
 - I. The rate of X is \ge 20/kg.
 - II. The rate of Y is ₹ 13/kg.
- **35.** By selling a product for ₹ 100, how much profit was earned?

[Bank PO]

- I. 20% profit would have been earned if it were sold for ₹ 90.
- II. The profit was one-third of the purchase price.
- **36.** What is the cost price of the article?
 - I. The profit earned on the article is one-third of the cost price.
 - II. The article is sold for ₹ 400.

37. What was the cost price of the suitcase purchased by Richard?

[Bank PO]

- I. Richard got 20% concession on the labelled price.
- II. Richard sold the suitcase for ₹ 2000 with 25% profit on the labelled price.
- 38. What would have been the selling price per kg of rice?

[Bank PO]

- I. 50 kg of rice was purchased for ₹ 3350 and ₹ 150 was spent on transport.
- II. Profit earned was 5%.
- **39.** By selling a product with 20% profit, how much profit was earned?

ISBI POI

- I. The difference between cost and selling price is $\stackrel{?}{\stackrel{?}{\sim}} 40$.
- II. The selling price is 120% of the cost price.
- 40. The average age of teacher and students in a class is 3 years more than the average age of students. What is the age of the class teacher? [Bank PO]
 - I. There are 11 students in the class.
 - II. The average age of teacher and students is 14 years.
- 41. How many children are there in the group?

[Bank PO]

- I. Average age of the children in the group is 15 years. The total age of all the children in the group is 240 years.
- II. The total age of all the children in the group and the teacher is 264 years. The age of the teacher is 9 years more than the average age of the children.
- 42. The average age of P, Q, R and S is 30 years. How old is R?

[RBI]

- I. The sum of ages of P and R is 60 years.
- II. S is 10 years younger than R.
- 43. What is the average age of children in the class?

[Bank PO]

- I. Age of the teacher is as many years as the number of children.
- II. Average age increased by 1 year if the teacher's age is also included.
- 44. How many candidates were interviewed everyday by panel A out of the three panels A, B and C?

 [Bank PO]
 - I. The three panels on an average interview 15 candidates everyday.
 - II. Out of a total of 45 candidates interviewed everyday by the three panels, the number of candidates interviewed by panel A is more by 2 than the candidates interviewed by panel C and is more by 1 than the candidates interviewed by panel B.
- 45. What will be the average weight of the remaining class?

[Bank PO

- I. Average weight of 30 children out of total 46 in the class is 22.5 kg and that of the remaining children is 29.125 kg. A child having weight more than 40 kg is excluded.
- II. Average weight of a class of 46 children is 23.5 kg. A child weighing 46 kg is dropped out.
- **46.** Rahul, Anurag and Vivek started a business together. In what proportion would the annual profit be distributed among them? [Bank PO]
 - I. Rahul got one-fourth of the profit.
 - II. Rahul and Vivek contributed 75% of the total investment.
- 47. Ravi, Gagan and Nitin are running a business firm in partnership. What is Gagan's share in the profit earned by them? [MBA]
 - I. Ravi, Gagan and Nitin invested the amounts in the ratio of 2:4:7.
 - II. Nitin's share in the profit is ₹8750.

- 48. A and B together can complete a work in 7 days. B alone can do it in 20 days. What part of the work was carried out by A?

 [MBA]
 - I. A completed the job alone after A and B worked together for 5 days.
 - II. Part of the work done by A could have been done by B and C together in 6 days.
- **49.** How long will machine Y, working alone, take to produce x candles?

[MBA]

- I. Machine X produces x candles in 5 minutes.
- II. Machine X and machine Y working at the same time produce x candles in 2 minutes.
- 50. What percentage of simple interest per annum did Anand pay to Deepak?

[IBPS]

- I. Anand borrowed ₹8000 from Deepak for four years.
- II. Anand returned ₹8800 to Deepak at the end of two years and settled the loan.
- **51.** The simple interest on a sum of money is ₹ 50. What is the sum?

[RBI]

- I. The interest rate is 10% p.a.
- II. The sum earned simple interest in 10 years.
- **52.** What is the rate of simple interest?

[Bank PO]

- I. The total interest earned was ₹ 4000.
- II. The sum was invested for 4 years.
- 53. What was the total compound interest on a sum after 3 years?

[Bank PO]

- I. The interest after one year was ₹ 100 and the sum was ₹ 1000.
- II. The difference between simple and compound interest on a sum of ₹ 1000 at the end of 2 years was ₹ 10.
- 54. What was the rate of interest on a sum of money?

[SBI PO]

- I. The sum fetched a total of ₹ 2522 as compound interest at the end of 3 years.
- II. The difference between the simple interest and the compound interest at the end of 2 years at the same rate was $\stackrel{?}{\sim} 40$.
- **55.** What will be the compounded amount?

[Bank PO]

- I. ₹ 200 was borrowed for 192 months at 6% compounded annually.
- II. ₹ 200 was borrowed for 16 years at 6%.
- **56.** What will be the cost of painting the inner walls of a room if the rate of painting is ₹ 20/ft²?

[Bank PO]

- I. Circumference of the floor is 44 ft.
- II. Height of the wall of the room is 12 ft.
- 57. The area of a rectangle is equal to the area of a right angled triangle. What is the length of the rectangle? [Bank PO]
 - I. The base of the triangle is 40 cm.
 - II. The height of the triangle is 50 cm.
- 58. Area of a square is equal to the area of a circle. What is the circumference of the circle?

[SBI PO]

- I. The diagonal of the square is x inches.
- II. The side of the square is y inches.
- **59.** The area of a playground is 1600 m². What is its perimeter?

[Bank PO]

- I. It is a perfect square playground.
- II. It costs $\stackrel{?}{\sim} 3200$ to put a fence around the playground at the rate of $\stackrel{?}{\sim} 20$ /m.
- **60.** What is the height of the triangle?

[Bank PO]

- I. The area of the triangle is 20 times its base.
- II. The perimeter of the triangle is equal to the perimeter of a square of side 10 cm.

61. What is the height of a circular cone?

[Bank PO]

- I. The area of the cone is equal to the area of a rectangle whose length is 33 cm.
- II. The area of the base of the cone is 154 cm².
- **62.** What is the capacity of a cylindrical tank?

IIBPSI

- I. Radius of the base is half of its height which is 28 metres.
- II. Area of the base is 616 m² and its height is 28 metres.
- 63. Is a given rectangular block, a cube?

[MAT]

- I. At least 2 faces of the rectangular block are squares.
- II. The volume of the block is 64.
- 64. What is the volume of 32 m high cylindrical tank?

[Bank PO]

- I. The area of its base is 154 m.
- II. The diameter of the base is 14 m.
- 65. What is the volume of a cube?

[Bank PO]

- I. The area of each face of the cube is 64 m².
- II. The length of one side of the cube is 8 m.

Q. (66 - 70) In this type of questions, you have to determine which statement(s) is/are necessary/sufficient to answer the question. Questions will be accompanied by the statements A, B and C.

- 66. What is the minimum marks required to pass an examination?
 - A. A student gets 25% marks and fails by 40 marks
 - B. Another student who appeared in the same examination gets 70% of the pass marks and fails by 42 marks.
 - C. The total marks of the examination is 400.
 - (a) Any two of them

(b) Either B alone or A and C together

(c) Only A and C together

(d) Any of them

- (e) None of these
- **67.** Find the length of the side of a square.
 - A. Total cost of flooring the square is ₹ 1000 at the rate of ₹ 40/m².
 - B. A rectangle whose perimeter is 20 m is equal to the perimeter of the square.
 - C. A square of area equal to 50 m² can be made on the diagonal of the given square.
 - (a) Any of them

(b) Any two of them

(c) Either B or C only

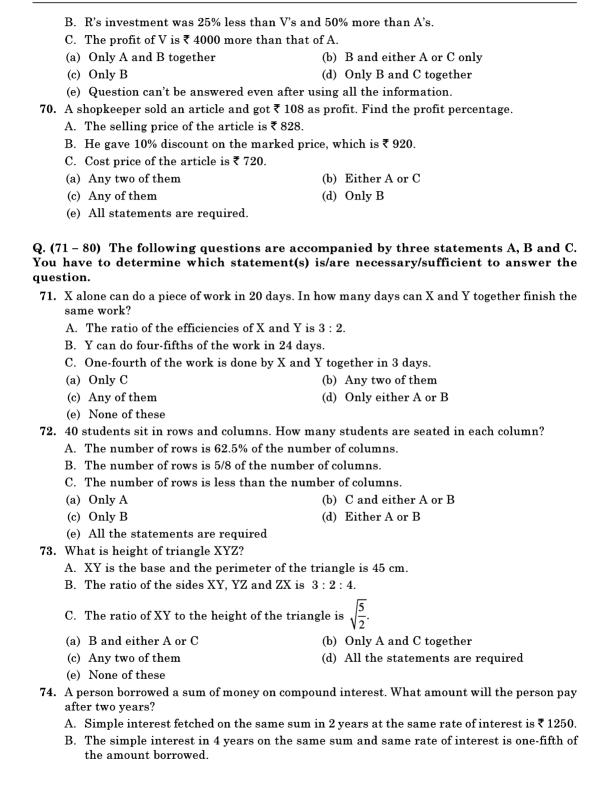
(d) Either A or B only

- (e) Either A or C only
- 68. In how many days can a work be completed by X, Y and, Z together?
 - A. The ratio of work efficiencies of X, Y and Z is 3:2:1.
 - B. Z works for 5 days and leaves the job and the remaining work is done by X and Y together in 5 days.
 - C. 60% of the work is done by X alone in 6 days
 - (a) Any two of them

(b) Only A and C together

(c) Only B

- (d) Either B alone or A and C together
- (e) Only A and either B or C
- **69.** What will be the share of R in the profit earned by V, R and A together?
 - A. They together invested an amount of ₹ 54000 for a period of one year.



two years is ₹ 31.25. (a) Only A and B together

(c) Only B and C together

direction in 9 seconds.

(e) All the statements are required

75. At what time will a train reach Lucknow from Patna?

C. The 97.50 m long train crosses a signal pole in 5 seconds.

	(a) Only A	(b) B and C together										
	(c) A and C together	(d) All statements are required										
	(e) Only B											
76.	Find the per cent profit on an article.											
	A. The marked price of the article is	₹ 6000.										
	B. The difference between the selling	g price and cost price of the article is ₹ 800.										
	C. A discount of 20% is given on the	marked price.										
	(a) A and B together											
	(b) A and C together											
	(c) B and C together											
	(d) Question can't be answered even	after using all the statements										
	(e) All statements are required.											
77.	What will be the sum of the present a	ges of P and Q?										
	A. Ratio of present age of P to that of	f Q is 4:5.										
	B. Q's present age is 25% more than	that of P.										
	C. Difference between the present ag	ges of Q and P is 10 years.										
	(a) Any two of them	(b) C and either A or B										
	(c) Any of them	(d) Only A and C together										
	(e) None of these											
78.	What is the length of a train?											
	A. The train crosses a pole in 5 secon											
	B. The train crosses a platform of ler											
	C. The speed of the train is 79.2 km/s											
	(a) Any two of them	(b) Only A and C together										
	(c) Only A and B together	(d) All statements are required										
	(e) A and either B or C											
79 .	How many ice cubes can be accommod											
	A. The length and the breadth of the container are 4 m and 5 m respectively.											
	B. The edge of the cube is 2 m.											
	C. The ratio of the volume of the container to that of the ice cube is 27:1.											
	(a) A and B together	(b) Only C is sufficient										
	(c) A and C together	(d) B and C together										
	(e) None of these											

C. The difference between compound interest and simple interest on the same sum after

A. The train crosses another train of equal length of 200 m and running in the opposite

B. The train leaves Patna at 11: 15 A.M. for Lucknow, which is at a distance of 567 km.

(b) Only A and C together

(d) Any two of them

- 80. The area of a rhombus is 1152 m². What is the side of rhombus?
 - A. One of its diagonal is 36 m.
 - B. The other diagonal is 32 m.
 - C. All the four sides are equal.
 - (a) A and B together are sufficient
- (b) Only A alone is sufficient
- (c) Only B alone is sufficient
- (d) Either A alone or B alone is sufficient
- (e) All A, B and C are necessary



1. (d)	2. (b)	3. (b)	4. (a)	5. (e)	6. (c)	7. (a)	8. (d)	9. (e)	10. (c)
11. (e)	12. (e)	13. (e)	14. (c)	15. (d)	16. (e)	17. (e)	18. (b)	19. (d)	20. (e)
21. (e)	22. (d)	23. (a)	24. (b)	25. (c)	26. (a)	27 . (e)	28. (e)	29. (e)	30. (c)
31. (e)	32. (a)	33. (b)	34. (d)	35. (c)	36. (e)	37. (e)	38. (e)	39. (a)	40. (e)
41. (a)	42. (d)	43. (d)	44. (b)	45. (b)	46. (e)	47 . (e)	48. (a)	49. (e)	50. (e)
51. (e)	52. (d)	53. (c)	54. (e)	55. (c)	56. (c)	57. (d)	58. (c)	59. (c)	60. (a)
61. (d)	62. (c)	63. (d)	64. (c)	65. (c)	66. (b)	67 . (d)	68. (b)	69. (d)	70. (c)
71. (c)	72. (d)	73. (e)	74. (d)	75. (b)	76 . (e)	77. (b)	78. (a)	79 . (b)	80. (d)



11. Let the unit digit of number be x and tenth digit of number be y.

$$\therefore \quad I \quad \rightarrow \quad x \sim y = 9 \tag{1}$$

$$II \rightarrow x + y = x \sim y \tag{2}$$

Solving (1) and (2), we get

$$x \sim y = 9$$
 and $x + y = 9$

$$\Rightarrow$$
 $x = 9 \text{ and } y = 0 \text{ or } y = 9 \text{ and } x = 0$

.. Required number is 90.

Thus both I and II are required to answer the question.

12. I
$$\rightarrow$$
 $x + y = 8$ (1)

II
$$\rightarrow \frac{1}{5}(10x + y) = (\frac{1}{2} \times 44 - 15)$$

$$\Rightarrow 10x + y = 35 \tag{2}$$

Solving (1) and (2), we get x = 3 and y = 5.

- .. Both I and II are required to answer the question.
- 13. Let the tenth place digit of number be a, and unit place digit of number be b.

Number = 10a + b

$$I \rightarrow 10a + b = 51 \times k \tag{1}$$

where k = 1, 2, 3 etc.

$$II \rightarrow a + b = 6 \tag{2}$$

When k = 1, solving (1) and (2), we get

$$a = 5$$
 and $b = 1$

.: Number is 51

Therefore I and II together are necessary to answer the question.

14. Let the age of Shruti be x years and that of Divya be 2x years.

2x + 5 : x + 5 = 9 : 5

II
$$\rightarrow$$
 $2x - 10 : x - 10 = 3 : 1$
i.e. $\frac{2x + 5}{x + 5} = \frac{9}{5}$ (1)

$$\frac{2x-10}{x-10} = \frac{3}{1} \tag{2}$$

From (1),

 \mathbf{or}

 $I \rightarrow$

$$\frac{2x+5}{x+5} = \frac{9}{5}$$

$$\Rightarrow \frac{(2x+5)-(x+5)}{x+5} = \frac{9-5}{5}$$

$$\Rightarrow \frac{x}{x+5} = \frac{4}{5}$$

$$\therefore \Rightarrow \frac{x+5}{x} = \frac{5}{4}$$

$$1 + \frac{5}{x} = \frac{1}{4} + 1$$
$$x = 5 \times 4 = 20$$

Difference in ages = x = 20 years From (2),

$$\Rightarrow \frac{2x-10}{x-10} = \frac{3}{1}$$

$$\Rightarrow \frac{(2x-10)-(x-10)}{x-10} = \frac{3-1}{1}$$

$$x = \frac{3}{1}$$

$$\Rightarrow \frac{x}{x-10} = \frac{2}{1}$$

$$\Rightarrow \qquad \frac{x-10}{x} = \frac{1}{2}$$

$$\Rightarrow \qquad 1 - \frac{10}{x} = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{10}{x} = \frac{1}{2}$$

$$x = 20 \text{ years}$$

- : Either I or II alone is sufficient to answer the question.
- 15. Here data in both statements I and II together are not sufficient to answer the question.
- 16. I \rightarrow Retirement age is 60 years

II \rightarrow There are 50 employees in the department.

Average age of 50 employees = 30 years.

or total age of 50 employees = $50 \times 30 = 1500$ years

Number of employees next year = 40

Total age of 40 employees next year = $50 \times 31 - 10 \times 60 = 1550 - 600 = 950$

$$\therefore$$
 Average age next year = $\frac{950}{40}$ = 23 $\frac{3}{4}$ years

:. I and II together are necessary to answer the question.

17. I \rightarrow Sonia's age = $5 \times$ Deepak's age

II \rightarrow (Sonia's age -5) = $25 \times$ (Deepak's age -5)

Sonia's age
$$-5 = 25 \times \left(\frac{\text{Sonia's age}}{5} - 5\right)$$

= $5 \times \text{Sonia's age} - 125$

or Sonia's age =
$$\frac{125 - 5}{4}$$
 = 30 years

:. I and II together are necessary to answer the question.

18. Let speed of boat upstream be x km/hr

$$I \rightarrow \text{Speed of boat downstream} = 3x \text{ km/hr}$$

$$II \rightarrow x + 3x = 12 \text{ km/hr}$$

Speed of boat in still water =
$$\frac{x + 3x}{2}$$

$$= \frac{12}{2} = 6 \text{ km/hr}$$

: II alone is sufficient to answer the question.

19. Let the distance covered be S km

$$\therefore I \rightarrow Downstream speed = \frac{S}{2} \, km/hr$$

II
$$\rightarrow$$
 Upstream speed = $\frac{S}{4}$ km/hr

∴ Speed of boat in still water =
$$\left(\frac{S}{2} + \frac{S}{4}\right)\frac{1}{2}$$

= $\frac{3}{8}S$ km/hr

:. I and II even together are not sufficient to answer the question.

20. I \rightarrow speed of current = 1 km/hr

$$II \rightarrow PQ = 4 \text{ km}$$

Let speed of boat in still water be S km/hr

$$\therefore \frac{PQ}{(S+1)} + \frac{PQ}{(S-1)} = 3$$

i.e.
$$\frac{4}{S+1} + \frac{4}{S-1} = 3$$

$$\Rightarrow$$
 $S = 3 \text{ km/hr}$

: I and II together are sufficient to answer the question.

21. I
$$\rightarrow$$
 Upstream speed = $\frac{48}{6}$ = 8 km/hr

II
$$\rightarrow$$
 Downstream speed = $\frac{48}{4}$ = 12 km/hr

Speed of boat in still water =
$$\frac{1}{2}(8+12) = 10 \text{ km/hr}$$

:. I and II together are necessary to answer the question.

22. I
$$\rightarrow$$
 $L_1 = L_2$; $u_1 = u_2$

II
$$\rightarrow$$
 Speed of train = $\frac{L_1}{5}$ m/s

Speed of train =
$$\frac{L_1 + L_2}{x}$$
 m/s = $\frac{L_1}{5}$

From the given details, it is not possible to find speed of train, i.e. I and II taken even together is not sufficient to answer the question.

23. I
$$\rightarrow$$
 L = 90 m; $u = \frac{90}{9} = 10$ m/s

II
$$\rightarrow u = \frac{90 + 180}{27} = 10 \text{ m/s}$$

:. I alone gives the correct answer.

24. II
$$\rightarrow$$
 Speed of train = 108 km/hr = $108 \times \frac{5}{18} = 30$ m/s

$$\therefore$$
 Length of train = Speed \times Time = $30 \times 10 = 300$ m

:. II alone is sufficient to answer the question.

25. II
$$\rightarrow$$
 Speed of train = 80 km/hr = $80 \times \frac{5}{18} = \frac{200}{9}$ m/s

I
$$\rightarrow$$
 Speed of train = $\frac{L+100}{y}$ m/s = $\frac{200}{9}$ m/s

Also speed of train =
$$\frac{L}{x}$$
 m/s = $\frac{200}{9}$ m/s

It is not possible to find length of train from I or II alone.

26. I
$$\rightarrow$$
 Speed of train = $\frac{280 \text{ m}}{18 \text{ sec}} = \frac{140}{9} \text{ m/s}$

II
$$\rightarrow$$
 Speed of train = $\frac{280 + L}{45} = \frac{140}{9}$ m/s

$$\Rightarrow$$
 $L = 420 \text{ m}$

I alone is sufficient to answer the question.

(B

27. I
$$\rightarrow$$
 Relative speed of two trains = 150 km/hr = 150 $\times \frac{5}{18}$ m/s = $\frac{125}{3}$ m/s

II
$$\rightarrow \frac{L+180}{9} = \text{Speed of train} = \frac{125}{3} \text{ m/s}$$

$$\Rightarrow$$
 $L = 195 \text{ m}$

Both I and II are necessary to answer the question.

28. I
$$\rightarrow \frac{l_1 + l_2}{u + v} = 10$$
 or $u + v = \frac{l_1 + l_2}{10} = 51$ m/s

$$\frac{210 + 300}{u + v} = 10$$
 or $\frac{210 + 300}{10} = u + v = 51$ m/s

II
$$\rightarrow \frac{l_1 + l_2}{u - v} = 30$$
 or $\frac{210 + 300}{\left(4 - 60 \times \frac{5}{18}\right)} = 30$

or
$$u = \frac{210 + 300}{30} + 60 \times \frac{5}{18} = 17 + \frac{50}{3} = 33 \frac{2}{3} \text{ m/s}$$

:. I and II are necessary to answer the question.

29. Let length of train be L metres

$$I \rightarrow Speed = \frac{L}{9} \text{ m/s}$$
 (1)

II
$$\rightarrow$$
 Speed = $\frac{L + 240}{24}$ m/s (2)

Solving (1) and (2), we get

$$8L = 3L + 720$$

$$\Rightarrow$$
 5L = 720

$$\Rightarrow$$
 $L = 144 \text{ m}$

:. I and II are necessary to answer the question.

30.
$$AB = 100 \text{ km}$$
; $AC = ?$

$$I \rightarrow AB = 1.25 BC$$

or
$$BC = \frac{1}{1.25} AB = 0.8 AB$$

or
$$AC = 0.2 AB$$

$$= 0.2 \times 100 = 20 \text{ km}$$

II
$$\rightarrow AC = \frac{1}{4}BC$$
 or $\frac{AC}{BC} = \frac{1}{4}$ or $AC = \frac{1}{1+4} \times 100 = 20 \text{ km}$

:. I or II alone is sufficient to answer the question.

31. Let usual speed be u km/hr

$$\therefore$$
 Time taken to cover 150 km = $\frac{150}{u}$ hour

I \rightarrow Time taken to cover the distance at increased speed = $\frac{150}{(u+25)}$ hour

II
$$\rightarrow$$
 $\frac{150}{u} - \frac{150}{u + 25} = \frac{30}{60}$
 $\Rightarrow \frac{1}{u} - \frac{1}{u + 25} = \frac{1}{300}$
 $\Rightarrow [(u + 25) - u] \times 300 = u(u + 25)$
 $\Rightarrow u^2 + 25u - 7500 = 0$
 $\Rightarrow (u + 100) (u - 75) = 0$
 \therefore Usual speed = 75 km/hr

- :. I and II together are necessary to answer the question.
- 32. I \rightarrow Relative speeds of faster car w.r.t. slower car = 135 km/hr

$$\therefore \text{ Time taken by them to be 500 km apart} = \frac{\text{Distance}}{\text{Speed}} = \frac{500 \, \text{km}}{135 \, \text{km/hr}} = \frac{100}{27} \, \text{hours} = 3 \, \frac{19}{27} \, \text{hours}$$

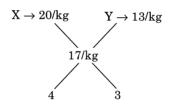
Hence I alone is sufficient to answer the question, but II alone is not sufficient to answer the question.

33. II
$$\rightarrow$$
 SP = ₹310; Profit = ₹70.

∴
$$CP = SP - Profit = 310 - 70 = ₹ 240$$

:. Profit % =
$$\frac{SP - CP}{CP} \times 100 = \left(\frac{310 - 240}{240}\right) \times 100\%$$

- :. II alone is sufficient to answer the question, but I alone is not sufficient.
- **34.** Both I and II taken together is not sufficient to answer the question as the ratio in which two types of rices X and Y are mixed is not given.



$$I \rightarrow If SP = ₹90$$
, Gain % = 20%

∴
$$CP = \frac{100}{120} \times SP = \frac{100}{120} \times 90 = ₹75$$

∴ If SP = ₹ 100, Profit % =
$$\frac{100 - 75}{75} \times 100 = 33\frac{1}{3}\%$$

and Profit =
$$100 - 75 = ₹25$$

II \rightarrow Profit is $\frac{1}{3}$ rd of purchase price or $33\frac{1}{3}\%$

Let
$$CP = ₹x; Profit = \frac{x}{3}$$

$$\therefore SP = CP + Profit = x + \frac{x}{3} = \sqrt[3]{\frac{4}{3}} x = 100$$

$$\therefore \qquad x = \frac{3}{4} \times 100 = 75$$

Profit =
$$\frac{75}{3}$$
 = ₹ 25

: I alone or II alone is sufficient to answer the question.

36. Let CP of article be ₹ x

∴ I → Profit earned =
$$\sqrt[3]{\frac{x}{3}}$$
; SP = $x + \frac{x}{3} = \frac{4}{3}x$;

$$∴ \frac{4}{3}x = 400 or x = \frac{3}{4} \times 400 = ₹300$$

.. Both I and II together are sufficient to answer the question. Hence answer is (e).

37. Let labelled price of suitcase be ₹ x

II
$$\rightarrow$$
 SP = ₹ 2000; SP = $\frac{125}{100} \times x = \frac{5}{4}x$

i.e.
$$\frac{5}{4}x = 2000$$

or
$$x = \frac{4}{5} \times 2000 = ₹ 1600$$

$$\therefore \text{ CP} = \frac{4}{5}x = \frac{4}{5} \times 1600 = ₹ 1280$$

:. I and II together are sufficient to answer the question.

38. I → CP of 50 kg of rice = 3350 + 150 = ₹3500

$$\therefore \qquad \text{CP/kg of rice} = \frac{3500}{50} = ₹70$$

II
$$\rightarrow$$
 Profit = 5%

∴
$$SP = \frac{105}{100} \times CP = \frac{105}{100} \times 70 = ₹73.50$$

:. Both I and II together are necessary to answer the question.

39. Profit % = 20%

$$I \rightarrow SP - CP = ₹40$$

i.e. Profit earned = ₹ 40

II
$$\rightarrow$$
 SP = $\frac{120}{100} \times CP$

II does not give the correct answer.

I alone is sufficient to answer the question.

40. Let the age of class teacher be x years.

I \rightarrow Number of students = 11

II
$$\rightarrow \frac{\text{Sum of ages of } 11 \text{ students} + x}{12} = 14 \text{ years}$$

Sum of ages of 11 students = $11 \times (14 - 3) = 121$

$$\therefore$$
 $x = 12 \times 14 - 121 = 47$ years

- :. Both I and II are necessary to answer the question.
- 41. Let the number of children in the group be x.

I
$$\rightarrow x = \frac{240}{15} = 16 \text{ numbers}$$

II \rightarrow Age of teacher = 15 + 9 = 24 years.

:. Age of x children = 264 - 24 = 240.

I alone gives the answer. II alone is not sufficient to answer the question.

42.
$$\frac{P+Q+R+S}{4}=30; R=?$$

$$I \rightarrow P + R = 60$$

II
$$\rightarrow R - S = 10$$

It is not possible to find the age of R.

Hence both I and II taken together are not sufficient to answer the question.

43. Let the average age of children be x years, and number of children be y.

I \rightarrow Age of teacher = y years.

II
$$\rightarrow$$
 New average = $x + 1 = \frac{xy + y}{y + 1} = \frac{(x + 1)y}{y + 1}$

It is not possible to find age of teacher.

Both I and II taken together are not sufficient to answer the question.

44. II \rightarrow Let the candidates interviewed by panel A per day be x, that by panel B be y and by C be 45 - (x + y)

Now,
$$x = 45 - (x + y) + 2$$
 (1)

$$x = y + 1 \tag{2}$$

Solving (1) and (2), we get

$$45 + 2 - (x + y) = y + 1$$

$$\Rightarrow 47 - x - 1 = 2y$$
or
$$46 - x = 2y$$
But
$$y = (x - 1), \text{ from (2)}$$

$$\therefore 46 - x = 2(x - 1)$$

$$46 - x = 2x - 2$$
or
$$46 + 2 = 3x$$
or
$$x = \frac{48}{3} = 16$$

Hence II gives the answer. I does not give the correct answer.

45. I \rightarrow Total weight of 46 children = $30 \times 22.5 + 16 \times 29.125 = 675 + 466 = 1141$

Weight excluded is not having correct value.

- :. Average of remaining class cannot be found.
- II \rightarrow Total weight of 46 children = $46 \times 23.5 = 1081$

$$\therefore$$
 New average = $\frac{1081 - 46}{45} = \frac{1035}{45} = 23$

:. II gives correct answer, but I is not sufficient.

46. Let total investment be P.

$$\therefore \quad I \rightarrow \qquad R = \frac{P}{4}$$

$$II \rightarrow \quad R + V = \frac{75}{100} \times P = \frac{3}{4} P$$

$$\Rightarrow \qquad A = \frac{1}{4} P$$

$$R: A: V = \frac{P}{4}: \frac{P}{4}: \frac{P}{2}$$
= 1:1:2

:. Both I and II together are necessary to answer the question.

47. I \rightarrow R: G: N = 2: 4: 7

$$\therefore$$
 Gagan's share = $\frac{4}{2+4+7}$ of Total investment = $\frac{4}{13} \times I$

II
$$\rightarrow \frac{7}{13} \times Investment amount = 8750$$

$$\therefore \frac{4}{13} \times \text{Investment amount} = \frac{8750}{7} \times 4 = 1250 \times 4$$

i.e. Gagan's share in the profit = ₹ 5000

:. Both I and II together are necessary to answer the question.

48. $A + B \rightarrow 7 \text{ days}$ $B \rightarrow 20 \text{ days}$

I \rightarrow Total contribution by A and B for 5 days = $\frac{5}{7}$

Part of work by A alone = $1 - \frac{5}{7} = \frac{2}{7}$

II \rightarrow B + C \rightarrow 6 days

I alone gives answer. II is not sufficient to answer the question.

49. I $\rightarrow X \rightarrow x$ candles/5 minutes

II $\rightarrow X + Y \rightarrow x$ candles/2 minutes.

$$X \rightarrow 5$$
 minutes

$$X + Y \rightarrow 2$$
 minutes

$$\therefore Y \rightarrow \frac{5 \times 2}{5 - 2} = \frac{10}{3}$$
 minutes

.. Both I and II together are necessary to answer the question.

50. I \rightarrow Loan amount = ₹ 8000 for 4 years

II
$$\rightarrow$$
 Simple interest for 2 years = $\frac{8800 - 8000}{8000 \times 2} \times 100 = 5\%$

:. Both I and II taken together are sufficient to answer the question.

51. Simple interest on an amount = ₹ 50

Let amount be P

I
$$\rightarrow$$
 $r = 10\%$ p.a.

II \rightarrow Interest earned in 10 years.

$$I = \frac{PNR}{100}$$
i.e.
$$50 = \frac{P \times 10 \times 10}{100}$$

$$P = ₹50$$

:. I and II together are necessary to answer the question.

52. I →
$$I = ₹ 4000$$

II
$$\rightarrow N = 4 \text{ years}$$

$$R = \frac{100 \ I}{P \times N}$$

But P is unknown. Therefore, R cannot be calculated. I and II together are not sufficient to answer the question.

53. I →
$$P = ₹ 1000;$$
 $P\left(1 + \frac{R}{100}\right) = 1100;$
∴ $R = 10\%$

∴ Sum after 3 years =
$$P\left(1 + \frac{R}{100}\right)^N$$

= $1000\left(1 + \frac{10}{100}\right)^3$
= $1000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = 1331$

.. I gives the answer

II
$$\rightarrow 1000 \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] - \frac{1000 \times 2 \times R}{100} = 10$$

$$1000 \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] - 20R = 10$$

$$1000 \left(\frac{2R}{100} + \frac{R^2}{100^2} \right) - 20R = 10$$

$$20R + \frac{R^2}{10} - 20R = 10$$
or
$$R^2 = 10 \times 10 = 100$$
or
$$R = 10\%$$

- :. Sum after 3 years = $1000 \times \left(1 + \frac{10}{100}\right)^3 = 1331$
- : II gives the answer
- :. I alone or II alone is sufficient to answer the question.

54. I \rightarrow Let P be the sum and R% be the interest.

$$P\left[\left(1 + \frac{R}{100}\right)^3 - 1\right] = 2522\tag{1}$$

II
$$\rightarrow P \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] - \frac{P \times 2R}{100} = 40$$
 (2)

Dividing (1) by (2), we get

$$\frac{P\left[\left(1 + \frac{R}{100}\right)^3 - 1\right]}{P\left[\left(1 + \frac{R}{100}\right)^2 - 1\right] - \frac{PR}{50}} = \frac{2522}{40} = \frac{1261}{20}$$

i.e.
$$\frac{\left(1 + \frac{R}{100}\right)^3 - 1}{\left(1 + \frac{R}{100}\right)^2 - 1 - \frac{R}{50}} = \frac{1261}{20}$$

$$\Rightarrow \frac{\frac{R^3}{100^3} + \frac{3R^2}{100^2} + \frac{3R}{100}}{\frac{R^2}{100^2} + \frac{2R}{100} - \frac{R}{50}} = \frac{\frac{R^3}{100^3} + \frac{3R^2}{100^2} + \frac{3R}{100}}{\frac{R^2}{100^2}} = \frac{1261}{20}$$

i.e.
$$\frac{R}{100} + 3 + \frac{300}{R} = \frac{1261}{20}$$

or
$$\frac{R}{100} + \frac{300}{R} = \frac{1261}{20} - 3 = \frac{1261 - 60}{20} = \frac{1201}{20}$$

$$\Rightarrow \frac{R^2 + 30000}{100R} = \frac{1201}{20}$$

$$\Rightarrow \qquad \qquad R^2 + 30000 = 1201 \times 5R$$

i.e.
$$R^2 - 6005R + 30000 = 0$$

∴
$$(R - 6000) (R - 5) = 0$$

∴ $R = 5\%$

I and II together are necessary to answer the question.

55.
$$I \to A = P \left(1 + \frac{R}{100} \right)^N = 200 \left(1 + \frac{6}{100} \right)^{192/12}$$
$$= 200 \left(1 + \frac{6}{100} \right)^{16}$$

II
$$\rightarrow A = 200 \left(1 + \frac{6}{100} \right)^{16}$$

:. I alone or II alone is sufficient to answer the question.

56. I
$$\rightarrow$$
 P = 44 ft

II
$$\rightarrow h = 12 \text{ ft}$$

Area =
$$P \times h = 44 \times 12 = 528 \text{ ft}^2$$

$$\therefore$$
 Cost of painting = 528 ft² × 20/ft² = ₹ 10560

I and II together are necessary to answer the question.

57.
$$l \times b = \frac{1}{2} b_1 h$$

$$I \rightarrow b_1 = 40 \text{ cm}$$

II
$$\rightarrow h = 50 \text{ cm}$$

To find l, b is also needed.

b is not given, it is not possible to find l.

58.
$$a^2 = \pi r^2$$
 (1)

$$p = 2\pi r = ?$$

$$I \rightarrow a\sqrt{2} = x$$
 inches:

II
$$\rightarrow$$
 $a = y$ inches.

(1)
$$\Rightarrow \pi r^2 = a^2 = y^2 \Rightarrow r = \frac{y}{\sqrt{\pi}}$$

Circumference of circle, $p = 2\pi r = 2\pi \times \frac{y}{\sqrt{\pi}} = 2y\sqrt{\pi}$

$$I \rightarrow \pi r^2 = \alpha^2 = \left(\frac{x}{\sqrt{2}}\right)^2 = \frac{x^2}{2}$$

$$\therefore \qquad r^2 = \frac{x^2}{2\pi} \quad \text{or} \quad r = \frac{x}{\sqrt{2\pi}}$$

$$\therefore \qquad p = 2\pi r = 2\pi \times \frac{x}{\sqrt{2\pi}} = \mathbf{x} \times \sqrt{2\pi}$$

:. I alone or II alone is sufficient to answer the question.

59. $A = 1600 \text{ m}^2$; P = ?

$$I \rightarrow a^2 = 1600 \Rightarrow a = 40 \text{ m}$$

$$P = 4a = 4 \times 40 = 160 \text{ m}$$

II
$$\rightarrow p = \frac{3200}{20} = 160 \text{ m}$$

:. I alone or II alone is sufficient to answer the question.

60. I
$$\rightarrow$$
 $A = \frac{1}{2}bh = 20b$ (1)

$$\Rightarrow$$
 $h = 2 \times 20 = 40$ units

II
$$\rightarrow$$
 $P = 4 \times 10 = 40 \text{ cm}$

I alone gives the answer.

61. I $\rightarrow \pi r^2 = 154$ gives value of radius of cone.

II does not give height of cone.

I $\rightarrow A = \pi r l$ = Area of rectangle with length = 33 cm

Area of cone cannot be found out as breadth of rectangle is not given.

Hence height of cone cannot be found out as statements I and II taken together are insufficient to answer the question.

62. I
$$\rightarrow r = \frac{h}{2} = \frac{28}{2} = 14 \text{ cm}$$

i.e.
$$r = 14 \text{ m}$$
; $h = 28 \text{ m}$

Capacity of cylindrical tank =
$$\pi r^2 h = \pi (14)^2 \times 28 = 17248 \text{ m}^2$$

II
$$\rightarrow \pi r^2 = 616 \text{ m}^2, h = 28 \text{ m}$$

Capacity of tank =
$$\pi r^2 h = 616 \times 28 = 17248 \text{ m}^2$$

63. I
$$\rightarrow$$
 Two quantities of l , b and h are equal.

II
$$\rightarrow lbh = 64$$
.

64. Given
$$h = 32 \text{ m}$$

$$I \rightarrow \pi r^2 = 154 \text{ m}^2$$
; Volume = $\pi r^2 h = 154 \times 32 = 4928 \text{ m}^2$

II
$$\rightarrow 2r = 14 \text{ m}$$
; Volume = $\pi \times \left(\frac{14}{2}\right)^2 \times 32 = 4928 \text{ m}^2$

i.e. I alone or II alone is sufficient to answer the question.

65. I
$$\rightarrow a^2 = 64 \text{ m}^2 \rightarrow a = 8 \text{ m}$$

$$\therefore$$
 Volume of cube = $a^3 = 8^3 = 512 \text{ m}^3$

II
$$\rightarrow a = 8 \text{ m}$$

Volume of cube =
$$a^3 = 8^3 = 512 \text{ m}^3$$

Data Analysis

Data analysis is an important topic in competitive examinations today. Usually, a table bar chart, pie-chart or a graph is given and candidates are asked questions to analyse the data given in order to test their ability.

In order to solve such questions you should have the following things in mind.

- 1. Try to understand quickly what the diagram represents.
- 2. If you are doing a table, make sure what the sum of all entries in a row and column individually represent. For example, if a table represents marks scored by 10 students in a class in three subjects (Physics, Chemistry and Biology), then the sum of entries is each row represents total marks scored by each student. The sum of entries in each column represents the total marks scored by all 10 students in three different subjects. Here, the sum of all entries in the last row is equal to the sum of all entries in the last column.

i.e.
$$\sum_{i=1}^{n} \text{total score by student}_i = \sum_{j=1}^{3} \text{total score for subject}_j$$
 where i represents the

number of students and *j* the number of subjects.

- 3. Do not make unnecessary steps. For example, if you are asked to find the subject in which all students have performed best, then look out for the total marks in each subject and locate the highest among them.
- 4. In the questions having diagrams, by just looking at the diagram, you can locate the most correct answer among the alternatives. Pen is not needed for answering such questions.

For example, you are given with a pie diagram showing the population of a town in 1991 and 2001 showing literacy of men, women and children in both the years. You have to find out by what percentage literacy of women has increased.

Now, you have to find out the percentage of literate women in both years and total women in both years and find the ratios separately.

```
i.e. percentage increase = \frac{\text{Literate women (2001)/Total women (2001)}}{\text{Literate women (1991)/Total women (1991)}} - 1
```

5. In some questions calculations are unavoidable. In this case short cuts can be applied if possible. Otherwise, some approximations can be made if it does not lead to wrong answers.

For example if you are asked to find the result on simplification like

$$\frac{1.2\times1.3\times1.5\times1.4}{2.1\times2.6}.$$
 Now you have to go through the question and look for any possible cancellation among numerators and denominators. Then after such

$$\frac{1.2 \times 1.3 \times 1.5 \times 1.4}{2.1 \times 2.6} = \frac{1.2}{2} = \mathbf{0.6}$$

6. In certain cases the result may not be finite or answer may be given as a value between 1.4 and 1.7 or 0.8 and 1.2. In that case approximation may be made and locate the answer from the given alternatives.

For example,
$$\frac{2}{13} + \frac{3}{14} = ?$$

By approximating

$$2 \times 0.0775 + 3 \times 0.071 = 0.155 + 0.213 = 0.368$$
 (approx.)

Given alternatives are (a) 0.3 (b) 0.35 (c) 0.36 (d) 0.37.

cancellation simplify the question and find the answer.

The answer will be (d) as it is the nearest value to approximated one.

TABLE: This section consists of questions that contain data (such as imports, exports, income of employees in a company, marks scored by students in a subject and so on) form of a table. You have to study the given information and then answer the given questions on the basis of comparative analysis of the data.

EXAMPLE 1 A class consists of 4 students and the marks obtained by them in Physics, Chemistry and Mathematics are given in the following table.

Students Subjects •	S_1	S_2	S_3	S_4
Physics	60	50	70	75
Chemistry	70	45	60	80
Mathematics	80	55	55	75

Study the table carefully and answer the questions that follow.

- 1. Find the average score in Physics by the students in the class.
- 2. In which subject student S_2 got the highest score?
- 3. In which subject average score in the class was highest?
- 4. Who among the students is ranked first in the class?
- 5. Who got the least marks in Chemistry in the class?

Solution

1. Average score in Physics in the class is given by
$$\frac{60 + 50 + 70 + 75}{4} = 63\frac{3}{4} \text{ marks}$$

2. S_2 scored the highest in **Mathematics**

3. Average score for Physics =
$$\frac{60 + 50 + 70 + 75}{4} = 63\frac{3}{4}$$

Average score for Chemistry =
$$\frac{70 + 45 + 60 + 80}{4} = 63\frac{3}{4}$$

Average score for Mathematics =
$$\frac{80 + 55 + 55 + 75}{4} = 66\frac{1}{4}$$

: Average score is highest for **Mathematics**.

4. Total score for $S_1 = 210$

Total score for $S_2 = 150$

Total score for $S_3 = 185$

Total score for $S_4 = 230$

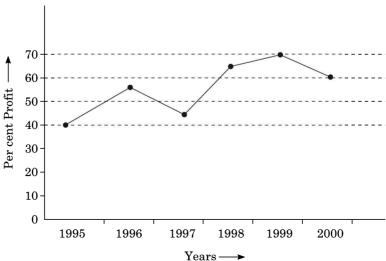
- $\therefore S_4$ is ranked first in the class.
- 5. S_2 got the least marks in **Chemistry**.

LINE-GRAPH: A line-graph displays data that change over time. Line graphs compare two variables plotted along vertical axis and horizontal axis.

EXAMPLE 2 The following line-graph gives the annual profit earned by a company during the period 1995–2000. Study the line graph and answer the questions that are based on it. [RBI]

Per cent Profit Earned by a Company over the Years

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



- 1. If the income in 1998 was ₹ 264 crores, what was the expenditure in 1998? (a) ₹ 104 crores (b) ₹ 145 crores (c) ₹ 160 crores (d) ₹ 185 crores (e) None of these 2. During which of the following years was the ratio of income to expenditure the minimum? (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000 3. What is the average profit earned for the given years? (b) $55\frac{5}{6}$ (c) $60\frac{1}{6}$ (d) $65\frac{1}{3}$ (a) $50\frac{2}{3}$ (e) None of these 4. During which year the ratio of percentage profit earned to that in the previous year is the minimum? (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- 5. If the expenditures in 1996 and 1999 are equal, then the approximate ratio of the income in 1996 and 1999 respectively is
 - (a) 1:1

(b) 2:3

(c) 9:10

(d) 13:14

- (e) Cannot be determined
- 6. If the expenditure in 2000 is 25% more than the expenditure in 1997, then the income in 1997 is what per cent less than the income in 2000?
 - (a) 22.5%
- (b) 25%
- (c) 27.5%
- (d) 31.25%
- (e) 32.5%
- 7. If the profit in 1999 was ₹ 4 crores, what was the profit in 2000?
 - (a) ₹ 4.2 crores
- (b) ₹ 6.6 crores
- (c) ₹ 6.8 crores
- (d) Cannot be determined (e) None of these 8. In which year was the expenditure minimum?
 - (a) 2000

(b) 1997

- (c) 1996
- (d) Cannot be determined (e) None of these

Solution

1. Let the expenditure in 1998 be \mathbb{Z} x.

$$\frac{264 - x}{x} \times 100 = 65$$

$$\Rightarrow \frac{264}{x} - 1 = \frac{65}{100}$$

$$\Rightarrow \frac{264}{x} = \frac{65}{100} + 1 = \frac{165}{100}$$

$$\Rightarrow x = \frac{264 \times 100}{165} = 160$$

- ∴ Expenditure in 1998 = ₹ 160 crores
- 2. Given % Profit = $\frac{Income Expenditure}{Expenditure} \times 100$

$$\Rightarrow \qquad \text{% Profit} = \left(\frac{\text{Income}}{\text{Expenditure}} - 1\right) \times 100$$

$$\Rightarrow \frac{\text{Income}}{\text{Expenditure}} = \frac{\% \text{ Profit}}{100} + 1$$

: Expenditure to be minimum, % profit should be minimum.

In the given years, % profit is minimum in 1997.

3. Average profit earned in the given years

$$= \frac{1}{6} (40 + 55 + 45 + 65 + 70 + 60)$$
$$= \frac{335}{6} = 55 \frac{5}{6}$$

4. Ratio of percentage profit earned to that in the previous year for the given years:

1996:
$$\frac{55}{40} = 1.375$$
; 1997: $\frac{45}{55} = 0.818$;
1998: $\frac{65}{45} = 1.414$; 1999 = $\frac{70}{65} = 1.077$;
2000: $\frac{60}{70} = 0.857$;

- ... Required ratio is minimum in 1997.
- 5. Let the expenditure be \mathbb{Z} *x* in 1996 and 1999. Let the income be \mathbb{Z} I_1 in 1996 and be \mathbb{Z} I_2 in 1999.

 $\frac{I_2}{r} = \frac{70}{100} + 1 = \frac{170}{100}$

 \Rightarrow

$$\therefore \qquad \% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$

$$\Rightarrow \qquad 55 = \frac{I_1 - x}{x} \times 100 \qquad \text{(in the year 1996)}$$

$$\Rightarrow \qquad 55 = \left(\frac{I_1}{x} - 1\right) \times 100$$

$$\Rightarrow \qquad \frac{I_1}{x} = \frac{55}{100} + 1 = \frac{155}{100}$$

$$\Rightarrow \qquad 70 = \frac{I_2 - x}{x} \times 100 \qquad \text{(in the year 1999)}$$

$$\Rightarrow \qquad 70 = \left(\frac{I_2}{x} - 1\right) \times 100$$

(2)

Dividing (1) by (2), we have

$$\frac{I_1}{I_2} = \frac{155}{170} = \frac{31}{34} = 9:10$$

6. Let expenditure in 1997 be $\stackrel{?}{\stackrel{?}{=}} x$

∴ Expenditure in 2000 = ₹ 1.25 xLet the income in 1997 be ₹ I_1 and that in 2000 be ₹ I_2 . In 1997,

$$45 = \left(\frac{I_1 - x}{x}\right) \times 100$$

$$\Rightarrow \frac{45}{100} = \frac{I_1}{x} - 1$$

$$\Rightarrow \frac{I_1}{x} = \frac{45}{100} + 1 = \frac{145}{100}$$

$$\Rightarrow I_1 = 1.45x \qquad (1)$$

$$60 = \left(\frac{I_2 - 1.25x}{1.25x}\right) \times 100$$

$$\Rightarrow \frac{60}{100} = \frac{I_2}{1.25x} - 1$$

$$\Rightarrow \frac{I_2}{1.25x} = \frac{60}{100} + 1 = \frac{160}{100}$$

$$\Rightarrow I_2 = \frac{160}{100} \times 1.25x$$

$$= 2x \qquad (2)$$

$$\therefore \frac{I_2 - I_1}{I_2} \times 100 = \frac{2x - 1.45x}{2x} \times 100\%$$

$$= \frac{0.55x}{2x} \times 100\% = 27.5\%$$

Required percentage = 27.5%.

7. Let the profit in 2000 be \mathbb{Z} x.

From the given line graph only % profit is available. Therefore, profit in 2000 cannot be determined.

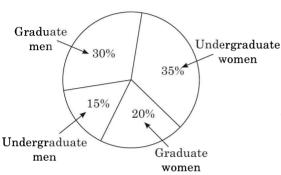
8. From the line graph, expenditure cannot be determined. Hence answer is (d).

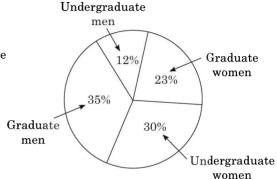
PIE-CHART: A pie-chart, also called a circle graph, is a circular chart that is divided into sectors. Each sector represents an item in a data to match the amount of item as a percentage of the total data. Pie-charts are useful to compare different parts of a whole amount.

EXAMPLE 3 Study the following pie-chart showing population of a town categorised according to their educational background.

Total population in 1991 = 40,000

Total population in 2001 = 1,20,000





- 1. What is the percentage increase of graduate men to total population from 1991 to 2001?
 - (a) 33%
- (b) 45%
- (c) 16.67%
- (d) 25%
- (e) None of these
- 2. What is the percentage increase in total population/year from 1991 to 2001?
 - (a) 1%
- (b) 2%
- (c) 3%
- (d) 4%
- (e) None of these
- 3. What is the total number of graduate men in 2001?
 - (a) 12000
- (b) 16000
- (c) 32000
- (d) 42000
- (e) None of these
- 4. What is the total number of undergraduate women in the year 2001?
 - (a) 16000
- (b) 32000
- (c) 36000
- (d) 42000
- (e) None of these
- 5. What is the ratio between graduate men in 1991 to that in 2001?
 - (a) 5:7
- (b) 6:7
- (c) 2:7
- (d) 3:7
- (e) None of these

Solution

1. % increase of graduate men to total population from 1991 to 2001

$$= \left(\frac{\frac{35\% \text{ of } 1,20,000}{100\% \text{ of } 1,20,000} - \frac{30\% \text{ of } 40,000}{100\% \text{ of } 40,000}}{\frac{30\% \text{ of } 40,000}{100\% \text{ of } 40,000}}\right) \times 100$$

$$= \left(\frac{\frac{35}{100} - \frac{30}{100}}{\frac{30}{100}}\right) \times 100 = \left(\frac{35 - 30}{30}\right) \times 100 = \frac{5}{30} \times 100$$

$$= \frac{100}{6} = 16 \frac{4}{6}\% = 16 \frac{2}{3}\% = 16.67\%$$

2. Required percentage =
$$\left(\frac{1,20,000-40,000}{40,000}\right) \times \frac{100}{10} = \frac{80,000}{40,000} \times 10 = 20\%$$

3. Total number of graduate men in 2001

= 35% of
$$120000 = \frac{35}{100} \times 1,20,000 = 42,000$$

4. Total number of undergraduate women in 2001

= 30% of
$$120000 = \frac{30}{100} \times 1,20,000 = 36,000$$

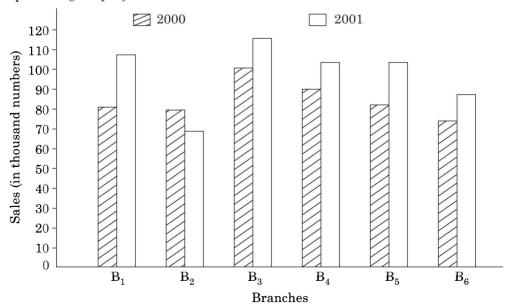
5.
$$\frac{\text{Number of graduate men in 1991}}{\text{Number of graduate men in 2001}} = \frac{30\% \text{ of } 40,000}{35\% \text{ of } 1,20,000} = \frac{\frac{30}{100} \times 40,000}{\frac{35}{100} \times 1,20,000}$$
$$= \frac{30}{35 \times 3} = \frac{10}{35} = \frac{2}{7} = 2:7$$

BAR DIAGRAM This section consists of questions in which data collected is represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted against horizontal axis and the other parameter is plotted against vertical axis.

You have to study the given information carefully and then answer the given questions on the basis of data analysis.

EXAMPLE 4 The bar-graph provided below gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph. [Bank PO]

Sales of Books (in thousand numbers) from six branches B_1 , B_2 , B_3 , B_4 , B_5 and B_6 of a publishing company in 2000 and 2001.



1.	Total sales	of	branches	В1,	B_3	and	B_5	together	for	both	the	years	(in	thousand
	numbers) i	s												

(a) 250

(b) 310

(c) 435

(d) 560

(e) 585

2. Total sales of branch B_6 for both the years is what per cent of the total sales of branch B_3 for both the years?

(a) 68.54%

(b) 71.11%

(c) 73.17%

(d) 75.55%

(e) 77.26%

3. What is the average sale of all the branches (in thousand numbers) for the year 2000?

(a) 73

(b) 80

(c) 83

(d) 88

(e) 96

4. What is the ratio of total sales of branch B_2 for both years to the total sales of branch B_4 for both years?

(a) 2:3

(b) 3:5

(c) 4:5

(d) 5:7

(e) 7:9

5. What per cent of the average sales of branches B_1 , B_2 and B_3 in 2001 is the average sales of branches B_1 , B_3 and B_6 in 2000?

(a) 75%

(b) 77.5%

(c) 82.5%

(d) 85%

(e) 87.5%

Solution

1. Total sales of branches B_1 , B_3 and B_5 together for both the years (in thousand numbers) = (80 + 105) + (95 + 110) + (75 + 95) = 560

2. Required percentage =
$$\frac{(70+80)}{(95+110)} \times 100\% = \frac{150}{205} \times 100\% = 73.17\%$$

3. Average sale of all the branches (in thousand numbers) for the year 2000

$$= \frac{1}{6} (80 + 75 + 95 + 85 + 75 + 70) = \frac{1}{6} \times 480 = 80$$

4. Required ratio =
$$\frac{(75+65)}{(85+95)} = \frac{140}{180} = \frac{7}{9}$$

5. Average sales of branches $B_1,\,B_2$ and B_3 in 2001 (in thousand numbers)

$$= \frac{1}{3}(105 + 65 + 110) = \frac{280}{3}$$

Average sales of branches B₁, B₃ and B₆ in 2000 (in thousand numbers)

$$= \frac{1}{3} (80 + 95 + 70) = \frac{245}{3}$$

:. Required percentage =
$$\frac{(245/3)}{(280/3)} \times 100 = \frac{245}{280} \times 100$$

= $\frac{49}{56} \times 100 = \frac{700}{8} = 87.5\%$



Directions (Q. 1-5): The following table contains the percentage of literate population with respect to the total population of different states over the years.

States	1997	1998	1999	2000	2001	2002
A	67	72	69	78	79	82
В	82	81	85	85	87	88
\mathbf{C}	78	81	84	87	89	91
D	56	65	69	71	75	77
${f E}$	89	93	94	95	95	97
\mathbf{F}	85	88	91	93	95	96

1.	The populations of state C in 1997 and 1998 were in the ratio 2:3. What was the ratio o
	heir literate population?

- (a) 51:82
- (b) 15:18
- (c) 9:13
- (d) 26:27
- (e) None of these
- 2. In 1998, all the six states had equal population of 12 lakhs. What was the average literate population of all the states in that year?
 - (a) 988000
- (b) 1056000
- (c) 960000
- (d) 1028000
- (e) None of these
- 3. The populations of state D in 1998 and 2001 were equal. Approximately what was the percentage increase in literate population from 1998 to 2001?
 - (a) 17
- (b) 15
- (c) 16
- (d) 18
- (e) 19
- 4. The population of state B increased every year by 10%. If its total population in 1999 was 10 lakhs, what was its literate population in 2001?
 - (a) 1052700

(b) 957000

(c) 1152400

- (d) Cannot be determined
- (e) None of these
- 5. If the average population of all the states in 1999 was 12.5 lakhs, then what was the average literate population in that year?
 - (a) 10.25 lakhs
- (b) 10.75 lakhs
- (c) 11 lakhs

- (d) Cannot be determined
- (e) None of these

Directions (Q. 6-10): Study the following table carefully and answer the given questions. The details about the strength of six management institutes and the number of students getting placement over the years.

Institute -	A		В		C		D		\mathbf{E}		F	
Year	S	P	S	P	S	P	S	P	S	P	\mathbf{S}	P
1998	480	350	525	470	350	300	560	500	380	320	400	300
1999	540	380	580	495	425	360	625	550	480	440	450	380
2000	580	400	700	600	410	370	580	540	450	420	480	420
2001	680	420	675	590	440	400	550	510	520	490	520	480
2002	650	450	750	680	520	480	600	530	500	470	550	500
2003	720	480	780	700	500	470	630	540	540	500	575	515
S = Strengt	h. <i>P</i>	= Num	ber of	studer	its geti	ing pl	acemei	nt.				

highest? (a) 1998

(a) 80

(a) F

(a) 12.59

the year 2002?

institutes together in year 2000?

(b) 1999

(b) 78

(b) E

(b) 13.12

(b) 709:574

six different so	hools.				·	J
	Schools	% of minor	students	Proportion of	boys and girls	
				Minor B : G	Major B : G	
	A	92		12:11	3:1	
	В	96		5:3	1:1	
	C	68		3:1	5:3	
	D	84		2:5	3:1	
	E	88		3:1	7:5	
	F	80		2:3	3:2	
	Minor: Stu	dents whose ag	e is less tha	n 18.		
	age height of	all minor stude (b) $5\frac{1}{2}$	nts of schoo		ainor boys is 5 ft 6 $\frac{2}{5}$ ft) in. Find
12. The nun 18.2 year	nber of stud		B is 1000		ge age of major ecomes 20 years.	•
(a) 36 ye	ears (b)	40 years ((c) 56 years	(d) 46 yea	rs (e) None	of these
13. The num school.	ber of minor	girls in school	C is 153. I	Find the total nu	ımber of student	s in that
(a) 800	(b)	850 ((c) 900	(d) 1000	(e) None	of these
14. There are school?	e 240 minor b	oys in school D.	What is the	number of girls	who are not mino	rs in that
(a) 45	(b)	40	(c) 60	(d) 75	(e) None	of these

6. During which year the percentage of students getting placement from Institute B was the

7. Approximately what is the overall percentage of students getting placement from all the

8. The percentage of students getting placement was the highest for which of the institutes in

9. What is the percentage increase in the percentage of students getting placement from Institute

(c) 12.89

(c) 574:709

Directions (Q. 11-15): Study the following table and answer the questions carefully. The following table shows the percentage of minor students and the proportion of boys and girls in

(d) 2002

(d) 89

(d) C

(d) 13.29

(d) 579:709

(e) None of these

(e) None of these

(e) None of these

(e) None of these

(c) 2001

(c) 75

(c) B

10. What is the ratio of the average strength of Institute E to that of Institute D?

F from 1998 to 1999 (rounded off to two digits after decimal)?

15. If the total number of students in school F is 1250, then what is the number of girls in that school? (b) 700 (a) 600 (c) 650 (d) 750 (e) None of these Directions (Q. 16-20): Study the following table and information to answer the questions. Number of boys participating in different games Games → Cricket Hockey Chess Badminton Football Classes 4 VII 22 18 24 6 12 VIII 20 16 16 12 8 IX24 8 2212 14 X 18 20 22 8 6 XI8 12 14 14 18 XII 6 24 12 20 8 Information: (a) Every student of each class participates in a game. (b) Each student participates in one game only. (c) The total number of boys of the given classes is 444. (d) In each game, from every class, the number of girls participating is 50% of the number of boys participating in each game from the same class. 16. The girls of class VII and the boys of class XII together form what percentage of total number of students of the school? (c) $16\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$ (e) None of these (a) 25% (b) 20% 17. Before starting a match, every girl hockey player of class X shakes hands with every other girl hockey players of that class. What would be the total number of hand shakes? (a) 66 (b) 55 (c) 231 (d) 253 (e) None of these 18. A committee of 5 students is to be formed out of the total badminton players of class XII. In how many ways can it be done so that a particular member is always included? (a) 70 (b) 56 (c) 126 (d) 66 (e) None of these 19. How many boys can't be paired for a game, if boys of classes IX and X are paired with girls of their classes? (a) 72 (b) 62 (c) 67 (d) 77 (e) None of these 20. How many students of the given classes play football? (b) 98 (c) 108 (d) 78 (e) None of these 21. In an office, the distribution of work hours is as shown in the following table. [IAS]

No. of staff members No. of hours worked 5 0-19 12 20-24 25 25-29 40 30-34 15 35-39 8 40-45		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	No. of staff members	No. of hours worked
25 25–29 40 30–34 15 35–39	5	0–19
40 30–34 15 35–39	12	20 – 24
15 35–39	25	25 - 29
	40	30–34
8 40–45	15	35-39
	8	40–45

Consider the following inferences drawn from the table.

- (i) The average number of hours worked by a staff member is about 30.
- (ii) The percentage of those who worked 35 or more hours is less than 25.
- (iii) At least 5 staff members worked more than 44 hours.

Which of these inference is valid?

- (a) (i) alone
- (b) (ii) alone
- (c) (i) and (ii)
- (d) (i), (ii), and (iii)
- 22. Consider the table given below providing details of traffic volume per hour for four locations.

[IAS]

Location	Total traffic volume	% of heavy vehicle	Average noise level in dB(A)	Noise pollution level in dB(A)
I	377	24.40	73.50	84.00
II	380	12.50	72.60	83.00
III	377	30.00	73.50	86.50
IV	225	12.50	72.98	80.90

When the total traffic volume is the same, the factor(s) which affect(s) the noise pollution level is/are:

- (a) % of heavy vehicles.
- (b) Noise pollution level and average noise level.
- (c) Average noise level and % of heavy vehicles.
- (d) Indeterminable on the basis of details given.

Directions (Q. 23–27): Study the following table, which contains classification of 100 students based on the marks obtained by them in physics and chemistry in an examination, carefully and answer the questions given below: [Bank PO]

Marks out of 50 Subject	40 and above	30 and above	20 and above	10 and above	0 and above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Average	7	27	73	87	100

- 23. The number of students scoring less than 40% marks in aggregate is
 - (a) 13
- (b) 19
- (c) 20
- (d) 27
- (e) 34
- 24. If at least 60% marks in physics is required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics?
 - (a) 27
- (b) 32
- (c) 34
- (d) 41
- (e) 68
- 25. What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate?
 - (a) 3
- (b) 4
- (c) 5
- (d) 6
- (e) 7
- 26. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately
 - (a) 21%
- (b) 27%
- (c) 29%
- (d) 31%
- (e) 34%

- 27. If it is known that at least 23 students were eligible for a symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to symposium would lie in the range
 - (a) 40-50

(b) 30-40

(c) 20-30

(d) Below 20

(e) Cannot be determined

Directions (Q. 28-30): Refer to the following table and answer the questions that follow: Distribution of marks obtained by 100 students in two papers (I and II) in Mathematics

Marks out of 50	40 and above	30 and above	20 and above	10 and above	0 and above
I	5	22	67	82	100
II	8	31	79	91	100
Aggregate (Average)	6	27	71	88	100

- 28. What should be the passing marks if minimum 80 students are required to be qualified with compulsory passing only in Paper I?
 - (a) Below 20
- (b) Above 20
- (c) Below 40
- (d) Above 40
- (e) None of these
- 29. What will be the difference between the number of students passed with 30 as cut-off marks in Paper II and the number of students passed with same cut-off marks in aggregate?
 - (a) 2
- (b) 4
- (c) 8
- (d) 3
- (e) None of these
- **30.** What is the approximate percentage of students who have obtained 60% and more marks in paper II over the number of students who obtained 40% and more marks in aggregate?
 - (a) 44

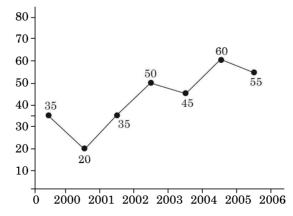
(b) 40

(c) 48

- (d) Data inadequate
- (e) None of these

Directions (Q. 31–35): The following graph shows the percentage net profit of a certain company during the given period. Study it carefully and answer the questions given below

[Bank PO]



- 31. If the expenditure in 2002 was 20% more than the expenditure in 2000, by what per cent the income in 2002 was more/less than the income in 2000?
 - (a) 25% less

- (b) 20% more
- (c) 27% more

- (d) Data inadequate
- (e) None of these
- 32. During which of the following years was the ratio of income to expenditure the minimum?
 - (a) 2000
- (b) 2003
- (c) 2004
- (d) 2005
- (e) None of these

- 33. During which year the ratio of percentage net profit earned to that in the previous year was the minimum?
 - (a) 2002

(b) 2003

(c) 2005

- (d) 2002 and 2003 both
- (e) 2002, 2003 and 2005
- 34. If the expenditure in 2004 and 2006 was equal and income in 2006 was 15.5 lakhs, what was the income in 2004?
 - (a) 12.5 lakhs
- (b) 13.5 lakhs
- (c) 14 lakhs

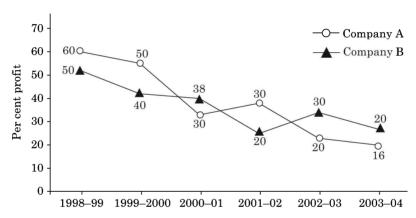
- (d) Data inadequate
- (e) None of these
- 35. If the income in 2003 was ₹ 25 lakhs, what was the expenditure in that year?
 - (a) 16 lakhs

- (b) 16.33 lakhs
- (c) 16.67 lakhs

- (d) 15.67 lakhs
- (e) None of these

Directions (Q. 36-40): Study the following graph to answer the given questions Per cent profit earned by two companies over the given years.

Per cent profit =
$$\frac{Income - Expenditure}{Expenditure} \times 100$$



- 36. In 2000-01, the profit of company B was ₹ 10.5 crore. What was its income in that year?
 - (a) ₹30 crores
- (b) ₹ 43.5 crores
- (c) ₹ 40.5 crores

- (d) Can't be determined
- (e) None of these
- 37. If the income of company A in 1999–2000 was equal to the expenditure of company B in the same year, then the ratio of their respective profits was
 - (a) 5:4

(b) 5:6

(c) 15:14

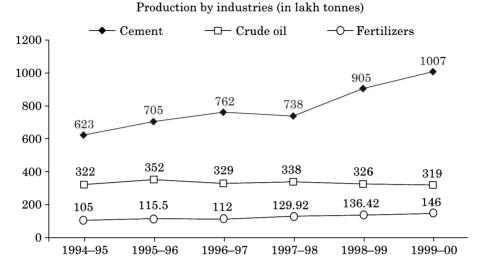
- (d) Can't be determined
- (e) None of these
- 38. In a certain year, the expenditure and income of company A were ₹ 80 crores and ₹ 92 crores respectively. This can be true for which of the following years?
 - (a) 2000-01
- (b) 2001-02
- (c) 2002-03
- (d) 2003-04
- (e) 1999–2000
- 39. In how many of the given years, the income of company A was more than that of company B?
 - (a) Three

(b) Two

(c) Four

- (d) Can't be determined
- (e) None of these
- **40.** In a certain year, the income and profit of company B were ₹ 52 crores and ₹ 12 crores respectively. This can be true for which of the following years?
 - (a) 1999-2000
- (b) 2000-01
- (c) 2001–02
- (d) 2002-03
- (e) None of these

Directions (Q. 41-45): Study the following graph and answer the questions carefully



- 41. The difference between the production of crude oil in 1995-96 and that in 1999-2000 was exactly equal to the difference between the production of cement in which of the following pairs of years?
 - (a) 1994-95 and 1997-98
- (b) 1994–95 and 1999–00
- (c) 1995-96 and 1997-98

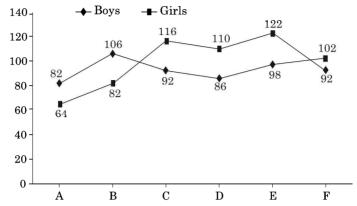
- (d) 1998-99 and 1999-00
- (e) None of these
- 42. In which of the following years was the percentage fluctuation (increase/decrease) in fertilizers the minimum from the previous year?
 - (a) 1994-95
- (b) 1996–97
- (c) 1999–2000
- (d) 1997–98
- (e) 1998-99
- 43. What was the difference between the average cement production and the average crude oil production during the given years (in lakh tonne)?
 - (a) 459
- (b) 460
- (c) 560
- (d) 469
- (e) 559
- 44. If there was 15% increase in the fertilizers production from 1999-2000 to 2000-01, what would be the difference between the cement and fertilizers production in 2000-01 (in lakh tonnes)?
 - (a) 839.10

(b) 990.15

(c) 1012.05

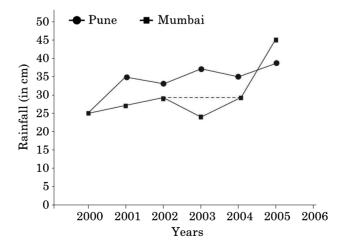
- (d) Can't be determined
- (e) None of these
- 45. Find the approximate percentage increase in cement production from 1994-95 to 1999-2000.
 - (a) 55%
- (b) 60%
- (c) 65%
- (d) 69%
- (e) 58%

Directions (Q. 46-50): Study the following graph carefully and answer the questions that follow. The following diagram shows the passed candidates of class X in annual examination of six schools in a city.



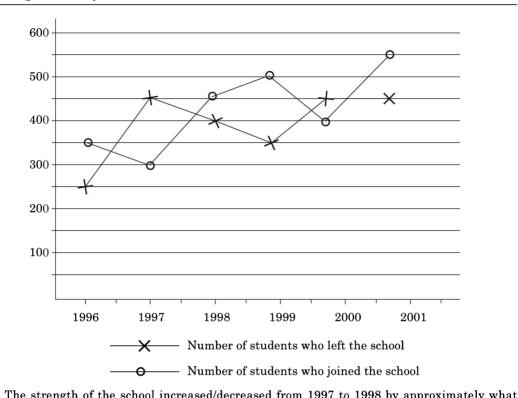
- 46. Three-fourths of the number of boys of school C who passed is approximately what per cent more than four-sevenths of boys of school E who passed?
 - (a) 18%
- (b) 20%
- (c) 23%
- (d) 26%
- (e) 32%
- 47. If the total number of candidates appearing from schools A and F was 550, then find the approximate percentage of students failed in schools A and F together.
 - (a) 28%
- (b) 32%
- (c) 36%
- (d) 38%
- (e) 42%
- 48. In how many schools the number of boys passed in above average of the number of boys passed in all the six schools?
 - (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5
- 49. If total number of boys and girls appeared in school B are 150 and 120 respectively, then find the per cent difference of their passing.
 - (a) 2.33%
- (b) 5.3%
- (c) 8%
- (d) 12%
- (e) 7%
- 50. What is the difference between average number of boys and girls passed for all the schools?
 - (a) 2.6
- (b) 3.3
- (c) 4.9
- (d) 6.8
- (e) 7.2

Directions (Q. 51–55): Study the following graph carefully and answer the questions given below. Rainfall (in cm) in the cities of Mumbai and Pune over the given years.



51. What is the ratio of rainfall in the city of Mumbai to that in Pune in the year 2002? (a) 13:15 (b) 15:19 (c) 15:17 (d) 13:17 (e) 15:34 **52.** Which year showed the maximum rainfall in both the cities? (a) 2002 (b) 2004 (c) 2001 (e) None of these 53. What is the average rainfall in the city of Mumbai over the years? (b) 28 cm (a) 30.5 cm (c) 35 cm (d) 26 cm (e) 38 cm 54. The rainfall in the city of Pune in the year 2001 is approximately, what per cent of the rainfall over the given years in the same city? (b) 17 (d) 25 (e) 11 55. What is the difference in the rainfall of the two cities in the year 2003? (a) 10 cm (b) 11 cm (c) 18 cm (d) 13 cm (e) 16 cm **Directions (Q. 56-60):** Answer the questions based on the line-graph below. Ratio of Exports to Imports (in terms of money in ₹ crores) of two companies over the years. 2.25 -2.00 1.75 1.50 1.25 1.00 0.750.50 0.25 0 1995 1996 1997 1998 1999 2000 Company A — Company B **56.** In how many of the given years were the exports more than the imports for company A? (b) 3 (e) 6 (a) 2 (c) 4 (d) 5 57. In which year(s) was the difference between imports and exports of Company B the maximum? (b) 1996 (c) 1998 and 2000 (d) Cannot be determined (e) None of these 58. If the exports of Company A in 1998 were ₹237 crores, what was the amount of imports in that year? (a) ₹ 189.6 crores (b) ₹ 243 crores (c) ₹ 281 crores (d) ₹ 316 crores (e) None of these 59. If the imports of Company A in 1997 were increased by 40 per cent, what would be the ratio of exports to the increased imports? (a) 1.20 (b) 1.25 (c) 1.30 (d) None of these (e) Cannot be determined 60. In 1995, the export of Company A was double that of Company B. If the imports of Company A during the year was ₹ 180 crores, what was the appropriate amount of imports of Company B during the year? (a) ₹ 190 crores (b) ₹ 210 crores (c) ₹ 225 crores (e) None of these (d) Cannot be determined

Directions (Q. 61-67): Study the following line-graph which gives the number of students who joined and left the school in the beginning of year for six years from 1996 to 2001. Initial strength of the school in 1995 = 3000. The questions given below the graph are based on the line-graph.



01.	per cent?	of the sensor mere	ascu/accreascu ire	m 1007 to 1000 t	y approximately w	пас
	(a) 1.2%	(b) 1.7%	(c) 2.1%	(d) 2.4%	(e) 2.6%	
62 .	The number o	of students studyin	g in the school dur	ing 1999 was:		
	(a) 2950	(b) 3000	(c) 3100	(d) 3150	(e) 3200	
63.	During which	of the following pa	airs of years, the st	rength of the scho	ool was same?	
	(a) 1999 and(d) 1996 and	2001 2000			(c) 1997 and 1	998
64.		of students studyin ying in the school :	U	1998 was what per	cent of the numbe	r of
	(a) 92.13%	(b) 93.75%	(c) 96.88%	(d) 97.25%	(e) 99%	
65.	Among the give	ven years, the larg	est number of stud	lents joined the sc	hool in the year.	
	(a) 1996	(b) 1998	(c) 1999	(d) 2000	(e) 2001	
66.	•	ar, the percentage he previous year, i		umber of student	s, who left the sch	nool
	(a) 1997	(b) 1998	(c) 1999	(d) 2000	(e) 2001	

Directions (Q. 68-72): The following line-graph gives the percentage of the number of candidates who qualified an examination out of the total number of candidates who appeared for the examination over a period of seven years from 1994 to 2000. Study the graph and answer the questions based on it.

[Bank PO]

(c) 3:4

67. The ratio of the least number of students who joined the school to the maximum number of

(d) 9:11

students who left the school in any of the years during the given period is:

(b) 4:5

Percentage of Candidates Qualified to Appear in an Examination Over the Year.



- 68. The difference between the percentages of candidates qualified to appear was maximum in which of the following pairs of years?
 - (a) 1994 and 1995
- (b) 1997 and 1998
- (c) 1998 and 1999

- (d) 1999 and 2000
- (e) 1994 and 1997
- 69. In which pair of years was the number of candidates qualified the same?
 - (a) 1995 and 1997
- (b) 1995 and 2000
- (c) 1998 and 1999

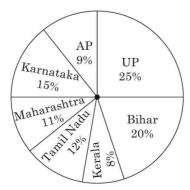
- (d) 1996 and 2000
- (e) Data inadequate
- 70. If the number of candidates qualified in 1998 was 21200, what was the number of candidates appeared in 1998?
 - (a) 32000
- (b) 28500
- (c) 26500
- (d) 25000
- (e) 24500
- 71. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates qualified in these two years together was:
 - (a) 34700

(b) 32100

(c) 31500

- (d) None of these
- (e) Data inadequate
- 72. The total number of candidates qualified in 1999 and 2000 together was 33500, and the number of candidates appeared in 1999 was 26500. What was the number of candidates appeared in 2000?
 - (a) 24500
- (b) 22000
- (c) 20500
- (d) 19000
- (e) 18500

Directions (Q. 73-79): Study the following graph and table and answer the following questions given below.



together?
(a) 4.5 lakh

(a) 20%

The total population of the different states in 1993 was 25 lakhs.

Sex-and-literacy wise population ratio

States	Sex M : F	Literacy Literate – Illiterate
UP	5:3	2 : 7
Bihar	3:1	1 : 4
AP	2:3	2 : 1
Karnataka	3:5	3 : 2
Maharashtra	3:4	5 : 1
Tamil Nadu	3:2	7 : 2
Kerala	3:4	9 : 4

73. Approximately what is the total number of literate people in Maharashtra and Karnataka

74. Approximately what will be the percentage of total male in UP, Maharashtra and Kerala of

(d) 3.5 lakh

(4) 30%

(e) 6 lakh

(a) 25%

(c) 3 lakh

(a) 28%

M = Male, F = Female

(b) 8.5 lakh

• Statewise distribution of candidates appeared. Total candidates appeared = 120000

the total population of the given states?

(b) 18%

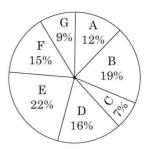
	(a) 20/0	(D) 1070	(C) 2070	(u) 5070	(E) 4070
75.	•				% of Bihar compared to Bihar in the year
	(a) 50:40	(b) 40:50	(c) 48:55	(d) 55:48	(e) None of these
76.	What was the app Nadu?	roximate percenta	age of women of An	ndhra Pradesh to t	the women of Tamil
	(a) 90%	(b) 110%	(c) 120%	(d) 85%	(e) 95%
77.	What is the ratio Kerala?	of the number of	f females in Tami	l Nadu to the nu	mber of females in
	(a) 1:2	(b) 1:1	(c) 2:1	(d) 2:3	(e) None of these
78.	In Tamil Nadu, if the total number of			75% of the males	are literate, what is
	(a) 75000	(b) 85000	(c) 71000	(d) 81000	(e) None of these
79 .	What is the ratio	of literates in And	lhra Pradesh to the	e literates in Biha	ır?

Directions (Q. 80-84): Study the following data carefully to answer these questions?

• Details of statewise distribution of candidates appeared in an examination.

Ratio of male and female candidates appeared

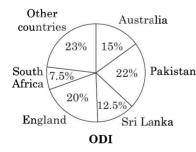
State	Ratio
	M : F
A	5:7
В	11:8
\mathbf{C}	3:4
D	9:7
${f E}$	5:6
\mathbf{F}	8:7
\mathbf{G}	4:5

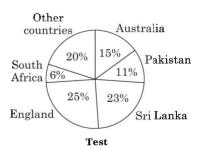


- 80. The number of female candidates appeared from state C is equal to the number of male candidates appeared in the examination from which state?
 - (a) A
- (b) B
- (c) D
- (d) E
- (e) G
- 81. What is the ratio of number of male candidates appeared in the examination from state A to that from state E?
 - (a) 1:2
- (b) 1:1
- (c) 2:1
- (d) 6:7
- (e) None of these
- 82. The total number of female candidates appeared from states E and G together is what per cent of the total number of candidates appeared in the examination from all the states?
 - (a) 13
- (b) 11
- (c) 17
- (d) 15
- (e) None of these
- 83. What is the total number of male candidates appeared in the examination from states B and F together?
 - (a) 16800
- (b) 18400
- (c) 13200
- (d) 22800
- (e) None of these
- 84. The number of female candidates appeared from state D is what per cent of the total number of candidates appeared in the examination from all the states?
 - (a) 9
- (b) 8
- (c) 7.5
- (d) 8.5
- (e) None of these

Directions (Q. 85-89): Study the following pie charts and answer the questions that follow:

The following pie charts show the runs scored by a batsman against different countries in One-Day Internationals (ODI) and Test matches:





The runs scored by the batsman in ODI and Test matches are 8000 and 6000 respectively.

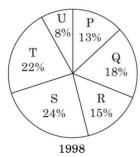
- 85. In the case of which of the following countries was there minimum difference in runs scored in ODI and that in Test matches?
 - (a) Australia

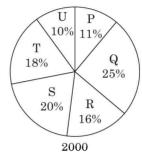
- (b) Sri Lanka
- (c) England

- (d) South Africa
- (e) Pakistan
- 86. Find the ratio of runs scored against South Africa in ODI to that against England in Test Matches
 - (a) 3:10
- (b) 2:5
- (c) 10:3
- (d) 3:4
- (e) None of these

- 87. If the batsman played 29 innings against Sri Lanka in ODI and remained not out in 4 innings, find his average runs scored against Sri Lanka in ODI.
- (b) 34.5
- (c) 34.48
- (d) 34.92
- (e) None of these
- 88. If the batsman played 30 innings against Australia in ODI, how many runs must be make in his next innings (against Australia) so as to raise his average to 42 against that country?
- (b) 62
- (c) 40
- (d) 122
- 89. Find the total runs scored by the batsman in Test matches and ODI against Pakistan
 - (a) 2240
- (b) 3080
- (c) 1540
- (d) 2420
- (e) None of these

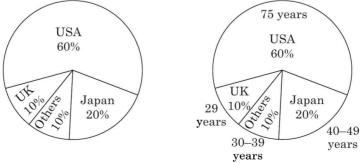
Directions (Q. 90-93): Study the given pie diagrams carefully and answer the questions that follow.





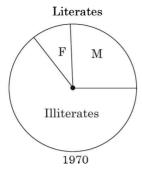
The given pie diagrams show the number of workers of different categories of a factory for the two different years. The total number of workers in 1998 and 2000 are 2000 and 2400 respectively.

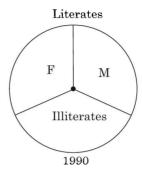
- 90. Find the percentage increase in the number of workers in category U in 2000.
 - (a) 25%
- (b) 50%
- (c) $33\frac{1}{3}\%$ (d) $66\frac{2}{3}\%$
- 91. What is the total number of increased workers for the categories in which the number of workers has increased?
 - (a) 468
- (b) 404
- (c) 382
- (d) None of these
- 92. Which category has shown decrease in the number of workers from 1998 to 2000?
- (b) R
- (c) S
- (d) None of these
- 93. Find the maximum difference between the number of workers of any two categories taken together for any one year and that of any two for the other year.
 - (a) 660
- (b) 636
- (c) 416
- 94. The distribution of 100000 tourists who visited India during a particular year is shown in the given charts. Based on this, the number of Japanese tourists below the age of 39 who visited India in the year concerned is. [IAS]



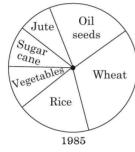
- (a) 10000
- (b) 8000
- (c) 6000
- (d) 4000

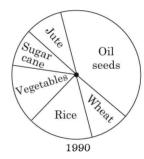
95. The given pie charts show the proportion of literates and illiterates in a country, in the years 1970 and 1990, and also the proportion of males (M) and females (F) among the literates. Which one of the following statements can be said to be beyond any doubt? [IAS]





- (a) In 1970 half of the illiterates were women.
- (b) The proportion of literate males to the total population of males remained the same over the years.
- (c) The male literacy did not improve over this period.
- (d) The ratio of female literates to male literates improved significantly over this period.
- 96. Consider the following pie chart:

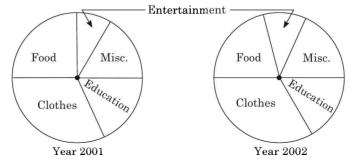




The pie charts given above depict the area under cultivation of different cash crops, for two years, 1985 and 1990. Which one of the following statement is FALSE? [IAS]

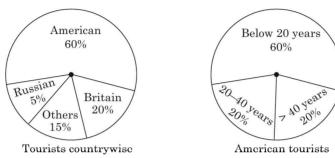
- (a) In 1985, the area under jute cultivation was less than that under rice cultivation.
- (b) More land was used for growing vegetables in 1990 than that of 1985.
- (c) In 1990, the area under rice and wheat cultivation taken together was less than 1/2 the total cultivated area.
- (d) The percentage area under jute cultivation was less in 1988 than in 1990.
- **97.** Which cannot be concluded from the following figures?

[IAS]



- (a) The expenditure towards entertainment is less.
- (b) The expenditure is same in both years.
- (c) The family spends more on clothes.
- (d) Each years expenditure on food is almost 25%.
- 98. The characteristics of foreign tourists in one year is given in the following pie chart [IAS]

 Total number of tourists = 100000

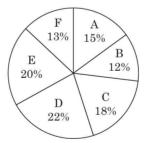


From the chart, the number of American tourists in the age group 20-40 who visited during the year is _____.

- (a) 12000
- (b) 20000
- (c) 40000
- (d) 60000

Directions (Q. 99-104): Study the following pie chart carefully and answer to the questions that follow.

There are six companies which are responsible for the production of two items. The pie chart shows the percentage of total production by six companies.



The cost of the total production (both items together) by six companies = ₹ 40 crores. The following table shows the ratio of production between items I and II and the per cent profit earned for the two items.

Company	Ratio of 1	production	% profit earned	
	Item I	Item II	Item I	Item II
A	2	1	25	30
В	7	5	35	15
\mathbf{C}	3	5	20	25
D	1	3	28	20
${f E}$	1	3	24	32
F	6	7	25	35

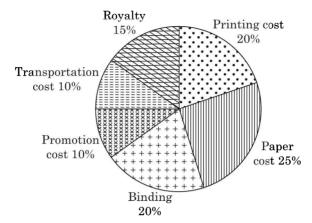
	(a) 1.166	(b) 1.665	(c)	1.125	(d)	1.765	(e) None of these
100.	What is the total	al cost of the pro	oduc	tion of item	I by	companies	B and F together
	(in ₹ crore)?						
	(a) 10	(b) 4.8		5.2	(d)		(e) 9
101.	The cost of production by company A on		comp	any D is what	t per c	ent more tha	in the profit earned
	(a) 550%	(b) 580%	(c)	560%	(d)	490%	(e) None of these
102.	What is the total p on item I (in ₹ cro		mpa	ny E on item l	II and	the profit ea	rned by company B
	(a) 2.80	(b) 1.90	` ′	3.10	(d)		(e) None of these
103.	What is the ratio of C?	of the cost of produ	ction	n of item I of co	ompar	y F to that of	fitem II of company
	(a) 15:8	(b) 8:13	(c)	4:7	(d)	48 : 65	(e) None of these
104.	Find the differentitem I.	ce in the profit ea	arne	d by company	D on	item II and	l by company A on
	(a) 32 lakhs	(b) 320 lakhs	(c)	$0.032 \mathrm{\ crore}$	(d)	3.2 crore	(e) None of these
Dire belov		9): Study the follo	owin	g pie-chart car	refully	and answer	the questions given
							and the candidates
(Out	of those enrolled)	who passed the ex	am 1	in different ins	stitut	es.	[Bank PO]
	Candidat	es enrolled = 8550)	(dates who pa e exam = 570	
	S 179 8%	P			S 16%		P 8%
105.	candidates enrolle	ed from the same i	insti	tute?			the total number of
	(a) 50%	(b) 62.5%		75%	(d)		
106.	What is the ratio						nstitute P?
105	(a) 9:11	(b) 14:17	• •	6:11		9:17	r · 0 1
107.	R together?						for institutes Q and
	(a) 68%	(b) 80%		74%	(d)		
108.				-	_		andidates enrolled?
100	(a) Q	(b) R	(c)		(d)		1 41 1 1
109.	candidates enrolle	ed from institutes	T ar	nd R together	by:		eeds the number of
	(a) 228	(b) 279	(c)	399	(d)	407	

99. What is the total profit earned by company C for items I and II together (in ₹ crore)?

Directions (Q. 110-118): The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and answer the questions based on it.

[Bank PO]

Various Expenditure (in percentage) incurred in publishing a book.



- 110. What is the central angle of the sector corresponding to the expenditure incurred on royalty?
 - (a) 15°
- (b) 24°
- (c) 54
- (d) 48°
- 111. Which two expenditures together have a central angle of 108°?
 - (a) Binding cost and transportation cost
- (b) Printing cost and paper cost

(c) Royalty and promotion cost

- (d) Binding cost and paper cost
- 112. If for an edition of the book, the cost of paper is ₹ 56250, then find the promotion cost for this edition.
 - (a) ₹ 20000
- (b) ₹ 22500
- (c) ₹ 25500
- (d) ₹ 28125
- 113. If the difference between the two expenditures are represented by 18° in the pie-chart, then these expenditures possibly are:
 - (a) Binding cost and promotion cost

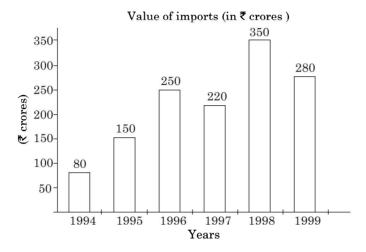
(b) Paper cost and royalty

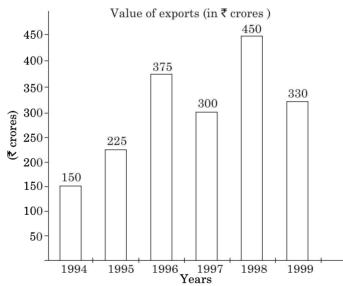
(c) Binding cost and printing cost

- (d) Paper cost and printing cost
- 114. If for a certain quantity of books, the publisher has to pay ₹ 30600 as printing cost, then what will be the amount of royalty to be paid for these books?
 - (a) ₹ 19450
- (b) ₹ 21200
- (c) ₹ 22950
- (d) ₹ 26150
- 115. The price of the book is marked 20% above the C.P. If the marked price of the book is ₹ 180, then what is the cost of paper used in a single copy of the book?
 - (a) ₹36
- (b) ₹37.50
- (c) ₹42
- (d) ₹ 44.25
- 116. For an edition of 12500 copies, the amount of royalty paid by the publisher is ₹ 281250. What should be the selling price of the book if the publisher desires a profit of 5%?
 - (a) ₹ 152.50
- (b) ₹ 157.50
- (c) ₹ 162.50
- (d) ₹ 167.50
- 117. If 5500 copies are published and the transportation cost on them amounts to ₹82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?
 - (a) ₹ 187.50
- (b) ₹ 191.50
- (c) ₹ 175
- (d) ₹ 180
- 118. Royalty on the book is less than the printing cost by:
 - (a) 5%
- (b) $33\frac{1}{3}\%$
- (c) 20%
- (d) 25%

Directions (Q. 119-123): Study the following graphs carefully and answer the questions that follow: [Bank PO]

The value of imports and exports by a company over the years.



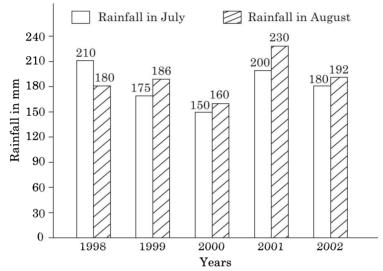


- 119. The value of exports in 1998 was, what percentage of the average value of imports in the years 1996, 1997 and 1999.
 - (a) 200
- (b) 100
- (c) 300
- (d) 150
- (e) None of these
- 120. The value of exports in 1996 was exactly, what percentage of the value of imports in the same year?
 - (a) 125
- (b) 160
- (c) 200
- (d) 75
- (e) None of these
- 121. What was the approximate difference between the value of average exports and the value of average imports of the given years?
 - (a) ₹85 cr.
- (b) ₹ 100 cr.
- (c) ₹75 cr.
- (d) ₹90 cr.
- (e) ₹80 cr.

- 122. In which of the following years was the difference between the value of exports and the value of imports exactly ₹ 100 crores?
 - (a) 1995
- (b) 1998
- (c) 1997
- (d) 1999
- (e) None of these
- 123. What was the percentage increase in the value of exports from 1997 to 1998?
 - (a) 150
- (b) 100
- (c) 75
- (d) 50
- (e) None of these

Directions (Q. 124-128): Study the following bar chart carefully and answer the questions that follow.

The bar chart shows the amount of rainfall (in mm) in a town in the months of July and August for different years.



- 124. In August 1999, the average rainfall of first 16 days was 5 mm and that of last 16 days was 8 mm. Find the rainfall (in mm) of 16 August 1999.
 - (a) 6
- (b) 16
- (c) 32
- (d) 22
- (e) None of these
- 125. For the given town, what is the percentage increase in the total rainfall of July and August from 2000 to 2001?
 - (a) 36.7
- b) 38.7
- (c) 32.6
- (d) 39.6
- (e) 37.6
- 126. The highest rainfall in a single month of August (among the given years) is how much more/less than the average rainfall of July for the given years?
 - (a) 37 mm more
- (b) 47 mm more
- (c) 37 mm less

- (d) 47 mm less
- (e) None of these
- 127. The ratio of rainfall in July to that in August is the same for which of the following years?
 - (a) 1998 and 2000
- (b) 2000 and 1999
- (c) 2000 and 2002

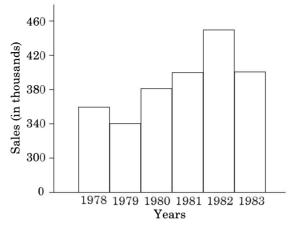
- (d) 2001 and 1999
- (e) 1999 and 2002
- 128. On the basis of the given data, which of the following can be inferred?
 - A. The town recorded the lowest rainfall in July 2000 among all the given months and years.
 - B. The average rainfall in July (for all the given years only) is 183 mm.
 - C. The average rainfall of the town during 2002 was 192 mm.
 - (a) All the above
- (b) Only A

(c) Only B and C

(d) Only B

(e) Only A and B

129. The following figure represents sales (in thousands) over the period 1978 to 1983.



The sales in 1982 exceeded that in 1979 by

(a) ₹ One hundred

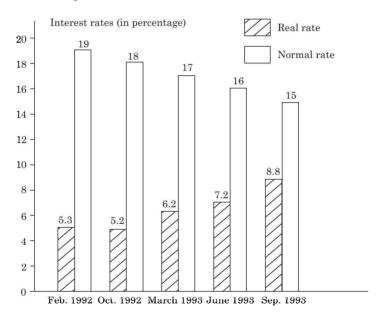
(b) ₹ Ten thousand

(c) ₹ One lakh

(d) ₹Ten lakh

130. Consider the following chart:

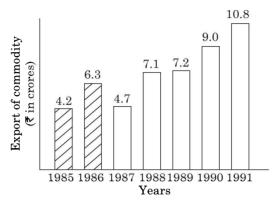
[IAS]



The above chart shows the movement of nominal interest rates and real interest rates (defined as nominal interest rate minus inflation). Which one of the following is the correct interpretation?

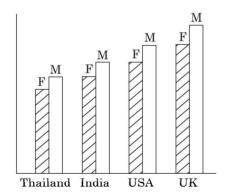
- (a) Nominal interest rates decreased at a constant rate in the period Feb. 1992-Sep. 1993.
- (b) Inflation has constantly decreased from Feb. 1992 to Sep. 1993.
- (c) Although the nominal rates of interest have constantly declined from Feb. 1992 to Sep. 1993, the real rates have been steadily increasing throughout the period.
- (d) The nominal rate of interest and real rate of interest are likely to become equal in 1994.

131. In which of the following pairs of years was the average export of the commodity around 6 crore? [IAS]

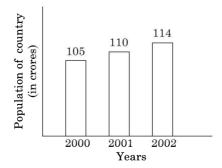


- (a) 1985 and 1986

- (b) 1985 and 1991
- (c) 1986 and 1987 (d) 1987 and 1988
- 132. The following figure corresponds to the heights of men and women of four different countries with standard deviation. Which of the following is true? [IAS]



- (a) All the women in the UK are taller than the men in the USA
- (b) All the men in Thailand are shorter than the men in India
- (c) All the men in UK are taller than the men in the USA
- (d) Most men in the UK are taller than the men in India
- 133. Study the following bar chart and state which of the given statements are true.



- (a) The population remains the same in 2000 and 2001
- (b) The population is increased by 10% from year 2000 to 2001
- (c) The increase in population from 2000 to 2002 is nearly 9%
- (d) The increase in population from 2001 to 2002 is more than 5%





- 1. Ratio of literate population of state C in the years 1997 and 1998 = $\frac{2 \times \frac{78}{100}}{3 \times \frac{81}{100}}$ = 52 :81. Hence answer is (e)
 - answer is (e).
- 2. The average literate population of all the states in the year of 1998

$$= 1200000 \times \frac{\left(\frac{72}{100} + \frac{81}{100} + \frac{81}{100} + \frac{65}{100} + \frac{93}{100} + \frac{88}{100}\right)}{6}$$
$$= \frac{1200000 \times 480}{600} = 960000$$

3. The required percentage increase in the literate population from 1998 to 2001 in the state 'D'

$$= \frac{(75 - 65)}{65} \times 100 = 15.38\% \approx 15\%$$

- 4. The literate population of state B in $2001 = 1000000 \times \frac{110}{100} \times \frac{110}{100} \times \frac{87}{100} = 1052700$
- 5. The ratio of populations of the given state is needed to calculate average literate population in the given year. Therefore, the answer is (d).
- 6. Percentage of students getting placement from institute B in the given years are as shown in the following table.

1998	1999	2000	2001	2002	2003
89.52%	85.34%	85.71%	87.40%	90.66%	89.74%

- .. Percentage of students getting placement from institute B was highest in 2002, i.e., 90.66%.
- 7. The overall percentage of students getting placement from all the institutes together the year 2000.

$$= \frac{400 + 600 + 370 + 540 + 420 + 420}{580 + 700 + 410 + 580 + 450 + 480} \times 100$$
$$= \frac{2750}{3200} \times 100 = 85.93 \approx 86\%$$

8. The percentage of students getting placement in the year 2002 can be seen in the following table.

A	В	C	D	E	F
69.23%	90.66%	92.30%	88.33%	94%	90.90%

- :. Institute E is the one with highest percentage of students getting placed.
- 9. Percentage of students getting placement in 1998 = $\frac{300}{400} \times 100 = 75\%$

Percentage of students getting placement in 1999 = $\frac{380}{450} \times 100 = 84.44\%$

Hence, the required percentage increase = $\left(\frac{84.44 - 75}{75}\right) \times 100 = 12.59\%$

10. The required ratio between the average strength of institutes

$$= \frac{380 + 480 + 450 + 520 + 500 + 540}{560 + 625 + 580 + 550 + 600 + 630} = \frac{2870}{3545} \approx 574 : 709$$

11. Ratio of minor boys to girls in school F = 2:3

Let the no. of minor boys = 2x

and the no. of minor girls = 3x

$$\therefore \text{ Average height of all minor students} = \frac{2x \times \frac{11}{2} + 3x \times 5}{2x + 3x} = \frac{26x}{5x} = 5\frac{1}{5} \text{ ft}$$

12. The number of major boys of the school = $1000 \times \frac{4}{100} \times \frac{1}{2} = 20$

Total age of 20 boys = $20 \times 18.2 = 364$ years

When principal's age is included, total age = $21 \times 20 = 420$ years

- \therefore Age of principal = 420 364 = 56 years
- 13. 900
- 14. 40
- 15. Minor girls in school F = $1250 \times \frac{80}{100} \times \frac{3}{5} = 600$

Major girls in school F = $1250 \times \frac{20}{100} \times \frac{2}{5} = 100$

:. Total girls = 600 + 100 = 700

16. Total number of girls of class 7th = 50% of (22 + 18 + 24 + 12 + 6) = 41

Total boys of class XII = 24 + 12 + 20 + 8 + 6 = 70

- \therefore Total number of students = 444 + 50% of 444 = 666
- $\therefore \text{ Required percentage} = \frac{70 + 41}{666} \times 100 = 16\frac{2}{3}\%$
- 17. Girl hockey players of class X = 50% of 22 = 11
 - \therefore Number of handshakes = ${}^{11}C_2 = 55$
- 18. Badminton players of class XII = 6 + 3 = 9
 - \therefore Number of ways of selection = ${}^{1}C_{1} \times {}^{8}C_{4} = \frac{1 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$
- 19. 77
- **20.** Total football players = 88 + 44 = 132. Hence answer is (e).
- 21. (i) and (ii). Hence answer is (c).
- **23.** 40% of 50 = $\frac{40}{100} \times 50 = 20$

Required number = 100 - Number of students scoring more than 20 marks

$$= 100 - 73 = 27$$

24. 60% of 50 = 30

Required number = Number of students scoring 30 and above marks in Physics

25. Number of students passed with 30 as cut-off marks in Chemistry = 21

Number of students passed with 30 as cut-off marks in aggregate = 27

- \therefore Required difference = 27 21 = 6
- 26. Number of students getting at least 60% marks in Chemistry

= Number of students scoring 30 marks and above in Chemistry = 21

Number of students getting at least 40% marks in aggregate

= Number of students scoring 20 marks and above in aggregate = 73

Required percentage =
$$\left(\frac{21}{73} \times 100\right)\% = 28.77\% \simeq 29\%$$

27. 23 students are to be selected for a symposium in Chemistry.

From the given ranges, only from 20-30 range, 66-21=45 or less number of students can be selected. Hence answer is (c).

- 28. Here 80 students are required to be qualified. Therefore, correct answer is either 10 and above or 0 and above thereby 82 students or 100 students qualify paper I. Hence answer is (e).
- **29.** Required difference = 31 27 = 4
- 30. Students obtained 60% and more in paper II = 31

Students who obtained 40% and more in aggregate = 71

- :. Required percentage = $\frac{31}{71} \times 100 = 43.66\% = 44\%$
- **31.** Let Income = I and Expenditure = E

$$\frac{I_{2002} - E_{2002}}{E_{2002}} = \frac{I_{2000} - E_{2000}}{E_{2000}} \qquad (\because \text{ net profit for both the years is the same})$$

$$\frac{I_{2002}}{E_{2002}} = \frac{I_{2000}}{E_{2000}}$$

$$\frac{I_{2002}}{120\%E_{2000}} = \frac{I_{2000}}{E_{2000}}$$

or

$$\frac{I_{2002}}{I_{2000}} = 120\%$$
 or 20% more

i.e. income in 2002 is 20% more than income in 2000.



Short cut: Given net profit 2000 = Net profit 2002

Expenditure 2002 =
$$\frac{120}{100} \times \text{Expenditure } 2000$$

Income 2002 =
$$\frac{120}{100} \times \text{Income } 2000$$

32. When the percentage profit is minimum ratio of income to expenditure is also the minimum.

i.e.
$$\%$$
 profit = $\frac{I-E}{E} \times 100 = \left[\frac{I}{E} - 1\right] \times 100$

% profit
$$\propto \frac{I}{F}$$

Here (e) is the answer.

33. In 2002, ratio of net profit = 35/20 = 1.75

In 2003, ratio of net profit = 50/35 = 1.43

In 2005, ratio of net profit = 60/45 = 1.33

: the answer is (c).

34. $E_{2004} = E_{2006} = k$; $I_{2006} = 15.5$ lakhs; $I_{2004} = ?$

$$\frac{I_{2006} - E_{2006}}{E_{2006}} = \frac{55}{100} = \frac{15.5 - k}{k} = \frac{15.5}{k} - 1 \tag{1}$$

$$\frac{I_{2004} - E_{2004}}{E_{2004}} = \frac{45}{100} = \frac{I_{2004} - k}{E_{2004}} = \frac{E_{2004}}{k} - 1 \tag{2}$$

(1)
$$\Rightarrow \frac{15.5}{k} = 1 + \frac{55}{100} = \frac{155}{100}$$
 (A)

(2)
$$\Rightarrow \frac{I_{2004}}{k} = 1 + \frac{45}{100} = \frac{145}{100}$$
 (B)

$$\frac{\text{(A)}}{\text{(B)}} \Rightarrow \frac{(15.5/k)}{(I_{2004}/k)}$$

$$\Rightarrow \frac{15.5}{I_{2004}} = \frac{155}{145} = \frac{31}{29}$$

$$\Rightarrow I_{2004} = \frac{29 \times 15.5}{31} = 14.5 \text{ lakhs}$$

Hence answer is (e).



Short cut: (By rule of fraction)

$$I_{2004} \left(\frac{100 + 45}{100 + 55} \right) I_{2006} = \frac{145}{155} \times 15.5 \text{ lakhs} = 14.5 \text{ lakhs}$$

Here expenditures are same in both years since the % profit is less in 2004 than in 2006, income in 2004 will also be less than that in 2006.

35. $I_{2003} = 25$ lakhs; $E_{2003} = ?$

$$\frac{I_{2003} - E_{2003}}{E_{2003}} = \frac{50}{100} = \frac{1}{2} \tag{1}$$

(1)
$$\Rightarrow$$
 $\frac{25 - E_{2003}}{E_{2003}} = \frac{25}{E_{2003}} - 1 = \frac{1}{2}$

$$\Rightarrow 1 + \frac{1}{2} = \frac{25}{E_{2003}} = \frac{3}{2}$$

⇒
$$E_{2003} = \frac{25 \times 2}{3} = 16.67 \text{ lakhs}$$



Short cut: Quicker Method

Income = Expenditure
$$\times \left[\frac{100 + \% \text{ profit}}{100} \right]$$

i.e. Expenditure = Income
$$\times \frac{100}{[100 + \% \text{ profit}]}$$

i.e.
$$\begin{split} \mathbf{E}_{2003} &= I_{2003} \times \frac{100}{100 + 50} \\ &= I_{2003} \times \frac{100}{150} = I_{2003} \times \frac{2}{3} \\ &= \frac{25 \times 2}{3} = \mathbf{16.67 \ lakhs} \end{split}$$

Here expenditures are same in both years since the % profit is less in 2004 than in 2006, income in 2004 will also be less than that in 2006.

- 36. Required income = $\frac{135}{35}$ × ₹ 10.5 crore = ₹ 40.5 crore
- 37. Given income of A = Expenditure of B

$$\mathbf{or}$$

Expenditure of A + Profit of A = Expenditure of B

or
$$\frac{100}{50} \times \text{Profit of A} + \text{Profit of A} = \frac{100}{40} \times \text{Profit of B}$$

or
$$\frac{\text{Profit of A}}{\text{Profit of B}} = \frac{5}{6} = 5:6$$

38. % profit of company A in that year =
$$\frac{92-80}{80} \times 100 = 15\%$$

From the graph, profit of company A = 15% during 2003-04.

40. Expenditure of company B in that year = 52 - 12 = ₹40 crores.

$$\therefore$$
 Profit = $\frac{52-40}{40} \times 100 = 30\%$

and profit of company B is 30% during 2002-03.

41. Difference in the production of crude oil from 1995–96 to 1999–2000 = 33 = Difference in production of cement from 1995–96 to 1997–98. Hence answer is (c).

45. Required % increase =
$$\frac{1007 - 623}{623} \times 100 = 60\%$$

46.
$$\frac{92 \times \frac{3}{4} - 98 \times \frac{4}{7}}{98 \times \frac{4}{7}} \times 100 = \frac{13}{56} \times 100 = 23\%$$

- 47. Total number of passed students = 82 + 64 + 102 + 92 = 340
 - \therefore Number of failed students = 550 340 = 210
 - ∴ Required percentage of failed students = $\frac{210}{550} \times 100 = 38.1\% \approx 38\%$
- **51.** The required ratio = 30:34 or 15:17
- **52.** 2005. Hence answer is (e).

53. The required average =
$$\frac{25 + 29 + 30 + 24 + 30 + 45}{6} = \frac{183}{6} = 30.5$$

54. The required per cent =
$$\frac{35}{(26+35+34+37+35+40)} \times 100 = \frac{35}{207} \times 100 = 16.9\%$$

- **55.** The required difference = 37 24 = 13 cm
- 56. Exports are more than imports for the years with exports to imports ratio greater than 1. Here for company A it is in 1995, 1996 and 1997, i.e. for 3 years. Hence answer is (b).
- 57. From the given line graph only exports to imports ratio is available. The difference between imports and exports cannot be determined. Hence answer is (d).
- **58.** In the year 1998, for company A, export to import ratio = 0.75;

Given that export in that year = $\overline{\epsilon}$ 237 crores.

Let import for the year be \mathbf{z} x crores.

$$\therefore \frac{237}{x} = 0.75$$

$$\Rightarrow$$
 $x = \frac{237}{0.75} = 79 \times 4 = ₹ 316 \text{ crores}$

59. In the year 1997, for company A, Export to Import ratio = 1.75;

i.e.
$$\frac{E}{I} = 1.75$$

New import
$$I_1 = I + \frac{40}{100}I = 1.4I$$
.

Required ratio =
$$\frac{E}{I_1} = \frac{1.75I}{1.4I} = 1.25$$

60. For company A in 1995,

$$\frac{E_A}{I_A} = 1.75 \tag{1}$$

For company B in 1995,

$$\frac{E_B}{I_B} = 0.75 \tag{2}$$

Given $I_A = ₹ 180 \text{ crores}$; $I_B = ?$

$$E_A = 2E_B \tag{3}$$

(1)
$$\Rightarrow$$
 $E_A = 1.75 I_A = 1.75 \times 180 = 315 \text{ crores}$

$$\therefore (3) \Rightarrow E_B = \frac{E_A}{2} = \frac{315}{2} = \text{crores}$$

(2)
$$\Rightarrow$$
 $I_B = \frac{E_B}{0.75} = \frac{315}{2 \times 0.75} = \frac{630}{3} = 210 \text{ crores}$

61. From the line graph,

In 1996: Number of students left school = 250

Number of students joined school = 350

In 1997: Number of students left school = 450

Number of students joined school = 300

In 1998: Number of students left school = 400

Number of students joined school = 450

In 1999: Number of students left school = 350

Number of students joined school = 500

In 2000: Number of students left school = 450

Number of students joined school = 400

In 2001: Number of students left school = 450

Number of students joined school = 550

 \therefore Strength of school in various years is:

1996: 3000 - 250 + 350 = 3100

1997: 3100 - 450 + 300 = 2950

1998: 2950 - 400 + 450 = 3000

1999: 3000 - 350 + 500 = 31502000: 3150 - 450 + 400 = 31002001: 3100 - 450 + 550 = 3200

Required percentage = $\left(\frac{3000 - 2950}{2950}\right) \times 100 = 1.69\% = 1.7\%$ decreased

- **62.** In 1999, strength = 3150.
- 63. In 1996 and 2000, strength was same. Hence answer is (d).
- **64.** Required percentage = $\frac{3000}{3200} \times 100 = 93.75\%$.
- 65. Largest number of students joined the school in 2001.
- **66.** Percentage rise/fall in the number of students left the school (compared to previous year) in given years are:

1997:
$$\left(\frac{450 - 250}{250}\right) \times 100 = 80\% \text{ (rise)}$$

1998:
$$\left(\frac{450 - 400}{450}\right) \times 100\% = 11.11\%$$
 (fall)

1999:
$$\left(\frac{400 - 350}{400}\right) \times 100\% = 12.5\%$$
 (fall)

2000:
$$\left(\frac{450 - 350}{350}\right) \times 100\% = 28.57\% \text{ (rise)}$$

$$2001: \left(\frac{450 - 450}{450}\right) \times 100\% = 0\%$$

Per cent rise/fall is maximum in 1997 = 80%. Hence answer is (a).

67. From the above calculations,

Required ratio =
$$\frac{300}{450} = \frac{2}{3}$$
 or 2:3

68. Difference between percentages of candidates qualified to appear for the given pair of years are:

1994 and 1995: 50 - 30 = 20%1997 and 1998: 80 - 50 = 30%1998 and 1999: 80 - 80 = 0%1999 and 2000: 80 - 60 = 20%1994 and 1997: 50 - 30 = 20%

Here required percentage is maximum during 1997-1998.

- **69.** Number of candidates qualified cannot be determined as the line graph gives only percentage. Hence answer is (e).
- 70. Number of candidates qualified in 1998 = 21200Let the number of candidates appeared be x.

$$\therefore \qquad \frac{21200}{x} = 0.8$$

$$\Rightarrow \qquad x = \frac{21200}{0.8} = 26500$$

- 71. From the total number of candidates appeared in 1996 and 1997, it is not possible to find out the number of candidates qualified in each year.
 - .. Data is inadequate. Hence answer is (e).
- 72. Let number of candidates appeared in 2000 be x

Number of candidates qualified in 1999 and 2000 = 33500

Number of candidates appeared in 1999 = 26500

Number of candidates qualified in $1999 = 0.8 \times 26500 = 21200$

 \therefore Number of candidates qualified in 2000 = 33500 - 21200 = 12300

$$\therefore x = \frac{12300}{0.6} = \frac{123000}{6} = 20500$$

73. Total number of literate people in Maharashtra and Karnataka.

$$= \left[\frac{5}{6} \times 11\% + \frac{3}{5} \times 15\%\right] 25 \text{ lakhs}$$

$$= \left[\frac{55}{6} + 9\right] \% \times 25 \text{ lakhs} = \left[\frac{55}{6} + 9\right] \times \frac{1}{4} \text{ lakhs}$$

$$= \left[\frac{55 + 54}{6}\right] \times \frac{1}{4} = \frac{109}{24} \text{ lakhs} = 4.54 \text{ lakh}$$

74. Total male in UP, Maharashtra and Kerala in percentage of total population

$$= \frac{5}{8} \times 25\% + \frac{3}{7} \times 11\% + \frac{3}{7} \times 8\%$$

$$= \left[\frac{125}{8} + \frac{33}{7} + \frac{24}{7}\right]\% = \left[\frac{128}{8} + \frac{57}{7}\right]\%$$

$$= [15.63 + 8.14]\% = 23.77\% \approx 25\%$$

.. Approximate percentage = 25%

75. Required ratio =
$$\frac{\frac{100}{110} \times \frac{25}{100} \times 25 \text{ lakhs}}{\frac{100}{112} \times \frac{20}{100} \times 25 \text{ lakhs}} = \frac{112}{110} \times \frac{25}{20} = \frac{112 \times 1}{11 \times 2 \times 4} = \frac{112}{88} = \frac{14}{11}$$

or 14:11. Hence answer is (e).

76. Approximate percentage of women (AP)/women (TN)

$$= \frac{\frac{3}{5} \times 9\% \times 25 \text{ lakhs}}{\frac{2}{5} \times 12\% \times 25 \text{ lakhs}} \times 100$$
$$= \frac{3 \times 9}{2 \times 12} \times 100 = \frac{9}{2 \times 4} \times 100 = \frac{9}{8} \times 100\% = 112.5\% \approx 110\%$$

77.
$$\frac{\text{Number of females (TN)}}{\text{Number of females (Kerala)}} = \frac{\frac{2}{5} \times \frac{12}{100} \times 25 \text{ lakhs}}{\frac{4}{7} \times \frac{8}{100} \times 25 \text{ lakhs}}$$
$$= \frac{\frac{2}{5} \times 12}{\frac{4}{7} \times 8} = \frac{24}{5} \times \frac{7}{32} = \frac{3 \times 7}{5 \times 4} = \frac{21}{20}$$

Hence answer is (e).

78. Total number of illiterates in Tamil Nadu =
$$\left(1 - \frac{70}{100}\right)$$
 females + $\left(1 - \frac{75}{100}\right)$ males
$$= 25 \text{ lakhs} \times \left[\frac{30}{100} \times \frac{2}{5} \times \frac{12}{100} + \frac{25}{100} \times \frac{3}{5} \times \frac{12}{100}\right]$$

$$= \frac{25 \text{ lakhs}}{100 \times 100} \left[\frac{30 \times 2 \times 12}{5} + \frac{25 \times 3 \times 12}{5}\right]$$

$$= \frac{25 \text{ lakhs}}{100 \times 100} \left[6 \times 24 + 15 \times 12\right] = \frac{25 \text{ lakhs}}{100 \times 100} \left[144 + 180\right]$$

$$= \frac{25 \times 324 \times 10^5}{10^4} = 25 \times 324 \times 10 = 81000$$

79. Required ratio =
$$\frac{\frac{2}{3} \times 9\% \times 25 \text{ lakhs}}{\frac{1}{5} \times 20\% \times 25 \text{ lakhs}} = \frac{6\%}{4\%} = \frac{6}{4} = \frac{3}{2} \text{ or } 3:2$$

80. No. of female candidates appeared from state C

$$= 120000 \times \frac{7}{100} \times \frac{4}{7} = 4800$$

No. of male candidates appeared from state G

$$= 120000 \times \frac{9}{100} \times \frac{4}{9} = 4800$$

i.e.
$$\left(\frac{7}{100} \times \frac{4}{7}\right) = \left(\frac{9}{100} \times \frac{4}{9}\right)$$

Hence answer is (e).

81. Required ratio =
$$\frac{12 \times \left(\frac{5}{12}\right)}{22 \times \left(\frac{5}{11}\right)} = 1:2$$

82. Total number of female candidates appeared from states E and G together

$$= 120000 \times \frac{22}{100} \times \frac{6}{11} + 120000 \times \frac{9}{100} \times \frac{5}{9}$$

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:. No. of female candidates = 17% of total candidates appeared from states E and G.



Short cut: Required percentage = $22 \times \frac{6}{5+6} + 9 \times \frac{5}{4+5} = 12+5 = 17\%$

83. The total no. of male candidates appeared in the examination from states B and F together

=
$$120000 \times \frac{19}{100} \times \frac{11}{19} + 120000 \times \frac{15}{1000} \times \frac{8}{15} = 13200 + 9600 = 22800$$

84. The percentage of female candidates appeared from state D with respect to the total no. of candidates appeared in the examination

$$=\frac{120000\times\frac{16}{100}\times\frac{7}{16}\times100}{120000}=7\%$$

From the pie chart this can be directly obtained.

i.e. total candidates from state D = 16%. As ratio of male to female is given as 9:7.

.. No. of female candidates = 7%. Hence answer is (e).

85. England

86. Runs against South Africa in ODI Runs against England in Test =
$$\frac{7.5\% \text{ of } 8000}{25\% \text{ of } 6000} = 2:5$$

87. His total runs against Sri Lanka in ODI = 12.5% of 8000 = 1000 runs

:. Average runs =
$$\frac{1000}{29-4} = \frac{1000}{25} = 40 \text{ runs}$$

Hence answer is (e).

88. His total runs against Australia in ODI = 15% of 8000 = 1200 runs

His runs after 31st innings = $31 \times 42 = 1302$ runs

 \therefore Required answer = 1302 - 1200 = 102 runs

89. Total runs scored against Pakistan

$$= 22\%$$
 of $8000 + 11\%$ of $6000 = 1760 + 660 = 2420$ runs

90. Required increase =
$$\frac{(10\% \text{ of } 2400) - (8\% \text{ of } 2000)}{(8\% \text{ of } 2000)} \times 100 = \frac{240 - 160}{160} \times 100 = 50\%$$

91. Workers have increased in categories of P, Q, R and U.

.. Total no. of increased workers

$$= \left(\frac{11}{100} \times 2400 - \frac{13}{100} \times 2000\right) + \left(\frac{25}{100} \times 2400 - \frac{18}{100} \times 2000\right) + \left(\frac{16}{100} \times 2400 - \frac{15}{100} \times 2000\right) + \left(\frac{10}{100} \times 2400 - \frac{8}{100} \times 2000\right)$$

$$= (264 - 260) + (600 - 360) + (384 - 300) + (240 - 160)$$
$$= 4 + 240 + 84 + 80 = 408$$

Hence answer is (d).

93. Case I: Two categories having the highest number of workers in 1998 and simultaneously the two categories having the least number of workers in 2000.

Difference =
$$(480 + 440) - (240 + 264) = 416$$

Case II: Two categories having the highest number of workers in 2000 and simultaneously the two categories having the least number or workers in 1998.

- \therefore Difference = (600 + 480) (160 + 260) = 660
- 94. Number of Japanese tourists below the age of 39 who visited India.

$$= 100000 \times \frac{20}{100} \times \frac{20}{100} = 4000$$

i.e. 20% of total strength in Japanese.

20% of total strength agewise lies below 39 years.

:. Required number =
$$100000 \times \frac{20}{100} \times \frac{20}{100} = 4000$$

98. Number of American tourists in the age group 20-40 years = $100000 \times \frac{60}{100} \times \frac{20}{100} = 12000$

100. Total cost =
$$\left(\frac{7}{12} \times \frac{12}{100} + \frac{6}{13} \times \frac{13}{100}\right) 40 = ₹ 5.2 \text{ crore}$$

102. Total profit =
$$\left(\frac{3}{4} \times \frac{20}{100} \times \frac{32}{100} + \frac{7}{12} \times \frac{12}{100} \times \frac{35}{100}\right)$$
 40 = ₹ 2.90 crore

105. Required percentage =
$$\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550}\right) \times 100$$

$$= \left(\frac{9 \times 5700}{8 \times 8550}\right) \times 100 = 75\%$$

106. Required ratio =
$$\left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550}\right)$$

$$= \left(\frac{18 \times 5700}{22 \times 8550}\right) = 6:11$$

107. Required percentage =
$$\left(\frac{30\% \text{ of } 5700}{25\% \text{ of } 8550}\right) \times 100$$

= $\left(\frac{30 \times 5700}{25 \times 8550}\right) \times 100 = 80\%$

108. Percentage of candidates passed to candidates enrolled

For Q:
$$\left(\frac{17\% \text{ of } 5700}{15\% \text{ of } 8550}\right) \times 100 = \left(\frac{17 \times 5700}{15 \times 8550}\right) \times 100 = \frac{17 \times 2}{15 \times 3} \times 100 = 75.56\%$$

For R:
$$\left(\frac{13\% \text{ of } 5700}{10\% \text{ of } 8550}\right) \times 100 = \left(\frac{13 \times 5700}{10 \times 8550}\right) \times 100 = \frac{13 \times 2}{10 \times 3} \times 100 = 86.67\%$$

For V:
$$\left(\frac{15\% \text{ of } 5700}{12\% \text{ of } 8550}\right) \times 100 = \left(\frac{15 \times 5700}{12 \times 8550}\right) \times 100 = \frac{15 \times 2}{12 \times 3} \times 100 = 83.33\%$$

For T:
$$\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550}\right) \times 100 = \left(\frac{9 \times 5700}{8 \times 8550}\right) \times 100 = \left(\frac{9 \times 2}{8 \times 3}\right) \times 100 = 75\%$$

Among these the institute R has the highest percentage of candidates passed to candidates enrolled. Hence answer is (b).

109. Number of candidates passed from institutes S and P

$$= (16 + 18)\% \text{ of } 5700 = 34\% \text{ of } 5700$$
 (1)

Number of candidates enrolled from institutes T and R

$$= (8 + 10)\% \text{ of } 8550 = 18\% \text{ of } 8550$$
 (2)

From (1) and (2),

Required difference =
$$\frac{34}{100} \times 5700 - \frac{18}{100} \times 8550$$

= $34 \times 57 - 18 \times 85.5$
= $57 (34 - 18 \times 1.5)$
= $57 (34 - 27) = 57 \times 7 = 399$

- 110. Central angle of sector corresponding to royalty = 15% of $360^{\circ} = 54^{\circ}$
- 111. Central angle of 108° corresponds to $\left(108^{\circ} \times \frac{100}{360^{\circ}}\right)\% = 30\%$ of total expenditure.

From the pie-chart, Binding cost + Transportation cost

$$= 20\% + 10\% = 30\%$$
 of total expenditure.

Hence answer is (a).

112. Let the promotion cost be $\not\in x$.

Given Paper cost: Promotion cost = 25:10=5:2

We have Paper cost = ₹ 56250

$$\therefore \frac{\text{Promotion cost}}{\text{Paper cost}} = \frac{x}{56250} = \frac{2}{5}$$
or
$$x = \frac{56250 \times 2}{5} = \text{ ? 22500}$$

113. Given difference in two expenditures corresponds to 18° in the pie-chart.

Percentage of expenditure corresponding to 18°

$$= \left(18^{\circ} \times \frac{100}{360^{\circ}}\right)\% = 5\%$$

From the given options difference between paper cost and printing cost is 5%.

114. From the pie-chart, $\frac{Printing cost}{Royalty} = \frac{20\%}{15\%} = \frac{4}{3}$

Given printing cost = $\stackrel{?}{\stackrel{?}{\sim}} 30600$. Let $\stackrel{?}{\stackrel{?}{\stackrel{?}{\sim}}} R$ be the royalty.

$$\therefore \frac{30600}{R} = \frac{4}{3}$$

$$\Rightarrow R = \frac{30600 \times 3}{4} = 7650 \times 3 = ₹ 22950$$

115. Marked price of book = ₹ 180

Cost price of book =
$$\frac{180 \times 100}{120}$$
 = ₹ 150

Cost of paper used/copy of book = 25% of 150 = ₹ 37.50

116. Let the selling price of book be $\not \in x$

.. Cost price of book =
$$x \times \frac{100}{105} = \frac{20x}{21}$$

Royalty paid/Copy of book = $\frac{281250}{12500} = \frac{15}{100} \times \frac{20x}{21}$
i.e. $\frac{281250}{12500} = \frac{x}{7}$
or $x = \frac{7 \times 281250}{12500} = ₹ 157.50$

117. Transportation cost/copy = $\frac{82500}{5500}$ = ₹ 15

Let selling price of book be $\not\in x$.

.. Cost price of book =
$$x \times \frac{100}{125} = \frac{4x}{5}$$

.. Transportation cost/book = $\frac{10}{100} \times \frac{4x}{5} = \frac{15}{125}$
i.e. $\frac{2x}{25} = 15$
or $x = \frac{25 \times 15}{2} = \frac{15}{125}$

118. Printing cost - Royalty = 5%

119. Average value of imports in the year 1996, 1997 and 1999 = $\frac{250 + 220 + 280}{3}$ = ₹ 250 cr

Value of exports in 1998 = ₹ 450 cr

$$\therefore \text{ Required percentage} = \frac{450}{250} \times 100 = 180\%$$

Hence answer is (e).

120. Value of exports in 1996 = ₹ 375 cr Value of imports in 1996 = ₹ 250 cr

$$\therefore$$
 Required percentage = $\frac{375}{250} \times 100 = 150\%$

Hence answer is (e).

121. Value of average exports =
$$\frac{150 + 225 + 375 + 300 + 450 + 330}{6}$$

$$= \frac{150 + 300 + 450 + 930}{6} = 25 + 50 + 75 + 155 = ₹305 \text{ cr}$$
Value of average imports =
$$\frac{80 + 150 + 250 + 220 + 350 + 280}{6}$$

$$= \frac{480 + 850}{6} = 80 + 141.67 = ₹221.67 \text{ cr}; ₹222 \text{ crores}$$

$$= \text{Value of average exports} - \text{Value of average imports}$$

$$= ₹305 \text{ cr} - ₹222 \text{ cr}$$

$$= 83 \text{ cr} ≈ 85 \text{ cr}$$

122. 1998

123. Required percentage =
$$\frac{450 - 300}{300} \times 100\% = \frac{150}{300} \times 100\% = 50\%$$

124. Total rainfall of 31 days in August 1999 = 186 cm Total rainfall of first 16 days = $16 \times 5 = 80$ mm Total rainfall of last 16 days = $16 = 16 \times 8 = 128$ mm Rainfall of 16th August 1999 = (128 + 80) - 186 = 22 mm

125. Percentage increase =
$$\frac{430 - 310}{310} \times 100 = 38.7\%$$

126. Highest rainfall in August = 230 mm

Average rainfall in July =
$$\frac{210 + 175 + 150 + 200 + 180}{5}$$
 = 183 mm

 \therefore Difference = 230 – 183 = 47 mm i.e., 47 mm more.

- 127. i.e. in 2000 and 2002
- 128. Only A and B can be inferred.

Mathematical Operations

ADDITION: In this type of problems, two factors taken into consideration are *speed* and *accuracy*.

Advantages of this method:

- 1. Save the labour of repeating all the work.
- 2. Locate the error, if any, in the column where it occurs.

EXAMPLE 1 Solve 3457 + 248 + 3746 + 5639 + 6854

[7 plus 8 is 15, mark a tick and add 5 to 6 which is 11, mark a tick and add 1 to 9 which is 10, mark a tick and add 0 to 4 which is 4 and write it down in the unit position].

[5 plus 3 (number of ticks in unit position) is 8; 8 plus 4 is 12; mark a tick and carry 2; 2 + 4 is 6; 6 + 3 is 9; 9 + 5 = 14; mark a tick and write down 4 in the tenth position].

[4 plus 2(number of ticks in tenth position) is 6; 6 + 2 is 8; 8 + 7 is 15; mark a tick and carry '5'. 5 + 6 is 11; mark a tick and carry '1'. 1 + 8 is 9; write down 9 in the hundredth position].

Step IV: 3457 248 3746 5639 6854

19944

[3+2] (number of ticks in hundredth position) is 5; 5+3 is 8; 8+5 is 13; 13+6 is 19.

ADDITION AND SUBTRACTION IN A SINGLE ROW

Find the value of $424 - 87 + 69 = \dots$? EXAMPLE 2

Solution For units digit of answer, add and subtract the digits at unit places according to the signs attached with respective numbers

$$4-7+9=6$$

 $424-87+69=---(6)$

Similarly, the value of tenth place is

$$2 - 8 + 6 = 0$$

So

424 - 87 + 69 = -(0)(6)Similarly, the value at hundredth place is 4.

$$\therefore$$
 424 - 87 + 69 = **406**

EXAMPLE 3 Find the value of $4124 - 729 + 631 - 535 = \dots$?

Solution Step I:
$$4-9+1-5=-9$$

 $2-2+3-3=0$
 $1-7+6-5=-5$
 $4-0+0-0=4$

 \therefore Temporary value is (4) (-5) (0) (-9).

Step II: Borrow 1 from 4 and add as 10 to -5 to get 5. Borrow 1 from 5 and add as 10 to 0 and borrow 1 from 10 and add as 10 to -9 to get 1.

 \therefore The number is **3491**.

EXAMPLE 4 Find the value of $89974 - 13568 - 46387 = \dots$?

Solution Step I:
$$4-8-7=-11$$

 $7-6-8=-7$
 $9-5-3=1$
 $9-3-6=0$
 $8-1-4=3$

 \therefore Temporary value is (3) (0) (1) (-7) (-11).

Step II: Borrow 1 from 1 and add it as 10 to - 7 to get 3. Borrow 2 from 3 and add it as 20 to -11 so as to get 9.

i.e. At unit place add 20 which is borrowed from the tenth place as 2.

$$\therefore$$
 -11 + 20 = 9

From hundredth place borrow 1 and add it as 10 to -7 to get 3.

i.e.
$$10-7-2=1$$

- \therefore The hundredth place digit is 0.
- \therefore Number = 30019.

DIGIT SUM METHOD

Rule: Add the digit sum of the numbers to be added till we get the result in single digit. Digit sum of the total should also be the same as the digit got after getting added the digit sums of individual numbers. For example,

$$23 + 48 + 61 + 15 = 147$$

Digit sum $5 + 3 + 7 + 6 = 12$
 $3 = 3$

MULTIPLICATION

Rule I: The last digit of the multiplicand (number multiplied) is put down as the right-hand figure of the answer.

Rule II: Each successive digit of the multiplicand is added to its neighbour at the right.

EXAMPLE 5 Find the value of $2479 \times 11 = ?$

Solution
$$02479 \times 11 = ----9 (9 \times 1 = 9)$$

= -----69 (7 + 9 = 16. Write 6, carry '1')
= -----269 (4 + 7 = 11 + '1' = 12, write 2, carry '1')
= ----7269 (2 + 4 = 6 + '1' = 7)
= 27269 (0 + 2 = 2)

MULTIPLICATION BY 12

Rule To multiply a number by 12, double each digit in turn and add its neighbour.

EXAMPLE 6 Find the value of $5328 \times 12 = \dots$?

Solution 05328×12

Step I:
$$=$$
 — — 6 (8 × 2 + 0 = 16. Write 6 and carry '1')

Step II: =
$$--36 (2 \times 2 + 8 + '1' = 13)$$
. Write 3 and carry '1')

Step III: =
$$--936 (3 \times 2 + 2 + 1) = 9$$

Step IV: =
$$-3936$$
 (5 × 2 + 3 = 13. Write 3 and carry '1')

Step V:
$$63936 (0 \times 2 + 5 + '1' = 6)$$

.: Required number is 63936.

MULTIPLICATION BY 13

Rule: To multiply by 13, treble each digit in turn and add its right neighbour.

EXAMPLE 7 Find the value of $9478 \times 13 = \dots$?

Solution $09478 \times 13 =$

Step I: = — — 4 (8 × 3 = 24. Write 4, carry '2')

Step II = -- 14 (7 × 3 + 8 = 29 + '2' = 31. Write 1, carry '3')

Step III: = $--214 (4 \times 3 + 7 = 19 + '3' = 22$. Write 2, carry '2')

Step IV: = -3214 (9 × 3 + 4 = 31 + '2' = 33. Write 3, carry '3')

Finally **123214** $(0 \times 3 + 9 = 9 + '3' = 12)$

EXAMPLE 8 Multiply 9876 by 13.

Solution $09876 \times 13 = \dots$

Step I: = ----8 (6 × 3 = 18; Write 8, carry '1')

Step II: = -- 88 (7 × 3 + 6 = 27 + '1' = 28. Write 8, carry '2')

Step III: = $--388 (8 \times 3 + 7 = 31 + '2' = 33)$. Write 3, carry '3')

Step IV: = $-8388 (9 \times 3 + 8 = 35 + '3' = 38$. Write 8, carry '3')

Finally **128388** $(0 \times 3 + 9 = 9 + '3' = 12)$

MULTIPLICATION BY 9

Step I: Subtract the right-hand figure of the long number from 10. This gives the right-hand figure of the answer.

Step II: Taking the next digit from right, subtract it from 9 and add the neighbour on its right.

Step III: Continue the process till the last number.

Finally, when you are under the zero in front of the long number, subtract one from the neighbour and use that as the left-hand figure of the answer.

EXAMPLE 9 Find the value of $9768 \times 9 = \dots$?

Solution $09768 \times 9 =$

Step I: Subtract 8 from 10, we get 2 as the right-end digit.

Step II: Subtract 6 from 9 and add neighbouring digit 8, then the result will be 11. Then write 1, carry '1'.

Step III: Subtract 7 from 9 and add neighbouring digit 6 and carry '1', then the result will be 9.

Step IV: Subtract 9 from 9 and add neighbouring digit 7, then the result will be 7.

Finally, subtract 1 from neighbouring digit 9 so that the result will be 8 and use that as the left-hand figure of the answer.

 \therefore The answer is 87912

EXAMPLE 10 Find the value of $1347 \times 9 = \dots$?

Solution $01347 \times 9 = \dots$?

Step I: Subtract 7 from 10, we get 3 as the right-hand figure.

Step II: Subtract 4 from 9 and add neighbouring digit 7 so that the answer will be 12. Write 2 and carry '1'.

Step III: Subtract 3 from 9 and add neighbouring digit 4 and carry '1' in the last step so that the answer will be 11. Write 1, carry '1'.

Step IV: Subtract 1 from 9 and add neighbouring digit 3 and carry '1' in the last step so that the answer will be 12. Write 2, carry '1'.

Step V: Subtract 1 from neighbouring digit 1 and add carry '1' in the last step to get answer '1'.

Write the numbers got in all steps from right to left.

.. The answer is 12123.

Method II

Step I: Put a zero at the right end of the number.

Here the number is 1347. So write it as 13470.

Step II: Subtract the original number from this number

i.e. 13470 - 1347 = 12123

MULTIPLICATION BY 25: Multiplication by 25 will be treated as multiplication by 100 followed by division by 4.

Step I: Put two zeros at the right end of the number.

Step II: Divide it by 4.

EXAMPLE 11 Multiply 1479 by 25.

Solution $\frac{147900}{4} = 36975$

GENERAL RULE FOR MULTIPLICATION: Multiplication by a two-digit number.

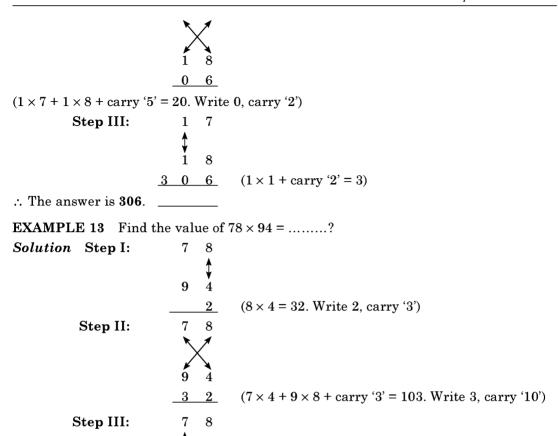
Step I: Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

Step II: Do cross multiplication i.e. multiply unit-digit of one-number with tenth place digit of the other and sum up those terms with carry over from step I.

Step III: Multiply the left-hand figures of both multiplicand and multiplier and add carry over from last step with the above product.

The number obtained through the above steps when arranged from right to left, we will get the required number.

EXAMPLE 12 Find the value of $17 \times 18 = ?$



 \therefore The answer is **7332**.

MULTIPLICATION OF A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER

EXAMPLE 14 Find the value of $347 \times 59 = \dots$? **Solution** Step I: 3 4 7

Step I: 3 4 7

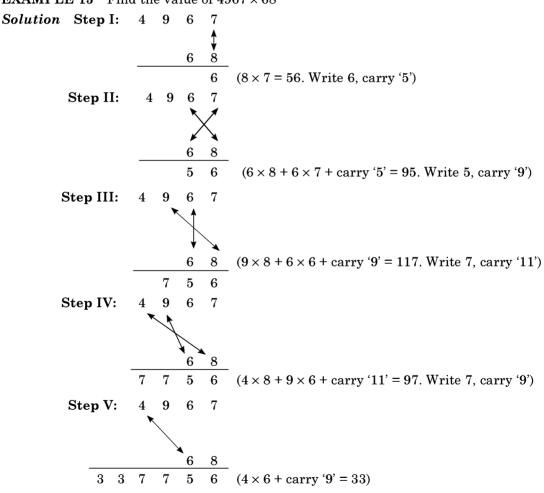
$$5 9$$

 3 (7 × 9 = 63. Write 3, carry '6')
Step II: 3 4 7
 $5 9$
 $7 3$ (4 × 9 + 5 × 7 + carry '6' = 77. Write 7, carry '7')

 $(7 \times 9 + 10 = 73)$

 \therefore The answer is 20473.

EXAMPLE 15 Find the value of 4967×68



 \therefore The answer is **337756**.

MULTIPLICATION BY A 3-DIGIT NUMBER

EXAMPLE 16 Find the value of 346×647

Solution Step I:

 $(6 \times 7 = 42. \text{ Write } 2, \text{ carry '4'})$

Step II:



 $(4 \times 7 + 4 \times 6 + \text{carry '4'} = 56. \text{ Write 6, carry '5'})$

Step III:



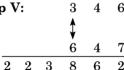
 $(3 \times 7 + 4 \times 4 + 6 \times 6 + \text{carry '5'} = 78$. Write 8, carry '7')

Step IV:



 $(3 \times 4 + 6 \times 4 + \text{carry '7'} = 43. \text{ Write 3, carry '4'})$

Step V:



$$(3 \times 6 + \text{carry } '4' = 22)$$

 \therefore The answer is **223862**.

EXAMPLE 17 Find the value of $4678 \times 376 = \dots$?

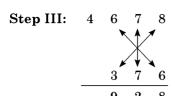
Solution Step I:

 $(8 \times 6 = 48. \text{ Write } 8, \text{ carry '4'})$

Step II:



 $(7 \times 6 + 7 \times 8 + \text{carry '4'} = 102$. Write 2, carry '10')



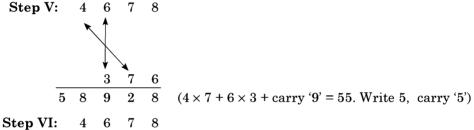
 $(6 \times 6 + 7 \times 7 + 3 \times 8 + \text{carry } 10 = 119. \text{ Write } 9, \text{ carry '11'})$

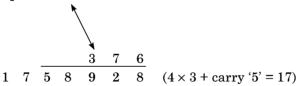
Step IV: 4 6 7 8

3 7 6

8 9 2 8

 $(4 \times 6 + 6 \times 7 + 7 \times 3 + \text{carry '11'} = 98$. Write 8, carry '9')

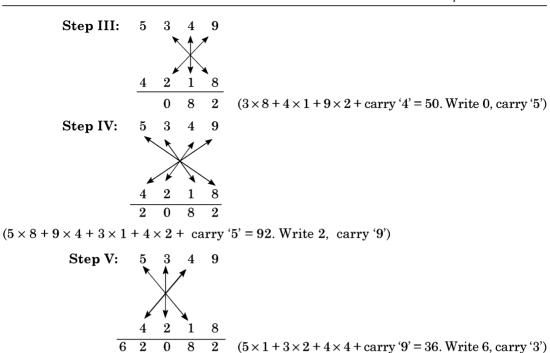




 \therefore The answer is 1758928.

MULTIPLICATION BY A 4-DIGIT NUMBER

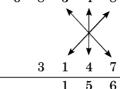
EXAMPLE 18 Find the value of $5349 \times 4218 = \dots$?



 \therefore The answer is **22562082**.

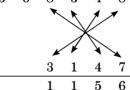
EXAMPLE 19 Find the value of 568348×3147 .

Step III: 5 6 8 3 4 8



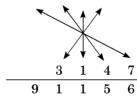
 $(3 \times 7 + 4 \times 4 + 8 \times 1 + \text{carry '6'} = 51$. Write 1, carry '5')

Step IV: 5 6 8 3 4 8



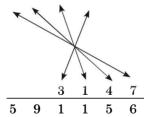
 $(8 \times 7 + 3 \times 4 + 4 \times 1 + 8 \times 3 + \text{carry '5'} = 101$. Write 1, carry '10')

Step V: 5 6 8 3 4 8



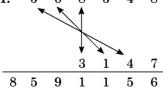
 $(6 \times 7 + 8 \times 4 + 3 \times 1 + 4 \times 3 + \text{carry '}10' = 99. \text{ Write 9, carry '}9')$

Step VI: 5 6 8 3 4 8



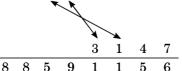
 $(5 \times 7 + 6 \times 4 + 8 \times 1 + 3 \times 3 + \text{carry '9'} = 85. \text{ Write 5, carry '8'})$

Step VII: 5 6 8 3 4 8

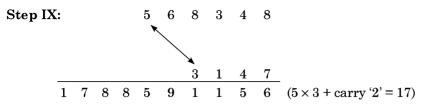


 $(5 \times 4 + 6 \times 1 + 8 \times 3 + \text{carry '8'} = 58$. Write 8, carry '5')

Step VIII: 5 6 8 3 4 8



88591156 (5×1+6×3+carry '5' = 28. Write 8, carry '2')



:. The answer is 1788591156.

DIGIT-SUM CHECK FOR MULTIPLICATION

EXAMPLE 20
$$27 \times 13 = 351$$
 Solution Digit-sum $9 \times 4 = 9$ $36 = 9$ $9 = 9$

Hence the calculation is correct.

It may be noted that if the calculation is correct, you will get digit-sums equal on both sides of the equation. (i.e., digit sum of product of two numbers on right-hand side and product of digit-sums of two numbers on left-hand side are equal). But if the digit-sums on both sides are equal, it does not mean that calculation is perfectly right.

EXAMPLE 21 Find the value of $343 \times 34 = 11662$

Solution Digit sum
$$1 \times 7 = 7$$

Hence the calculation is correct.

The above check shows that our answer is correct. But digit-sum check predicts that if the answer is correct, the digit-sum check will be correct. If the calculation is wrong, digit-sum check need not be wrong.

EXAMPLE 22 Find the value of $12 \times 13 = 516$

Solution Digit Sum
$$3 \times 4 = 12$$

 $12 = 3$

3 = 3

Hence the digit-sum check is correct. But the answer is false. Here the actual answer is 156. But we got it mistakenly as 516. Since digit-sum of 156 and 516 are the same, you will get digit-sum check as correct.

Therefore, if the answer is correct, digit-sum check will be correct. But we can't predict that if the digit-sum check is correct, the answer will be correct.

? EXERCISES

1.	23145×11	2. 89068×11	3. 5776900×11	4. 1122334607 × 11
5.	35609×12	6. 457689×12	7. 22233344×12	8. 4488669911×12
9.	12768×13	10. 13489×13	11. 21438×13	12. 61346×13
13.	11434×13	14. 13922×13	15. $569 \times 9 = \dots$?	16. $1328 \times 9 = \dots$?

17.	56493 × 9 =?	18. 89273528 × 9 =	? 19. $79 \times 9 = \dots$?	20. 56×78
21.	79×84	22. 47×53	23. 92×86	24. 93×32
25 .	89×32	26. 76×59	27. 87×67	28. 37×46
29.	68 imes 47	30. 338×579	31. 974×687	32. 4374×346
33.	54678×243	34. 13478×3487	35. 387459×246	36. 984653×1347
37.	8652423×3145			

Solutions with Necessary Explanation

1.	254595	2. 979748	3. 63545900	4. 12345680677
5.	427308	6. 5492268	7. 266800128	8. 53864038932
9.	165984	10. 175357	11. 278694	12. 797498
13.	148642	14. 180986	15. 5121	16. 11952
17.	508437	18. 803461752	19. 711	20. 4368
21.	6636	22. 2491	23. 7912	24. 2976
25 .	2848	26. 4484	27. 5829	28. 1702
29 .	3196	30. 195702	31. 669138	32. 1513404
33.	13286754	34. 46997786	35. 95314914	36. 1326327591
37.	27211870335			

Divisibility

DIVISIBILITY BY 2 A number whose digit in the unit place is either even or zero is divisible by 2. For example,

- (a) 12786 and 6789580 are divisible by 2.
- (b) 18453 and 8678477 are not divisible by 2.

DIVISIBILITY BY 3: If the sum of the digits of a number is divisible by 3, then number is divisible by 3. For example,

- (a) 312: 3 + 1 + 2 = 6 is divisible by 3;
 - ∴ 312 is divisible by 3.
- (b) 6894: 6+8+9+4=27; 2+7=9;
 - ... The number is divisible by 3.
- (c) 98562: 9 + 8 + 5 + 6 + 2 = 30; 3 + 0 = 3;
 - ... The number is divisible by 3.
- (d) 58763: 5 + 8 + 7 + 6 + 3 = 29; 2 + 9 = 11 is not divisibly 3.
 - .. The number is not divisible by 3.

DIVISIBILITY BY 4: If the last two digits of a number are divisible by 4, the number is divisible by 4. The number having two or more zeros at the end is also divisible by 4. For example,

- (a) 5848: 48 is divisible by 4. Hence the number is divisible by 4.
- (b) 4600: Last two digits are zero. Hence the number is divisible by 4.
- (c) 64000: There are more than 2 zeros at the end of the number. Therefore the number is divisible by 4.
- (d) 1426: 26 is not divisible by 4. Therefore the number is not divisible by 4.

DIVISIBILITY BY 5: If a number ends with '5' or '0', then the number is divisible by 5. For example,

- (a) 1365: The last digit is 5.
 - .. The number is divisible by 5.

- (b) 1480: Last digit is '0'.
 - .. The number is divisible by 5.
- (c) 1573: Last digit is neither 5 nor zero.
 - .. The number is not divisible by 5.

DIVISIBILITY BY 6: If a number is divisible by both 2 and 3, then the number is divisible by 6.

For a number to be divisible by 6.

- (i) The number should end with either an even number or a zero.
- (ii) The sum of its digits should be divisible by 3.

For example,

- (a) 546: 5+4+6=15; 1+5=6; the last digit is an even number and sum of digits is divisible by 3.
 - ... Therefore number is divisible by 6.
- (b) 58614: 4 is an even number. 5+8+6+1+4=24 is divisible by 3. Therefore the number is divisible by 6.
- (c) 371: 1 is not an even number. Therefore the number is not divisible by 6.

DIVISIBILITY BY 8: If the last three digits of a number are divisible by 8, then the number is divisible by 8. Otherwise if the last three digits are zero, number is divisible by 8. For example,

- (a) 2448: : 448 is divisible by 8. Therefore the number is divisible by 8.
- (b) 247952: : 952 is divisible by 8; therefore the number is divisible by 8.
- (c) 34000: ∴ the last three digits of the number are zeros; therefore the number is divisible by 8.
- (d) 1246: : 246 is not divisible by 8; therefore the number is not divisible by 8.

DIVISIBILITY BY 9: If the sum of all the digits of a number is divisible by 9, the number is divisible by 9. For example,

(a) 36846: 3+6+8+4+6=27 is divisible by 9 and hence the number is divisible by 9.

DIVISIBILITY BY 11: If the sum of digits at odd and even places are equal or differ by a number divisible by 11, then the number is divisible by 11. For example,

- (a) 27618701: The sum of odd positions, $S_o = 2 + 6 + 8 + 0 = 16$ Sum of even positions, $S_e = 7 + 1 + 7 + 1 = 16$ $S_o = S_e$; therefore, the number is divisible by 11.
- (b) 264935: $S_o = 2 + 4 + 3 = 9$; $S_e = 6 + 9 + 5 = 20$; $S_o \sim S_e = 11$; therefore the number is divisible by 11.
- (c) 62849182: $S_o = 6 + 8 + 9 + 8 = 31;$ $S_e = 2 + 4 + 1 + 2 = 9;$ $S_o - S_e = 31 - 8 = 22;$

Therefore, the number is divisible by 11.

DIVISIBILITY BY 12: Any number which is divisible by both 3 and 4 is also divisible by 12.

To check the divisibility, by 12,

- (i) First check the last two digits of the number for divisibility by 4. If it is not divisible by 4, then the number is not divisible by 12.
- (ii) If it is divisible by 4, check whether the number is divisible by 3. For example,
 - (a) 246864: 64 is divisible by 4 and so the number is divisible by 4. 2+4+6+8+6+4=30 is divisible by 3. Therefore, the number is divisible by 12.

DIVISIBILITY BY 14: Any number which is divisible by both 2 and 7 is divisible by 14.

To check the divisibility by 14

- (i) The last digit should be checked whether it is even or zero.
- (ii) The number should be divisible by 7.

For example,

(a) 1834: 4 is divisible by 2 and therefore number is divisible by 2.

 $183 - 4 \times 2 = 175$;

 $17 - 5 \times 2 = 7$; therefore number is divisible by 7.

Hence 1834 is divisible by 14.

DIVISIBILITY BY 15: Any number which is divisible by both 5 and 3 is divisible by 15.

To check divisibility by 15,

- (i) First check whether the last digit of number is 5 or zero. If not, then the number is not divisible by 15.
- (ii) Check for divisibility by 3, if it is divisible by 5. If the number is divisible by 3, then the number is divisible by 15.

For example,

(a) 3645: Last digit of the number is 5 and therefore number is divisible by 5. Digit-sum of number = 3 + 6 + 4 + 5 = 18. Since digit-sum is divisible by 3, number is divisible by 3. Since the given number is divisible by both 3 and 5, the number is divisible by 15.

DIVISIBILITY BY 16: Any number whose last 4 digit number is divisible by 16 or number having last 4 digits as zeros is divisible by 16. For example,

(a) 31264: Since the last 4 digit of the number is divisible by 16, number is divisible by 16.

DIVISIBILITY BY 18: Any number which is divisible by 9 and has its unit-digit even or zero is divisible by 18. For example,

- (a) 18612: Unit digit 2 is even; therefore the number is divisible by 2. 1 + 8 + 6 + 1 + 2 = 18, a multiple of 9. Hence the number is divisible by 18.
- (b) 298728: The unit-digit (8) is even, the digit sum is 36, a multiple of 9. Hence the number is divisible by 18.

SPECIAL CASES: Rules for divisibility by 7, 13, 17, 29 etc. are unique. For that you should know terms like *one-more osculator* and *negative osculator*.

One-more osculator means the number needs one more to be a multiple of 10. For example,

- (a) 19 needs 1 to become 20, a multiple of 10. Therefore the osculator for 19 is 2 (i.e. $2 \times 10 = 20$). Similarly, the osculator for 49 is 5 (i.e., $5 \times 10 = 50$).
- (b) Negative osculator for 21 is 2 (i.e. $21 1 = 20 = 2 \times 10$). Similarly, the negative osculator for 51 is 5 (i.e. $51 1 = 50 = 5 \times 10$).

Negative osculator for 7

 $7 \times 3 = 21$, one more than a multiple of 10.

$$\therefore 21 - 1 = 20 = 2 \times 10.$$

Hence the negative osculator for 7 is 2.

One-more osculator for 7

 $7 \times 7 = 49$, which is one less than 50, a multiple of 10.

$$7 \times 7 = 49 = 50 - 1$$
, $50 = 5 \times 10$.

Therefore, one-more osculator for 7 is 5.

One-more osculator for 13

 $13 \times 3 = 39$, which is one less than 40, a multiple of ten.

Therefore, $13 \times 3 = 39 = 40 - 1$; $40 = 4 \times 10$;

One more osculator for 13 is 4.

One more osculator for 19

 $19 \times 1 = 19$, which is one less than 20, a multiple of ten,

Therefore, $19 \times 1 = 19 = 20 - 1$; $20 = 2 \times 10$.

One more osculator for 19 is 2.

Negative osculator for 17

 $17 \times 3 = 51$, which is one more than a multiple of ten.

$$17 \times 3 = 51$$
; = $51 - 1 = 50 = 5 \times 10$

Hence the negative osculator for 17 is 5.

DIVISIBILITY BY 7: The negative osculator for 7 is 2. This is used to check divisibility of a number by 7.

EXAMPLE 1 Is 112 divisible by 7?

Solution Step I: $11 \ 2 : 11 - 2 \times 2 = 7$

As 7 is divisible by 7, the number 112 is divisible by 7.

EXAMPLE 2 Is 3871 divisible by 7?

Solution Step I: $387 \cdot 1:387 - 1 \times 2 = 385$

Step II: $385:38-5\times2=28$

As 28 is divisible by 7, the given number is divisible by 7.

DIVISIBILITY BY 13: The osculator for 13 is 4, which is one more osculator. Therefore instead of subtraction done in the previous case, you have to add here.

EXAMPLE 3 Is 143 divisible by 13?

Solution $143: 14+3\times 4=26$

Since 26 is divisible by 13, the given number divisible by 13.

EXAMPLE 4 Is 24167 divisible by 13?

Solution Step I: $24167: 4 \times 7 \text{ (from } 24167) + 6 \text{ (from } 24167) = 34$

Step II: $4 \times 4 \text{ (from } 34) + 3 \text{ (from } 34) + 1 \text{ (from } 24\underline{1}67) = 20$

Step III: 4×0 (from $2\underline{0}$) + 2(from $\underline{2}0$) + 4 (from $2\underline{4}167$) = 6

Step IV: $4 \times 6 \text{ (from } \underline{6}) + 2 \text{(from } \underline{2}4167) = 26$

Since 26 is divisible by 13, the number is divisible by 13.

DIVISIBILITY BY 17: The negative osculator for 17 is 5. Therefore the working principle is same as in the case of 7.

EXAMPLE 5 Is 1904 divisible by 17?

Solution $1904:190-4\times5=170$

Since 170 is divisible by 17, the number is divisible by 17.

EXAMPLE 6 Is 8971563 divisible by 17?

Solution 897

Since 51 is divisible by 17, the number is divisible by 17.

DIVISIBILITY BY 19: One more osculator for 19 is 2; the working principle is same as in the case of 13.

EXAMPLE 7 Is 149283 divisible by 19?

Solution Step I: 2×3 (from 149283) + 8 (from 149283) = 14

Step II: $2 \times 4 \text{ (from } 14) + 1 \text{ (from } 14) + 2 \text{ (from } 149283) = 11$

Step III: 2×1 (from $1\underline{1}$) + 1(from $\underline{1}$ 1) + 9(from $14\underline{9}$ 283) = 12

Step IV: $2 \times 2 \text{ (from } 1\underline{2}) + 1 \text{ (from } \underline{1}2) + 4 \text{ (from } 1\underline{4}9283) = 9$

Step V: $2 \times 9 \text{ (from } \underline{9}) + 1 \text{ (from } \underline{1}49283) = 19$

which is divisible by 19. Hence the number is divisible by 19.

SURDS

The roots of those quantities which cannot be exactly obtained are called surds; for example, $\sqrt{3}$, $\sqrt{6}$, etc.

MIXED SURDS: A rational factor and a surd multiplied together produce a mixed surd, for example, $2\sqrt{3}$, $4\sqrt{5}$, etc.

ORDER OF SURDS: $a^{1/n}$ is called a surd of the *n*th order.

Changing the surds into that of the same order:

EXAMPLE 1 Express $4^{1/3}$, $2^{1/5}$ and $5^{1/6}$ as surds of the same order and arrange them in the ascending order of magnitude.

Solution The LCM of 3, 5 and 6 (root indices) is 30. Reducing all surds into the 30th order, we get,

$$4^{1/3} = (4^{10})^{1/30} = (16^5)^{1/30}$$
$$2^{1/5} = (2^6)^{1/30} = (64)^{1/30}$$
$$5^{1/6} = (5^5)^{1/30} = (3125)^{1/30}$$

Hence, the order of magnitude is $((16)^5)^{1/30} > (3125)^{1/30} > (64)^{1/30}$ or $4^{1/3} > 5^{1/6} > 2^{1/5}$

EXAMPLE 2 Express $4^{1/3}$, $3^{1/6}$ and $6^{1/4}$ as surds of the same order and arrange them in the ascending order of magnitude.

Solution The LCM of orders 4, 3 and 6 is 12. Now reduce them to the 12th order.

$$\begin{aligned} &4^{1/3}=4^{4/12}=(4^4)^{1/12}=(256)^{1/12}\\ &3^{1/6}=3^{2/12}=(3^2)^{1/12}=(9)^{1/12}\\ &6^{1/4}=6^{3/12}=(6^3)^{1/12}=(216)^{1/12} \end{aligned}$$

Hence $(9)^{1/12} < (216)^{1/12} < (256)^{1/12}$ or $3^{1/16} < 6^{1/4} < 4^{1/3}$.

ADDITION AND SUBTRACTION OF SURDS: Similar surds like $3\sqrt{7}$, $5\sqrt{7}$, and $11\sqrt{7}$ can be added but dissimilar surds like $3\sqrt{5}$, $5\sqrt{2}$, $7\sqrt{3}$ cannot be added.

EXAMPLE 3 Simplify
$$\sqrt{75} + \sqrt{48}$$

Solution
$$\sqrt{75} + \sqrt{48} = \sqrt{25 \times 3} + \sqrt{16 \times 3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$$

EXAMPLE 4 Simplify
$$\sqrt{48} + \sqrt{147}$$

Solution
$$\sqrt{48} + \sqrt{147} = \sqrt{16 \times 3} + \sqrt{49 \times 3} = 4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$$

MULTIPLICATION OF SURDS

EXAMPLE 5 Find the product of $4^{1/3}$, $6^{1/6}$ and $\sqrt{5}$.

Solution
$$4^{1/3} \times 6^{1/6} \times 5^{1/2} = 4^{2/6} \times 6^{1/6} \times 5^{3/6} = (4^2 \times 6^1 \times 5^3)^{1/6} = (12000)^{1/6}$$

EXAMPLE 6 Find the product of $4^{1/3}$, $3^{1/6}$ and $6^{1/4}$.

Solution
$$4^{1/3} \times 3^{1/6} \times 6^{1/4} = 4^{4/12} \times 3^{2/12} \times 6^{3/12} = (4^4 \times 3^2 \times 6^3)^{1/12} = (497664)^{1/12}$$

DIVISION OF SURDS

EXAMPLE 7 Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$.

Solution
$$\frac{12 \times 4^{1/3}}{3 \times 2^{1/2}} = 4 \times \frac{(4^2)^{1/6}}{(2^3)^{1/6}} = 4 \times \left(\frac{16}{8}\right)^{1/6} = 4 \times 2^{1/6}$$

EXAMPLE 8 Divide $8 \times 6^{1/3}$ by $2\sqrt{2}$.

Solution
$$\frac{8 \times 6^{1/3}}{2\sqrt{2}} = \frac{8 \times 6^{1/3}}{2 \times 2^{1/2}} = 4 \times \frac{6^{2/6}}{2^{3/6}} = 4 \times \left(\frac{6^2}{2^3}\right)^{1/6} = 4 \times \left(\frac{36}{8}\right)^{1/6} = 4 \times \left(\frac{9}{2}\right)^{1/6}$$
$$= 2 \times 2^{5/6} \times 9^{1/6} = 2^{11/6} \times 9^{1/6}.$$

Important Results

1.
$$\sqrt{a} \times \sqrt{a} = a$$

$$2. \quad \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

3.
$$\sqrt{a^2 \times b} = a\sqrt{b}$$

4.
$$(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$$

5.
$$(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$$

6.
$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

SOME USEFUL ROOTS TO BE REMEMBERED

$$\sqrt{2} = 1.41421;$$
 $\sqrt{3} = 1.73205;$ $\sqrt{5} = 2.23607$

$$\sqrt{6} = 2.44949;$$
 $\sqrt{7} = 2.64575;$ $\sqrt{8} = 2.82842$

EXAMPLE 9 Find the value of $\sqrt{300}$.

Solution
$$\sqrt{300} = \sqrt{10 \times 10 \times 3} = 10\sqrt{3} = 10 \times 1.732 = 17.32$$

EXAMPLE 10 Find the value of $\sqrt{500}$.

Solution
$$\sqrt{500} = \sqrt{10 \times 10 \times 5} = 10\sqrt{5} = 10 \times 2.236 = 22.36$$

EXAMPLE 11 Evaluate the following:

(a)
$$\sqrt{75} + \sqrt{147}$$
 (b) $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$

Solution

(a)
$$\sqrt{75} + \sqrt{147} = \sqrt{5 \times 5 \times 3} + \sqrt{7 \times 7 \times 3} = 5\sqrt{3} + 7\sqrt{3} = 12\sqrt{3} = 20.7846$$

(b)
$$\sqrt{80} + 3\sqrt{245} - \sqrt{125} = \sqrt{4 \times 4 \times 5} + 3\sqrt{7 \times 7 \times 5} - \sqrt{5 \times 5 \times 5}$$

= $4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5} = 44.7214$

EXAMPLE 12 Evaluate the following:

(a)
$$\sqrt{108} + \sqrt{192}$$

(a)
$$\sqrt{108} + \sqrt{192}$$
 (b) $\sqrt{45} + 2\sqrt{405} - \sqrt{180}$

Solution

(a)
$$\sqrt{108} + \sqrt{192} = \sqrt{6 \times 6 \times 3} + \sqrt{8 \times 8 \times 3}$$

= $6\sqrt{3} + 8\sqrt{3} = 14\sqrt{3} = 14 \times 1.732 = 24.248$

(b)
$$\sqrt{45} + 2\sqrt{405} - \sqrt{180} = \sqrt{3 \times 3 \times 5} + 2\sqrt{9 \times 9 \times 5} - \sqrt{6 \times 6 \times 5}$$

= $3\sqrt{5} + 2 \times 9\sqrt{5} - 6\sqrt{5} = 3\sqrt{5} + 18\sqrt{5} - 6\sqrt{5}$
= $(3 + 18 - 6)\sqrt{5} = 15\sqrt{5}$

EXAMPLE 13 Evaluate the following:

(a)
$$\sqrt{2} \times \sqrt{3}$$

(b)
$$\sqrt{6} \times \sqrt{150}$$

(c)
$$\sqrt{242} \div \sqrt{72}$$

Solution

(a)
$$\sqrt{2} \times \sqrt{3} = \sqrt{6} = 2.4495$$

(b)
$$\sqrt{6} \times \sqrt{150} = \sqrt{6 \times 150} = \sqrt{900} = 30$$

(c)
$$\sqrt{242} \div \sqrt{72} = \frac{\sqrt{121 \times 2}}{\sqrt{36 \times 2}} = \frac{11}{6} = 1\frac{5}{6}$$

EXAMPLE 14 Evaluate the following:

(a)
$$\sqrt{2} \times \sqrt{3}$$

(b)
$$\sqrt{8} \times \sqrt{288}$$

(c)
$$\sqrt{243} \div \sqrt{75}$$

Solution

(a)
$$\sqrt{2} \times \sqrt{3} = \sqrt{6} = 2.44949$$

(b)
$$\sqrt{8} \times \sqrt{288} = \sqrt{8 \times 288} = \sqrt{16 \times 144} = 4 \times 12 = 48$$

(c)
$$\sqrt{243} \div \sqrt{75} = \sqrt{9 \times 9 \times 3} \div \sqrt{5 \times 5 \times 3} = \frac{9\sqrt{3}}{5\sqrt{3}} = \frac{9}{5} = 1\frac{4}{5}$$

EXAMPLE 15 Find the values of

(a)
$$\frac{1}{\sqrt{2}}$$

(b)
$$\frac{1}{\sqrt{3}}$$

(c)
$$\frac{1}{\sqrt{5}}$$

(b)
$$\frac{1}{\sqrt{3}}$$
 (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$

(e)
$$\frac{1}{\sqrt{7}}$$

Solution

(a)
$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} = \mathbf{0.7071}$$

(b)
$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5773$$

(c)
$$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5} = \mathbf{0.4472}$$

(d)
$$\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{\sqrt{6}}{6} = \mathbf{0.4082}$$

(e)
$$\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7} = \mathbf{0.3779}$$

EXAMPLE 16 Evaluate the following:

(a)
$$\frac{1}{\sqrt{3}-1}$$

(b)
$$\frac{16}{2 + \sqrt{3}}$$

(c)
$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

(d)
$$\frac{\sqrt{5} - 1}{\sqrt{5} + 1}$$

(a)
$$\frac{1}{\sqrt{3}-1}$$
 (b) $\frac{16}{2+\sqrt{3}}$ (c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (e) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$(f) \quad \frac{4+\sqrt{2}}{\sqrt{2}+1}$$

(f)
$$\frac{4+\sqrt{2}}{\sqrt{2}+1}$$
 (g) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Solution

(a)
$$\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2} = 1.366$$

(b)
$$\frac{16}{2+\sqrt{3}} = \frac{16(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{16(2-\sqrt{3})}{4-3}$$
$$= 16(2-\sqrt{3}) = 16(2-1.732) = 16 \times 0.268 = 4.288$$

(c)
$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3} = 3.732$$

(d)
$$\frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}-1)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{(\sqrt{5}-1)^2}{5-1} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2} = \frac{3-2.236}{2} = \mathbf{0.382}$$

(e)
$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{4-3} = \frac{7+4\sqrt{3}}{1}$$

$$= 7 + 4 \times 1.732 = 7 + 6.928 = 13.928$$

(f)
$$\frac{4+\sqrt{2}}{\sqrt{2}+1} = \frac{(4+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{2-4+3\sqrt{2}}{2-1} = \frac{3\sqrt{2}-2}{1} = 4.242-2 = 2.242$$

(g)
$$\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = \frac{(\sqrt{3} - \sqrt{2})^2}{3 - 2} = \frac{5 - 2\sqrt{6}}{1}$$
$$= 5 - 2(2.449) = 5 - 4.899 = 0.101$$



Short cut: (Direct Formula)

(i)
$$\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$$
 and (ii) $\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}$

Solution

(i)
$$\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{(\sqrt{a} - \sqrt{b})^2}{(\sqrt{a})^2 - (\sqrt{b})^2} = \frac{a + b - 2\sqrt{ab}}{a - b} = \frac{a + b}{a - b} - \frac{2\sqrt{ab}}{a - b}$$

(ii)
$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a})^2 - (\sqrt{b})^2} = \frac{a + b + 2\sqrt{ab}}{a - b} = \frac{a + b}{a - b} + \frac{2\sqrt{ab}}{a - b}$$

In the above expressions, the first and second terms are the same for both cases. They are added when numerator of surd has a '+' sign and subtracted when the numerator of the surd has a '-' sign.

EXAMPLE 17 Find the value of (a) $\frac{\sqrt{8} - \sqrt{7}}{\sqrt{8} + \sqrt{7}}$ (b) $\frac{\sqrt{8} + \sqrt{7}}{\sqrt{8} - \sqrt{7}}$

Solution (a)
$$\frac{\sqrt{8} - \sqrt{7}}{\sqrt{8} + \sqrt{7}} = \frac{8+7}{8-7} - \frac{2\sqrt{8\times7}}{8-7} = 15 - 2\sqrt{56} = 15 - 4\sqrt{14}$$

(b)
$$\frac{\sqrt{8} + \sqrt{7}}{\sqrt{8} - \sqrt{7}} = \frac{8+7}{8-7} - \frac{2\sqrt{8\times7}}{8-7} = 15 + 2\sqrt{56} = 15 + 4\sqrt{14}$$

EXAMPLE 18 Find the value of the following expressions correct up to 3 decimal places.

(a)
$$\frac{1}{\sqrt{6}}$$

(b)
$$\frac{1}{\sqrt{11}}$$

(c)
$$\frac{1}{\sqrt{2}-1}$$

(b)
$$\frac{1}{\sqrt{11}}$$
 (c) $\frac{1}{\sqrt{2}-1}$ (d) $\frac{1}{\sqrt{7}-1}$ (e) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

(e)
$$\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

(f)
$$\frac{\sqrt{5} + 1}{\sqrt{5} - 1}$$

(g)
$$\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

(f)
$$\frac{\sqrt{5}+1}{\sqrt{5}-1}$$
 (g) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ (h) $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ (i) $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$

$$(i) \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$$

Solution

(a)
$$\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} = 0.4082$$

(b)
$$\frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11} = \mathbf{0.302}$$

(c)
$$\frac{1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = 2.414$$

(d)
$$\frac{1}{\sqrt{7}-1} = \frac{(\sqrt{7}+1)}{7-1} = \frac{3.64575}{6} = \mathbf{0.608}$$

(e)
$$\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)^2}{2-1} = 3 + 2\sqrt{2} = 5.828$$

(f)
$$\frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)^2}{5-1} = \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2} = 2.618$$

(g)
$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})^2}{4-3} = \frac{7+4\sqrt{3}}{1} = 13.928$$

(h)
$$\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}} = \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}} = \frac{(\sqrt{5}+1)}{2} = 1.618$$

(i)
$$\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 3.732$$

EXAMPLE 19 Arrange the following in the ascending order

$$\sqrt{7} - \sqrt{6}$$
, $\sqrt{6} - \sqrt{5}$, $\sqrt{9} - \sqrt{7}$, $\sqrt{11} - \sqrt{9}$

Solution
$$\sqrt{7} - \sqrt{6} = \frac{(\sqrt{7} - \sqrt{6})(\sqrt{7} + \sqrt{6})}{(\sqrt{7} + \sqrt{6})} = \frac{7 - 6}{\sqrt{7} + \sqrt{6}} = \frac{1}{\sqrt{7} + \sqrt{6}}$$

$$\sqrt{6} - \sqrt{5} = \frac{(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})}{(\sqrt{6} + \sqrt{5})} = \frac{6 - 5}{\sqrt{6} + \sqrt{5}} = \frac{1}{\sqrt{6} + \sqrt{5}}$$

$$\sqrt{9} - \sqrt{7} = \frac{(\sqrt{9} - \sqrt{7})(\sqrt{9} + \sqrt{7})}{(\sqrt{9} + \sqrt{7})} = \frac{9 - 7}{\sqrt{9} + \sqrt{7}} = \frac{2}{\sqrt{9} + \sqrt{7}}$$

$$\sqrt{11} - \sqrt{9} = \frac{(\sqrt{11} - \sqrt{9})(\sqrt{11} + \sqrt{9})}{(\sqrt{11} + \sqrt{9})} = \frac{11 - 9}{\sqrt{11} + \sqrt{9}} = \frac{2}{\sqrt{11} + \sqrt{9}}$$

$$\therefore \frac{1}{\sqrt{7} + \sqrt{6}} < \frac{1}{\sqrt{6} + \sqrt{5}}; \frac{2}{\sqrt{11} + \sqrt{9}} < \frac{2}{\sqrt{9} + \sqrt{7}} \text{ and } \frac{1}{\sqrt{6} + \sqrt{5}} < \frac{2}{\sqrt{11} + \sqrt{9}}$$

$$\therefore \frac{1}{\sqrt{7} + \sqrt{6}} < \frac{1}{\sqrt{6} + \sqrt{5}} < \frac{2}{\sqrt{11} + \sqrt{9}} < \frac{2}{\sqrt{9} + \sqrt{7}}$$

When the denominator increases, the value of fraction decreases.

Number System

NUMBER SYSTEM

CYCLIC NUMBER: This is an integer of n digits which on being multiplied by any number from 1 through n, gives a product containing the same n digits as in the original number and these digits are in the same cyclic order in the product. For example,

- (a) $142857 \times 1 = 142857$
- (b) $142857 \times 2 = 285714$
- (c) $142857 \times 3 = 428571$
- (d) $142857 \times 4 = 571428$

REAL NUMBER: Any number having a meaningful value is called a real number. It can be either rational or irrational.

PRIME NUMBER: A number which is divisible by one and the number itself is called a prime number. For example, 2, 3, 5, 7, 11, etc.

PERFECT NUMBER: If the sum of the divisors of a number excluding the number itself is equal to the number, then the number is called perfect number. For example, 6, 28, 496, 8128, etc.

i.e.
$$6 = 1 + 2 + 3$$
; $28 = 1 + 2 + 4 + 7 + 14$.

GOLDEN RATIO: The average of 1 and $\sqrt{5}$ is called the golden ratio.

i.e.
$$\frac{1+\sqrt{5}}{2} = 1.61803.$$

The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2. For example,

for 6,
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2$$

for 28,
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = 2$$

Every even perfect number is of the form $2^{n-1}(2^n-1)$ where (2^n-1) and n are prime numbers.

AMICABLE NUMBERS: If the sum of divisors of N_1 is N_2 and the sum of divisors of N_2 is N_1 , then N_1 and N_2 are amicable numbers. For example, 220 and 284.

The sum of divisors of 220

$$= 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$$

The sum of divisor of 284

$$= 1 + 2 + 4 + 71 + 142 = 220$$

: 220 and 284 are amicable numbers.

SOCIABLE: If the chain that leads back to the original number has more than two links the number is called *sociable*. For example, 12496, 14288, 15472, and 14536, 14264.

- (i) If n is odd, $n(n^2 1)$ is divisible by 24 and if n is an odd prime number greater than 3, $n^2 1$ is divisible by 24. For example, n = 11, $n^2 1 = 11^2 1 = 120 = 24 \times 5$, divisible by 24
- (ii) If n is odd, $2^n + 1$ is divisible by 3 and if n is even, $2^n 1$ is divisible by 3.
- (iii) If n is prime to 5, then $n(n^4 1)$ is divisible by 30.
- (iv) If n is an odd number $2^{2n} + 1$ is divisible by 5 and if n is an even number, $2^{2n} 1$ is divisible by 5.
- (v) If n is odd, $5^{2n} + 1$ is divisible by 13 and if n is even, $5^{2n} 1$ is divisible by 13.
- (vi) If a > b, then |a b| = a b and if a < b, then |a b| = b a.
- (vii) If a < b, then $a < \frac{a+b}{2} < b$. This is because $\frac{a+b}{2}$ is the average of a and b.

The average of two numbers always lies between the two numbers. There are infinite number of rational numbers between any two rational numbers.

- (viii) $x^n + y^n$ is divisible by x + y when n is odd and $x^n + y^n = (x + y)(x^{n-1} x^{n-2}y + x^{n-3}y^2 + \dots + y^{n-1})$.
 - (ix) $x^n y^n$ is divisible by x + y when n is even and by x y when n is either even or odd and $x^n y^n = (x + y) (x^{n-1} x^{n-2} y + x^{n-3} y^2 \dots + y^{n-1})$ when n is even and $x^n y^n = (x y) (x^{n-1} + x^{n-2} y + \dots + y^{n-1})$ when n is either odd or even.

The fifth power of any single digit number has the same right hand digit as the number itself.

$$x^3 - x$$
 is divisible by 3

 $x^5 - x$ is divisible by 5

 $x^7 - x$ is divisible by 7

 $x^{11} - x$ is divisible by 11

 $x^{13} - x$ is divisible by 13

i.e. if n is any prime number, then for any whole number $x^n - x$ is divisible by n.

Difference between the squares of two consecutive numbers is always an odd number and is equal to the sum of those two consecutive numbers.

When two numbers are divided by a third number, leave the same remainder, the difference of those two numbers must be perfectly divisible by the third number.

$$Quotient = \frac{Dividend - Remainder}{Divisor}$$

$$Divisor = \frac{Dividend - Remainder}{Quotient}$$

or

To find the product of two numbers when the sum and the difference of the two numbers are given.

Product of numbers =
$$\frac{(Sum + Difference)(Sum - Difference)}{4}$$

EXAMPLE 1 The given sum and difference of two numbers are respectively 50 and 30. Find the product of those numbers.

Solution Product of numbers =
$$\frac{(50+30)(50-30)}{4} = \frac{80\times20}{4} = 400$$

The numbers can also be found by the following formula

First number =
$$\frac{Sum + Difference}{2}$$

Second number =
$$\frac{Sum - Difference}{2}$$

The sum of the digits of a 2-digit number is S. If the digits are reversed, the number is decreased by N, then the number is $5\left(S + \frac{N}{9}\right) + \frac{1}{2}\left(S - \frac{N}{9}\right)$.

The sum of digits of a two-digit number is S. If the digits are reversed, the number is increased by N. Then the number is $5\left(S-\frac{N}{9}\right)+\frac{1}{2}\left(S+\frac{N}{9}\right)$.

EXAMPLE 2 The sum of digits of a two-digit number is 6 and if the digits are reversed the number is increased by 36. Find the number.

Solution Required number =
$$5\left(S - \frac{N}{9}\right) + \frac{1}{2}\left(S + \frac{N}{9}\right)$$

= $5\left(6 - \frac{36}{9}\right) + \frac{1}{2}\left(6 + \frac{36}{9}\right)$

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$$= 5(6-4) + \frac{1}{2}(6+4)$$
$$= 5 \times 2 + \frac{1}{2} \times 10 = 15$$

If the ratio of sum and difference of two numbers is x : y, then the ratio of these two numbers is $\frac{x+y}{x-y}$.

EXAMPLE 3 The ratio of sum and difference of two numbers is 9:5. Find the ratio of these two numbers.

Solution Required ratio =
$$\frac{9+5}{9-5} = \frac{14}{4} = 7:2$$

If the sum of a number and its square is
$$P$$
, then the number is $\frac{\sqrt{1+4P}-1}{2}$

EXAMPLE 4 If the sum of a number and its square is 2070, find the number.

Solution Let the number be x.



Short cut: Required number =
$$\frac{\sqrt{1+4P}-1}{2} = \frac{\sqrt{1+4\times2070}-1}{2} = \frac{91-1}{2} = 45$$

The ratio between a two-digit number and the sum of the digits of that number is x : y. If the digit in the unit place is P more than the digit in the tenth place, then

Number =
$$\left(\frac{9x}{11y - 2x}\right)P$$

Digit in unit place = $\left(\frac{10y - x}{11y - 2x}\right)P$
Digit in the tenth place = $\left(\frac{x - y}{11y - 2x}\right)P$

EXAMPLE 5 The ratio between a two-digit number and the sum of the digits of that number is 4:1. If the digit in the unit place is 4 more than digit in the tenth place, then find the number.

Solution Let the number be xy.

$$\therefore \frac{10 \times x + y}{x + y} = \frac{4}{1}$$

$$\Rightarrow 10x + y = 4x + 4y$$

$$\Rightarrow 6x = 3y \text{ or } 2x = y$$
But
$$y = x + 4$$

$$\therefore 2x = y \Rightarrow 2x = x + 4 \text{ or } x = 4$$

$$\therefore y = x + 4 = 4 + 4 = 8$$

.. The required number is 48.



Short cut: Required number =
$$\left(\frac{9x}{11y - 2x}\right)P$$

where x: y = ratio of two-digit number to sum of digits. P = amount by which unit place is more than tenth place.

$$\therefore \text{ Required number} = \left(\frac{9 \times 4}{11 \times 1 - 2 \times 4}\right) 4 = \frac{9 \times 4 \times 4}{3} = 48$$

To find the difference of the two digits of a two digit number, when the difference between two-digit number and the number obtained by interchanging the digits is given.

Solution Difference of two digits = $\frac{\text{Difference in original and interchanged numbers}}{9}$

The product of two numbers is P and their difference is q. Then the number are

$$\frac{\sqrt{q^2+4P}+q}{2}$$
 and $\frac{\sqrt{q^2+4P}-q}{2}$

EXAMPLE 6 The product of 2 numbers is 368 and the difference of these two numbers is 7. Find the numbers.

Solution Let the numbers be x and y. Given $x \times y = 368$; x - y = 7; x = ?; y = ?

Required numbers are $\frac{\sqrt{7^2 + 4 \times 368} + 7}{2}$ and $\frac{\sqrt{7^2 + 4 \times 368} - 7}{2} = 23$ and 16.

The product of 2 numbers is x and the sum of these 2 numbers is y then the numbers

are
$$\frac{y + \sqrt{y^2 - 4x}}{2}$$
 and $\frac{y - \sqrt{y^2 - 4x}}{2}$.

EXAMPLE 7 The product of two numbers is 204 and their sum is 29. Find the numbers.

Solution Required numbers are $\frac{29 + \sqrt{29^2 - 4 \times 204}}{2}$ and $\frac{29 - \sqrt{29^2 - 4 \times 204}}{2} = 17$ and 12.

The sum of squares of two numbers is x and the square of their difference is y, then the product of the two numbers is $\frac{x-y}{2}$.

EXAMPLE 8 The sum of squares of two numbers is 90 and the square of their difference is 36. Find the product of the numbers.

Solution Let the two numbers be x and y.

Given
$$x^2 + y^2 = 90$$
; $(x - y)^2 = 36$

$$\therefore \text{ Product of number } xy = \frac{(x^2 + y^2) - (x - y)^2}{2} = \frac{90 - 36}{2} = 27$$

EXAMPLE 9 The product of two numbers is 120. The sum of their squares is 289. Find the sum of the two numbers and also find the difference of the two numbers.

Solution Sum of two numbers =
$$\sqrt{289 + 2 \times 120} = 23$$

Difference of numbers =
$$\sqrt{289 - 2 \times 120} = \sqrt{49} = 7$$

The denominator of a rational number is 'D' more than its numerator. If the numerator is increased by x and the denominator is decreased by y, the result obtained is N. Then the

rational number is
$$\frac{x - N(D - y)}{x + (yN - D)}$$
.

EXAMPLE 10 The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 6 and denominator is decreased by 2, we obtain 2. Find the rational number.

Solution Required answer =
$$\frac{6 - 2(3 - 2)}{6 + (2 \times 2 - 3)} = \frac{4}{7}$$

The sum of two numbers is P and their difference is q then the difference of their squares is Pq.

EXAMPLE 11 The sum of 2 numbers is 85 and their difference is 28. Find the difference of their squares.

Solution Required answer = $85 \times 28 = 2380$

The difference between the squares of two consecutive numbers is x. Then the numbers

are
$$\frac{x-1}{2}$$
 and $\frac{x+1}{2}$.

EXAMPLE 12 The difference between the squares of two consecutive numbers is 59. Find the numbers.

Solution The numbers are $\frac{59+1}{2}$, $\frac{59-1}{2} = 30,29$. If two consecutive numbers are

p and q, then the difference of their squares = p + q. If the sum of 2 numbers is p and sum of their squares is q. Then

(i) Product of numbers =
$$\frac{p^2 - q}{2}$$

(ii) Numbers are
$$\left(\frac{p-\sqrt{2q-p^2}}{2}\right)$$
 and $\left(\frac{p+\sqrt{2q-p^2}}{2}\right)$

EXAMPLE 13 The sum of two numbers is 15 and sum of their squares is 113. Find the numbers

Solution Required numbers are
$$=\frac{15 - \sqrt{2 \times 113 - 15^2}}{2}$$
 and $\frac{15 + \sqrt{2 \times 113 - 15^2}}{2}$
 $=\frac{15 - 1}{2}$ and $\frac{15 + 1}{2} = 7$ and 8

To find the number of zeros at the end of the product.

$$(2^m)$$
 (5^n) has n zeros if $m > n$ or m zeros if $m < n$.

To find the numbers in the unit place for odd numbers.

$$(...1)^n$$
, $(...3)^{4n}$, $(...7)^{4n}$, $(...9)^{2n}$ ends with 1.

For example,

$$(21)^1 = 21$$
, ends with 1
 $(21)^2 = 441$, ends with 1.
 $(13)^4 = (169)^2 = 28561$, ends with 1
 $(17)^4 = (289)^2 = 83521$, ends with 1
 $(9)^2 = 81$ ends with 1.

BINARY SYSTEM

A number system represents different numbers in different ways. There are many number systems. The number system with Base 2 is known as *binary number system*. The base denotes the number of symbols in the number system.

In the decimal system, for example, the base is 10 and it has 10 symbols (0, 1, 2, ..., 9). Some other examples are:

Number system	Base	Representation of symbols	
Octal number system	8	0, 1, 2, 3,, 7	
Hexadecimal number system	16	0, 1, 2,, 9, A(10), B (11), C(12) F(15)	
Binary number system	2	0 and 1.	

The binary number system was introduced by JV Newman in 1946.

CONVERSION OF DECIMAL NUMBERS INTO ITS BINARY EQUIVALENT: To find the binary equivalent of a decimal number, divide the given number by constant divisor 2 successively till last quotient 1 is obtained. For example convert 97 into its binary equivalent.

 $q_1, q_2, q_3, \dots q_6$ are the first/second/third ... quotients and r_1, r_2, \dots are the first/second/... remainders. For all stages of division the common divisor 2 remains unchanged and the quotient obtained becomes next dividend. The process continues till the last quotient (1) is obtained.

Here 97 is the first dividend. After dividing it by 2, the first quotient $(q_1 = 48)$ becomes the next dividend. Now after dividing 48 by 2, the second quotient $(q_2 = 24)$ becomes the next dividend, and so on. Every time the remainder is noted. When the last quotient (1) is obtained, note down the remainders including the last quotient, 1, starting from the last quotient, then the last remainder, then the second last remainder ... and finally the first remainder.

That is binary equivalent of 97 is 1100001. i.e. $(97)_{10} = (1100001)_2$

CONVERSION OF BINARY NUMBER TO ITS DECIMAL EQUIVALENT

Convert 1011001 into its decimal equivalent.

The above number can be written inside each box in the following way and value of each box is written above it

Now $(1011001)_2$

$$= 1 \times 2^{6} + 0 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 0 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0}$$

$$= 64 + 0 + 16 + 8 + 0 + 0 + 1$$

$$= 89$$

EXAMPLE 14 Convert 95 into its binary equivalent.

Solution 2
$$95 \ 1 \ (r_1)$$

$$47 \ 1 \ (r_2)$$

$$23 \ 1 \ (r_3)$$

$$11 \ 1 \ (r_4)$$

$$5 \ 1 \ (r_5)$$

$$2 \ 0 \ (r_6)$$

$$1 \ (last quotient)$$
Here $(95)_{10} = (1011111)_2$

$$Check: \ (1011111)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 \times 1 \times 2^1 + 1 \times 2^0$$

$$= 64 + 0 + 16 + 8 + 4 + 2 + 1$$

$$= 95$$

EXAMPLE 15 Convert 58 into its binary equivalent.

Solution 2 | 58 0

29 1

| 14 0

7 1

3 1 1

 $(58)_{10} = (111010)_2$

= 58

 $(111010)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

- **EXAMPLE 16** Convert the following numbers into their binary equivalents. (a) 3
 - (b) 4
- (c) 5

= 32 + 16 + 8 + 0 + 2 + 0

- (d) 6
- (e) 7
- (f) 8

Solution

Check:

(a) 2 | 3 1

i.e. $(3)_{10} = (11)_2$ 1

(b) 2 | 4 0

2 0 1

i.e. $(4)_{10} = (100)_2$

(c) 2 | 5 1

2 0

i.e. $(5)_{10} = (101)_2$ 1

(d) 2 | 6 0

3 1

i.e. $(6)_{10} = (110)_2$ 1

(e) 2 | 7 1

3 1 1

i.e. $(7)_{10} = (111)_2$

(f) 2 | 8 0

4 0

2 0

1 i.e. $(8)_{10} = (1000)_2$

EXAMPLE 17 Convert the following binary numbers into their decimal equivalent.

- (a) 11001

- (b) 100010 (c) 110010 (d) 1010110 (e) 1101101

Solution

(a)
$$(11001)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

= $16 + 8 + 0 + 0 + 1 = 25$

(b)
$$(100010)_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

= $32 + 0 + 0 + 0 + 2 + 0 = 34$

(c)
$$(110010)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

= $32 + 16 + 0 + 0 + 2 + 0 = \mathbf{50}$

(d)
$$(1010110)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

= $64 + 0 + 16 + 0 + 4 + 2 + 0 = 86$

(e)
$$(1101101)_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

= $64 + 32 + 0 + 8 + 4 + 0 + 1 = 109$



EXERCISES

1.	A certain number of two digits is three times the sum of its digits, and if 45 be	added to it,
	the digit will be reversed. Then the number is:	[LIC AAO]

- (a) 23
- (b) 27
- (c) 32
- (d) 72

2.	A number of two digits has 3 for its unit's digit, and the sum of digits is	$\frac{1}{7}$ of the number
	itself. The number is	[LIC]

- (a) 43
- (b) 53
- (c) 63
- (d) 73

3. The sum of two numbers is 25 and their difference is 13. Find their product. [LIC]

- (a) 104
- (b) 114
- (c) 315
- (d) 325

4. Find the number which when multiplied by 15 is increased by 196. [LIC]

- (a) 1*4*
- (b) 20
- (c) 26
- (d) 28

5. The numerator and denominator of a fraction are in the ratio of 2:3. If 6 is subtracted from the numerator, the result is a fraction that has a value $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is:

[SSC]

- (a) 6
- (b) 18
- (c) 27
- (d) 36

6. A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of digits in the number is: [SSC]

- (a) 6
- (b) 7
- (c) 8
- (d) 9

7. The sum of squares of three consecutive natural numbers is 2030. What is the middle number? [SSC]

- (a) 25
- (b) 26
- (c) 27
- (d) 28

8. If the sum of two numbers is 22 and the sum of their squares is 404, then the product of the numbers is:

[SSC]

- (a) 40
- (b) 44
- (c) 80
- (d) 88

9.	If the sum and d their squares is:	ifference of t	wo number	rs are 20 a	nd 8 res	spectively,	then the difference of [SSC]
	(a) 12	(b) 28	(c)	160	(d)	180	
10.	The difference be sum of the numb		ositive integ	gers is 3. If	the sun	n of their s	quares is 369, then the [SSC]
	(a) 25	(b) 27	(c)	33	(d)	81	
11.	Two numbers diff	fer by 5. If th	eir product	is 336, the	n the su	ım of the t	wo numbers is [SSC]
	(a) 21	(b) 28	(c)	37	(d)	51	
12.	The product of tw	o natural nu	mbers is 17	. Then the	sum of t	the recipro	cals of their squares is: [SSC]
	-07	(b) $\frac{289}{290}$, ,	289	
13.	If doubling a nun number by 8 and						wer as multiplying the [SSC]
	(a) 2	(b) 3	(c)	4	(d)	6	
14.	Thrice the square than the number			ecreased by	y 4 time	s the num	ber is equal to 50 more [SSC]
	(a) 4	(b) 5	(c)	6	(d)	10	
15.	second number, results are equal numbers?	the third is	multiplied difference l	by 3 and	the four	rth is divident the s	is subtracted from the ded by 3, then all the smallest of the original [SBI PO]
	(a) 21		(b) 27			(c) 32	
	(d) Cannot be de		(e) None				
16.	by 1. If the digits	in the unit's and the origi	place and t	he ten's pl	ace are i	nterchang	the digit in ten's place ed, difference between mber by 1. What is the [Bank PO]
	(a) 25	(b) 37	(c)	49	(d)	52	(e) 73
17.	-	umber thus o	btained is	equal to th			digit in the ten's place d by interchanging the [Bank PO]
	(a) Sum of the d	igits is a two	o-digit numl	ber			
	(b) Digit in the u	ınit's place i	s twice the	digit in th	e ten's p	olace	
	(c) Digit in the u	=	_	_			
	(d) Digit in the u	ınit's place i	s half of the	e digit in t	he ten's	place	
	(e) None of these						
18.	The sum of three is the middle nur		odd numbei	rs is 20 mo	re than	the first o	f these numbers. What [SBI PO]
	(a) 7		(b) 9			(c) 11	
	(d) Data inadequ	uate	(e) None	of these			
19.	One-fifth of a nur	nber is equal	l to $\frac{5}{8}$ of an	other num	ber. If 3	5 is added	to the first number, it
	becomes four tim				ond num	ber is:	[Bank PO]
	(a) 25	(b) 40	(c)	70	(d)	125	

[Bank PO]

	(a) 15	(b) 26	(c) 28	(d) 91	(e) None of these
21.		creased by 4 and dom the number an	· ·		ould be the result if [Bank PO]
	(a) $9\frac{2}{3}$	(b) 10	(c) $10\frac{1}{5}$	(d) $11\frac{1}{5}$	(e) None of these
22 .		ber is such that the digits are reverse		digits is 8. When	18 is added to the [MBA]
	(a) 18	(b) 24	(c) 42	(d) 81	
23.	digits is 36. Wha	•	between the sum	and the difference	interchanging the of the digits of the [MAT]
	(a) 4	(b) 8	(c) 16	(d) None of these	e
24.	Three times the fithird integer is:	first of three conse	cutive odd integer	rs is 3 more than t	wice the third. The [MBA]
	(a) 9	(b) 11	(c) 13	(d) 15	
25 .	The sum of the di What is the two-o	digit number?			veen the digits is 3. [BSRB]
	(a) 69(e) None of these	, ,	(c) 96	(d) Cannot be de	termined
26.	The product of tw	o numbers is 45 a	nd the sum of thei	r squares is 106. T	he numbers are: [RRB]
	(a) 3 and 5		(c) 5 and 19		
27.	The product of two numbers is:	wo numbers is 120	0 and the sum of	their square is 28	39. The sum of the [RRB]
	(a) 20	(b) 23	(c) 169	(d) None of these	
28.		numbers is 264. If the first, then the se		be twice the second	d and third number [RRB]
	(a) 48	` '	(c) 72	(d) 84	
29.		mber. What is the	number?		sum of 40 and one- [RRB]
	(a) 174	(b) 216	(c) 246	(d) 336	
30.	If $2\frac{1}{2}$ is added to	a number and the	sum is multiplied	by $4\frac{1}{2}$ and 3 is ac	dded to the product
		g the sum by $1\frac{1}{5}$, t			e number? [RRB]
		(b) $3\frac{1}{2}$			
31.			the sum of its digi	its is equal to 144, t	digit by 2, and that then the number is: [CBI]
	(a) 24	(b) 26	(c) 42	(d) 46	
32.	bigger of the two	numbers is:		_	mes the other. The [CBI]
	(a) 10	(b) 12	(c) 15	(d) 16	

20. If the sum of a number and its square is 182, what is the number?

33. If the sum of two numbers is 33 and their difference is 15, the smaller number is: [CBI]

- (a) 9
- (b) 12
- (c) 15
- (d) 18

34. If the sum of one-half and one-fifth of a number exceeds one-third of that number by $7\frac{1}{3}$, the number is:

- (a) 15
- (b) 18
- (c) 20
- (d) 30

35. Two-third of a positive number and $\frac{25}{216}$ of its reciprocal are equal. The number is: [SSC]

- (a) $\frac{5}{12}$
- (b) $\frac{12}{5}$
- (c) $\frac{25}{144}$
- (d) $\frac{144}{25}$



1. (b) **2.** (c) **3.** (b) **4.** (a) **5.** (b) **6.** (b) **7.** (b) **8.** (a) **9.** (c) **10.** (b)

11. (c) 12. (c) 13. (c) 14. (b) 15. (c) 16. (b) 17. (b) 18. (b) 19. (b) 20. (e)

21. (b) 22. (b) 23. (b) 24. (d) 25. (d) 26. (b) 27. (b) 28. (c) 29. (b) 30. (b)

31. (a) **32.** (b) **33.** (a) **34.** (c) **35.** (a)

T.

${f S}$ olutions with Necessary Explanation

1. Let the number be 10x + y

Now, 10x + y = 3(x + y)

 \Rightarrow 7x = 2y

 $\therefore \frac{x}{y} = \frac{2}{7}$

 $\therefore \qquad \text{Required number} = 27$

2. Let the number be 10x + 3

$$x + 3 = \frac{1}{7}(10x + 3)$$

$$\Rightarrow 7x + 21 = 10x + 3$$

$$\Rightarrow 3x = 18$$

 \Rightarrow x = 6

 \therefore Required number = 63

3. Let the number be x and y.

$$\therefore \qquad \qquad x + y = 25 \tag{1}$$

$$x - y = 13 \tag{2}$$

Now, $(25)^2 - (13)^2 = 4xy$

 $xy = \frac{(25)^2 - (13)^2}{4}$ $= \frac{(25 + 13)(25 - 13)}{4}$ $= \frac{38 \times 12}{4} = 19 \times 6 = 114$

4. Let number be x.

$$15x = x + 196$$

$$14x = 196$$

$$x = 14$$

5. Let the fraction be $\frac{p}{q}$.

Then
$$\frac{p}{q} = \frac{2}{3} \tag{1}$$

$$\frac{p-6}{q} = \frac{4}{9} \tag{2}$$

$$\Rightarrow (1) 3p = 2q (A)$$

$$\Rightarrow (2) \qquad 9(p-6) = 4q \tag{B}$$

From (A) and (B), we get

$$9(p-6) = 2 \times 3p$$
$$9p - 54 = 6p$$
$$3p = 54$$
$$p = 18$$

 \therefore 6. Let the number be 10x + y.

Then
$$x = y - 2 \tag{1}$$

$$3(10x + y) + \frac{6}{7}(10y + x) = 108$$

$$\Rightarrow 3\{10(y-2)+y\} + \frac{6}{7}\{10y+(y-2)\} = 108$$

$$\Rightarrow 3\{10y - 20 + y\} + \frac{6}{7}\{10y + y - 2\} = 108$$

$$\Rightarrow \qquad 33y - 60 + \frac{66}{7}y - \frac{12}{7} = 108$$

$$\Rightarrow \qquad \left(33 + \frac{66}{7}\right)y = 108 + 60 + \frac{12}{7}$$

$$\frac{297y}{7} = \frac{168 \times 7 + 12}{7}$$
$$y = \frac{168 \times 7 + 12}{297}$$

$$= \frac{56 \times 7 + 4}{99} = \frac{396}{99} = 4$$

:.
$$x = y - 2 = 4 - 2 = 2$$

Sum of digits of number = $x + y = 2 + 4 = 6$

7. Let the numbers be (x-1), x and (x+1)

:.

Then
$$(x-1)^2 + x^2 + (x+1)^2 = 2030$$

 $\Rightarrow 3x^2 + 2 = 2030$
 $\Rightarrow 3x^2 = 2028$

$$x^{2} = \frac{2028}{3}$$

$$x^{2} = 676$$

$$x = \sqrt{676}$$

$$= 26$$
∴ Middle number = $x = 26$

8. Let the numbers be x and y.

$$\therefore \qquad \qquad x + y = 22 \tag{1}$$

$$x^2 + y^2 = 404 \tag{2}$$

$$(1)^2 - (2) \Rightarrow (x + y)^2 - (x^2 + y^2) = (22)^2 - 404$$

$$\Rightarrow \qquad 2xy = 484 - 404$$

$$\Rightarrow$$
 $2xy = 80$

$$\Rightarrow \qquad xy = \frac{80}{2} = 40$$

9. Let the numbers be x and y.

Then
$$x + y = 20 \tag{1}$$

$$x - y = 8 \tag{2}$$

$$\therefore x^2 - y^2 = (x + y) (x - y) = (1) \times (2)$$

$$(1) \times (2) \quad \Rightarrow \quad (x+y) \ (x-y) = 20 \times 8$$

$$x^2 - y^2 = 160$$

10. Let the two positive integers be x and y.

$$\therefore \qquad x - y = 3 \tag{1}$$

$$x^2 + y^2 = 369 \tag{2}$$

$$(x + y) = ?$$

$$(x-y)^2 = (x^2 + y^2) - 2xy$$

i.e.
$$9 = 369 - 2xy$$

$$\Rightarrow$$
 $2xy = 360$

$$(x + y)^{2} = (x^{2} + y^{2}) + 2xy$$
$$= 369 + 360$$
$$= 729$$

$$x + y = \sqrt{729} = 27$$

11. Let the numbers be x and y.

Given
$$x - y = 5$$
 (1)

$$xy = 336 \tag{2}$$

$$x + y = \sqrt{(x - y)^2 + 4xy}$$

$$= \sqrt{5^2 + 4 \times 336}$$

$$= \sqrt{1369} = 37$$

12. Let the two natural numbers be x and y.

Given
$$xy = 17$$

 \therefore $x = 1$ and $y = 17$

$$\therefore \frac{1}{x^2} + \frac{1}{v^2} = \frac{1}{1^2} + \frac{1}{17^2} = 1 + \frac{1}{289} = \frac{290}{289}$$

13. Let the number be x.

Then
$$2x + 20 = 8x - 4$$

$$\Rightarrow 8x - 2x = 20 + 4$$

$$\Rightarrow 6x = 24$$

$$\Rightarrow x = \frac{24}{6} = 4$$

14. Let the natural number be x.

Then
$$3x^2 - 4x = x + 50$$

⇒ $3x^2 - 5x - 50 = 0$
⇒ $(3x + 10)(x - 5) = 0$
∴ $x = \frac{-10}{3}$ or 5

15. Let numbers be a, b, c and d.

Now,
$$a+3=b-3=3\times c=\frac{d}{3}=k$$

$$\therefore \qquad a=k-3; \quad b=k+3; \quad c=\frac{k}{3}; \quad d=3k;$$

$$\therefore \qquad a+b+c+d=64$$

$$\Rightarrow \qquad k-3+k+3+\frac{k}{3}+3k=64$$

$$\Rightarrow \qquad \frac{15k+k}{3}=64$$

$$\Rightarrow \qquad k=\frac{64\times 3}{16}=12$$

$$\therefore \qquad a = k - 3 = 9; \quad b = k + 3 = 12 + 3 = 15; \quad c = \frac{k}{3} = \frac{12}{3} = 4; \quad d = 3k \times 12 = 36$$

.. Numbers are 9, 15, 4 and 36.

Required difference = (36 - 4) = 32

16. Let the two digit number be 10x + y.

Then
$$y = 2x + 1$$
 (1)
 $(10y + x) - (10x + y) = (10x + y) - 1$
 $\Rightarrow \qquad (10y + x) + 1 = 2(10x + y)$
 $\Rightarrow \qquad 10y - 2y + x - 20x + 1 = 0$
 $\Rightarrow \qquad 8y - 19x + 1 = 0$ (2)
 \therefore Substituting value of y from (1), we get
 $8(2x + 1) - 19x + 1 = 0$

$$\Rightarrow \qquad -3x + 9 = 0$$

$$\Rightarrow \qquad x = 3$$

$$\therefore \qquad y = 2x + 1 = 2(3) + 1 = 7$$

 $\therefore \qquad \text{Original number} = 10x + y = 10 \times 3 + 7 = 37$

17. Let the number be 10x + y.

Then
$$10y + x = 10(2x) + \frac{y}{2}$$

$$\Rightarrow \qquad 10y + x = 20x + \frac{y}{2}$$

$$\Rightarrow \qquad 20y + 2x = 40x + y$$

$$\Rightarrow \qquad 38x = 19y$$

$$\Rightarrow \qquad 2x = y$$

18. Let the three consecutive odd numbers be (x-2), x and (x+2).

Then
$$(x-2) + x + (x+2) = (x-2) + 20$$

$$\Rightarrow 3x = x + 18$$

$$\Rightarrow 2x = 18$$

$$\Rightarrow x = 9$$

19. Let the numbers be x and y.

Now
$$\frac{1}{5}x = \frac{5}{8}y$$
 (1)
$$x + 35 = 4y$$
 (2) From (1),
$$x = \frac{25}{8}y$$

Substituting value of x in (2), we get

$$\frac{25}{8}y + 35 = 4y$$

$$\Rightarrow \qquad \left(4 - \frac{25}{8}\right)y = 35$$

$$\Rightarrow \qquad \frac{7}{8}y = 35$$

$$\Rightarrow \qquad y = \frac{35 \times 8}{7} = 40$$
∴ Second number = 40.

20. Let the number be x.

21. Let the number be x.

Then
$$\frac{x-4}{6} = 8$$

$$\Rightarrow \qquad x = 8 \times 6 + 4 = 52$$

$$\therefore \qquad \frac{x-2}{5} = \frac{52-2}{5} = 10$$

22. Let the number be 10x + y.

Now,
$$xy = 8$$
 (1)

$$(10x + y) + 18 = 10y + x$$
 (2)

$$\Rightarrow \qquad \qquad 9y - 9x = 18$$

$$\Rightarrow \qquad \qquad y - x = 2$$

$$y = 4 \text{ and } x = 2$$

$$xy = 8$$
Required number is $10x + y = 10(2) + 4 = 24$

23. Let the number be 10x + y.

so that

٠.

$$\therefore (10x + y) \sim (10y + x) = 36$$
i.e.
$$9x \sim 9y = 36$$

$$\Rightarrow x \sim y = 4$$
Given
$$y : x = 1 : 2$$

$$\therefore \text{ When } y = 4, x = 8$$
so that
$$x \sim y = 4$$

$$\therefore (x + y) \sim (x - y) = 2y = 8$$

24. Let the three consecutive odd numbers be (x-2), x and (x+2).

$$3(x-2) = 2(x+2) + 3$$

$$3x - 6 = 2x + 4 + 3$$

$$x = 4 + 3 + 6 = 13$$

 $\therefore \qquad \qquad \text{Third integer} = x + 2 = 15$

25. Let the digits of two-digit number be x and y.

$$\begin{array}{ll}
\therefore & x + y = 15 \\
x - y = 3
\end{array} \tag{1}$$

$$(1) + (2) \Rightarrow 2x = 18$$

$$\Rightarrow x = 9$$

From (1), y = 15 - 9 = 6

: 96 and 69 are possible answers for the required number, and both values are given as answer, correct answer is "cannot be determined". Hence answer is (d).

26. Let the numbers be x and y.

.. Required numbers are 5 and 9.

27. Let the numbers be x and y.

$$xy = 120$$

$$x^2 + y^2 = 289$$

$$(2) + 2 \times (1) \implies x^2 + y^2 + 2xy = 289 + 240 = 529$$

$$\therefore (x + y)^2 = 529$$

$$\Rightarrow x + y = \sqrt{529} = 23$$

$$(1)$$

$$(2)$$

$$x^2 + y^2 + 2xy = 289 + 240 = 529$$

$$\therefore (x + y)^2 = 529$$

28. Let the second number be x.

$$\therefore \qquad \qquad \text{First number} = 2x$$

$$\text{Third number} = \frac{2}{3}x$$

$$\therefore \qquad \qquad 2x + x + \frac{2}{3}x = 264$$

$$\Rightarrow \frac{11}{3}x = 264$$

$$\Rightarrow x = \frac{264 \times 3}{11}$$

$$= 24 \times 3 = 72$$

29. Let the numbers be x.

$$\frac{2}{3}x - 50 = \frac{x}{4} + 40$$

$$\Rightarrow \qquad \frac{2}{3}x - \frac{x}{4} = 50 + 40 = 90$$

$$\Rightarrow \qquad \frac{8x - 3x}{12} = 90$$

$$\Rightarrow \qquad 5x = 12 \times 90$$

$$x = 12 \times \frac{90}{5}$$

$$= 12 \times 18 = 216$$

30. Let the numbers be x.

Then
$$\left[\left(2\frac{1}{2} + x \right) 4\frac{1}{2} + 3 \right] \div 1\frac{1}{5} = 25$$

$$\Rightarrow \qquad \left[\left(\frac{5}{2} + x \right) \frac{9}{2} + 3 \right] \div \frac{6}{5} = 25$$

$$\Rightarrow \qquad \frac{25}{12} \times \frac{9}{2} + 3 \times \frac{5}{6} + x \times \frac{9}{2} \times \frac{5}{6} = 25$$

$$\Rightarrow \qquad \frac{75}{8} + \frac{5}{2} + \frac{15}{4} x = 25$$

$$\Rightarrow \qquad \frac{15}{4} x = 25 - \frac{75}{8} - \frac{5}{2}$$

$$\Rightarrow \qquad \frac{15}{4} x = \frac{25 \times 8 - 75 - 5 \times 4}{8}$$

$$\therefore \qquad x = \frac{4}{15} \times \frac{(200 - 75 - 20)}{8}$$

$$= \frac{4}{15} \times \frac{105}{8} = \frac{7}{2} = 3\frac{1}{2}$$

31. Let the number be 10x + y.

Given
$$x + 2 = y \tag{1}$$

$$(10x + y)(x + y) = 144 (2)$$

Substituting value of y in terms of x in (2), we get

$$(10x + x + 2) (x + x + 2) = 144$$

$$\Rightarrow (11x + 2) (2x + 2) = 144$$

$$\Rightarrow (11x + 2) (x + 1) = 72$$

$$\Rightarrow 11x^{2} + 13x + 2 = 72$$

i.e.
$$11x^2 + 13x - 70 = 0$$

 $\Rightarrow (11x + 35)(x - 2) = 0$
 $\Rightarrow x = 2 \text{ or } \frac{-35}{11}$
 $\therefore \text{ From (1)} \qquad y = x + 2 = 2 + 2 = 4$
 $\therefore \text{ Required number } = 10x + y = 10(2) + 4 = 24$

32. Let the numbers be x and y.

Then
$$x + y = 22$$
 (1)
$$5x = 6y$$
 (2) From (2),
$$x = \frac{6}{5}y$$

From (2),

Substitute value of y in terms of x in (1), we get

$$x + \frac{5}{6}x = 22$$

$$\Rightarrow \frac{11x}{6} = 22$$

$$\Rightarrow x = \frac{6 \times 22}{11} = 12$$

$$\therefore \text{ Bigger number} = 12$$

33. Let numbers be x and y.

Let numbers be
$$x$$
 and y .

Then
$$x + y = 33$$

$$x - y = 15$$

$$(1) - (2) \Rightarrow 2y = 33 - 15 = 18$$

$$y = 9$$

$$\therefore \qquad Smaller number = 9$$
(1)

34. Let the number be x.

$$\frac{x}{2} + \frac{x}{5} = \frac{x}{3} + 7\frac{1}{3}$$
i.e.
$$\frac{x}{2} + \frac{x}{5} - \frac{x}{3} = \frac{22}{3}$$

$$\frac{15x + 6x - 10x}{30} = \frac{22}{3}$$

$$\Rightarrow \qquad \frac{11x}{30} = \frac{22}{3}$$

$$\Rightarrow \qquad x = \frac{22}{3} \times \frac{30}{11} = 20$$

35. Let number be x.

Then
$$\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x}$$

$$\Rightarrow \qquad x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144}$$

$$\Rightarrow \qquad x = \sqrt{\frac{25}{144}} = \frac{5}{12}$$

Arithmetical Reasoning

This chapter deals with questions of different types which require some typical arithmetical calculations to solve them. Some of these types of questions have already been included in Chapters, namely *Elementary Algebra*, *Problem on Ages*, *Venn Diagram*, etc. Therefore, this chapter includes some typical examples and exercises for readers.

TYPE I CALCULATION BASED PROBLEMS

EXAMPLE 1 How many different words of 5 letters can be formed with the letters of the word 'EVOLUTION', so that the vowels occupy odd places?

Solution The three odd places can be occupied by 4 vowels in ${}^4P_3 = 4 \times 3 \times 2 = 24$ ways. The remaining two places can be occupied by 4 consonants in ${}^4P_2 = 4 \times 3 = 12$ ways. \therefore Required number of words = $24 \times 12 = 288$

EXAMPLE 2 Five players take part in a tennis match. Each player has to play with each other. How many match must they play?

Solution

- (i) The first player should play with 4 other players.
- (ii) The second player should play with 3 remaining players.
- (iii) The third player should play with 2 remaining players.
- (iv) The fourth player should play with 1 player other than first three players.

 \therefore Total number of matches = 4 + 3 + 2 + 1 = 10

EXAMPLE 3 In a class, 1/3 of the students are girls. 1/2 of the girls passed with distinction and 1/3 of the girls passed with distinction got admission to professional courses. If 3/4 of the boys passed with distinction and 2/3 of the boys passed with distinction got admission to professional courses, what part of the students are not admitted to professional courses?

Solution Let the number of students in the class be x.

 \therefore Number of girls = x/3; Number of boys = 2x/3

Number of girls got admission to professional courses =
$$\frac{1}{3}$$
 of $\frac{1}{2}$ of $\frac{x}{3}$
= $\frac{x}{18}$
Number of boys got admission to professional courses = $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2x}{3}$

Number of boys got admission to professional courses = $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2x}{3}$ $= \frac{x}{3}$

- .. The number of students got admission to professional courses = $\frac{x}{18} + \frac{x}{3} = \frac{7x}{18}$
- \therefore The number of students who are not admitted to professional courses = $x \frac{7x}{18} = \frac{11x}{18}$

TYPE II DATA-BASED QUESTIONS

Examples (4–6) The following questions are based on the given data of an examination with three subjects.

(A) Candidates appeared : 1000

(B) Candidates passed in all the three subjects: 460

(C) Candidates passed in two subjects only : 280

(D) Candidates passed in one subject only : 240

(E) Candidates failed in Physics only : 80

(F) Candidates failed in Chemistry only : 60

(G) Candidates failed in Maths only : 140

EXAMPLE 4 How many candidates failed in all the subjects?

Solution Candidates failed in all the subjects

= Candidates appeared – (Candidates passed in 1, 3 subject + Candidates failed in 1 subject only)

$$= 1000 - [240 + 460 + (80 + 60 + 140)]$$

$$= 1000 - [240 + 460 + 280]$$

$$= 1000 - 980 = 20$$

EXAMPLE 5 How many candidates passed in at least two subjects?

Solution Candidates passed in at least two subjects

- = Candidates passed in 2 subjects
 - + Candidates passed in all the three subjects
- = Candidates failed in 1 subject
 - + Candidates passed in all the three subjects

$$= [(80 + 60 + 140) + 460] = 280 + 460 = 740$$

EXAMPLE 6 How many candidates failed because of having failed in two or less subjects?

Solution Candidates failed in two or less subjects

= Candidates failed in 1 subject + Candidates failed in two subjects

= Candidates failed in 1 subject + Candidates passed in one subject

= [(80 + 60 + 140) + 240] = 280 + 240 = 520

TYPE III PROBLEMS ON AGES

EXAMPLE 7 The age of a father is 4 times that of his son. Four years ago, the father was six times as old as his son at that time. What is the present age of father?

Solution Let the age of son be x

∴ age of father =
$$4x$$

∴
$$\frac{4x-4}{x-4} = 6$$

⇒
$$4x-4 = 6x-24$$

⇒
$$2x = 20$$

⇒
$$x = 10 \text{ years}$$
i.e. age of son = 10 years

 $\therefore \qquad \qquad \text{Present age of father = } 4x = 40 \text{ years}$

EXAMPLE 8 The ratio of ages of Sanal and Suresh is 3:4. Four years earlier, the ratio was 5:7. Find the present ages of Sanal and Suresh.

Solution Let the age of Sanal be x

∴ Age of Suresh =
$$\frac{4}{3}x = y$$

Given
$$\frac{x-4}{\frac{4}{3}x-4} = \frac{5}{7}$$

$$\Rightarrow \frac{x-4}{4\left(\frac{x}{3}-1\right)} = \frac{5}{7}$$

$$\frac{3(x-4)}{4(x-3)} = \frac{5}{7}$$

$$\Rightarrow 21(x-4) = 20(x-3)$$

$$\Rightarrow x = 21 \times 4 - 20 \times 3 = 84 - 60 = 24 \text{ years}$$
∴ Age of Suresh = $\frac{4}{3} \times 24 = 32 \text{ years}$

EXAMPLE 9 The ratio of ages of Ajith and Sreejith at present is 6:1. After 5 years the ratio will become 7:2. What is the present age of Sreejith?

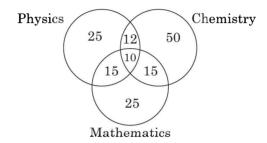
Solution Ajith : Sreejith
Present age 6 : 1
After 5 years 7 : 2

$$\therefore \qquad \text{Age of Sreejith} = 1 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 5 \text{ years}$$

$$\text{Age of Ajith} = 6 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 30 \text{ years}$$

TYPE IV VENN-DIAGRAM BASED QUESTIONS

EXAMPLE 10 Consider the following Venn diagram.



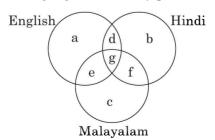
Three hundred candidates appeared in an examination with three subjects, Physics, Chemistry and Mathematics. The Venn diagram gives the number of candidates who failed in different subjects. What is the percentage of candidates who failed in at the most two subjects?

Solution Number of candidates who failed in at the most two subjects

= Candidates failed in one subject
+ Candidates failed in two subjects
=
$$(25 + 50 + 25) + (12 + 15 + 15)$$

= $100 + 42 = 142$
∴ Required percentage = $\left(\frac{142}{300} \times 100\right)\%$
= $47\frac{1}{3}\% = 47.33\%$

EXAMPLE 11 A group of persons is attending a conference. Ten persons can speak English, fifteen persons can speak Hindi, twenty five persons can speak Malayalam. None of them can speak any other language. If two persons in the group can speak two languages and one person can speak all the three languages, how many persons are there in the group?



Solution Number of persons speaking English = a + d + g + e = 10 (1)

Number of persons speaking Hindi =
$$b + d + f + g = 15$$
 (2)

Number of persons speaking Malayalam =
$$c + e + f + g = 25$$
 (3)

Persons speaking 2 languages =
$$d + e + f = 2$$
 (4)

Persons speaking all three languages =
$$g = 1$$
 (5)

From Eqs. (1) and (5), we have
$$a + d + e = 9$$
 (6)

From Eqs. (2) and (5), we have
$$b + d + f = 14$$
 (7)

From Eqs. (3) and (5), we have
$$c + e + f = 24$$
 (8)

$$(6) - (4) \quad \Rightarrow \quad a - f = 7 \tag{9}$$

Teachers

Engineers

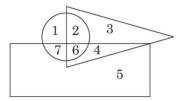
$$(7) + (8) \Rightarrow b + c + d + e + 2f = 38$$
 (10)

(9) + (10)
$$\Rightarrow$$
 a + b + c + d + e + f = 45
a + b + c + d + e + f + g = 46

or

 \therefore Total number of persons in the group = 46

In the figure given below, the circle represents government employed persons, the triangle represents lecturers and rectangle represents doctors.



Study the figure given above and answer the following questions.

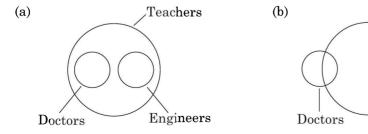
EXAMPLE 12 Which region represents lecturers working in non governmental institutions who are not professional?

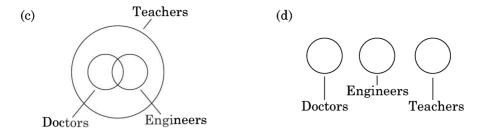
Solution Regions 3 and 4 represent lecturers in which 4 represents lecturer belonging to professional category. Therefore correct region is 3.

EXAMPLE 13 Which region represents government doctors, but not a lecturer?

Solution Here regions 7 and 6 denote government doctors. But region 6 represents a government doctor and a lecturer. So the region which represents a government doctor but not a lecturer is 7.

EXAMPLE 14 A group of doctors, engineers and teachers can be represented by Venn diagram as shown below:





(e) None of these

Solution Here doctors and engineers are two different professionals. Therefore, they can be represented by independent circles. Teacher is a different profession. But some of the doctors and engineers can be teachers. So it can be represented with a common intersection with circles representing doctors and engineers as given in (b).

Hence the answer is **(b)**.



EXERCISES

1.	There are some cows and	parrots in a group.	The number of	legs is 1	4 more tha	an twice tl	16
	number of heads. The nu	mber of cows is					

- (a) 5
- (b) 7
- (c) 10
- (d) 12
- (e) None of these
- 2. A certain number of bullocks and an equal number of men are walking through a road. If the number of legs walking on the ground is 72, how many bullocks are there?
 - (a) 10
- (b) 12
- (c) 14
- (d) 16
- (e) None of these
- 3. The sum of ages of Ajeesh, Abhishek, and Sajith is 74 years. What was the total of their ages three years ago?
 - (a) 63 years
- (b) 65 years
- (c) 67 years
- (d) 69 years
- (e) None of these
- 4. Bineesh is 3 years older to Dinesh and 3 years younger to Gireesh, while Dinesh and Manoj are twins. How many years older is Gireesh to Manoj?
 - (a) 2
- (b) 3
- (d) 12
- (e) None of these
- 5. A person told his son, "I was of your present age when you were born". If the father is 42 now, how old was the boy five years back?
 - (a) 15
- (b) 16
- (c) 18
- (d) 20
- (e) None of these



1. (b)

2. (b)

3. (b)

4. (c)

5. (b)

Solutions with Necessary Explanation

1. Let the number of cows and parrots be x and y.

Number of heads =
$$x + y$$
 (1)

$$\therefore \qquad \text{Number of legs} = 2(x+y) + 14 \tag{2}$$

i.e.
$$4x + 2y = 2(x + y) + 14$$

$$\Rightarrow 2x = 14$$
or
$$x = 7$$
(3)

- .. The number of cows is 7.
- **2.** Let the number of bullocks and men be x each.

$$\therefore \qquad 2x + 4x = 72$$

$$\Rightarrow \qquad x = \frac{72}{6} = 12$$

- \therefore Number of bullocks = 12
- 3. Let the ages of Ajeesh, Abhishek and Sajith be x, y and z.

$$Sum of ages = x + y + z = 74 \tag{1}$$

- \therefore 3 years ago, sum of their ages is given by (x-3)+(y-3)+(z-3)=74-9=65 years.
- 4. Let the ages of Bineesh, Dinesh, Gireesh and Manoj be b, d, g and m respectively.

$$b = d + 3 \tag{1}$$

$$b = g - 3 \tag{2}$$

$$d = m \tag{3}$$

Equating Eqs. (1) and (2), we get

$$d+3=g-3$$
$$d=g-6$$

or

- :. Gireesh is 6 years older to Manoj and Dinesh as Manoj and Dinesh are twins.
- 5. Here the age of father = 42 years
 - \therefore Age of son = $\frac{1}{2} \times 42 = 21$ years
 - \therefore Five years ago, the age of son was 21 5 = 16 years.

Simplification

This chapter deals with order of precedence of various arithmetical operators on numbers.

VBODMAS: VBODMAS is the rule applied to a given expression for its simplification. Here V stands for 'Virnaculum', B stands for 'Bracket', O stands for 'Of', D stands for 'Division', M stands for 'Multiplication', A stands for 'Addition' and S stands for 'Subtraction'.

Brackets must be removed in the order (), {} and []. After removing brackets, remaining operands are to be applied in the order.

(a) Of (b) Division (c) Multiplication (d) Addition and (e) Subtraction.

Virnaculum (or Bar): If an expression contains. Virnaculum, simplify the expression under Virnaculum before applying 'BODMAS'.

EXAMPLE 1 Simplify
$$0.6 \times 0.6 + 0.6 \div 6$$

[Bank PO]

Solution Given expression =
$$0.6 \times 0.6 + \frac{0.6}{6}$$

= $0.36 + 0.1 = 0.46$

EXAMPLE 2 Simplify
$$2004 - 1984 \div 2$$

Solution Given expression =
$$2004 - \frac{1984}{2}$$

= $2004 - 992 = 1012$

EXAMPLE 3 Simplify 3660 ÷ 5 + 18

Solution Given expression =
$$\frac{3660}{5} + 18$$

= $732 + 18 = 750$

EXAMPLE 4 What will be the value in place of question mark in the given expression?

$$3\frac{4}{9} + 6\frac{2}{5} + ? = 5\frac{2}{3} + 3\frac{4}{5}$$

Solution Let the value in place of question mark be x.

$$3\frac{4}{9} + 6\frac{2}{5} + x = 5\frac{2}{3} + 3\frac{4}{5}$$

$$x = 5\frac{2}{3} + 3\frac{4}{5} - 6\frac{2}{5} - 3\frac{4}{9}$$

$$= \left(5\frac{2}{3} + 3\frac{4}{5}\right) - \left(6\frac{2}{5} + 3\frac{4}{9}\right)$$

$$= \left(8 + \frac{10 + 12}{15}\right) - \left(9 + \frac{18 + 20}{45}\right)$$

$$= 9\frac{7}{15} - 9\frac{38}{45}$$

$$= \frac{7}{15} - \frac{38}{45}$$

$$= \frac{21 - 38}{45}$$

$$= \frac{-17}{45}$$

Hence value in place of question mark = $\frac{-17}{45}$

EXAMPLE 5 Find the value of
$$\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)...\left(1 - \frac{1}{100}\right)$$
 [SSC]

Solution Given expression =
$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{99}{100}$$

= $\frac{2}{100} = \frac{1}{50}$

EXAMPLE 6 Two pens and three pencils costs ₹ 86. Four pens and a pencil cost ₹ 112. Find the cost of a pen and that of a pencil. [Bank PO]

Solution Let cost of pen and pencil be $\not\in p$ and $\not\in q$.

$$\therefore \qquad 2p + 3q = 86 \tag{1}$$

$$4p + q = 112 \tag{2}$$

[SSC]

[SSC]

[RRB]

[SSC]

[SSC]

[RRB]

Solving (1) and (2), we get

$$(2) - 2 \times (1) \Rightarrow q - 6q = 112 - 172$$

-5q = -60
 $q = ₹ 12$

From (2),

 \Rightarrow

 \Rightarrow

$$p = \frac{112 - 12}{4} = \mathbf{₹25}$$

Cost of pen = ₹ 25; Cost of pencil = ₹ 12



EXERCISES

1. The value of 1001 ÷ 11 of 13 is:	
-------------------------------------	--

- (b) 91 (c) 143 (d) 169
- (a) 7
- **2.** The value of $25 5[2 + 3\{2 2(5 3) + 5\} 10] \div 4$ is: (a) 5
- (b) 23.25 (c) 23.75 (d) 25
- 3. What mathematical operation should come at the place of "?" in the equation:
 - $2?6-12 \div 4+2=11$
 - (a) +
- $(c) \times$ (d) ÷
- 4. $3640 \div 14 \times 16 + 340 = ?$
- [LIC]

- (a) 0.70
- (c) 4480 (b) 3525
- (d) 9600
- (e) None of these [SSC]

5. $(8 \div 88) \times 8888088 = ?$

(d) 8008008

6. $\frac{180 \times 15 - 12 \times 20}{140 \times 8 + 2 \times 55} = ?$

(a) 808008

[BSRB]

- (a) $\frac{1}{7}$ (b) $\frac{4}{5}$

(b) 808080

(c) 2

(c) 808088

- (d) 4
- (e) None of these

- 7. Evaluate $\frac{8 [5 (-3 + 2)] \div 2}{|5 3| |5 8| \div 3}$
- (c) 4 (d) 5
- 8. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$ is equal to:

(a) 2

- (b) 2.5
- (c) 3
- (d) 3.5

9. $1\frac{3}{4} + 5\frac{1}{3} + 3\frac{2}{5} = ?$

- [Bank PO]
- (a) $9\frac{2}{5}$ (b) $9\frac{29}{60}$ (c) $10\frac{2}{5}$ (d) $10\frac{29}{60}$

- 10. $\frac{1}{\left(2\frac{1}{3}\right)} + \frac{1}{\left(1\frac{3}{4}\right)}$ is equal to:

- (a) $\frac{7}{14}$ (b) $\frac{12}{49}$
- (c) $4\frac{1}{12}$ (d) None of these

11.
$$\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$$
. Then $x =$ [MBA]

(a) $\frac{5}{18}$ (b) $\frac{6}{19}$ (c) $\frac{18}{5}$ (d) $\frac{24}{11}$

12. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{21}{24}$ of $504 = ?$ [Bank PO]

(a) 63 (b) 69 (c) 96 (d) 109 (e) None of these

13. $\frac{3}{8}$ of $168 \times 15 \div 5 \div ? = 549 \div 9 + 235$ [SBI PO]

(a) 107 (b) 174 (c) 189 (d) 296 (e) None of these

14. When $\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$ is divided by $\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{3} - \frac{7}{18}\right)$, the result is: [SSC]

(a) $2\frac{1}{18}$ (b) $3\frac{1}{6}$ (c) $3\frac{3}{10}$ (d) $5\frac{1}{10}$

15. Difference between $1\frac{3}{16}$ and its reciprocal is: [MAT]

(a) $1\frac{1}{8}$ (b) $\frac{4}{3}$ (c) $\frac{15}{16}$ (d) None of these

16. Two-fifth of one-fourth of three-seventh of a number is 15. What is half of that number? [Bank PO]

(a) 94 (b) 96 (c) 188 (d) 196 (e) None of these

17. If $x \times y = x^2 + y^2 - xy$, then the value of 9×11 is: [SSC]

(a) 93 (b) 103 (c) 113 (d) 121

18. If $a \times b = 2a - 3b + ab$, then $3 \times 5 + 5 \times 3$ is: [SSC]

(a) 2 (b) 24 (c) 26 (d) 28

19. $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} + 5\frac{2}{3} = ?$ [Bank PO]

(a) $\frac{7}{17}$ (b) $1\frac{33}{34}$ (c) 8 (d) $18\frac{1}{34}$

20. $\frac{1}{3} + \frac{1}{3} \times \frac{1}{3} - \frac{1}{9} =$ [SSC]

(a) 0 (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) 1

21. The value of $0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004)$ is: [SSC]

(a) 9.2 (b) 9.56 (c) 27.2 (d) 27.56

23. If $\frac{x}{y} = \frac{4}{5}$, then the value of $\left(\frac{4}{7} + \frac{2y - x}{2y + x}\right)$ is: [RRB]

(a) $\frac{3}{7}$ (b) 1 (c) $1\frac{1}{7}$ (d) 2

[Bank PO]

[SBI PO]

[SSC]

(c) x > y

	$2 - \frac{1}{2}$			
(a) $\frac{3}{8}$	(b) $\frac{19}{8}$	(c) $\frac{8}{3}$	(d) $\frac{8}{19}$	
27. If $2 = x + \frac{1}{1 + -3}$	$\frac{1}{1+\frac{1}{4}}$, then the va	lue of x is:		[SSC]
(a) $\frac{12}{17}$	(b) $\frac{13}{17}$	(c) $\frac{18}{17}$	(d) $\frac{21}{17}$	
28. If $4x + 5y = 83$	and $\frac{3x}{2y} = \frac{21}{22}$, the	n y - x = ?		[Bank PO]
	•	(c) 7	(d) 11	
29. If $2x + y = 17$, $y = 1$	y + 2z = 15 and $x + 2z = 15$	y = 9, then the va	alue of $4x + 3y + z$ equals:	[SBI PO]
(a) 41	(b) 43	(c) 45		None of these
		()($\frac{1}{3}\left(1-\frac{1}{4}\right)\left(1-\frac{1}{n}\right)$ equa	ls: [SSC]
(a) $\frac{1}{n}$	(b) $\frac{2}{n}$	(c) $\frac{2(n-1)}{n}$	(d) $\frac{2}{n(n+1)}$	
31. The value of 99	$9 \frac{995}{999} \times 999 \text{ is:}$			[SSC]
(a) 990809	(b) 998996	(c) 998999	(d) 999824	
32. The value of $\frac{3}{1^2}$.	$\frac{5}{2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 3^2}$	$\frac{9}{4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6}$	$\frac{13}{6^2 \cdot 7^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2}$	$\frac{1}{9^2 \cdot 10^2}$ is:
(a) $\frac{1}{100}$	(b) $\frac{99}{100}$	(c) 1	(d) $\frac{101}{100}$	
			at equal distances, one tree consecutive trees?	being at each [RRB]
(a) 8 metres	(b) 9 metres	(c) 10 metres	(d) 15 metres	
	sts₹7 each. A wa apples purchased		each. X spends ₹ 38 on the	se fruits. The [SSC]
(a) 2	(b) 3	(c) 4	(d) Data inadequate	

(b) x = y, if a < 1

(e) y > x, if a < 1

(b) $8\frac{1}{4}$ (c) $8\frac{1}{2}$ (d) Cannot be determined

24. If $x = \frac{a}{a-1}$ and $y = \frac{1}{a-1}$, then:

26. The value of $\frac{1}{2 + \frac{1}{2 + \frac{1}{1}}}$ is:

25. If $3x + 7 = x^2 + P = 7x + 5$, what is the value of *P*?

(a) x = y

(a) $\frac{1}{2}$

(d) x > y, if a < 1

- 35. A printer numbers the pages of a book starting with 1 and uses 3189 digits in all. How many pages does the book have? [MAT]
 - (a) 1000
- (b) 1074
- (c) 1075
- (d) 1080
- 36. A sum of ₹ 1360 has been divided among A, B and C such that A gets $\frac{2}{3}$ of what B gets and

B gets $\frac{1}{4}$ of what C gets. B's share is

[MAT]

- (a) ₹ 120
- (b) ₹ 160
- (c) ₹ 240
- (d) ₹ 300

37.
$$\frac{(469 + 174)^2 - (469 - 174)^2}{469 \times 174} = ?$$

[MBA]

- (a) 2
- (b) 4
- (c) 295
- (d) 643
- 38. Eight persons are planning to share equally the rent of a car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by

 [MBA]
 - (a) $\frac{1}{7}$
- (b) $\frac{1}{8}$
- (c) $\frac{1}{9}$
- (d) $\frac{7}{8}$
- 39. In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. If he attempts in all 60 questions and secures 130 marks, the number of questions be attempts correctly is:

 [LIC AAO]
 - (a) 35
- (b) 38
- (c) 40
- (d) 42
- 40. A man has some hens and cows. If the number of heads be 48 and the number of legs equal 140, then the number of hens will be: [RRB]
 - (a) 22
- (b) 23
- (c) 24
- (d) 26



- 1. (a) 2. (c) 3. (c) 4. (e) 5. (a) 6. (c) 7. (d) 8. (a) 9. (d) 10. (d)
- 11. (b) 12. (e) 13. (a) 14. (d) 15. (d) 16. (e) 17. (b) 18. (a) 19. (d) 20. (a)
- 21. (a) 22. (d) 23. (b) 24. (d) 25. (b) 26. (d) 27. (d) 28. (b) 29. (e) 30. (a)
- 31. (b) 32. (b) 33. (b) 34. (c) 35. (b) 36. (c) 37. (b) 38. (a) 39. (b) 40. (d)

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Solutions with Necessary Explanation

1.
$$1001 \div 11 \text{ of } 13 = 1001 \div (11 \times 13)$$

$$= 1001 \div 143 = 7$$

2.
$$25 - 5[2 + 3\{2 - 2(5 - 3) + 5\} - 10] \div 4$$

= $25 - 5[2 + 3\{2 - 2 \times 2 + 5\} - 10] \div 4$
= $25 - 5[2 + 3 \times 3 - 10] \div 4$
= $25 - 5 \times 1 \div 4$
= $25 - \frac{5}{4} = 23.75$

3. Operator X in place of question mark satisfies the given equation according to order of precedence of operators.

i.e.
$$2 \times 6 - 12 \div 4 + 2 = 2 \times 6 - 3 + 2 = 11$$

4.
$$3640 \div 14 \times 16 + 340 = \frac{3640}{14} \times 16 + 340$$

= $260 \times 16 + 340 = 4500$

5.
$$(8 \div 88) \times 8888088 = \frac{1}{11} \times 8888088$$

6.
$$\frac{180 \times 15 - 12 \times 20}{140 \times 8 + 2 \times 55} = \frac{2700 - 240}{1120 + 110}$$

$$=\frac{2460}{1230}=2$$

7.
$$\frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| \div 3} = \frac{8 - [5 - (-1)] \div 2}{2 - 3 \div 3}$$
$$= \frac{8 - 6 \div 2}{2 - 3 \div 3} = \frac{8 - 3}{2 - 1} = \frac{5}{1} = 5$$

8.
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = 1 + \frac{14 + 7 + 4 + 2 + 1}{28}$$

$$= 1 + \frac{28}{28} = 1 + 1 = 2$$

9.
$$1\frac{3}{4} + 5\frac{1}{3} + 3\frac{2}{5} = (1+5+3) + \left(\frac{3}{4} + \frac{1}{3} + \frac{2}{5}\right)$$

= $9 + \left(\frac{3 \times 15 + 1 \times 20 + 2 \times 12}{60}\right)$

$$=9+\frac{45+20+24}{60}=9+1\frac{29}{60}=10\frac{29}{60}$$

10.
$$\frac{1}{\left(2\frac{1}{3}\right)} + \frac{1}{\left(1\frac{3}{4}\right)} = \frac{1}{\frac{7}{3}} + \frac{1}{\frac{7}{4}}$$

$$=\frac{3}{7}+\frac{4}{7}=\frac{7}{7}=1$$

11.
$$\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$$

$$\Rightarrow \frac{1}{x} = 4 - \left(\frac{1}{3} + \frac{1}{2}\right)$$
$$= 4 - \left(\frac{2+3}{6}\right) = 4 - \frac{5}{6} = \frac{19}{6}$$

$$\therefore \qquad x = \frac{6}{19}$$

12.
$$\frac{3}{5}$$
 of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{21}{24}$ of $504 = \frac{3}{5} \times \frac{4}{7} \times \frac{5}{9} \times \frac{21}{24} \times 504 = \frac{1}{6} \times 504 = 84$

13.
$$\frac{3}{8}$$
 of $168 \times 15 \div 5 + ? = 549 \div 9 + 235$

Let number in place of question mark be x.

$$\therefore \frac{3}{8} \times 168 \times 15 \div 5 + x = 549 \div 9 + 235$$

i.e.
$$\frac{3}{8} \times 168 \times \left(\frac{15}{5}\right) + x = \frac{549}{9} + 235$$

$$x = 61 + 235 - 3 \times 21 \times 3$$
$$= 296 - 189 = 107$$

14.
$$\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \left(\frac{1}{2} + \frac{1}{5}\right) - \left(\frac{1}{4} + \frac{1}{6}\right)$$
$$= \frac{7}{10} - \frac{5}{12} = \frac{42 - 25}{60} = \frac{17}{60}$$
$$\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18} = \left(\frac{2}{5} + \frac{3}{5}\right) - \left(\frac{5}{9} + \frac{7}{18}\right)$$
$$= 1 - \frac{17}{18} = \frac{1}{18}$$

$$\therefore$$
 Required result = $\frac{17/60}{1/18} = \frac{17}{60} \times 18 = \frac{51}{10} = 5\frac{1}{10}$

15. Required difference =
$$1\frac{3}{16} \sim \frac{1}{1\frac{3}{16}}$$

= $\frac{19}{16} \sim \frac{16}{19} = \frac{19^2 \sim 16^2}{19 \times 16}$
= $\frac{(19+16)(19-16)}{19 \times 16} = \frac{35 \times 3}{19 \times 16}$

16.
$$\frac{2}{5}$$
 of $\frac{1}{4}$ of $\frac{3}{7}$ of $x = 15$; $\frac{x}{2} = ?$

$$\frac{2}{5} \times \frac{1}{4} \times \frac{3}{7} \times x = 15$$

$$\Rightarrow x = 15 \times \frac{5 \times 4 \times 7}{2 \times 1 \times 3}$$

$$2 \times 1 \times 3$$

$$\Rightarrow \qquad x = 5 \times 5 \times 2 \times 7$$

$$\Rightarrow \frac{x}{2} = 5 \times 5 \times 7 = 175$$

17. Given
$$x * y = x^2 + y^2 - xy$$

$$9 * 11 = 9^2 + 11^2 - 9 \times 11$$

$$= 81 + 121 - 99$$

18.
$$a * b = 2a - 3b + ab$$

$$3*5+5*3 = [(2\times3)-(3\times5)+(3\times5)]+[(2\times5)-(3\times3)+(5\times3)]$$
$$= 6+16=22$$

19.
$$4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3} = ?$$

Let the required value in place of question mark be x.

$$x = \frac{9}{2} \times \frac{13}{3} - \left(\frac{25}{3} \div \frac{17}{3}\right) = \frac{9}{2} \times \frac{13}{3} - \frac{25}{17}$$

$$= \frac{39}{2} - \frac{25}{17} = 19\frac{1}{2} - 1\frac{8}{17}$$

$$= (19 - 1) + \left(\frac{1}{2} - \frac{8}{17}\right)$$

$$= 18 + \left(\frac{17 - 16}{34}\right)$$

$$= 18\frac{1}{34}$$

20.
$$\frac{\frac{1}{3} \div \frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \div \frac{1}{3} \text{ of } \frac{1}{3}} - \frac{1}{9} = \frac{1 \times \frac{1}{3}}{\frac{1}{3} \div \frac{1}{9}} - \frac{1}{9}$$
$$= \frac{\frac{1}{3}}{\frac{1}{2} \times 9} - \frac{1}{9} = \frac{1}{9} - \frac{1}{9} = \mathbf{0}$$

21.
$$0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004) = \frac{0.008 \times 0.01 \times 0.0072}{0.12 \times 0.0004}$$

= $0.001 \times 2 \times 1 \times 6 = 0.012$

22.
$$11.6 + 9.28 \div 0.464 - 0.2828 \div 0.07 = 11.6 + \frac{9.28}{0.464} - \frac{0.2828}{0.07}$$

i.e. Required value = 11.6 + 20 - 4.04= 31.6 + 4.04 = 27.56

23. Given
$$\frac{x}{y} = \frac{4}{5}$$

$$\therefore \frac{4}{7} + \frac{2y - x}{2y + x} = \frac{4}{7} + \frac{2 - (x/y)}{2 + (x/y)}$$

$$= \frac{4}{7} + \frac{2 - (4/5)}{2 + (4/5)}$$
$$= \frac{4}{7} + \frac{(6/5)}{(14/5)} = \frac{4}{7} + \frac{3}{7} = \mathbf{1}$$

24. Given
$$x = \frac{a}{a-1} = \frac{a-1+1}{a-1} = 1 + \frac{1}{a-1} = 1 + y$$

$$\therefore x > y$$

25.
$$3x + 7 = 7x + 5$$

$$\Rightarrow 7x - 3x = 7 - 5 = 2$$

$$\Rightarrow 4x = 2$$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

$$3x + 7 = 3\left(\frac{1}{2}\right) + 7 = \frac{17}{2}$$

$$\therefore \qquad x^2 + P = \frac{17}{2}$$

$$P = \frac{17}{2} - \left(\frac{1}{2}\right)^2 = \frac{17}{2} - \frac{1}{4} = \frac{34 - 1}{4} = \frac{33}{4} = 8\frac{1}{4}$$

26.
$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}} = \frac{1}{2 + \frac{1}{2 + \frac{1}{3}}}$$
$$= \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} = \frac{1}{2 + \frac{1}{8}} = \frac{1}{2 + \frac{3}{8}} = \frac{1}{\frac{19}{8}} = \frac{8}{19}$$

27.
$$\frac{1}{1+\frac{1}{3+\frac{1}{4}}} = \frac{1}{1+\frac{1}{\frac{13}{4}}} = \frac{1}{1+\frac{4}{13}}$$
$$= \frac{1}{\frac{17}{13}} = \frac{13}{17}$$

$$\therefore x = 2 - \frac{13}{17} = \frac{34 - 13}{17}$$
$$= \frac{21}{17}$$

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28.
$$4x + 5y = 83$$
 (1)

$$\frac{3x}{2y} = \frac{21}{22} \tag{2}$$

From (2),
$$\frac{x}{y} = \frac{7}{11}$$
 (A)

From (1),
$$4 \times \frac{7}{11} y + 5y = 83$$

$$\left(\frac{28}{11} + 5\right)y = 83$$

$$y = \frac{83 \times 11}{83} = 11$$

Substituting y = 11 in (A), we obtain

$$x = 7$$
$$y - x = 11 - 7 = 4$$



Short cut:

$$\therefore \frac{x}{y} = \frac{7}{11}, \quad y - x = (11 - 7)k = 4k.$$

i.e. (y-x) is a constant multiple of 4.

From the given options, we are having only one multiple of 4.

29.
$$2x + y = 17$$
 (1)

$$y + 2z = 15 \tag{2}$$

$$x + y = 9 \tag{3}$$

$$(1) \Rightarrow 2x = (17 - y)$$

$$\Rightarrow \qquad 4x = 34 - 2y$$
(2)
$$\Rightarrow \qquad 2z = 15 - y$$

or
$$z = \frac{15 - y}{2} \tag{B}$$

$$\therefore \quad (A) + (B) + 3y = 4x + 3y + z$$

$$= (34 - 2y) + \left(\frac{15 - y}{2}\right) + 3y$$
$$= \frac{83}{2} + \frac{y}{2}$$

Solving (1) and (3), we get

$$(1) - (3) \Rightarrow x = 17 - 9 = 8$$
 and

$$\therefore$$
 $y=1$

$$\therefore \qquad \text{Required value} = \frac{83}{2} + \frac{1}{2} = 42$$

30.
$$\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{n}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \left(n - \frac{1}{n}\right) = \frac{1}{n}$$

31.
$$999 \frac{995}{999} \times 999 = 999^2 + 995$$

= $(1000 - 1)^2 + 995$
= $1000000 - 2000 + 1 + 995$
= $998000 + 996$
= 998996

32.
$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2} + \frac{19}{9^2 \cdot 10^2}$$
$$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \dots + \left(\frac{1}{9^2} - \frac{1}{10^2}\right) = \frac{1}{1^2} - \frac{1}{10^2} = \frac{99}{100}$$

- 33. Required distance = $\frac{225}{25}$ = 9 metres.
- **34.** Let number of pineapples be x and watermelon be y

$$\therefore \qquad 7x + 5y = 38 \quad \Rightarrow \quad x = 4; \quad y = 2$$

35. Number of pages with single digit (i.e. from 1 to 9) = 9

Number of pages with two digit (i.e. from 10 to 99) = 90

Number of pages with three digit (i.e. from 100 to 999) = 900

Total number of pages = 9 + 90 + 900 = 999

$$\therefore Total number of digits in page numbers = 1 \times 9 + 2 \times 90 + 3 \times 900 \\ = 9 + 180 + 2700 = 2889$$

Balance number of digtis printed =
$$3189 - 2889 = 300$$

$$\therefore \qquad \text{Required number of 4-digit page numbers} = \frac{300}{4} = 75$$

$$\therefore$$
 Total number of pages = $9 + 90 + 900 + 75 = 1074$

36. Leth the share with C be \mathbb{Z} x.

$$\therefore \text{ C's share + B's share + A's share} = x + \frac{x}{4} + \frac{2}{3} \left(\frac{x}{4} \right) = 1360$$

i.e.
$$x + \frac{x}{4} + \frac{x}{6} = 1360$$

$$\Rightarrow \frac{x(12+3+2)}{12} = 1360$$

$$\Rightarrow \qquad x = \frac{1360 \times 12}{17} = 960$$

∴ B's share =
$$\frac{1}{4}x = \frac{960}{4} = ₹240$$

37.
$$\frac{(469+174)^2-(469-174)^2}{469\times174} = \frac{4\times469\times174}{469\times174} = 4$$
 [:: $(a+b)^2-(a-b)^2=4ab$]

38. Rent/person (if persons are 8) = $\frac{x}{8}$

Rent/person (if persons are 7) = $\frac{x}{7}$

Difference in rent/person =
$$\frac{x}{7} - \frac{x}{8} = \frac{x}{56}$$

$$\therefore \quad \text{Increase in rent/person} = \frac{x/56}{x/8} = \frac{8}{56} = \frac{1}{7}$$

39. Let number of correct answers be x.

$$\therefore x \times 4 + (60 - x)(-1) = 130$$

$$\Rightarrow 4x + x - 60 = 130$$

$$\Rightarrow \qquad \qquad x = \frac{130 + 60}{5} = 38$$

40. Let number of hens be x.

$$2 \times x + 4(48 - x) = 140$$

$$\Rightarrow 192 - 2x = 140$$

$$\Rightarrow$$
 $x = 26$

 \therefore Number of hens = 26.

Races and Games

RACES: A contest of speed in running, driving, sailing or rowing and so on is called a race.

RACE COURSE: The place, ground, field, path or track on which a contest is done is called the race course.

DEAD HEAT RACE: If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.

START: In a race with two contestants A and B, if before the start of the race, A is at the starting point and B is ahead of a A by 10 metres, then we say that 'A gives B, a start of 10 metres'.

To cover a race of 100 m, A will have to cover 100 metres while B will have to cover only (100-10) = 90 metres.

Here, we can say that in a 100 m race, 'A can give B 10 m' or 'A can give B a start of 10 m' or 'A beats B by 10 m' means while A runs 100 m, B runs (100 - 10) = 90 m.

GAMES: In a game if A scores 100 points while B scores only 75 points, then we say that A can give B 25 points.

Important Facts

1. A is n times as fast as B. A gives B a start of x m. How long should the race course be so that both reach the winning post at the same time?

A's speed : B's speed = n : 1

A gains (n-1) metres in a race of n metres.

 $\therefore \text{ A will gain } x \text{ metres in a race of } \frac{nx \text{ metres}}{(n-1)} = \frac{x}{\left(1 - \frac{1}{n}\right)} \text{ metres}$

$$\therefore \text{ Length of race course} = x \left[\frac{1}{\left(1 - \frac{1}{n}\right)} \right] \text{ metres}$$

2. A can run x km race in t_1 minutes and B in t_2 minutes. Then A beats B by a distance $= \frac{x}{t_2} \times (t_2 - t_1) \text{ km}.$



Short cut: A beats B by a distance

 $= \frac{\text{Length of the race course}}{\text{Time taken by B to complete the race}} \times (\text{Difference between the race timings of A and B})$

EXAMPLE 1 In a one-kilometre race A, B, and C are the three participants. A can give B a start of 50 m and C a start of 69 m. How many metres start can B give C?

Solution When A covers 1000 m, B covers (1000 - 50) = 950 m, and C covers (1000 - 69) = 931 m

When B covers 950 m, C covers 931 m

:. When B covers 1000 m, C will cover $\frac{931}{950} \times 1000 = 980$ m

Therefore B can give C a start of 20 m.

EXAMPLE 2 In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. A beats C by 180 m. Find the time taken by each to run a kilometre.

Solution A beats B by 30 seconds and B beats C by 15 seconds.

Therefore, A beats C by (30 + 15) = 45 seconds.

Also, A beats C by 180 m.

Time taken by C to run 1 km = $\frac{45}{180} \times 1000$ = 250 seconds

 \therefore Time taken by B to run 1 km = 250 - 15 = 235 seconds Time taken by A to run 1 km = 250 - 45 = 205 seconds.

EXAMPLE 3 A can run 1 km in 3 min 10 sec and B can cover the same distance in 3 min 20 sec. By what distance can A beat B?

Solution Here A beat B by 10 sec.

Distance covered by B in 10 sec = $\left(\frac{1000}{200} \times 10\right) = 50 \text{ m}$

:. A beats B by 50 m.

EXAMPLE 4 P, Q and R are three contestants in a kilometre race. If P can give Q a start of 50 m and P can give R a start of 69 m, how many metre's start can Q give R?

Solution When P covers 1000 m, Q covers 950 m and R covers (1000 - 69) = 931 m When Q covers 950 m, R covers 931 m.

.: When Q covers 1000 m, R covers

$$\left(\frac{931}{950} \times 1000\right)$$
 m = 49 × 20 = 980 m

 \therefore Q can give R a start of (1000 - 980) = 20 m

EXAMPLE 5 In a game of 100 points, P can give Q 10 points and R 20 points. Then how many points Q can give R in a game of 90 points?

Solution P: Q = 100: 90

$$P: R = 100:80$$

$$\frac{Q}{R} = \frac{Q}{P} \times \frac{P}{R} = \frac{90}{100} \times \frac{100}{80} = \frac{90}{80}$$

:. In a game of 90 points, Q can give R 10 points.



:.

EXERCISES

- 1. In a game of 100 points, P can give Q 10 points and R 28 points. Then Q can give R:
 - (a) 10 points
- (b) 14 points
- (c) 16 points
- (d) 20 points
- 2. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 280 m, C will beat B by:
 - (a) 18 m
- (b) 20 m
- (c) 22 m
- (d) 25 m
- 3. In a race of 100 m, A beats B by 10 m and C by 20 m. In a race of 180 m, B will beat C by:
 - (a) 18 m
- (b) 20 m
- (c) 21 m
- (d) 22 m
- 4. In a 500 m race, ratio of speeds of two contestants A and B is 4:5. A has a start of 140 m. Then A wins by
 - (a) 60 m
- (b) 50 m
- (c) 40 m
- (d) 30 m
- 5. A can run 23 m, while B runs 25 m. In a kilometre race, B beats A by:
 - (a) 40 m
- (b) 60 m
- (c) 80 m
- (d) 100 m



- 1. (d)
- **2.** (b)
- **3.** (b)
- **4.** (b)
- **5.** (c)

V

Solutions with Necessary Explanation

1. P : Q = 100 : 90; P : R = 100 : 72

$$\frac{Q}{R} = \left(\frac{Q}{P} \times \frac{P}{R}\right) = \frac{90}{100} \times \frac{100}{72} = \frac{10}{8} = \frac{100}{80}$$

.. Q can give R 20 points.

2. A: B = 200:169; A: C = 200:182

$$\frac{C}{B} = \left(\frac{C}{A} \times \frac{A}{B}\right) = \frac{182}{200} \times \frac{200}{169} = \frac{182}{169}$$

i.e., when C covers 182 m, B covers 169 m

:. When C covers 280 m, B covers
$$\left(\frac{169}{182} \times 280\right)$$
 m = 260 m

- .. C will beat B by 20 metres
- **3.** A : B = 100 : 90; A : C = 100 : 80

$$\therefore \frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{80} = \frac{90}{80}$$

.: When B runs 90 m, C runs 80 m

:. When B runs 180 m, C covers
$$\left(\frac{80}{90} \times 180\right) = 160 \text{ m}$$

- .. B will beat C by 20 metres
- 4. To win, A has to cover a distance of 500 140 = 360 m. When A covers 4 m, B covers 5 m.

:. When A covers 360 m, B covers
$$\left(\frac{5}{4} \times 360\right)$$
 m = 450 m

- ∴ A wins by 50 m
- 5. When B runs 25 m, A runs 23 m.

When B runs 1000 m, A runs
$$\left(\frac{23}{25} \times 1000\right)$$
 m = 920 m

∴ B beats A by 80 m

Stocks and Shares

STOCK-CAPITAL: Total amount needed to run a company is called stock-capital.

SHARES OR STOCK: The whole capital is divided into small units, called shares or stock.

For each investment, company issues a share-certificate, showing the value of each share and the number of units held by a person.

The person who invests in shares or stock is called share holder or stock holder.

DIVIDEND: Annual profit of a company distributed among shareholders is called dividend. Dividend is paid annually as per share or as a percentage.

FACE VALUE: The value of a share or stock printed on the share-certificate is called its face value or nominal value.

MARKET VALUE: The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (stock) is said to be:

- (i) At premium or above par: If its market value is more than its face value.
- (ii) At par : If the market value is same as its face value.
- (iii) At discount or below par: If its market value is less than its face value.

Example: If a ₹ 100 stock is quoted at a premium of 8, then market value of the stock = ₹ (100 + 8) = ₹ 108

Similarly if a ₹ 100 stock is quoted at a discount of 6, then market value of stock = ₹ (100-6) = ₹ 94

BROKERAGE: Broker's charge is called brokerage.

- (i) When stock is purchased, brokerage is added to the cost price.
- (ii) When stock is sold, brokerage is subtracted from the selling price.



Notes:

- (a) Face value of a share is always a constant.
- (b) Market value of a share varies from time to time.
- (c) Dividend is always paid on the face value of a share.
- (d) Number of shares held by a person = $\frac{\text{Total investment}}{\text{Investment in one share}}$

By a ₹ 50; 8% stock at 60, we mean that

- (i) Face value of stock = $\mathbf{\xi}$ 50.
- (ii) Market value of stock = ₹ 60.
- (iii) Annual dividend on 1 share = 8% of face value = 8% of $\stackrel{?}{\stackrel{?}{$}}$ 50 = $\stackrel{?}{\stackrel{?}{$}}$ 4.
- (iv) An investment of $\stackrel{?}{\sim} 50$ gives an annual income of $\stackrel{?}{\sim} 4$.
- (v) Rate of interest per annum = $\left(\frac{4}{60} \times 100\right)\% = 6\frac{2}{3}\%$

EXAMPLE 1 Find the cost of 96 shares of $\stackrel{?}{=}$ 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share.

Solution Cost of 1 share =
$$\left[\left(10 - \frac{3}{4} \right) + \frac{1}{4} \right] =$$
₹ $\frac{19}{2}$

∴ Cost of 96 shares =
$$₹\left(\frac{19}{2} \times 96\right) = ₹912$$

EXAMPLE 2 Find the annual income derived by investing ₹ 7200 in 10% stock at 144. **Solution** By investing ₹ 144, income obtained = ₹ 10.

By investing ₹ 7200, income obtained = $\left(\frac{10}{144} \times 7200\right)$ = ₹ **500**.



EXERCISES

- 1. A man buys ₹ 50 shares in a company which pays 10% dividend. If the man gets 12.5% on his investment, at what price did he buy the shares? [LIC AAO]
 - (a) ₹ 37.50
- (b) ₹ 40
- (c) ₹48
- (d) ₹ 52
- 2. How many shares of market value ₹ 25 each can be purchased for ₹ 12750, brokerage being 2%?

 [MAT]
 - (a) 450
- (b) 500
- (c) 550
- (d) 600
- 3. The cost price of a ₹ 100 stock at 4 discount, when brokerage is 1/4% is
 - (a) ₹ 95.75
- (b) ₹96
- (c) ₹96.25
- (d) ₹ 104.25

4. A man bought 20 shares of ₹ 100 at 4 discount the rate of dividend being 12%. The rate of interest obtained is:

- (a) $11\frac{1}{2}\%$
- (b) 12%
- (c) 12.5%
- (c) 13%

5. A man invested ₹ 14,400 in ₹ 100 shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get?

[Hotel Management]

- (a) ₹500
- (b) ₹600
- (c) ₹650
- (d) ₹720



- **2.** (b)
- **3.** (c) **4.** (c)



Solutions with Necessary Explanation

1. Dividend on each share = $\frac{10}{100} \times ₹50 = ₹5$

Let the price at which he bought the share be \overline{x} .

$$\therefore \frac{12.5}{100} \times x = 5$$

$$\therefore \qquad x = \frac{5 \times 100}{12.5} = \text{ } \mathbf{740}$$

2. Selling price of each unit of share = $25 \times 1.02 = 725.50$

:. Number of shares that can be purchased

$$= \frac{712750}{725.50} = 500$$

3. Cost price of stock at 4 discount and $\frac{1}{4}\%$ brokerage = $\Re\left(100 - 4 + \frac{1}{4}\right) = \Re\left(96.25\right)$

4. Cost price of 1 unit of share = (100 - 4) = ₹ 96.

Dividend =
$$\left(\frac{12}{100} \times 100\right) \times 20 = ₹ 240$$

 \therefore Rate of interest obtained = $\frac{240}{1920} \times 100 = 12.5\%$

5. Price of 1 unit share = ₹ 100 + 20 = ₹ 120.

Dividend =
$$\frac{5}{100}$$
 ×100 = ₹ 5 per unit

Number of units =
$$\frac{14400}{120}$$
 = **120**

∴ Total dividend he got = $120 \times 5 = ₹600$

Discount

Let a person has to pay ₹ 140 after 5 years at an interest rate of 8% per annum ₹ 100 at 8% p.a. will amount to ₹ 140 in 5 years. Therefore we may say that:

Sum due = ₹ 140

Present worth = ₹ 100

Therefore, true discount may be defined as the interest on present worth. True discount is calculated on the amount.

Let R% be the rate of interest and T years be the period. Then,

(a) Present worth (P.W.) =
$$\frac{100 \times \text{Amount}}{100 + R \times T} = \frac{100 \times \text{T.D.}}{R \times T}$$

(b) True discount (T.D.) =
$$\frac{(P.W.) \times R \times T}{100} = \frac{Amount \times R \times T}{100 + (R \times T)}$$

(c) Sum =
$$\frac{S.I. \times T.D.}{(S.I. - T.D.)}$$

(d)
$$S.I. - T.D. = S.I.$$
 on $T.D.$

(e) If the sum is put at compound interest, then P.W. =
$$\frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$$

EXAMPLE 1 Find the present worth of ₹ 1341 due 4 years hence at 9% per annum.

Solution P.W. =
$$\frac{100 \times \text{Amount}}{100 + (R \times T)}$$
 = ₹ $\frac{100 \times 1341}{100 + (9 \times 4)}$
= ₹ $\frac{100 \times 1341}{136}$ = ₹ 950

EXAMPLE 2 The true discount on a bill due 10 months hence at 9% p.a. is ₹ 600. Find the amount of the bill and its present worth.

Solution Let the amount be \mathbb{Z} A.

$$\therefore T.D. = \frac{A \times R \times T}{100 + (R \times T)} = \frac{A \times 9 \times \frac{10}{12}}{100 + (9 \times \frac{10}{12})}$$
i.e.
$$600 = \frac{A \times \frac{15}{2}}{100 + \frac{15}{2}}$$

$$\Rightarrow 600 \left(100 + \frac{15}{2}\right) = \frac{15}{2} \times A$$

$$\Rightarrow A = 600 \left(100 + \frac{15}{2}\right) \times \frac{2}{15}$$

$$= 80 \left(100 + \frac{15}{2}\right)$$

$$= 8000 + 600 = ₹ 8600$$
P.W. = Amount – T.D.
$$= 8600 - 600 = ₹ 8000$$

EXAMPLE 3 True discount on a certain sum of a money due 3 years is $\stackrel{?}{\underset{?}{?}}$ 300, and the simple interest on the same sum for the same time and at the same rate is $\stackrel{?}{\underset{?}{?}}$ 450. Find the sum and the rate per cent.

Solution T.D. = 300 and S.I. = 3450

∴ Sum due =
$$\frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \frac{450 \times 300}{450 - 300} = ₹900$$
Rate = $\frac{\text{S.I.} \times 100}{\text{Sum due} \times T} = \frac{450 \times 100}{900 \times 3} = \frac{50}{3} = 16\frac{2}{3}\%$

EXAMPLE 4 The difference between the simple interest and true discount on a certain sum of money for 6 months at 10% p.a. is ₹ 15. Find the sum.

Solution Let the sum be ξx

$$\therefore \qquad \text{T.D.} = \frac{x \times 10 \times \frac{1}{2}}{100 + \left(10 \times \frac{1}{2}\right)} = \frac{x \times 5}{105} = \frac{x}{21}$$

$$\text{S.I.} = x \times 10 \times \frac{1}{2} \times \frac{1}{100} = \frac{x}{20}$$

$$\therefore \frac{x}{20} - \frac{x}{21} = 15$$

$$\Rightarrow \frac{x}{420} = 15$$

$$\Rightarrow x = 15 \times 420 = \text{₹ } 6300$$

$$\therefore \text{Sum} = \text{₹ } 6300$$

BANKER'S DISCOUNT: If trader A buys goods worth ₹ 20,000 from another trader B at a credit of 6 months, then A must allow B to withdraw the amount from bank account of A exactly after 6 months. The bill so prepared by B is called the bill of exchange.

The date exactly after 6 months is called nominally due date. Three days are added (grace days) to it to get a date, known as legally due date.

If B wishes to have the money before legally due date, he can get it from banker or broker, who deducts SI on the face value (here ₹ 20,000) for the period from date on which bill is discounted to the legally due date. This amount is called Banker's discount (B.D.).

i.e. B.D. = S.I. on face value for the period from date on which bill was discounted to the legally due date.

Banker's gain (B.G.) = B.D. - T.D. for the unexpired time

Important Formulae

1. B.D. = S.I. on bill for unexpired time

2. B.G. = (B.D.) – (T.D.) = S.I. on T.D. =
$$\frac{(T.D.)^2}{P.W}$$

3. T.D. =
$$\sqrt{P.W. \times B.G.}$$

4. B.D. =
$$\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}\right)$$

5. T.D. =
$$\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}\right)$$

6. Amount =
$$\left(\frac{B.D. \times T.D.}{B.D. - T.D.}\right)$$

7. T.D. =
$$\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$$

EXAMPLE 5 The banker's discount on ₹ 2400 due certain time hence is ₹ 240. Find the true discount and banker's gain.

Solution Amount =
$$\frac{B.D. \times T.D.}{B.D. - T.D.} = \frac{B.D. \times T.D.}{B.G.}$$

$$\therefore \frac{T.D.}{B.G.} = \frac{Amount}{B.D.} = \frac{2400}{240} = \frac{10}{1}$$

If B.G. is $\stackrel{?}{\sim}$ 1, T.D. = $\stackrel{?}{\sim}$ 10 and if B.D. = $\stackrel{?}{\sim}$ 12, T.D. = B.D. – B.G. = $12 - 1 = \stackrel{?}{\sim}$ 11.

∴ If B.D. = ₹ 240, T.D. = ₹
$$\left(\frac{11}{12} \times 240\right)$$
 = ₹ 220.

B.G. = B.D - T.D. =
$$240 - 220 = ₹ 20$$

EXAMPLE 6 The banker's discount and the true discount on a sum of money due 6 months hence are $\stackrel{?}{\stackrel{?}{\stackrel{?}{$}}}$ 130 and $\stackrel{?}{\stackrel{?}{\stackrel{?}{$}}}$ 120 respectively. Find the sum and the rate per cent.

Solution Sum =
$$\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = ₹ \left(\frac{130 \times 120}{130 - 120} \right) = \frac{130 \times 120}{10} = ₹ 1560$$

But Banker's discount = S.I. on sum due.

 \therefore S.I. on ₹ 1320 for 6 months = 130.

:. Rate =
$$\left(\frac{100 \times 130}{1560 \times \frac{1}{2}}\right)\% = \frac{100}{6}\% = 16\frac{2}{3}\%$$

EXAMPLE 7 The present worth of a bill due certain time hence is ₹ 1300 and the true discount on the bill is ₹ 130. Find the banker's discount and the banker's gain.

Solution T.D. = $\sqrt{P.W. \times B.G.}$

B.G. =
$$\frac{(\text{T.D.})^2}{\text{P.W.}} = \frac{₹(130)^2}{1300} = ₹13$$

B.D. = T.D. + B.G. =
$$130 + 13 = ₹ 143$$
.

∴ Banker's discount = ₹ 143
 Banker's gain = ₹ 13.



EXERCISES

- 1. The true discount on a bill due 10 months hence at 15% per annum is ₹225. The amount of the bill is
 - (a) ₹ 1500
- (b) ₹ 1750
- (c) ₹ 1800
- (d) ₹ 2000
- 2. True discount on ₹3024 due 6 months hence is ₹144. The rate per cent is
 - (a) 6%
- (b) 8%
- (c) 10%
- (d) 12%
- 3. The simple interest and the true discount on a certain sum for a given time and at a given rate are ₹ 85 and ₹ 80 respectively. The sum is
 - (a) ₹ 6800
- (b) ₹ 3400
- (c) ₹ 1700
- (d) ₹ 1360
- 4. A person bought an article for ₹ 2500 and sold it the same day for ₹ 3410, allowing him a credit of 2 years. If the rate of interest is 12% p.a., then the gain of man is:
 - (a) 0%
- (b) 5%
- (c) 8%
- (d) 10%
- 5. ₹ 20 is the true discount on ₹ 210 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same?
 - (a) ₹ 10
- (b) ₹ 10.20
- (c) ₹ 10.40
- (d) ₹ 10.50

6.	The banker the same ti				money is₹	84 and	true disco	ount on	the sam	e sum for
	(a) ₹ 350		₹ 380		₹ 400	(d)	₹ 420			
7.	The banker	's gain of a	certain sum	due 2 y	ears hence	at $12\frac{1}{2}$	% per an	num is	₹ 36. Th	e present
	worth is					2				
	(a) ₹ 280	(b)	₹ 350	(c)	₹ 576	(d)	₹ 630			
8.	The presen ₹ 170. The			m due	$\mathbf{sometimes}$	hence i	s ₹ 1700	and the	e true di	scount is
	(a) ₹ 20	(b)	₹ 18	(c)	₹ 17	(d)	₹ 16			
9.	The true di	scount on a	a bill of₹72	0 is ₹ 8	0. The ban	ker's di	scount is	:		
	(a) ₹80	(b)	₹ 90	(c)	₹ 100	(d)	₹ 120			
10.	The banker	's gain on a	a bill due 1	year he	nce at 15%	p.a. is	₹ 9. The t	rue dis	count is:	
	(a) ₹50	(b)	₹ 54	(c)	₹ 60	(d)	₹ 72			
11.	The banker same time					same a	s true di	scount	on ₹ 252	0 for the
	(a) 4 mont	hs (b)	5 months	(c)	6 months	(d)	8 month	s		
12.	The banker				2	ears is₹	837 and	the tru	e discou	nt on the
	same sum f	for 2 years	is ₹ 900. Th	e rate p	er cent is:					
	(a) 10%	` ,	11%	` '	12%	` '	13%			
13.	The present true discou		sum due ce	rtain ti	me hence is	s₹ 1024	and the b	anker's	s gain is	₹ 36. The
	(a) ₹ 144	(b)	₹ 164	(c)	₹ 172	(d)	₹ 192			
14.	The banker	's gain on a	a sum due 2	years l	nence at 10)% p.a. i	s ₹ 120. T	he ban	ker's dis	scount is:
	(a) ₹ 240	(' /	₹ 360	, ,	₹ 540	` '	₹ 720			
15.	A bill for ₹ 10%. Find the bill rece	he banker (_							-
	(a) ₹ 7600	(b)	₹ 7820	(c)	₹ 7840	(d)	₹ 7880			
R	WERS									
1.	(c) 2. (d	e) 3. (d)	4. (d)	5. (d) 6. (d)	7.	(c) 8	. (c)	9. (b)	10. (c)
11.	, ,	e) 13. (d)	, ,	15 . (c	e)		` ,	, ,	. ,	
h4		. ,	. ,	`	•					
	Solution	ns with l	Necessar	y Expl	anation					
1	I at D W ha	Ŧ								

1. Let P.W. be $\overline{\xi}$ x.

∴ S.I. on ₹x for 10 months at 15% p.a. = ₹225

i.e.
$$x \times \frac{15}{100} \times \frac{10}{12} = 225$$

⇒ $x = \frac{225 \times 1200}{150} = 15 \times 120 = ₹ 1800$

2. P.W. = ₹
$$(3024 - 144) = ₹ 2880$$

$$\therefore \frac{144 \times 100}{2880 \times \frac{1}{2}} \% = 10\%$$

3. Sum =
$$\frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \frac{85 \times 80}{85 - 80} = ₹ 1360$$

4. C.P. = ₹ 2500; S.P. = ₹
$$\left[\frac{3410 \times 100}{100 + (12 \times 2)} \right]$$
 = ₹ 2750

$$\therefore \quad \text{Gain} = \frac{2750 - 2500}{2500} \times 100\% = 10\%$$

5. S.I on ₹ (210 – 20) for a given time = ₹ 20.

S.I. on ₹ 190 for half the time = ₹ 10

∴ T.D. on ₹ 210 =
$$\left(\frac{10}{200} \times 210\right)$$
 = ₹ 10.50

$$\frac{B.D. \times T.D.}{B.D. - T.D.} = \frac{84 \times 70}{84 - 70} = ₹ 420$$

7. Given B.G. =
$$\sqrt[7]{36}$$
; $R = 12\frac{1}{2}\%$ p.a.; $T = 2$ years;

P.W. =
$$\frac{100 \times \text{T.D.}}{R \times T}$$
;
T.D. = $\frac{\text{B.G.} \times 100}{R \times T} = \frac{36 \times 100}{12\frac{1}{2} \times 2} = ₹ 144$;

∴ P.W. =
$$\frac{100 \times 144}{12\frac{1}{2} \times 2} = ₹576$$

8. Given P.W. = $\overline{\xi}$ 1700; T.D. = $\overline{\xi}$ 170

∴ B.G. =
$$\frac{(T.D.)^2}{P.W.} = \frac{(170)^2}{1700} = ₹ 17$$

9. P.W. = ₹ (720 - 80) = ₹ 640;

S.I. on ₹
$$640 = ₹ 80$$
;

S.I. on ₹ 720 =
$$\left(\frac{80}{640} \times 720\right)$$
 = ₹ 90

10. B.G. = ₹ 9; T = 1 year; R = 15% p.a.

T.D. =
$$\frac{\text{B.G.} \times 100}{R \times T}$$
 = $\frac{9 \times 100}{15 \times 1}$ = ₹ 60

11. S.I. on ₹ 2400 = T.D. on ₹ 2520

∴ ₹ 2400 is the P.W. of ₹ 2520

i.e. ₹ 120 is S.I. on ₹ 2400 at 12%

:. Time =
$$\frac{100 \times 120}{2400 \times 12} = \frac{5}{12}$$
 years = 5 months

12. Given B.D. for $1\frac{1}{2}$ years = ₹837;

$$\therefore B.D. \text{ for 2 years} = ₹837 \times \frac{2}{3} \times 2 = ₹1116$$

T.D. for 2 years = 900;

Sum =
$$\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{1116 \times 900}{1116 - 900} = \frac{1116 \times 900}{216} = ₹ 4650$$

∴ ₹ 1116 is S.I. on ₹ 4650

$$\therefore$$
 Rate = $\left(\frac{1116 \times 100}{4650 \times 2}\right) = \frac{1116}{93} = 12\%$

13. Given P.W. = ₹ 1024; B.G. = ₹ 36;

∴ T.D. =
$$\sqrt{P.W. \times B.G.} = \sqrt{1024 \times 36} = ₹ 192$$

14. B.G. = ₹ 120; R = 10%; T = 2 years;

T.D. =
$$\frac{\text{B.G.} \times 100}{R \times T} = \frac{120 \times 100}{10 \times 2} = ₹600$$

(:: B.D. = T.D. + B.G.)

15. Face value of bill = ₹ 8000;

Date on which bill was drawn = September 19 at 6 months.

Nominally due date = March 19

Legally due date = March 22

Date on which bill was discounted = January 8.

Unexpired time:

$$23 + 28 + 22 = 73 \text{ days} = \frac{1}{5} \text{ years}.$$

∴ B.D. = S.I. on ₹8000 for
$$\frac{1}{5}$$
 years
= $8000 \times 10 \times \frac{1}{5} \times \frac{1}{100} = ₹160$

T.D. =
$$\frac{\text{B.D.} \times 100}{100 + R \times T} = \frac{160 \times 100}{100 + 10 \times \frac{1}{5}}$$

$$=\frac{16000}{102} = ₹ 156.86$$

B.G. = B.D. – T.D. = ₹
$$160 - ₹ 156.86 = ₹ 3.14$$

∴ Money received by holder of bill = ₹ (8000 - 160) = ₹ 7840

Logarithm

DEFINITION: If x is a positive real number other than 1 and $x^m = n$, then we write, m = n $\log_x n$, and we say that value $\log n$ to the base x is m.

Example:

1.
$$10^2 = 100 \implies \log_{10} 100 = 2$$

2.
$$5^4 = 625$$
 \Rightarrow $\log_5 625 = 4$

1.
$$10^2 = 100$$
 \Rightarrow $\log_{10} 100 = 2$
2. $5^4 = 625$ \Rightarrow $\log_5 625 = 4$
3. $3^{-2} = \frac{1}{9}$ \Rightarrow $\log_3 \frac{1}{9} = -2$

4.
$$(0.1)^3 = 0.001$$
 \Rightarrow $\log_{(0.1)} 0.001 = 3$

PROPERTIES OF LOGARITHM:

1.
$$\log_r(ab) = \log_r a + \log_r b$$

2.
$$\log_x \left(\frac{a}{b}\right) = \log_x a - \log_x b$$

$$3. \log_x x = 1$$

$$4. \log_x 1 = 0$$

5.
$$\log_x a^m = m \log_x a$$

$$6. \ \log_x a = \frac{1}{\log_a x}$$

$$7. \ \log_x a = \frac{\log_b a}{\log_b x} = \frac{\log a}{\log x}$$

COMMON LOGARITHM: Logarithm to the base 10 is known as common logarithm. If base is not mentioned in logarithm, by default, it is 10.

NATURAL LOGARITHM: Logarithm to the base e is called natural logarithm where $e = 2.718 \dots$

CHARACTERISTIC: The integral part of the logarithm of a number is called its characteristic.

Case I. When the number is greater than 1.

Here, characteristic is a number which is one less than the number of digits to the left of decimal point in the given number.

Case II. When the number is less than 1.

Here, characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number, and it is negative.

For such negative number like -1, -2, etc. we will write $\overline{1}$ (one bar) $\overline{2}$ (two bar) etc.

Example:

Number	Characteristic	Number	Characteristic
394.625	2	0.3162	1
4963.849	3	0.0459	$\overline{2}$
27.348	1	0.0025	$\overline{3}$

MANTISSA: The decimal part of the logarithm of a number is known as its mantissa. For mantissa, you have to refer to logarithm table.

EXAMPLE 1 Evaluate $\log_2 64$

Solution Let $\log_2 64 = x$

$$\therefore \qquad \qquad 2^x = 64 = 2^6$$

$$\therefore$$
 $x = 6$

$$\therefore \qquad \log_2 64 = \mathbf{6}$$

EXAMPLE 2 Evaluate $\log_9\left(\frac{1}{729}\right)$

Solution Let $\log_9\left(\frac{1}{729}\right) = x$

$$9^x = \frac{1}{729} = \frac{1}{9^3} = 9^{-3}$$

∴
$$x = -3$$

$$\log_9\left(\frac{1}{729}\right) = -3$$

EXAMPLE 3 Evaluate 49^{log, 6}

Solution We have $x^{\log_x a} = a$

$$\therefore 49^{\log_7 6} = (7^2)^{\log_7 6} = 7^{2\log_7 6} = 7^{\log_7 6^2} = 7^{\log_7 36} = 36$$

Solution

EXAMPLE 4 Simplify
$$\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}$$
 [SSC]

 $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

Solution
$$\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9}\right)^2 + \log \frac{32}{243}$$

$$= \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243}$$
$$= \log \left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}\right) = \log 2$$

EXAMPLE 5 Find the value of *x* which satisfies the relation:

$$\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$$

⇒
$$\log[3(4x+1)] = \log_{10}(x+1) + \log_{10} 10$$

⇒ $\log[3(4x+1)] = \log_{10}[10(x+1)]$
⇒ $3(4x+1) = 10(x+1)$
⇒ $2x = 10 - 3 = 7$

$$\Rightarrow \qquad \qquad x = \frac{7}{2}$$

EXAMPLE 6 If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$

[CBI]

Solution
$$\log_{10} 50 = \log_{10} \left(\frac{100}{2} \right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897$$

EXERCISES

- 1. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is
- [SSC]

[CDS]

- (b) 1.4471
- (c) 2.4471
- (d) None of these

2. If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to:

- (b) 3

- 3. If $\log_{10} 7 = \alpha$, then $\log_{10} \left(\frac{1}{70}\right)$ is equal to: [CDS]
 - (a) -(1+a) (b) $(1+a)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

4. If $\log_a(ab) = x$, then $\log_b(ab)$ is

[MAT]

- (a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$ (c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

				_	
5.	$\frac{\log \sqrt{8}}{\log 8}$ is equal to	o:			[MBA]
	(a) $\frac{1}{\sqrt{8}}$	(b) $\frac{1}{4}$	(c) $\frac{1}{2}$	(d) $\frac{1}{8}$	
6.	If $\log_x y = 100$ and	$\log_2 x = 10$, then t	the value of y is:		[SSC]
	(a) 2^{10}	(b) 2^{100}	(c) 2^{1000}	(d) 2 ¹⁰⁰⁰⁰	
7.	Value of log ₂ 16 is	s			[MBA]
	(a) $\frac{1}{8}$	(b) 4	(c) 8	(d) 16	
8.	If $\log_{10} 2 = 0.3010$, the numbr of digi	its in 2^{64} is:		[CBI]
	(a) 18	(b) 19	(c) 20	(d) 21	
9.	If $\log 2 = 0.3010$ a	and $\log 3 = 0.4771$,	then the value of le	og ₅ 512 is:	[MAT]
	(a) 2.870	(b) 2.967	(c) 3.876	(d) 3.912	
10.	$2\log_{10} 5 + \log_{10} 8$ -	$-\frac{1}{2}\log_{10} 4 = ?$			[MAT]
	(a) 2	(b) 4	(c) $2 + 2 \log_{10} 2$	(d) $4-4\log_{10}2$	
11	If $\log 2 = x \log 2 =$	- w and log 7 - a th	nen the value of log	- —	[SSC]
11.	$11 \log 2 - x, \log 3 -$	$-y$ and $\log t - z$, th	ien the value of logi	(4 (03) 18	[DBC]
	(a) $2x + \frac{2}{3}y - \frac{1}{3}z$	3	(b) $2x +$	$\frac{2}{3}y + \frac{1}{3}z$	
	(c) $2x - \frac{2}{3}y + \frac{1}{3}z$	Z	(d) $-2x$	$+\frac{2}{3}y + \frac{1}{3}z$	
12.	If $\log 3 = 0.477$ and	ad $(1000)^x = 3$, then	n <i>x</i> equals:		[SSC]
	(a) 0.0159	(b) 0.0477	(c) 0.159	(d) 10	
13.	If $a^x = b^y$, then:			[Hotel Manage	ement]
	(a) $\log \frac{a}{b} = \frac{x}{y}$	(b) $\frac{\log a}{\log b} = \frac{x}{y}$	(c) $\frac{\log a}{\log b} = \frac{y}{x}$	(d) None of these	
14.	$\mathbf{If} \log_4 x + \log_8 x =$	5, then x is equal t	to:		
	(a) 16	(b) 32	(c) 48	(d) 64	
15.	$\mathbf{If} \log_{10} 50 + \log_{10} 3$	20 = x, then x is eq	ual to:		
	(a) $\frac{1}{3}$	(b) -3	(c) 3	(d) 0.3	
_					



2. (b)

12. (c)

3. (a)

13. (c)

4. (d)

14. (d)

5. (c)

15. (c)

6. (c)

7. (b)

8. (c)

9. (c)

10. (a)

1. (a)

11. (b)

V

Solutions with Necessary Explanation

1.
$$\log_{10} 2.8 = \log_{10} \left(\frac{28}{10}\right) = \log_{10} \left(\frac{2^2 \times 7}{10}\right)$$

 $= \log_{10} 2^2 + \log_{10} 7 - \log_{10} 10$
 $= 2 \times 0.3010 + 0.8451 - 1$
 $= 0.602 + 0.8451 - 1 = 0.4471$

2.
$$\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$$

 $\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} (x + 5) + \log_{10} 10$
 $\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} [10 (x + 5)]$
 $\Rightarrow 5(5x + 1) = 10(x + 5)$

$$\Rightarrow 25x - 10x = 50 - 5$$

$$\Rightarrow 15x = 45$$

$$\Rightarrow x = 3$$

3. Given $\log_{10} 7 = a$;

$$\log_{10}\left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70$$

$$= \log_{10} 1 - \log_{10} (7 \times 10)$$

$$= 0 - [\log_{10} 7 + \log_{10} 10]$$

$$= -(a+1) = -(1+a)$$

4. Given $\log_a(ab) = x$;

5.
$$\frac{\log \sqrt{8}}{\log 8} = \frac{\frac{1}{2} \log 8}{\log 8} = \frac{1}{2}$$

6. Given $\log_x y = 100$; $\log_2 x = 10$;

$$\begin{array}{lll} \therefore & \log_2 x \times \log_x y = 10 \times 100 \\ \text{i.e.} & \log_2 y = 1000 & (\because \log_a x \times \log_x b = \log_a b) \\ \therefore & y = 2^{1000} \end{array}$$

7. $\log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4 \times 1 = 4$

8.
$$\log(2^{64}) = 64 \log 2 = 64 \times (0.3010) = 19.26592$$

 \therefore Number of digits = Characteristic + 1 = 20

9. Given
$$\log 2 = 0.3010$$
; $\log 3 = 0.4771$

$$\log_5 512 = \log_5 (16^2 \times 2)$$

$$= \log_5 ((2^4)^2 \times 2^1) = \log_5 (2^9) = 9 \log_5 2$$

$$= 9 \frac{\log_{10} 2}{\log_{10} 5} = \frac{9 \log_{10} 2}{\log 10 - \log 2}$$

$$= \frac{9 \times 0.3010}{1 - 0.3010} = \frac{2.709}{0.699} = 3.876$$

10.
$$2\log_{10} 5 + \log_{10} 8 - \frac{1}{2}\log_{10} 4 = \log_{10} \left(\frac{5^2 \times 8}{2}\right) = \log_{10} (100) = 2$$

11. Given
$$\log 2 = x$$
; $\log 3 = y$; $\log 7 = z$;

$$\log(4\sqrt[3]{63}) = \log(2^2 \times \sqrt[3]{3^2 \times 7}) = \log 2^2 + \frac{1}{3}\log(3^2 \times 7)$$
$$= 2\log 2 + \frac{1}{3}[2\log 3 + \log 7] = 2x + \frac{2}{3}y + \frac{1}{3}z$$

12. Given
$$\log 3 = 0.477$$
; $(1000)^x = 3$;

$$\begin{array}{ccc} \therefore & \log_{10}{(1000)^{x}} = \log_{10}{3} \\ \Rightarrow & \log_{10}{(10^{3})^{x}} = \log_{10}{3} \\ \Rightarrow & 3x = \log_{10}{3} \\ \Rightarrow & x = \frac{1}{3}\log_{10}{3} = \frac{1}{3} \times 0.477 = \mathbf{0.159}$$

13. Given
$$a^x = b^y$$

Applying logarithm on both sides.

$$\log a^{x} = \log b^{y}$$

$$\Rightarrow x \log a = y \log b$$

$$\Rightarrow \frac{\log a}{\log b} = \frac{y}{x}$$

14. Given $\log_4 x + \log_8 x = 5$

i.e.
$$\frac{\log x}{\log 4} + \frac{\log x}{\log 8} = 5$$

$$\Rightarrow \frac{\log x}{2 \log 2} + \frac{\log x}{3 \log 2} = 5$$

$$\Rightarrow 5 \log x = 5 \times 6 \log 2$$

$$\Rightarrow \log x = 6 \log 2 = \log 2^{6}$$

$$\therefore x = 64$$

15.
$$\log_{10} 50 + \log_{10} 20 = x$$

$$\Rightarrow \log_{10} (50 \times 20) = x$$

$$\Rightarrow 10^{x} = 1000 = 10^{3}$$

$$\therefore x = 3$$



SECTION A GENERAL MENTAL ABILITY

Chapter 1

Analogy

In the question based on analogy, there will be a particular relationship between the given two words. You have to identify the pair of words which are having the similar relationship among them.

Some useful points on the basic knowledge required for these tests are given below.

STUDY AND TOPIC RELATIONSHIP

Example: Anthropology: Man

We know that 'anthropology' is the study of man. Similar examples are:

1. Astrology	:	Future	19.	Occultism	:	Supernatural
2. Botany	:	Plants	20.	Onomatology	:	Names
3. Bryology	:	Bryophytes	21.	Ontology	:	Reality
4. Cardiology	:	Heart	22.	Oology	:	Eggs
5. Concology	:	Shells	23.	Ornithology	:	Birds
6. Craniology	:	Skull	24.	Orography	:	Mountains
7. Eccrinology	:	Secretions	25 .	Paleography	:	Writings
8. Entomology	:	Insects	26.	Paleontology	:	Fossils
9. Ethnology	:	Human races	27 .	Pathology	:	Diseases
10. Hematology	:	Blood	28.	Pedology	:	Soil
11. Herpetology	:	Amphibians	29.	Phycology	:	Algae
12. Histology	:	Tissues	30.	Seismology	:	Earthquakes
13. Ichthyology	:	Fishes	31.	Selenography	:	Moon
14. Malacology	:	Molluscs	32.	Semantics	:	Language
15. Mycology	:	Fungi	33.	Taxonomy	:	Classification
16. Nephrology	:	Kidney	34.	Virology	:	Viruses
17. Neurology	:	Nerves	35 .	Zoology	:	Animals
18. Nidology	:	Nests				

WORKER AND TOOL RELATIONSHIP

Example: Carpenter: Saw

A saw is a tool used by the carpenter.

Similar examples are:

1. Astronomer Telescope 13. Labourer Spade 2. Author Pen 14. Lumberjack Axe 3. Barber Scissors 15. Mason Plumbline 4. Blacksmith Anvil 16. Painter Brush 5. Bricklaver Chisel Trowel 17. Sculptor 6. Butcher Chopper 18. Soldier Gun 7. Chef Knife 19. Surgeon Scalpel 8. Cobbler Awl 20. Tailor Needle 21. Violinist Bow 9. Doctor Stethescope 22. Warrior Sword

23. Wood cutter

Axe

10. Farmer : Plough 11. Gardener : Harrow

12. Jockey : Tack

TOOL AND ACTION RELATIONSHIP

Example: Spade: Dig A spade is used for digging.

Similar examples are:

10. Needle Saw 1. Auger Bore 2. Axe Grind 11. Oar Row 3. Chisel. 12. Pen Write Carve 4. Filter 13. Shield Purify Guard 5. Gun Shoot 14. Showel Scoop 6. Knife Cut 15. Spanner Grip 7. Loudspeaker: 16. Spoon Amplify Feed 17. Sword 8. Mattock Dig Slaughter 9. Microscope Magnify 18. Steering Drive

WORK AND WORKING PLACE

Example: Actor: Stage An actor performs on a stage. Similar examples are:

8. Farmer

9. Gambler Casino 1. Artist Theatre 10. Grocer Shop 2. Beautician Parlour 11. Lawyer Court 3. Chef Kitchen 12. Mechanic Garage 4. Clerk Office 13. Painter Gallery 5. Doctor Hospital 14. Pilot Cockpit Cabin 6. Driver 15. Sailor Ship 7. Engineer Site 16. Scientist Laboratory

Field

17.	Servant	:	House	20 .	Waiter	:	Restaurant
18.	Teacher	:	School	21.	Warrior	:	Battle field
19.	Umpire	:	Pitch	22.	Worker	:	Factory

WORKER AND PRODUCT

Example: Producer: Film A producer makes movies.
Similar examples are:

1. Architect Design 10. Farmer Crop 2. Author Book 11. Goldsmith **Ornaments** 3. Butcher Meat 12. Hunter Prev 4. Carpenter Furniture 13. Judge Justice 5. Chef Food Wall 14. Mason Ballet 6. Choreographer: 15. Poet Poem 7. Cobbler Shoes 16. Tailor Clothes 8. Dramatist Plav 17. Teacher Education

9. Editor : Newspaper

PRODUCT AND RAW MATERIAL

Example: Paper: Pulp Paper is made from pulp. Similar examples are:

1. Book 11. Omelette Paper Egg 2. Butter Milk 12. Prism Glass 13. Pullover Wool 3. Cloth Fibre 4. Fabric 14. Road Asphalt Yarn 5. Furniture Wood 15. Rubber Latex 16. Sack Jute 6. Jaggery Sugarcane 17. Shoes Gold Leather 7. Jewellery 18. Wall Brick 8. Linen Flax 19. Wine 9. Metal Ore Grapes

10. Oil : Seed

INSTRUMENT AND MEASUREMENT

Example: Scale: Length Similar examples are:

1. Ammeter : Current 2. Anemometer : Wind vane 3. Balance : Mass : Pressure 4. Barometer 5. Hygrometer : Humidity 6. Odometer : Speed : Rain 7. Rain gauge : Thickness 8. Screw gauge

9. Seismography : Earthquakes 10. Sphygmomanometer : Blood pressure

11. Taseometer : Strains12. Thermometer : Temperature

QUANTITY AND UNIT

Example: Time: Seconds
Second is the unit of time.
Similar examples are:

Radians 10. Mass Kilogram 1. Angle 2. Area Hectare 11. Potential · Volt 3. Conductivity Mho 12. Power : Watt : Pascal 4. Current Ampere 13. Pressure : Ohm 5. Energy Joule 14. Resistance 6. Force Newton 15. Temperature : Degrees 7. Length Metre 16. Volume : Litre 8. Luminosity Candela 17. Work Joule

Oersted

ANIMALS AND YOUNG ONES

Example: Man: Child

The child is the young one of man.

9. Magnetic field:

Similar examples are:

8. Horse 1. Butterfly Catterpillar Ponv 2. Cat. Kitten 9. Insect Larva 3. Cow Calf 10. Lion Cub 4. Dog 11. Pig Farrow Puppy 12. Sheep 5. Duck Duckling Lamb Tadpole 6. Frog 13. Stallion Colt 7. Hen Chicken 14. Tortoise Turtle

MALE AND FEMALE

Example: Horse: Mare Mare is the female horse. Similar examples are:

1. Dog Bitch 6. Son Daughter 2. Drone Bee 7. Sorcerer Sorceress 3. Gentleman: Lady 8. Stag Doe 4. Lion Lioness 9. Tiger **Tigress** 5. Nephew Niece

WORD AND INTENSITY

Example: Anger: Rage

'Rage' is of higher intensity than 'anger'.

Similar examples are:

1. Crime Sin War 6. Quarrel 2. Error Blunder 7. Refuse Denv 3. Famous Renowned 8. Sink Drown 4. Kindle Sad Burn 9. Unhappy 5. Moist Drench 10. Wish Desire

WORD AND SYNONYM

Example: Abode: Dwelling

'Abode' means almost the same as 'dwelling'. Thus, 'dwelling' is the synonym of 'abode'. Similar examples are:

1. Abduct 11. Flaw Defect Kidnap 12. Haughty Proud 2. Alight Descend 3. Assign Allot 13. House : Home 4. Ban Prohibition 14. Mend Repair : Predict 5. Blend Mix 15. Presage 6. Brim Edge 16. Presume : Assume 7. Dearth 17. Sedate Calm Scarcity 8. Dissipate Squander 18. Solicit Request 9. Fallacv Illusion 19. Substitute : Replace 10. Fierce Violent 20. Vacant **Empty**

WORD AND ANTONYM

Example: Gradual: Abrupt

'Abrupt' means sudden, which is the opposite of 'gradual'. Thus, 'abrupt' is the antonym of 'gradual'.

Similar examples are:

1.	Advance	:	Retreat	11.	Fresh	:	Stale
2.	Affirm	:	Deny	12 .	Gentle	:	Harsh
3.	Attack	:	Defend	13.	Ignore	:	Notice
4.	Best	:	Worst	14.	Initial	:	Final
5 .	Chaos	:	Peace	15.	Kindle	:	Extinguish
6.	Condense	:	Expand	16.	Lend	:	Borrow
7.	Cordial	:	Hostile	17.	Lethargy	:	Alertness
8.	Create	:	Destroy	18.	Mourn	:	Rejoice
9.	Cruel	:	Kind	19.	Robust	:	Weak
10.	Deep	:	Shallow	20.	Sink	:	Float

TYPE I: In this type of questions, two words are given. These words are related to each other in some way. Another word is also given. You have to find out the relationship between the first two words and choose the word from the given alternatives, which bear the same relationship to the third word.

EXAMPLE 1 Doctor : Diagnosis : : Judge : ?

- (a) Court (b) Punishment (c) Lawyer (d) Judgement

Solution As a doctor diagnoses a disease. Similarly, a judge give judgement. So, the answer is (d).

EXAMPLE 2 Bombay: Maharashtra:: Trivandrum?

(a) Calcutta

(b) Gujarat

(c) Rajasthan

(d) Kerala

Solution Bombay is the capital of Maharashtra. Similarly, Trivandrum is the capital of Kerala. So the answer is (d).

EXAMPLE 3 Conference: Chairman:: Newspaper:?

(a) Reporter

(b) Distributor

(c) Printer

(d) Editor

Solution The chairman is the highest authority in a conference. Similarly, the editor is the highest authority in a newspaper agency. So the answer is (d).

EXERCISE 1

Directions: In each of the following questions, there is a certain relation between two words given on one side of:: and one word is given on the side of:: while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear, choose the correct alternative.

1.	Moon: Satellite:: Earth:?							
	(a) Sun	(b) Planet	(c)	Solar system	(d)	Asteroid		
2.	Melt: Liquid:: Freeze	e:?						
	(a) Ice	(b) Condense	(c)	Solid	(d)	Crystal		
3.	Influenza : Virus : : Ty	yphoid:?						
	(a) Bacillus	(b) Parasite	(c)	Protozoa	(d)	Bacteria		
4.	Eye: Myopia:: Teeth	: ?						
	(a) Pyorrhoea	` '	(c)	Trachoma	(d)	Eczema		
5.	Scribble: Write::Sta	mmer:?						
	(a) Walk	(b) Play	(c)	Speak	(d)	Dance		
6.	Muslims: Mosque:: 8	Sikhs : ?						
	(a) Golden temple	(b) Medina	(c)	Fire temple	(d)	Gurudwara		
7.	7. Acting : Theatre : : Gambling : ?							
	(a) Casino	(b) Club	(c)	Bar	(d)	Gym		
8.	$\mathbf{Breeze}: \mathbf{Cyclone}:: \mathbf{Dr}$	rizzle:?						
	(a) Earthquake	(b) Storm	(c)	Flood	(d)	Downpour		
9.	Water : Convection : :	Space:?						
	(a) Conduction	` '	(c)	Vacuum	(d)	Radiation		
10.	Grain: Stock:: Stick	?						
	(a) Heap	(b) Bundle	(c)	Collection	(d)	String		
11.	Orygen: Burn:: Carb	oon dioxide : ?						
	(a) Isolate	(b) Foam	(c)	Extinguish	(d)	Explode		
12.	Planet : Orbit : : Proje	ctile:?						
	(a) Trajectory	(b) Track	(c)	Milky way	(d)	Path		
13.	Cobbler: Leather:: C	Sarpenter : ?						
	(a) Furniture	(b) Wood	(c)	Hammer	(d)	Chair		

14	Genuine : Authentic :	· Mi	rage · ?				
11.	(a) Image		Transpiration	(c)	Reflection	(d)	Illusion
15	Cub: Lion:: Colt:?	(0)	Transpiration	(0)	itelicetion	(4)	masion
10.	(a) Doe	(h)	Stag	(c)	Leopard	(d)	Stallion
16	Drill: Bore:: Sieve:	, ,	Dtag	(0)	Беораги	(u)	Stanion
10.	(a) Thresh		Sift	(c)	Pry	(d)	Rinse
17	Country: President::	` ′		(0)	119	(u)	Tillise
17.	(a) Governor		Minister	(a)	Chief Minister	(4)	Citizen
10	Car : Garage : : Aerop			(0)	Omer winister	(u)	Oluzen
10.	(a) Airport			(a)	Hangar	(4)	Harbour
10	Bread: Yeast:: Curd		Depot	(0)	Hangai	(u)	Harbour
19.			Ractoria	(a)	Germs	(4)	Virus
20	(a) Fungi Ruby : Red : : Sapphir		Dacteria	(0)	Germs	(u)	virus
20.	(a) Blue		White	(0)	Green	(4)	Silver
91	House : Garbage : : Or		vv IIIte	(0)	Green	(u)	Bilvei
41.	(a) Rubbish		Canqua	(a)	Sand	(4)	Dregs
99	Steel: Rails: Alnico		Gangue	(0)	Danu	(u)	Diegs
22.			Machinery	(a)	Silver ware	(4)	Magnets
23	Naphthalene : Coal ta			(0)	bliver ware	(u)	Magnets
20.	(a) Petroleum			(c)	Chemicals	(d)	Carbon
24	Darwin: Evolution::	` '		(0)	Onemicais	(u)	Carbon
27.			Lubrication	(c)	Buoyancy	(d)	Liquids
25	Flower : Butterfly : : I			(0)	Buoyancy	(u)	Diquius
20.	(a) Rats		Fly	(c)	Bugs	(d)	Sweeper
26	Glucose : Carbohydrat		•	(0)	Dugs	(u)	Dweeper
20.	(a) Proteins		Vitamins	(c)	Minerals	(d)	Legumes
27.	Engineer: Map:: Brid			(0)	Willier aris	(4)	Degumes
	(a) Design			(c)	Mould	(d)	Cement
28	Microphone : Loud : : 1		=	(0)	Would	(4)	cement
20.	(a) Increase			(c)	Examine	(d)	Magnify
29.	Matricide: Mother::		•	(0)		(α)	magning
	(a) Human			(c)	Father	(d)	Apes
30.	Scrap: Food:: Lees:			(0)	T delici	(ω)	Tipos
00.	(a) Bread		Tea	(c)	Wine	(d)	Rice
31.	Quartz : Radio : : Gyps			(0)	***************************************	(42)	20200
0 2.	(a) Glass		Porcelain	(c)	Cement	(d)	Powder
32.	Cattle : Herd : : Sheep			(0)	000	()	1011401
	(a) Flock		Swarm	(c)	Crowd	(d)	Shoal
33.	Newspaper: Press::0	` '		(-)		()	
	(a) Tailor		Mill	(c)	Fibre	(d)	Factory
34.	Oceans : Deserts : : W			(-)		(/	
	(a) Sea		Dust	(c)	Sand dunes	(d)	Ripples
35.	Giant : Dwarf : : Geni			\-/		\/	11
	(a) Wicked		Gentle	(c)	Idiot	(b)	Tiny
	(/	()		(0)		()	J

ANSWERS

- 1. (b) 3. (d) **4.** (a) **5.** (c) 2. (a) **6.** (d) 7. (a) 8. (d) **9.** (d) 10. (b) 11. (c) 20. (a) 12. (a) **13.** (b) 14. (d) **15.** (d) **16.** (b) 17. (a) 18. (c) **19.** (b)
- 21. (b) 22. (d) 23. (a) 24. (c) 25. (b) 26. (a) 27. (b) 28. (d) 29. (a) 30. (c)
- **31.** (c) **32.** (a) **33.** (b) **34.** (c) **35.** (c)

Explanations

- 1. The moon is a satellite and earth is a planet.
- 2. The first is the process of formation of the second.
- 3. The first is the disease caused by the second.
- 4. The second is a disease of the first.
- 5. The first is an improper form of the second.
- 6. Muslims worship in a mosque and Sikhs worship in a gurudwara.
- 7. The second is the place for performing the first.
- 8. The second is more intense than the first.
- 9. The second is the mode of transfer of heat by the first.
- 10. The second is a collection of the first.
- 11. Oxygen helps in burning whereas carbon dioxide extinguishes fire.
- 12. The second is the path traced by the first.
- 13. The second is the raw material used by the first.
- 14. The given words are opposites of each other.
- 15. The first is a young one of the second.
- 16. The second denotes the function performed by the first.
- 17. The President is the head of the country. Similarly, the Governor is the head of the state.
- 18. The first is temporarily parked in the second.
- 19. The first is produced by the action of the second.
- 20. Ruby is a red precious stone and sapphire is a blue precious stone.
- 21. The waste of the house is called garbage. Similarly, the impurities in the ore are called gangue.
- 22. The first is used to make the second.
- 23. The first is obtained from the second.
- 24. Darwin gave the theory of evolution. Likewise, Archimedes gave the principle of buoyancy.
- 25. The first attracts the second.
- 26. Glucose is a source of carbohydrates and soyabean is a source of proteins.
- **27.** The second gives the pattern to be followed by the first.
- 28. A microphone makes the sound louder and a microscope magnifies an object.
- 29. The first implies killing the second.
- **30.** The first is the left over of the second.
- 31. The first is used to make the second.
- 32. The second is the group or collection of the first.
- **33.** The second is the place where first is made.
- 34. If oceans were deserts, waves would be sand dunes.
- 35. The given words are opposites of each other.

EXERCISE 2

Directions: In each of the following questions, there is a certain relation between two given words on one side of:: and one word is given on the other side of:: while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear. Choose the best alternative.

1.	Tea: Cup:: Tobacco:	?				
	(a) Leaves	(b) Hookah	(c)	Toxin	(d)	Cheroot
2.	Pesticide: Crop::Ant	tiseptic:?				
	(a) Wound	(b) Clotting	(c)	Bandage	(d)	Bleeding
3.	Atom: Matter::Parti	icle:?				
	(a) Proton	(b) Electron	(c)	Molecule	(d)	Dust
4.	${\bf Disease: Pathology::}$	Planet:?				
	(a) Astrology	(b) Geology	(c)	Astronomy	(d)	Palaeontology
5.	Fire: Ashes:: Explosi	ion:?				
	(a) Sound	(b) Debris	(c)	Fury	(d)	Flame
6.	Tuberculosis : Lungs :	: Cataract : ?				
	(a) Ears	(b) Throat	(c)	Skin	(d)	Eyes
7.	Professor : Lecture : :	$\operatorname{Doctor}: ?$				
	(a) Hospital	(b) Disease	(c)	Medicine	(d)	Patient
8.	Victory: Encourageme	ent : : Failure : ?				
	(a) Sadness	(b) Defeat	(c)	Anger	(d)	Frustration
9.	Fossils: Creatures::	Mummies : ?				
	(a) Egypt	(b) Human beings	(c)	Animals	(d)	Martyrs
10.	Ornithologist: Birds:					
	(a) Plants	(b) Animals	(c)	Mankind	(d)	Environment
11.	Ocean : Pacific : : Islan					
	(a) Greenland	(b) Netherlands	(c)	Island	(d)	Borneo
12.	Meat : Vegetarian : : I	_				
	(a) Insane	(b) Introvert	(c)	Teetotaller	(d)	Foolish
13.	${\bf Accomodation: Rent:}$					
	(a) Expense	(b) Octroin	(c)	Freight	(d)	Fare
14.	Head : Cap : : Finger :					
	(a) Glove	(b) Thimble	(c)	Nail	(d)	Thumb
15.	Proteins: Growth:: C					
	(a) Energy	(b) Strength	(c)	Resistance	(d)	Diseases
16.	Cells : Tissues : : Aton					
_	(a) Elements	(b) Molecules	(c)	Electrons	(d)	Organs
17.	Girl: Beautiful:: Boy					
	(a) Smart	(b) Heroic	(c)	Courageous	(d)	Handsome
18.	Taxonomy: Classifica	 -		~		
	(a) Nature	(b) Farming	(c)	Soil	(d)	Mountain
19.	Aeroplane : Cockpit : :				/ 3 >	.
	(a) Wagon	(h) Coach	(c)	Compartment	(d)	Engine

20.	Palaeography : Writin	gs:	: Ichthyology : ?							
	(a) Fishes	(b)	Whales	(c)	Oysters	(d)	Mammals			
21.	Bullet: Gun:: Smoke	: ?								
	(a) Factory	(b)	Cigarette	(c)	Chimney	(d)	Fire			
22 .	22. Wrist: Elbow:: Ankle:?									
	(a) Heel	(b)	Fingers	(c)	Foot	(d)	Knee			
23.	Ottawa : Canada : : Ca	nbe	rra:?							
	(a) Argentina	(b)	Switzerland	(c)	Sri Lanka	(d)	Australia			
24.	$\mathbf{Leather}: \mathbf{Cobbler}:: \mathbf{W}$	⁷ ood	:?							
	(a) Furniture	(b)	Cottage	(c)	Carpenter	(d)	Mason			
25.	Roentgen: X rays::B	ecqı	ierel : ?							
	(a) Uranium	(b)	Radioactivity	(c)	Fission	(d)	Super conductivity			

ANSWERS											
1. (d)	2. (a)	3. (d)	4. (c)	5. (b)	6. (d)	7. (c)	8. (d)	9. (b)	10. (c)		
11. (a)	12. (c)	13. (d)	14. (b)	15. (a)	16. (b)	17. (d)	18. (c)	19. (d)	20. (a)		
21. (c)	22. (d)	23. (d)	24. (c)	25. (b)							

Explanations

- 1. Tea is contained in the cup. Similarly, tobacco is contained in cheroot.
- 2. Pesticide protects crops from insects and antiseptic protects wounds from germs.
- 3. The smallest unit of matter is atom and that of dust is particle.
- 4. Pathology is the study of diseases. Similarly, astronomy is the study of planets.
- 5. Ashes are the remains after a fire. Similarly, debris is the remains after an explosion.
- 6. Tuberculosis is a disease of lungs. Similarly, cataract is a disease of eyes.
- 7. Professor delivers lecture to his students. Similarly, Doctor gives medicine to his patients.
- 8. Victory leads to encouragement and failure brings frustration.
- 9. Fossils are the remains of creatures. Similarly, mummies are the remains of human beings.
- 10. Ornithologist specialises in the study of birds. Similarly, anthropologist specialises in the study of mankind.
- 11. The largest ocean is pacific ocean. Similarly, the largest island is greenland.
- 12. A vegetarian never eats meat. Similarly, a teetotaller never drinks liquor.
- 13. Money paid for accommodation is called rent. Similarly, the money paid for a journey is called fare.
- 14. Head is covered by a cap. Similarly, finger is covered by a thimble.
- 15. Proteins are essential for growth. Similarly, carbohydrates are essential for energy.
- 16. Cells constitute tissues and atoms constitute molecules.
- 17. 'Beautiful' describes the quality of prettiness in girls while 'handsome' describes the quality of prettiness in boys.
- 18. Taxonomy is the science dealing with classification. Similarly, pedology deals with the study of soil.
- 19. The pilot of an aeroplane sits in the cockpit. Likewise, the driver of a train works in the engine.

- 20. Palaeography is the study of ancient writings. Similarly, ichthyology is the study of fishes.
- 21. The first comes out of the second.
- 22. Wrist is the lower part of elbow. Similarly, ankle is the lower part of knee.
- 23. Ottawa is the capital of Canada and Canberra is the capital of Australia.
- 24. The first is the raw material required by the second.
- 25. Roentgen discovered X-rays. Similarly, Becquerel discovered radioactivity.

EXERCISE 3

Directions: In each of the following questions, there is a certain relation between two given words on one side of: and one word is given on the other side of: while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear. Choose the best alternative.

1.	Tectonics: Building:: Taxidermy:?									
	(a) Classification	(b) Conserving	(c)	Stuffing	(d)	Collecting				
2.	Visitor: Invitation::	Witness:?								
	(a) Subpoena	(b) Permission	(c)	Assent	(d)	Document				
3.	$\mathbf{Mash}:\mathbf{Horse}::\mathbf{Mast}$: ?								
	(a) Cow	(b) Monkey	(c)	Chimpanzee	(d)	Pig				
4.	Penology: Punishmen	it : : Seismology : ?								
	(a) Law	(b) Liver	(c)	Earthquakes	(d)	Medicine				
5.	Wine: Grapes::Perry	y:?								
	(a) Whisky	(b) Pears	(c)	Almonds	(d)	Pomegranates				
6.	${\bf Pericardium: Heart:}$: Rind : ?								
	(a) Body	(b) Head	(c)	Trees	(d)	Teak				
7.	Eccrinology: Secretion	ns : : Selenography : ?								
	(a) Sun	(b) Moon	(c)	Crust	(d)	Mantle				
8.	. Anaemia: Blood::Anarchy:?									
	(a) Disorder	(b) Monarchy	(c)	Government	(d)	Lawlessness				
9.	Anatomy: Zoology::]	Paediatrics : ?								
	(a) Chemistry	(b) Medicine	(c)	Palaeontology	(d)	Mechanics				
10.	Winter: Hibernation:	::Summer:?								
	(a) Survival	(b) Activation	(c)	Aestivation	(d)	Cache				
11.	Vegetable: Chop::Bo	ody:?								
	(a) Cut	(b) Amputate	(c)	Peel	(d)	Prune				
12.	${\bf Misogamy: Marriage}$:: Misogyny:?								
	(a) Children	(b) Husband	(c)	Relations	(d)	Women				
13.	Eye: Wink:: Heart:	?								
	(a) Move	(b) Throb	(c)	Pump	(d)	Quiver				
14.	Virology : Virus : : Ser	nantics:?								
	(a) Amoeba	(b) Language	(c)	Nature	(d)	Society				
15.	Mattock : Digging : : S	Shovel:?								
	(a) Break	(b) Push	(c)	Scoop	(d)	Whittle				

16.	Yaws : Skin : :	Thrush:?								
	(a) Legs	(b)	Eyes	(c)	Belly	(d)	Throat			
17.	Pituitary : Bra	in : : Thym	ıs : ?							
	(a) Larynx		Spinal cord	(c)	Throat	(d)	\mathbf{Chest}			
18.	${\bf Roster:Duty:}$: Inventory	: ?							
	(a) Furnace		Exports	(c)	Goods	(d)	Produce			
19.	Xylograph: En	graving::	Diorama : ?							
	(a) Painting	(b)	Exhibition	(c)	Colouring	(d)	Staging			
20 .	Funk: Vitamir	ns : : Curie :	?							
	(a) Uranium	(b)	Radium	(c)	Radioactivity	(d)	Photograp	hy		
21.	Karnataka : Go	old : : Madh	ya Pradesh :	: ?						
	(a) Diamond	(b)	Iron	(c)	\mathbf{Copper}	(d)	Gems			
22 .	Dum Dum : Ko	lkata : : Pa	lam:?							
	(a) Kerala	(b)	Delhi	(c)	Chennai	(d)	Mumbai			
23 .	Seismography	: Earthqual	ces : : Taseo	meter:?						
	(a) Volcanoes	, ,	Resistances	s (c)	Landslides	(d)	Strains			
24.	Pig:Farrow::	Dog: ?								
	(a) Mare		Puppy	(c)	Bitch	(d)	Colt			
25 .	Line: Square:	: Arc : ?								
	(a) Ring		Sphere	(c)	Circle	(d)	Ball			
26 .	Calendar : Dat		nary:?							
	(a) Vocabulary	(b)	Language	(c)	Words	(d)	Book			
27 .	Shark: Fish::	Lavender :	?							
	(a) Shrub	()	Tree	(c)	Herb	(d)	Climber			
28.	Applique : Orn	ament : : In	ipound : ?							
	(a) Confiscate	` '	Powder	(c)	Grab	(d)	Snatch			
29 .	Oriel: Room::	Tendon : ?								
	(a) Blood	` ,	Muscles	(c)	Veins	(d)	Liver			
30.	Probe : Wound	_	-							
	(a) Humidity	(b)	Rainfall	(c)	Force	(d)	Pressure			
	ANSWERS									
1.	(c) 2. (a)	3. (d)	4. (c) 5.	(b) 6. (c) 7. (b)	8. (c)	9. (b)	10. (c)		
11.	(b) 12. (d)	13. (b) 1	4. (b) 15.	(c) 16. (d) 17. (d)	18. (c)	19. (b)	20. (b)		
21.	(a) 22. (b)	23. (d) 2	4. (b) 25.	(c) 26. (c) 27. (a)	28. (a)	29. (b)	30. (c)		
			Exi	olanation	as l					

Explanations

- 1. Tectonics is the science dealing with the art of building. Similarly, taxidermy is the art of stuffing animals.
- 2. A visitor is given an invitation to attend an occassion. Similarly, a witness is delivered a subpoena providing for attendance at the court.
- **3.** The first is a food for the second.
- 4. Penology is the study of punishment. Similarly, seismology is the study of earthquakes.

- 5. Wine is made from grapes and perry is made from pears.
- **6.** The first is a thick covering over the second.
- 7. Eccrinology is the study of secretions and Selenography is the study of moon.
- 8. Anaemia is the lack of blood. Similarly, Anarchy is the lack of government.
- 9. Anatomy is a branch of zoology. Likewise, Paediatrics is a branch of medicine.
- 10. Winter sleep of animals is called hibernation and summer sleep is called aestivation.
- 11. Cutting of vegetables is called chopping. Cutting of a body part is called amputating.
- 12. The first is a hatred for the second.
- 13. The second is the movement of the first.
- 14. Virology deals with the effect of virus. Similarly, semantics deals with the effects of language.
- 15. Mattock is a tool for digging hard ground. Similarly, showel is a tool to scoop.
- 16. The first is a disease of the second.
- 17. Pituitary is a gland in the brain. Similarly, thymus is a gland of the chest.
- 18. Roster is a list of duties and inventory is a list of goods.
- 19. The given words are synonyms of each other.
- 20. Funk discovered vitamins and Curie discovered radium.
- 21. Gold is mined in Karnataka. Likewise, diamonds are mined in Madhya Pradesh.
- 22. Dum Dum is an Airport in Kolkata and Palam is an Airport in Delhi.
- 23. Seismograph is an instrument to measure the intensity of an earthquake. Similarly, taseometer is an instrument to measure strains.
- 24. The second is the young one of the first.
- 25. The first is a part of the second.
- 26. Calendar is a list of dates. Likewise, dictionary is a collection of words.
- 27. Shark is a fish and lavender is a shrub.
- 28. The given words are synonyms of each other.
- **29.** The first is a part of the second.
- **30.** Probe is an instrument to examine a wound. Similarly, anemograph is an instrument for recording force.

TYPE II: In this type of questions, a pair of words is followed by four pairs of words as alternatives. You have to choose the pair of words in which the words bear the same relationship to each other.

EXAMPLE 4 Balance: Weigh

(a) Aeroplane : Height

(b) Radar: Detection

(c) Satellite: Revolution

(d) Television: Picture

Solution A balance is used to weigh. Similarly, a radar is used for detection.

Hence the answer is (b).

EXAMPLE 5 Cells : Cytology

 $(a) \ \ Worms: Ornithology$

(b) Insects: Entomology

(c) Diseases: Physiology

(d) Tissues: Morphology

Solution The study of cells is called cytology. Similarly, the study of insects is called entomology.

Hence the answer is (b).

EXAMPLE 6 Chisel: Sculptor

(a) Scooter: Rider

(c) Scalpel: Surgeon

(b) Brush: Teeth(d) Knife: Woodcutter

Solution Chisel is a tool used by a sculptor. Similarly, scalpel is a tool used by a surgeon. Hence the answer is (c).

EXERCISE 4

Directions: The following questions consist of two words each that have a certain relationship to each other, followed by four lettered pairs of words. Select the lettered pair that has the same relationship as the original pair of words.

1. Birds: Aves

(a) Fish: Water

(c) Lizard: Insect

2. Dusk: Night

(a) Afternoon: Evening

(c) Walk: Run

3. Triangle: Hexagon

(a) Cone: Sphere

(c) Pentagon: Heptagon

4. Shield: Soldier

(a) Stethescope: Doctor

(c) Advocate: Court

5. Pesticide: Plant

(a) Injection: Disease

(c) Medicine: Cure

6. Cloth: Texture

(a) Body: Weigh

(c) Wood: Grains

7. House: Ceiling

(a) Hut: Roof

(c) Bed: Bedsheet

8. Fish: Aquarium

(a) Teacher: Hostel

(c) Bird: Nest

9. Dog: Kennel

(a) Horse: Carriage

(c) Cow: Barn

10. Paper: Ream

(a) Eggs: Dozen

(c) Twigs: Bush

11. Explosion: Destruction

(a) Talk: Exaggeration

(c) Success: Failure

(b) Whale: Fish

(d) Man: Homosapiens

(b) Infant: Child

(d) Day: Light

(b) Rectangle: Octagon

(d) Angle: Quadrilateral

(b) Book: Author

(d) Helmet: Rider

(b) Vaccination: Body

(d) Teacher: Student

(b) Silk: Cloth

(d) Ornaments: Gold

(b) Building: Floor

(d) Grapes: Wine

(b) Bee: Apiary

(d) Child: School

(b) Sheep: Flock

(d) Sports: Stadium

(b) Books: Pile

(d) Food: Packet

(b) Girl: Woman

(d) Engagement: Marriage

12. Identity: Anonymity

(a) Flaw: Perfection

(c) Truth: Lie

(a) Tear: Joy

13. Sigh: Relief

(c) Carelessness: Accident

14. Ecstasy: Pleasure

(a) Hatred: Affection

(c) Rage: Anger

15. Range: Mountain

(a) Point : Line

(c) School: Class

16. Spring: Summer

(a) Adolescence: Youth

(c) Stagger: Walk

17. Teeth: Dentist

(a) Legs: Philanthropist

(c) Operation: Surgeon

18. Textile: Mill

(a) Eggs: Hen

(c) Food: Agriculture

19. Shoes: Cobbler

(a) Spectacles: Optician

(c) Oxygen: Plant

20. Symphony: Music

(a) Mural: Painting

(c) Preface: Book

21. Numismatist : Coins

(a) Philatelist : Stamps

(c) Cartographer: Maps

22. Embroider: Cloth

(a) Patch: Quilt

(c) Carve: Knife

23. Curtain: Drapery

(a) Cockroach: Insect

(c) Pillow: Cushion

24. Theft: Confess

(a) Fight: Dare

(c) Murder: Commit

25. Crown: Royal

(a) Throne: Regal

(c) Pen: Author

(b) Careless: Mistake

(d) Fear: Joy

(b) Trembling: Fear

(d) Sweat: Hot

(b) Joy: Grief

(d) Mumble: Speak

(b) Bouqet: Flower

(d) String: Bead

(b) Fight: Battle

(d) Read: Learn

(b) Eyes: Occulist

(d) Sight: Spectator

(b) Coal: Mine

(d) Brick: Kiln

(b) Education: Teacher

(d) Food: Kitchen

(b) Ode: Prose

(d) Editorial: Journal

(b) Jeweller: Jewels

(d) Geneticist: Chromosomes

(b) Stain: Glass

(d) Chase: Metal

(b) Bedsheet: Bed

(d) Mat: Floor

(b) Fault: Admit

(d) Mistake: Agree

(b) Wrap: Ermine

(d) Crucifix: Religion

26. Cattle: Drove

(a) Soldier : Crew(c) Chicken : Brood

27. Meadow : Sheep

(a) Stable: Horse

(c) Grass: Grasshopper

28. Friendly: Inimical

(a) Lithosphere: Hydrosphere

(c) Abstain: Refrain

29. Modesty: Arogance

(a) Passion : Emotion(c) Cause : Purpose

30. Traitor: Disloyalty

(a) Executioner: Reliability

(c) Manager: Administration

(b) Grain: Bundle

(d) Bees: Heap

(b) Hay: Insect

(d) Pasture: Cattle

(b) Condemnation: Approval

(d) Disappointment: Embarrasment

(b) Practice: Perfection

(d) Debility: Strength

(b) Rebel: Defiance

(d) Hope: Pessimism

ANSWERS

1. (d) 2	2. (b)	3. (b)	4. (d)	5. (b)	6. (c)	7. (a)	8. (b)	9. (c)	10. (a)
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11. (d) 12. (a) 13. (b) 14. (c) 15. (d) 16. (a) 17. (b) 18. (d) 19. (a) 20. (a)

21. (a) 22. (d) 23. (a) 24. (b) 25. (d) 26. (c) 27. (d) 28. (b) 29. (d) 30. (b)

Explanations

- 1. Birds belong to the class of Aves. Similarly, man belongs to the class of Homasapiens.
- 2. Dusk is the initial stage of night. Similarly, infant is the initial stage of child.
- 3. The number of sides in the second figure in both the pairs is twice that in the first.
- 4. Shield is a defensive instrument for a soldier. Likewise, helmet is a defensive instrument for a rider.
- 5. Pesticide is meant to protect a plant from diseases. Similarly, vaccination is meant to protect the body from diseases.
- 6. The quality of cloth is identified by its texture and that of wood by grains.
- 7. Ceiling is the top part of a house. Likewise, roof is the top part of a hut.
- 8. Fishes are kept and reared in an aquarium. Similarly, bees are reared in an apiary.
- 9. Kennel is the home for a dog. Likewise, barn is the home for cows.
- 10. The paper is bought in reams. Similarly, eggs are bought in dozen.
- 11. Explosion is followed by destruction. Similarly, engagement is followed by marriage.
- 12. A lack of identity is anonymity. Similarly, a lack of flaw is perfection.
- 13. Sigh is a sign of relief. Similarly, trembling is a sign of fear.
- 14. The first is the higher intensity of second.
- 15. The first is the name given to a continuous chain of the second.
- 16. Spring is followed by summer. Similarly, adolescence is followed by youth.
- 17. The teeth are examined by a dentist. Likewise, eyes are examined by an occulist.
- 18. Textiles are manufactured in a mill. Similarly, bricks are manufactured in kiln.

- 19. Shoes are made by a cobbler. Similarly, spectacles are designed by an optician.
- 20. Symphony is a type of music. Similarly, mural is a type of painting.
- 21. A numismatist collects coins. Similarly, a philatelist collects stamps.
- 22. A pattern is embroidered on a cloth and chased on a metal.
- 23. Curtain belongs to the class of drapery. Similarly, cockroach belongs to the class of insects.
- 24. Theft is confessed and fault is admitted.
- 25. Crown is a symbol of royalty. Similarly, crucifix is a mark of religion.
- 26. Drove is a group of cattle. Similarly, brood is a group of chickens.
- 27. Sheep graze in a meadow. Similarly, cattle graze in a pasture.
- 28. The words in each pair are antonyms of each other.
- 29. The words in each pair are antonyms of each other.
- 30. The second is the defining characteristic of the first.

EXERCISE 5

Directions: Each of the following questions consists of two words which have a certain relationship to each, followed by four pairs of words. Select the pair which has the same relationship as the original pair of words.

1. Magazine: Periodical

(a) Gun: Soldier

(c) Pun: Joke

2. Heart: Cardiology

(a) Brain: Psychology

(c) Civics: Polity

3. Doctor: Hospital

(a) Plumber: Wrench

(c) Water: Reservoir

4. Flag: Nation

(a) Emblem: Prosperity

(c) Wealth: Prestige

5. Coin: Mint

(a) Grain: Field

(c) Wine: Brewery

6. Infection: Illness

(a) Satisfaction: Appetite

(c) Antidote: Disease

7. Sheep: Mutton

(a) Duck: Roast

(c) Deer: Venison

8. Yen: Currency

(a) Brass: Metal

(c) Paper: Book

9. Cricket: Football

(a) Solid: Liquid

(c) Prose: Poetry

(b) Harvesting: Agriculture

(d) Truck: Transport

(b) History: Histology

(d) Fossils: Palaeontology

(b) Chef: Kitchen

(d) Farmer: Village

(b) Insignia: Rank

(d) Honour: Status

(b) Hay: Stable

(d) Book: Publisher

(b) Applause: Audience

(d) Rehearsal: Performance

(b) Hen: Poultry

(d) Lamb: Veal

(b) Flower: Fragrance

(d) Karnataka: State

(b) Hockey: Ball

(d) Shoe: Gloves

10. Geology: Earth

(a) Architect: Building

(c) Aquarium: Fish

11. Carpenter: Furniture

(a) Teacher: Teaching

(c) Mason: Wall

12. Traveller: Destination

(a) Beggar: Donation

(c) Teacher: Education

13. Horns: Bull

(a) Mane: Lion

(c) Hoofs: Horse

14. Taxonomist: Classify

(a) Haggler: Bargain

(c) Kind: Alms

15. Horse: Equine

(a) Lion: Carnivorous

(c) Table: Furniture

16. Wife: Marriage

(a) Bank: Money

(c) Service: Qualification

17. Sprain: Fracture

(a) Cool : Cold

(c) Pneumonia: Fever

18. Ampere: Current

(a) Sound: Waves

(c) Distance: Kilometre

19. Eyes: Tears

(a) Sea: Water

(c) Heart: Astery

20. Bread: Flour

(a) Train: Wagon

(c) Road : Asphalt

21. Jupiter: Planet

(a) Sparrow: Bird

(c) Chilka: Lake

22. Kitchen: Bedroom

(a) Woman: Man

(c) Botany: Zoology

23. Buoy: Channel

(a) White line: High way

(c) Red light: Street

(b) Biology: Science

(d) Archaeology: Artifacts

(b) King: Empire

(d) Farmer: Agriculture

(b) Accident: Hospital

(d) Refugee: Shelter

(b) Antlers: Stag

(d) Wattles: Turkey

(b) Doctor: Medicine

(d) Engineer: Building

(b) Cat: Feline

(d) Dog: Vulpine

(b) Nationality: Citizenship

(d) Attendance: Register

(b) Accident: Death

(d) Fall: Slip

(b) Speed: Time

(d) Ohm: Resistance

(b) Volcano: Lava

(d) Hunger: Bread

(b) Car: Engine

(d) House: Wall

(b) Yamuna: River

(d) Everest: Peak

(b) Vegetable: Fruit

(d) Gas: Cylinder

(b) Light house: Ship

(d) Road map: Travel

24. Knowledge: Ignorance

(a) Cure: Health

(c) Breath: Suffocation

25. Dove: Peace

(a) Crow: Scavenge

(c) Lull: Storm

26. Calligraphy: Writing

(a) Music: Song

(c) Drama: Prose

27. Conciliatory: Friendliness

(a) Cache: Hide

(c) Timid: Bold

28. Wick: Candle

(a) Lead: Pencil

(c) Light: Darkness

29. Gypsy: Caravan

(a) Hare: Byre

(c) Monk: Temple

30. Plaintiff : Defendant

(a) Judge: Jury

(c) Attorney: Lawyer

(b) Conceal: Hide

(d) Construction: War

(b) Knife: Cut

(d) Pearl: Purity

(b) Lyric: Poem

(d) Chapter: Stanza

(b) Garrulous: Old

(d) Obvious: Explain

(b) Thread: Wool

(d) Quick: Rapid

(b) Knight: Mansion

(d) Convict: Cell

(b) Court: Law

(d) Injured: Accused

ANSWERS

1. (c)	2. (d)	3. (b)	4. (b)	5. (c)	6. (d)	7. (c)	8. (d)	9. (c)	10. (d)
11. (c)	12. (d)	13. (b)	14. (a)	15. (b)	16. (c)	17. (a)	18. (d)	19. (b)	20. (c)

21. (d) 22. (c) 23. (a) 24. (c) 25. (d) 26. (b) 27. (a) 28. (a) 29. (b) 30. (d)

Explanations

- 1. Magazine is a type of periodical. Likewise, pun is a type of joke.
- 2. The study of heart is called cardiology. Similarly, the study of fossils is palaeontology.
- 3. A doctor works in a hospital. Similarly, a chef works in a kitchen.
- 4. Flag is the symbol of nation. Similarly, insignia is the symbol of rank.
- 5. Coin is made in a mint. Similarly, wine is made in a brewery.
- 6. Infection is followed by illness. Similarly, rehearsal is followed by performance.
- 7. The flesh of sheep is called mutton. Similarly, the flesh of a deer is called venison.
- 8. Yen is a currency. Likewise, Karnataka is a state.
- 9. Both cricket and football belongs to the same class of sports. Similarly, both prose and poetry belong to the same class of literature.
- 10. Geology is the study of earth. Likewise, archaeology is the study of artifacts.
- 11. A carpenter makes furniture. Similarly, a mason builds a structure.
- 12. A traveller seeks destination. Similarly, a refugee seeks shelter.
- 13. A bull bears horns on its head. Likewise, a stag bears antlers on its head.
- 14. A taxonomist classifies and a haggler bargains.

- 15. Equine is a horse like animal. Similarly, feline is a cat-like animal.
- 16. The second is necessary to acquire the first.
- 17. The second is more intensive form of the first.
- 18. Ampere is the unit of current. Similarly, Ohm is the unit of resistance.
- 19. Tears come out of the eye. Similarly, lava comes out of a volcano.
- 20. Bread is made of flour. Similarly, road is made of asphalt.
- 21. Jupiter is the largest planet. Similarly, Everest is the highest peak.
- 22. Both are parts of the same. Kitchen and bedroom are the parts of house. Similarly, botany and zoology are the branches of science.
- 23. A buoy indicates channel or a path for a ship to follow, white line shows the path on a highway to be followed by vehicles.
- 24. One is the opposite of the other.

8. Hitler: Germany

(c) Tulsidas: India

(a) Shakespeare: England

- 25. Dove is a symbol of peace. Similarly, pearl is a symbol of purity.
- 26. Calligraphy is a type of writing. Similarly, lyric is a type of poem.
- 27. The words in both the pairs are synonyms of each other.
- 28. The uppermost working part of a candle is the wick. Likewise, the working part of a pencil is lead.
- 29. Gypsy stays in a caravan. Similarly, a knight stays in a mansion.
- 30. Injured is the plaintiff and the accused is the defendant.

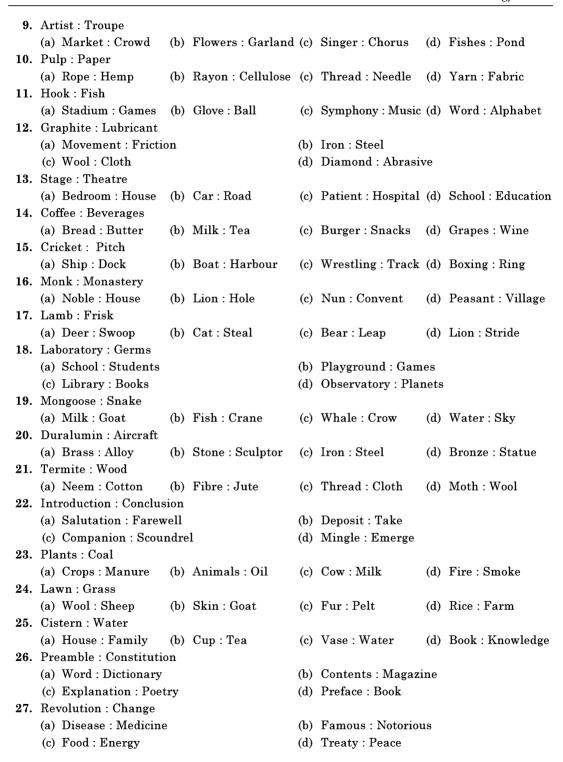
EXERCISE 6

Directions: The following questions consist of two words each that have a certain relationship to each other followed by four lettered pairs of words. Select the lettered pair that has the same relationship as the original pair of words.

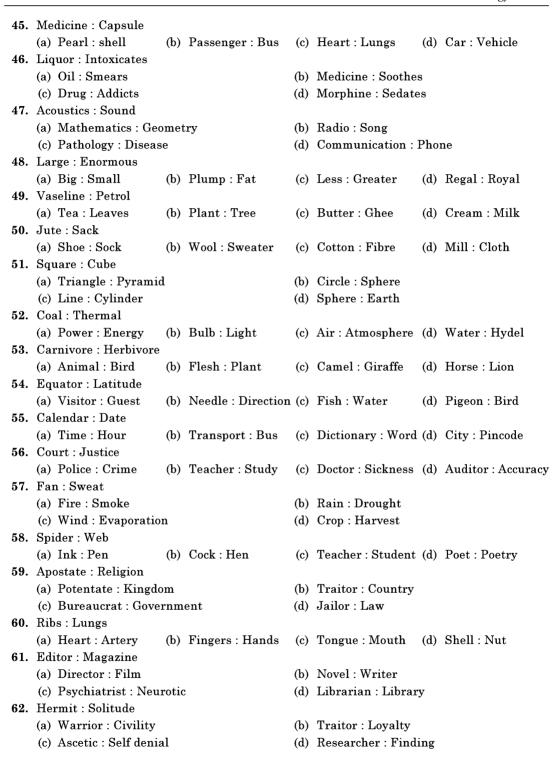
erat	ionsnip as the original	pan	oi woras.						
1.	Read : Legible								
	(a) Hear : Audible		(b)	Qualify : Eligible					
	(c) See : Illegible		(d)	Require : Admissible					
2.	Locks : Warble								
	(a) Crows : Cackle	(b)	Tiger : Yelp	(c)	Owls: Hoot	(d)	Camel : Bleat		
3.	Horse: Mare								
	(a) Duck : Geese	(b)	Dog: Puppy	(c)	Donkey : Pony	(d)	Fox: Vixen		
4.	Water: Thirst								
	(a) Ripe: Harvest	(b)	Book: Ignorance	(c)	Needle: Stitch	(d)	Rain : Drought		
5.	Iodine : Goitre								
	(a) Insuline : Diabetes	s		(b)	Mango : Anaemia				
	(c) Hormones: Haem	ophi	ilia	(d)	Fat : Obesity				
6.	Waiter : Tip								
	(a) Student: Marks		(b)	Worker : Bonus					
	(c) Employee: Wages		(d)	Clerk : Bribe					
7.	Kangaroo : Australia								
	(a) Whale: River		(b)	Elephant : Russia					
	(c) Penguin: Antartic	a		(d)	India : Peacock				
_									

(b) Mussolini: Italy

(d) Boris Yeltsin: Russia



28.	Coronation : Reign								
	(a) Vaccination: Imm	unit	.y	(b)	Sculptor : Statue				
	(c) Degree : Graduate		•		Summer : Rain				
29.	Leather : Milk			()					
	(a) Cat: Mouse	(b)	Curd : Bacteria	(c)	Fruit : Rubber	(b)	Sun : Moon		
30.	Loud : Cape	()		(0)		()			
	(a) House : Gate	(h)	Foot : Toe	(c)	Hand : Glove	(d)	Finger : Nails		
31	Sailor : Pirate	(~)	1000.100	(0)	Tiuliu . Giove	(α)	ingo: . i ano		
01.	(a) Police : Robbers	(h)	Lion : Lamb	(c)	Plant : Fungus	(d)	Major : Sepoy		
32	Pharaohs : Egypt	(~)	21011 / 241110	(0)	1 14417 / 1 441 9 46	(4)	1.1ajo1 (Sopo)		
52.	(a) Socrates : Greece				(b) Kings : India				
	(c) Imperator : Rome				Government : Sta	te			
33.	Basement : Attic			(4)					
00.	(a) Nadir : Zenith	(b)	Zenith : Apex	(c)	Zenith : Root	(d)	Apex : Pinnacle		
34.	Lok Sabha : Legislatu			(0)		(02)			
01.	(a) President: Executive				Minister : Meetin	ø			
	(c) Judge : Court				People : Election	•			
35.	Blister : Skin			(4)	1 copie : 21cccion				
	(a) Sore: Toe	(b)	Sty: Eye	(c)	Ball : Pitcher	(d)	Wound : Arm		
36.	Energy : Dissipate	(~)	20j · 2j c	(0)		(44)			
	(a) Power : Generator			(b)	Food : Hunger				
	(c) Money : Squander				Battery : Charge				
37.	Optimistic : Pessimist			()					
	(a) Difficult : Impossi			(b)	Study : Play				
	(c) Tolerating : Disgu		g		Export : Import				
38.	Acquire : Inherit		0	` '					
	(a) Profit : Loss			(b)	Learn : Discover				
	(c) Instinct : Habit				Hierarchial : Succession				
39.	Protagonist : Characte	er		` ´					
	(a) Earth: Moon		Termite : Insect	(c)	Lizard : Housefly	(d)	Whale : Fish		
40.	Bird : Wings								
	(a) Whale: Water	(b)	Dog : Lungs	(c)	Car : Wheel	(d)	Pen : Paper		
41.	War : Destruction								
	(a) Fire: Burn	(b)	Court : Justice	(c)	Water : Drown	(d)	Food : Hunger		
42 .	Elevation : Depression	1							
	(a) Pen : Notebook				Intelligence : Stuj	pidit	y		
	(c) School: Education	Į		(d)	Doctor : Medicine				
43.	Dawn : Morning								
	(a) Morning: Evening	S		(b)	Sun : Rain				
	(c) Lamp: Light			(d)	Autumn: Winter				
44.	Question: Question m	ark							
	(a) Remark : Colon			(b)	Aside: Parenthes	es			
	(c) Sentence : Period			(d)	Clause : Semicolo	n			



63. Taj Mahal : Agra

(a) Eiffel Tower: Paris(b) Algevia: Africa(c) Cotton: Ahmedabad(d) Turkey: Asia

64. Jews: Synagogue

(a) Parsis: Temple(b) Jains: Fire Temple(c) Buddhists: Pagoda(d) Hindus: Vedas

65. Script: Drama

(a) Article: Magazine (b) Chapter: Book (c) Lyrics: Note (d) Score: Symphony

(ANSWERS)										
1. (a)	2. (c)	3. (d)	4. (d)	5. (a)	6. (b)	7. (c)	8. (b)	9. (c)	10. (d)	
11. (b)	12. (d)	13. (a)	14. (c)	15. (d)	16. (c)	17. (b)	18. (d)	19. (b)	20. (d)	
21. (d)	22. (a)	23. (b)	24. (c)	25. (b)	26. (d)	27. (d)	28. (d)	29. (c)	30. (b)	
31. (c)	32. (b)	33. (a)	34. (a)	35. (b)	36. (c)	37. (d)	38. (d)	39. (b)	40. (c)	
41. (a)	42. (b)	43. (d)	44. (b)	45. (a)	46. (d)	47. (c)	48. (b)	49. (d)	50. (b)	
51. (a)	52. (d)	53. (b)	54. (d)	55. (c)	56. (d)	57. (b)	58. (d)	59. (b)	60. (d)	
61. (a)	62. (c)	63. (a)	64. (c)	65. (d)						

- 1. Legible means able to be read. Similarly, audible means able to be heard.
- 2. The second is the sound produced by the first.
- 3. The second is the female of the first.
- 4. The lack of water is thirst. Similarly, the lack of rain is drought.
- 5. The lack of first causes the second.
- 6. The second is the additional money given to the first for good service.
- 7. Kangaroo is the native of Australia. Similarly, penguin is the native of Antartica.
- 8. Hitler was a tyrant belonging to Germany. Similarly, Mussolini was a tyrant belonging to Italy.
- 9. A group of artists is called a troupe. Likewise, a group of singers is called chorus.
- 10. Pulp is used to make paper. Likewise, yarn is used to make fabric.
- 11. The first helps to catch the second.
- 12. Graphite is used as a lubricant. Similarly, diamond is used as an abrasive.
- 13. The first is a part of the second.
- 14. The first one is specific and the second is the class to which it belongs.
- 15. The game of cricket is played on a pitch. Similarly, the game of boxing is performed in a ring.
- 16. The second is the dwelling place of the first.
- 17. The second denotes the manner of walking of the first.
- 18. A laboratory is a place to study germs. Similarly, observatory is a place of studying planets.
- 19. The first is eaten by the second.
- 20. The first is used to make the second.
- 21. The first damages the second.
- 22. The given words are opposite to each other.
- 23. Plants on decay yield coal. Similarly, animals on decay produce oil.

- 24. The second grows on the first.
- **25.** The first is used to hold the second.
- 26. Preamble is the introduction to the constitution and mentions its main ideals and objectives. Similarly, preface is the introduction to a book.
- 27. The first brings the second.
- 28. The first is followed by the second.
- 29. Leather and milk are both obtained from animals. Similarly, fruit and rubber are both obtained from trees.
- 30. Cape is the projection of land. Similarly, toe is the projection of foot.
- 31. Both belong to the same species but first one is useful and the second one is harmful.
- 32. The rulers of Egypt were known as pharaohs. Similarly, the rulers of India were known as kings.
- 33. Nadir (lowest point) and zenith (highest point) can be compared to basement and attic respectively.
- 34. Lok Sabha is a part of the Legislature. Likewise, President is a part of the Executive.
- 35. Blister is an infection of the skin. Similarly, sty is an infection of the eye.
- **36.** 'Dissipate' and 'squander' are synonyms of 'waste'. Just as energy can be dissipated , money can be squandered.
- 37. The given words are opposite to each other.
- 38. Each pair consists of synonyms.
- 39. Protagonist is a character. Similarly, termite is an insect.
- 40. The second is used by the first for movement.
- 41. The first one causes the second.
- 42. The given words are opposites to each other.
- 43. Dawn is followed by morning. Similarly autumn is followed by winter.
- 44. The second is the symbol used at the end of first.
- 45. Medicine is contained in capsule. Similarly, pearl is contained in a shell.
- 46. The second is the effect of the first after consumption.
- 47. Acoustics is the science of sound. Similarly, pathology is the study of diseases.
- 48. Enormous is the extreme of large. Similarly, fat is the extreme of plump.
- 49. Vaseline is extracted from petrol. Similarly, cream is extracted from milk.
- 50. Sack is made up of jute. Similarly, sweater is made up of wool.
- 51. Each face of a cube is a square and each face of a pyramid is a triangle.
- **52.** Coal produces thermal energy. Similarly, water produces hydel energy.
- 53. Flesh eating creatures are carnivore and plant eating creatures are herbivore.
- 54. The first one is specific and the second one is the class to which the first belongs. Equator is a line of latitude; pigeon is a bird.
- 55. Just as calendar consists of dates in an organised way; dictionary consists of words in an organised way.
- 56. The duty of court is to dispense justice. The duty of an auditor is to provide accuracy in financial matters.
- **57.** The second is ceased due to the first.
- 58. As spider makes web, poet makes poetry.
- 59. The first rebels against the second.
- 60. Ribs are the bones, which protect the lungs Similarly, shell protects the nut.

- **61.** Just as all the articles of a magazine are interpreted by the editor, in the same way all the actions of film are interpreted by director.
- 62. A hermit leads a life of solitude. Likewise, an ascetic leads a life of self-denial.
- 63. The Taj Mahal is located in Agra. Similarly, Eiffel Tower is located in Paris.
- 64. The Jews worship in a synagogue. Likewise, the Buddhists worship in a pagoda.
- 65. Script is a written instruction of drama. Similarly, score is a written piece of symphony (type of music).

			EXERCI	SE	7		
1.	A sword is related to s	slauį	ghter in the same w	ay.	scalpel is related to		
	(a) murder	(b)	stab	(c)	surgery	(d)	chopping
2.	Ecology is related to e	nvir	onment in the same	e wa	ay <i>histology</i> is relate	ed t	0
	(a) fossils	(b)	history	(c)	tissues	(d)	hormones
3.	Auger is related to car	rpen	<i>ter</i> in the same way	, au	vl is related to		·
	(a) sculptor	(b)	$\operatorname{cobbler}$	(c)	chef	(d)	mason
4.	Sirius is related to sto	ır in	the same way cygn	us	is related to		.
	(a) constellation	(b)	asteroid	(c)	galaxy	(d)	meteor
5.	Island is related to ar	chip	elago in the same v	vay	chapter is related t	o	·
	(a) book	(b)	mystery	(c)	author	(d)	content
6.	Catalogue is related to	o lib	rary books in the sa	ame	way index is relate	d to)
	(a) chapters	(b)	books	(c)	preface	(d)	contents
7.	Bald is related to blor	ιd in	the same way bar	ren	is related to		·
	(a) vegetation	(b)	farm	(c)	fertile	(d)	inhabited
8.	Lion is related to prov	vl in					·
	(a) frisk	(b)	lumber	(c)	stride	(d)	bound
9.	Aflatoxin is related to	food	poisoning in the sai	me v	way histamind is re	late	d to
	(a) allergy	(b)	headache	(c)	anthrax	(d)	contamination
l 0.	Ballworm is related to	cot	ton in the same way	y gh	<i>undibug</i> is related	to_	•
	(a) wheat	(b)	rice	(c)	millet	(d)	tomato
l 1.	Naphthalene is relate	d to	woollen in the same	e wa	ay <i>antibiotics</i> is rela	ated	to
	(a) germs	(b)	immunity	(c)	diseases	(d)	\mathbf{body}
l 2.	Drama is related to so	ene	in the same way bo	ok i	is related to		·
	(a) story	(b)	page	(c)	chapter	(d)	author
l 3.	Tungsten is related to	fila	ment in the same w	vay	bronze is related to		
	(a) copper	(b)	ships	(c)	tin	(d)	ornaments
l 4.	Tobacco is related to r	ierve	es in the same way	alch	$hohol$ is related to $_$		•
	(a) liver	(b)	liquor	(c)	intoxication	(d)	head
l 5.	Man is related to show	ιt in	the same way crou	is t	related to		·
	(a) caw	(b)	chirp	(c)	mutter	(d)	\mathbf{mob}
l 6.	Gill is related to lame	llae	in the same way lu	ng:	is related to		.
	(a) ribs	(b)	trachea	(c)	alveoli	(d)	pharynx
l 7.	Dwell is related to der	nize	n in the same way	inh	<i>erit</i> is related to		
	(a) acquire	(b)	successor	(c)	outcast	(d)	heir

18.	Gardener is related to	o trowel in the same wa	ay seams	stress is related	to _	,
	(a) saw	(b) scissors	(c) sn	ieakers	(d)	crowbar
19.	Wince is related to po	ain in the same way pro	ostration	n is related to $_$		
	(a) discomfiture	(b) frustration	(c) su	ıbmissiveness	(d)	strained
20 .	Solicitous is related t	o concern in the same v	way verl	bose is related to		
	(a) tiredness	(b) wordiness	(c) sp	eech	(d)	deafness
21.	$Physiology \ {\rm is} \ {\rm related}$	to biology in the same	way me	taphysics is rela	ted	to
	(a) Physics	(b) Statistics	(c) M	athematics	(d)	Philosophy
22 .	Birds are related to a	wiary in the same way	bees are	e related to		·
	(a) aquarium	(b) hive	(c) br	rewery	(d)	apiary
23 .	Anthropology is relat	ed to man in the same	way An	thology is relate	d to	.
	(a) nature	(b) trees	(c) ap	oes	(d)	poems
24.	$Starvation \ {\rm is} \ {\rm related}$	to nutrition in the sam	e way <i>e</i> :	xhaustion is rela	ated	to
	(a) energy	(b) bravery	(c) fre	eshness	(d)	courage
25 .	Snowfall is related to	precipitation in the sa	me way	grotto is related	d to	
	(a) throat	(b) castle	(c) ca	ve	(d)	\mathbf{fort}
26 .	Condolence is related	to loss in the same wa	y congre	atulation is rela	ted t	to
	(a) praise	(b) achievement	(c) ac	cusation	(d)	reward
27 .	Lumberjack is related	d to <i>axe</i> in the same wa	y chef is	s related to		,
	(a) bow	(b) poker	(c) ch	isel	(d)	$\operatorname{colander}$
28.	Scrupulous is related	to <i>principles</i> in the same	me way	ethical is relate	d to	·
	(a) morals	()				
29 .	Annotate is related to	text in the same way o				.
	(a) novel	(b) law	(c) fil	m	(d)	photograph
30 .	Retirement is related	to $service$ as $dismissal$	is relate	ed to		
	(a) agreement	(b) communication	(c) en	${f nployment}$	(d)	adoption
31.	Betel is related to che	ew in the same way foot	tball is 1	related to		
	(a) play	` '				kick
32.		book in the same way ch				
		(b) ballet		_		_
33.	Book is related to ma	gazine in the same way				
	(a) journal	` '				
34.	Indolence is related t	o <i>work</i> in the same way				
	(a) observe	` ' -				act
35.		anium in the same way				·
	(a) box	(b) oyster	(c) sa		` '	shore
36.		ase in the same way mi				
	(a) drink	(b) ghee	(c) cu			protein
37.		lls in the same way mu				
	(a) carbohydrates	(b) minerals		tamins		proteins
38.		atient in the same way				
	(a) customer	(b) accused	(c) m	agistrate	(d)	client

39	Chef is related to restaurant	in the same way druggist is related to	

- (a) medicine
- (b) pharmacy
- (c) store
- (d) chemist
- **40.** Mirror is related to reflection in the same way water is related to ______
 - (a) conduction
- (b) dispersion
- (c) immersion
- (d) refraction

	ANSWERS											
1. (c)	2. (c)	3. (b)	4. (a)	5. (a)	6. (d)	7. (c)	8. (b)	9. (a)	10. (b)			
11. (d)	12. (c)	13. (d)	14. (a)	15. (d)	16. (c)	17. (d)	18. (b)	19. (c)	20. (b)			
21. (d)	22. (d)	23. (d)	24. (a)	25. (c)	26. (b)	27. (d)	28. (a)	29. (d)	30. (c)			
31. (d)	32. (b)	33. (a)	34. (b)	35. (b)	36. (c)	37. (d)	38. (d)	39. (b)	40. (d)			

- 1. The second denotes the purpose for which the first is used.
- 2. Ecology deals with the study of environment. Similarly, histology deals with the study of tissues.
- **3.** The first is the tool used by the second.
- 4. Sirius is a star and cygnus is a constellation.
- 5. The first is a part of the second.
- 6. Catalogue is an arranged list to find the names of the library books. Similarly, index is an arranged list of contents.
- 7. The given words are opposite to each other.
- 8. The second is the manner of walking of the first.
- 9. The first causes the second.
- 10. The first damages the second.
- 11. The first is used to protect the second from attack by germs and insects.
- 12. The second is a unit of the first.
- 13. The first is used to make the second. Thus tungsten is used to make filament and bronze is used to make ornaments.
- 14. The consumption of the first affects the second.
- 15. The second is the noisy sound made by the first.
- 16. The second is the oxygen absorbing part of the first.
- 17. When denized dwells, he occupies the place. When heir inherits, he occupies the throne.
- 18. The second is the tool used by the first.
- 19. The first is the sign of the second.
- 20. A solicitous exhibits concern and a verbose exhibits wordiness.
- 21. Physiology is a branch of biology. Similarly, metaphysics is a branch of philosophy.
- 22. Birds are reared in an aviary. Similarly, bees are kept and reared in an apiary.
- 23. Anthropology deals with the study of man. Similarly, anthology deals with collection of poems.
- 24. A lack of nutrition is starvation. Similarly, a lack of energy is exhaustion.
- 25. Snowfall is a type of precipitation. Similarly, grotto is a type of cave.
- 26. Others offer condolence in a state of loss and congratulations when one make an achievement.
- 27. The second is the tool used by the first.

- 28. When one abides by the second, he is said to be the first by nature.
- 29. The first is a comment on the second.
- **30.** The first interrupts the second.
- 31. The first is the object and the second is the action to be performed on it.
- **32.** The first composes the second.
- 33. The second contains smaller articles of the same nature as the first.
- 34. The given words are antonyms to each other.
- **35.** The first is enclosed by the second.
- **36.** The first is used to prepare the second.
- 37. Hooke discovered the cells. Similarly, Mulder discovered the proteins.
- 38. The first works for the second.
- 39. The second is the working place for the first.
- 40. Light rays falling on mirror undergo reflection and those falling on water undergo refraction.

	EXERCISE 8									
1.	Trigonometry is relate	d to <i>triangles</i> in the sam	ne way <i>mensuration</i> is r	elate	d to					
	(a) geometry	(b) circles	(c) areas		polygons					
2.	Lotus is related to cut	icle in the same way fis	h is related to		_•					
	(a) scales	(b) gills	(c) tail		wings					
3.	Tapeworm is related t	o taeniasis in the same	way plasmodium is re	elated	to					
	(a) malaria	(b) constipation	(c) diphtheria	(d)	diarrhoea					
4.	Leaf is related to sap i	in the same way bone is	s related to							
	(a) fluid	(b) blood	(c) marrow	(d)	calcium					
5 .	Chlorophyll is related	to chloroplast in the sa	ame way <i>vulture</i> is rela	ated to	0					
	(a) flesh	(b) wings	(c) air	(d)	bird					
6.	Waves are related to a	air in the same way $ripp$	oles are related to							
	(a) wind	(b) water	(c) tree	(d)	bud					
7.	Rung is related to lade	der in the same way tw	ig is related to							
	(a) leaf	(b) flower	(c) tree	(d)	bud					
8.	Circle is related to circ	cumference in the same	$way \ square \ is \ related$	to	·					
	(a) area	(b) volume	(c) diagonal	(d) j	perimeter					
9.	Grain is related to gra	anary in the same way o	curios is related to		.					
	(a) archives	(b) museum	(c) library	(d)	Z 00					
10.	Cat is related to kitten	in the same way fish i	s related to	·						
	(a) fry	(b) fawn	(c) fin	(d) :	foal					
11.	Orthopaedist is related	d to bones in the same v	way <i>chiropodist</i> is rela	ted to)					
	(a) nails	(b) sounds	(c) feet	(d)	heart					
12 .	Formula is related to	$constituent \ { m in \ the \ same}$	way equation is relate	ed to .						
	(a) numbers	(b) variables	(c) term	(d)	constant					
13.	Dog is related to kenn	el in the same way fowl	<i>l</i> is related to							
	(a) barn	(b) cottage	(c) nest	(d)	coop					
14.	Honey is related to wa	ux in ther same way mil	lk is related to							
	(a) cow	(b) leather	(c) eggs	(d)	butter					

15.	Helm is related to rud	lder in the same way br	rain is related to		
	(a) heart	(b) ribs	(c) limbs	(d)	body
16.	Crumb is related to be	read in the same way m	norsel is related to		.
	(a) fruit	(b) biscuit	(c) food	(d)	cake
17.	Door is related to ban	g in the same way <i>chai</i>	n is related to		_·
	(a) thunder	(b) clinch	(c) tinkle	(d)	clank
18.	Hong Kong is related	to <i>China</i> in the same w	yay <i>Vatican</i> is related t	0	
	(a) Canada	(b) Mexico	(c) North America	(d)	Rome
19.	Horse is related to haz	y in the same way cow			
	(a) leaves	(b) fodder			
20.	Earth is related to axi	is in the same way whe	el is related to		 ·
	(a) tyre	(b) car	` '	` '	
21.		oull in the same way mo			
	_	(b) separation			-
22 .		to composer in the same			
			(c) singer		
23.		ught in the same way as			
2.4			(c) farm	()	fodder
24.		t in the same way fist i			1 1
0.5	(a) blow	(b) hand	. , .	` ′	
25.		to <i>India</i> in the same wa			
96	- · · ·	(b) Canada	() 001		
26.		wages in the same way			
97	(a) loan	(b) interest	• •		
21.		e in the same way wate			
98	(a) strait	(b) lagoon	, ,		
40.	(a) light	orain in the same way a (b) glare	c) stare		
29		loan in the same way b	• •	` ′	· ·
20.	(a) alms		(c) money		
30.	` '	heatre in the same way		, ,	•
00.	(a) hotel		(c) feast		
31.	` '	ge in the same way ma			
	(a) house		(c) room		
32 .	` '	is related to plants in			
	to	•	·		
	(a) poultry	(b) rubber	(c) animals	(d)	forests
33.	${\it Charcoal}$ is related to	wood in the same way	coke is related to		.
	(a) plastic	(b) graphite	(c) soot	(d)	coal
34.	Joule is related to ene	ergy in the same way Po	ascal is related to		·
	(a) volume	(b) pressure	(c) density	(d)	purity
35.	Kindle is related to bu	urn in the same way an			·
	(a) annoyed	(b) determined	(c) resentful	(d)	furious

36.	6. Ostrich is related to antelope in the same way egret is related to						
	(a) cow	(b) buffalo	(c) camel	(d)	zebra		
37.	Blood is related to circ	culation in the same wa	y <i>hormone</i> is related t	o			
	(a) egestion	(b) control	(c) co-ordination	(d)	digestion		
38.	Man is related to arm	s in the same way cockr	roach is related to		·		
	(a) wings	(b) pseudopodia	(c) legs	(d)	antennae		
39.	Transistor is related to	o <i>radio</i> in the same way	television is related t	o			
	(a) entertainment	(b) cinema	(c) video	(d)	cassette		
40.	cobra is related to snake in the same way leopard is related to						
	(a) tiger	(b) lion	(c) cat	(d)	zebra		

(ANSWERS)

1. (c)	2. (a)	3. (a)	4. (c)	5. (d)	6. (b)	7. (c)	8. (d)	9. (b)	10. (a)
11. (c)	12. (c)	13. (d)	14. (b)	15. (d)	16. (c)	17. (d)	18. (d)	19. (b)	20. (d)
21. (c)	22. (a)	23. (b)	24. (d)	25. (c)	26. (d)	27. (c)	28. (b)	29. (b)	30. (a)
31. (d)	32. (c)	33. (d)	34. (b)	35. (d)	36. (b)	37. (c)	38. (d)	39. (b)	40. (c)

- 1. Trigonometry is the study of triangles. Similarly, mensuration is the study of areas.
- 2. The second is the means to protect the body of the first from water.
- 3. The second is the disease caused by the first.
- 4. The second is the fluid contained in the first.
- **5.** Chlorophyll is a type of chloroplast. Similarly, vulture is a type of bird.
- 6. Waves travel in air; ripples travel in water.
- 7. The first is a part of the second.
- 8. The second is the measure of boundary of the first.
- 9. Grain is stored in a granary. Similarly, curios (rare things to be collected) are kept in a museum.
- 10. The second is the young one of the first.
- 11. The first is a specialist of the second.
- 12. The second is a unit of the first.
- 13. The second is the living place of the first.
- 14. Honey and wax are both obtained from the same organism i.e. bee. Similarly, milk and leather both are obtained from buffalo.
- 15. Helm regulates the rudder and brain regulates the body.
- 16. The first is a part of the second.
- 17. The second is the sound made by the first.
- 18. Hong Kong is a city of China. Similarly, Vatican is a city of Rome.
- 19. The second is the food for the first.
- 20. The first rotates about the second.
- 21. The first draws things nearer through the second.
- 22. The first is prepared by the second.
- 23. The bull is a draught animal (beast of burden) and the cow is a milch animal (milk yielding).

- 24. The hold of tooth is called grit and the hold of the fist is called clench.
- 25. Charminar is situated in India. Similarly, Sphinx is a monument of Egypt.
- 26. The first earns in the form of the second.
- 27. The cape is the land projected into the water and the bay is the portion of water body projected into land.
- 28. The first provides protection from the second.
- 29. The first gets money in the form of the second.
- **30.** The second is the place where the first is held.
- **31.** The first is locked up in the second.
- 32. The first is the name given to increase in the production of the second.
- **33.** The first is obtained from the second.
- 34. Joule is the unit of energy and Pascal is the unit of pressure.
- **35.** The second is the larger intensity of the first.
- 36. Both live together to derive benefits from each other.
- **37.** The second is the function of the first.
- 38. The first uses the second for the purpose of holding.
- **39.** The second is the enlarged form of the first.
- 40. The first belongs to the family of the second.

EXERCISE 9

Directions: In each of the following questions, the first two words (given in italics) have a definite relationship. Choose one word out of the given four alternatives which will fill in the blank space and show the same relationship with the third word as between the first two.

1.	Hygrometer is to humidity as sphygmomanometer is to								
	(a) pressure	(b)	blood pressure	(c)	precipitation	(d)	heart beat		
2.	Steel is to $Bokaro$ as h	osie	<i>ry</i> is to	<u></u> .					
	(a) Chennai	(b)	Patna	(c)	Vishakhapatnam	(d)	Ludhiana		
3.	Milk is to water as ghe	e is	to						
	(a) vanaspati	(b)	mustard oil	(c)	argemome	(d)	cream		
4.	${\it Insulin} \ {\rm is} \ {\rm to} \ {\it hormone}$	as ti	ypsin is to		<u></u>				
	(a) juice	(b)	liver	(c)	enzyme	(d)	digestion		
5.	6. Ploughing is to aeration as manuring is to								
	(a) fertile	(b)	replenishment	(c)	earthing	(d)	agriculture		
6.	Infestation is to food a	s in	fection is to		<u></u>				
	(a) germs	(b)	diseases	(c)	body	(d)	microbes		
7.	Book is to publisher as	s fili	<i>n</i> is to						
	(a) writer	(b)	editor	(c)	director	(d)	producer		
8.	Bicycle is to pedal as b	oat	is to	.•					
	(a) steering	(b)	water	(c)	oar	(d)	sail		
9.	Latex is to rubber as fe	lax i	s to						
	(a) linen	(b)	wool	(c)	jute	(d)	cotton		
10.	Cattle is to fodder as f	ish i	s to						
	(a) hay	(b)	insects	(c)	feed	(d)	plankton		

11	Algae is to water as virus is to				
11.			surroundings	(4)	soil
12	Insomnia is to lead as minamata is to			(u)	5011
12.	(a) tobacco (b) mercury			(4)	chromium
13	Orange is to peel as tooth is to		arconor	(4)	ciiroiiituii
10.	(a) gums (b) clove		enamel	(A)	joints
14	Hear is to deaf as speak is to		chamer	(u)	joinus
	(a) quiet (b) silent		mumb	(A)	dumb
15	Exercise is to obesity as water is to			(4)	damo
10.		(c)		(d)	purity
16	Food is to fad as religion is to		di iiik	(4)	pullty
10.	(a) crucification (b) notion		superstition	(b)	mythology
17	Sulphur is to vulcanisation as chlorine		_	(4)	my unology
	(a) extraction (b) bleaching			(b)	allotropy
18.	Magnalium is to aluminium as brass is			(4)	шин
				(b)	copper
19.	Infrared is to heat as ultraviolet is to _			(4)	copper
	(a) cancer (b) blisters			(d)	ozone
20.	Zinc is to galvanisation as nickel is to			(/	
	(a) aircraft (b) corrosion			(d)	filament
21.	Memory is to amnesia as movement is t			(/	
	(a) lubrication (b) lethargy			(d)	hermit
22.	Liquid is to fluidity as comedian is to _			` '	
	(a) ridicule (b) humour			(d)	companion
23.	Kilometre is to distance as poundal is t			` '	•
	(a) density (b) acceleration			(d)	force
24.	Truthfulness is to liar as loyalty is to _			` '	
	(a) worker (b) traitor			(d)	faithful
25 .	Preface is to book as overture is to		_	, ,	
	-		novel	(d)	symphony
26.	Aluminium is to bauxite as iron is to _		.		
	(a) pyrite (b) magnesite			(d)	haematite
27 .	Tempest is to storm as slim is to				
			slender	(d)	beautiful
28.	Amorphousness is to definition as lass	<i>itude</i> is to			
	(a) energy (b) awareness		uniformity	(d)	companionship
29.	Tiff is to battle as frugal is to	·			
	(a) sprint (b) vague	(c)	miserly	(d)	vital
30.	Exculpate is to acquit as precise is to _				
	(a) concise (b) conceal	(c)	brief	(d)	particular
31.	Burma is to Pagodas as Pakistan is to				
	(a) Rivers (b) Canals	(c)	Agriculture	(d)	Dams
32.	Bhakra is to Sutlej as Aswan is to				
	(a) Indus (b) Damodar	(c)	Volga	(d)	Nile

33.	Sparrow is to seed as	silkworm is to				
	(a) silk	(b) maple	(c)	mulberry	(d)	pune
34.	Pineapple is to jelly a	s tomato is to				
	(a) jam	(b) pury	(c)	squash	(d)	pickles
35.	Aseel is to poultry as	salmon is to				
	(a) cow	(b) camel	(c)	fish	(d)	horse

				ANSV	VERS				
1. (b)	2. (d)	3. (a)	4. (c)	5. (b)	6. (c)	7. (d)	8. (c)	9. (a)	10. (d)
11. (b)	12. (b)	13. (c)	14. (d)	15. (a)	16. (c)	17. (b)	18. (d)	19. (a)	20. (c)
21. (c)	22. (b)	23. (d)	24. (b)	25. (a)	26. (d)	27 . (c)	28. (a)	29. (c)	30. (d)
31. (b)	32. (d)	33. (c)	34. (b)	35. (c)					

- 1. The first is an instrument used to measure the second.
- 2. Bokaro is famous for steel industry and Ludhiana is famous for hosiery works.
- 3. The first is adulterated by using the second.
- **4.** The second denotes the class to which the first belongs. Thus, insulin is a hormone and *trypsin* is an enzyme.
- 5. Ploughing is done for the aeration of soil and manuring is done for the replenishment of soil.
- 6. The contamination of food by germs is called infestation. Similarly, attack on body by germs is called infection.
- 7. The production of the first is done by the second.
- 8. The second is the tool which is acted upon to move the first.
- **9.** The first is the raw material used to obtain the second.
- 10. The second is the food eaten by the first.
- 11. The second is the dwelling place for the first.
- 12. The poisoning by the second causes the first.
- 13. The second is the protective covering over the first.
- 14. One who cannot hear is deaf. Similarly, one who cannot speak is dumb.
- 15. The first eliminates the second.
- 16. The second is the name given to wrong notions about the first.
- 17. Sulphur is used for vulcanisation of rubber. Likewise, chlorine is used for bleaching.
- 18. Magnalium is an ore of aluminium and brass is an ore of copper.
- 19. The second is the effect produced by the first.
- 20. The second is the purpose for which first is used.
- 21. The lack of memory is amnesia and lack of movement is paralysis.
- 22. The second is the defining characteristic of the first.
- 23. Kilometre is a unit of distance and poundal is a unit of force.
- 24. The lack of first is the defining characteristic of the second.
- 25. The first is an opening comment on the second.
- **26.** The second is the ore used for extraction of the first.

- 27. The first is of the higher intensity than the second.
- 28. The given words are opposite to each other.
- 29. The second is of higher intensity than the first.
- 30. The given words are synonyms of each other.
- 31. Burma is famous for pagodas and Pakistan is famous for canals.
- 32. Bhakra is a dam situated on the Sutlej river. Similarly, Aswan is a dam situated on Nile river.
- 33. The first feeds on the second.
- **34.** The first is preserved in the form of the second.
- 35. Aseel is a breed of poultry and salmon is a breed of fish.

Classification

Classification means 'to assort the items of a given group on the basis of a certain common quality they possess and then spot the stranger out'.

In this test, a group of five items will be given, out of which four are similar to one another in the same manner and one is different. You have to find out this item which does not fit into the group.

Directions: Choose the word which is least like the other words in the group.

Directions. Choose the word which is least like the other words in the group

EXAMPLE 1 (a) Copper (b) Tin (c) Brass (d) Platinum (e) Zinc

Solution Here, all except brass are metal while brass is an alloy. Hence the answer is (c). **EXAMPLE 2** (a) Calf (b) Cub (c) Piglet (d) Duckling (e) Hireling

Solution Here, all except Hireling, are young ones of animals. Hence the answer is (e).

EXAMPLE 3 (a) Curd (b) Butter (c) Oil (d) Cheese (e) Cream

Solution Here, all except, oil are products obtained from milk. Hence the answer is (c).

EXAMPLE 4 (a) Magnalium (b) Germanium (c) Duralumin

(d) Bronze (e) Brass

Solution All, except germanium, are alloys, while germanium is a metal. So, the answer is (b).

EXAMPLE 5 (a) Garnet (b) Ruby (c) Graphite (d) Emerald (e) Topaz **Solution** All, except graphite, are precious stones. So the answer is (c).

EXERCISES

Directions: In each of the following questions, five words are given, out of which four are alike in some manner and one is different. Choose the odd one out.

1. (a) Nephrology (b) Entomology (c) Astrology (d) Mycology (e) Pathology (2. (a) Whale (b) Dolphin (c) Shark (d) Cod (e) Starfish

3. (a) Indigo (b) Orange (c) Yellow (d) Pink (e) Green

4.	(a)	Tarapur	(b)	Kota	(c)	Kalpakkam	(d)	Paradeep	(e)	Narora
5.	(a)	Brick	(b)	Heart	(c)	Diamond	(d)	Spade	(e)	Club
6.	(a)	Cataract	(b)	Hypermetropia	(c)	Trachoma	(d)	Eczema	(e)	Glaucoma
7.	(a)	Radium	(b)	Thorium	(c)	Sodium	(d)	Polonium	(e)	Uranium
8.	(a)	Bishop	(b)	Knight	(c)	Pawn	(d)	Rook	(e)	Jockey
9.	(a)	Mongolia	(b)	China	(c)	Burma	(d)	Afghanistan	(e)	Bangladesh
10.	(a)	Wise	(b)	Gentle	(c)	Honest	(d)	Rude	(e)	Arrogance
11.	(a)	Pituitary	(b)	Pancreas	(c)	Thalamus	(d)	Adrenal	(e)	Testis
12.	(a)	Nun	(b)	Knight	(c)	Monk	(d)	Priest	(e)	Padre
13.	(a)	Granite	(b)	Lignite	(c)	Peat	(d)	Anthracite	(e)	Bituminous
14.	(a)	Gasoline	(b)	Methane	(c)	Asphalt	(d)	Paraffin wax	(e)	Diesel
15.	(a)	Galileo	(b)	Coppernicus	(c)	Columbus	(d)	Bhaskara	(e)	Aryabhatta
16.	(a)	Cricket	(b)	Baseball	(c)	Football	(d)	Billiards	(e)	Badminton
17.	(a)	Rigveda	(b)	Yajurveda	(c)	At harva ved a	(d)	Ayurveda	(e)	Samaveda
18.	(a)	Producer	(b)	Director	(c)	Investor	(d)	Financier	(e)	${\bf Entrepreneur}$
19.	(a)	Flute	(b)	Guitar	(c)	Sitar	(d)	Violin	(e)	Veena
20.	(a)	Kiwi	(b)	Eagle	(c)	Emu	(d)	Penguin	(e)	Ostrich
21.	(a)	Tortoise	(b)	Snail	(c)	Turtle	(d)	Spider	(e)	Oyster
22 .	(a)	Epicentre	(b)	Seismology	(c)	Focus	(d)	Crater	(e)	Richter Scale
23.	(a)	Arrow	(b)	Missile	(c)	Sword	(d)	Bullet	(e)	Spear
24.	(a)	Nerves	(b)	Auricle	(c)	Artery	(d)	Valve	(e)	Aorta
		Konark	(b)	Madurai	(c)	Ellora	(d)	Khajuraho	(e)	Dilwara
26.	(a)	Akbar	(b)	Jahangir	(c)	Shah Jahan	(d)	Vikramaditya	(e)	Aurangazeb
27 .	(a)	Manipur	(b)	Sikkim	(c)	Maharashtra	(d)	Haryana	(e)	Lakshadweep
28.	(a)	Sial	(b)	Mantle	(c)	Core	(d)	Sima	(e)	Pengia
29 .	(a)	Turtle	(b)	Lamb	(c)	Colt	(d)	Bitch	(e)	Farrow
30.	(a)	Mettur	(b)	Aswan	(c)	Hirakund	(d)	Sutlej	(e)	Pong
					AN	SWERS				
-		- / \					_	/ \	_	

1. (c)	2. (a)	3. (d)	4. (d)	5. (a)	6. (d)	7. (c)	8. (e)	9. (a)	10. (e)
11. (c)	12. (b)	13. (a)	14. (b)	15. (c)	16. (d)	17. (d)	18. (b)	19. (a)	20. (b)
21. (d)	22. (d)	23. (c)	24. (a)	25. (c)	26. (d)	27. (e)	28. (e)	29. (d)	30. (d)

- 1. All except astrology are concerned with biology.
- 2. All except whale belong to the family of fishes, while whale is a mammal.
- 3. All except pink are colours seen in a rainbow.
- 4. All except Paradeep are atomic power stations, whereas Paradeep is a port.
- 5. All except brick are suits of cards.
- 6. All except eczema are eye infections, whereas eczema is a skin infection.
- 7. All except sodium are radio isotopes, whereas sodium is a metal.
- 8. All except jockey are chessmen, whereas jockey is a professional horse rider.

- 9. All except Mongolia are neighbouring countries of India.
- 10. All except arrogance are adjectives, whereas arrogance is a noun.
- 11. All except thalamus are hormone secreting glands.
- 12. All except knight are religious persons, whereas knight is a warrior.
- 13. All except granite are different types of coal, whereas granite is a rock.
- 14. All except methane are products obtained from petroleum.
- 15. All except Columbus were astronomers, whereas Columbus was an explorer.
- 16. All except billiards are outdoor games.
- 17. All except Ayurveda are names of holy scriptures, the four vedas, Ayurveda is a branch of medicine.
- 18. All except director spend money.
- 19. All except flute are string instruments.
- 20. All except eagle are flightless birds.
- 21. All except spider have hard protective sheels.
- 22. All except crater are terms associated with the earthquake.
- 23. All except sword strike the target at a distance.
- 24. All except nerves are parts of the heart.
- 25. All except Ellora are famous for temples whereas Ellora is famous for caves.
- 26. All except Vikramaditya were Mughal rulers.
- 27. All except Lakshadweep are states of India, whereas Lakshadweep is a Union Territory.
- 28. All except pengia are layers of earth.
- 29. All except bitch are young ones of animals, whereas bitch is a female dog.
- 30. All except Sutlej are dams, whereas Sutlej is a river.

Series Completion

This chapter deals with the questions in which a series of numbers or alphabetical letters is given, which are called *terms of the series*. These terms follow a certain pattern. You have to recognize this pattern and either complete the given series with the most suitable alternative or find the wrong term in the series.

TYPE 1: NUMBER SERIES

Case I: Completing the Given Series

EXAMPLE 1 Tick the number that will come next in the following sequence:

4, 6, 12, 14, 28, 30, (...)

EXAMPLE 2 Complete the series: 4, -8, 16, -32, 64, (...)

Solution Each number in the series is got by multiplying the preceding number by -2. Missing term = $64 \times (-2) = -128$

Hence, the answer is (b).

EXAMPLE 3 Which number would replace the question mark in the series 2, 7, 14, 23, ?, 47.

Solution The given sequence is +5, +7, +9, ... i.e. 2+5=7; 7+7=14; 14+9=23. \therefore Missing number =23+11=34. Hence, the answer is (b).

EXERCISE 1

Directions: In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and fill in the blank spaces.

oran	k spaces.						
1.	2, 6, 11, 17, (), 32						[CBI Exam]
	(a) 22	(b)	23	(c)	24	(d)	28
2.	3, 10, 20, 33, 49, 68, (.)				[Ho	tel Management]
	(a) 75	(b)	85	(c)	90	(d)	91
3.	3, 7, 15, 31, 63, ()						
	(a) 92	(b)	127	(c)	115	(d)	131
4.	6, 24, 12, (), 18, 8, 2	4					[Railways]
	(a) 4	(b)	8	(c)	16	(d)	6
5 .	212, 179, 146, 113, ()					[Railways]
	(a) 91	(b)	79	(c)	112	(d)	80
6.	2, 6, 3, 4, 20, 5, 6, ()	, 7					[Railways]
	(a) 25	(b)	30	(c)	42	(d)	28
7.	2, 5, 9, 19, 37, ()						[Railways]
	(a) 76	(b)	75	(c)	74	(d)	72
8.	2, 6, 12, 20, 30, 42, 56	, ())				[Railways]
	(a) 60	(b)	64	(c)	70	(d)	72
9.	8, 24, 12, 36, 18, 54, (.)					
	(a) 27	(b)	68	(c)	72	(d)	108
10.	165, 195, 255, 285, 34	5, (.)				
	(a) 375	` '	420	(c)	435	(d)	390
11.	71, 76, 69, 74, 67, 72,	()					
	(a) 65	(b)	76	(c)	77	(d)	80
12.	9, 12, 11, 14, 13, (),	15					
	(a) 12	(b)	16	(c)	10	(d)	17
13.	3, 15, 35, (), 99, 143						
	(a) 48	(b)	63	(c)	80	(d)	95
14.	3, 10, 20, 33, 49, 68, (.)					
	(a) 75	(b)	85	(c)	90	(d)	91
15.	1, 3, 4, 8, 15, 27, ()						
	(a) 37	(b)	44	(c)	50	(d)	55
16.	66, 36, 18, ()						
	(a) 9	(b)	3	(c)	6	(d)	8
17.	11, 13, 17, 19, 23, 25,	()					
	(a) 25	(b)	27	(c)	29	(d)	31
18.	2, 4, 7, 11, 16, ()						
	(a) 18	(b)	20	(c)	22	(d)	25
19.	0, 2, 6, (), 20, 30, 42						
	(a) 8	(b)	10	(c)	12	(d)	14

20.	5, 16, 13, 26, 29, 58, 6	1, (.)				
	(a) 122	(b)	125	(c)	128	(d)	64
21.	2, 9, 28, 65, 126, ()						
	(a) 137	(b)	223	(c)	217	(d)	199
22 .	4, 9, 13, 22, 35, ()						
	(a) 57	(b)	70	(c)	63	(d)	75
23.	1, 8, 27, 64, 125, 216,	()					
	(a) 354	(b)	343	(c)	392	(d)	245
24.	1, 2, 3, 6, 9, 18, (), 5	4					
	(a) 18	(b)	36	(c)	81	(d)	27
25.	11, 13, 17, 19, 23, 29,	31, 3	37, 41, ()				
	(a) 43	(b)	47	(c)	51	(d)	53
26.	2, 5, 11, 23, 47, ()						
	(a) 49	(b)	52	(c)	95	(d)	106
27 .	4, 9, 5, 12, 7, 15, 8, (.), 10					
	(a) 25	(b)	18	(c)	21	(d)	24
28.	10, 5, 13, 10, 16, 20, 19	9, (.)				
	(a) 22	(b)	23	(c)	38	(d)	40
29.	2, 6, 12, 20, 30, 42, 56,	, ())				
	(a) 60	(b)	64	(c)	70	(d)	72
30.	2, 3, 5, 7, 11, (), 17						
	(a) 12	(b)	13	(c)	14	(d)	15

ANSWERS

1. (c)	2. (c)	3. (b)	4. (c)	5. (d)	6. (c)	7. (b)	8. (d)	9. (a)	10. (c)
11. (a)	12. (b)	13. (b)	14. (c)	15. (c)	16. (d)	17. (c)	18. (c)	19. (c)	20. (a)
21. (c)	22. (a)	23. (b)	24. (d)	25. (a)	26. (c)	27. (b)	28. (d)	29. (d)	30. (b)

- 1. The sequence is +4, +5, +6 ...
 - \therefore Missing term = 17 + 7 = 24.
- **2.** The sequence is +7, +10, +13, +16, +19 ...
 - \therefore Missing number = 68 + 22 = 90
- 3. Each number in the series is the preceding number multiplied by 2 and then increased by 1. Thus $(3 \times 2) + 1 = 7$; $(7 \times 2) + 1 = 15$; $(15 \times 2) + 1 = 31$ and so on.
- 4. The given sequence is a combination of two series 6, 12, 18, 24, and 24, (...), 8. The first series consists of consecutive multiples of 6 and the second series consists of multiples of 8. Thus, the missing number is a multiple of 8 which lies between 8 and 24, which is 16.
- 5. 33 is subtracted from each term of the series to obtain the next term.
 - :. Missing term = 113 33 = 80
- 6. The arrangement of series is as follows: $2 \times 3 = 6$; $4 \times 5 = 20$, and so on. Here from each three consecutive term, middle term = product of two end terms. So the missing number = $6 \times 7 = 42$.

- 7. Here, the numbers in the even position are equal to the numbers in the odd position multiplied by 2 and added by 1. The numbers in the odd positions are got by multiplying numbers in the even position by 2 and subtracting 1 from it. So the answer is $37 \times 2 + 1 = 75$.
- 8. The sequence is 1×2 , 2×3 , 3×4 , 4×5 , 5×6 , 6×7 , 7×8 .
 - \therefore Next number = $8 \times 9 = 72$.
- 9. This series is got by alternatively multiplying by 3 and dividing by 2.

So,
$$8 \times 3 = 24$$
; $24 \div 2 = 12$, $12 \times 3 = 36$; $36 \div 2 = 18$ and so on.

- \therefore The missing term = $54 \div 2 = 27$.
- 10. This series is a multiple of consecutive prime numbers.

```
i.e. 15 \times 11, 15 \times 13, 15 \times 17, 15 \times 19, 15 \times 23.
```

- \therefore The missing term = $15 \times 29 = 435$.
- 11. Here the series is got by addition of 5 and subtraction of 7 to terms alternately.

Here,
$$71 + 5 = 76$$
; $76 - 7 = 69$; $69 + 5 = 74$; $74 - 7 = 67$ and so on.

So the missing term = 72 - 7 = 65.

12. Alternately, add 3 and subtract = 1.

So
$$9 + 3 = 12$$
; $12 - 1 = 11$; $11 + 3 = 14$; $14 - 1 = 13$.

So the missing term = 13 + 3 = 16.

13. The terms of the series are $2^2 - 1$, $4^2 - 1$, $6^2 - 1$, ..., $10^2 - 1$, $12^2 - 1$.

So, the missing number =
$$8^2 - 1 = 64 - 1 = 63$$
.

14. The sequence is +7 + 10, +13, +16, +19, ...

So, the missing number = 68 + 22 = 90.

15. The sum of any three consecutive terms of the series gives the next term.

So,
$$1 + 3 + 4 = 8$$
; $3 + 4 + 8 = 15$; $4 + 8 + 15 = 27$ and so on.

- \therefore The missing number = 8 + 15 + 27 = 50.
- 16. Each number in the series is the product of the digits of the preceding number.

i.e.
$$6 \times 6 = 36$$
; $3 \times 6 = 18$ and so on.

- \therefore The missing number = $1 \times 8 = 8$
- 17. The sequence is +2, +4, +2, +4, +2, ...
 - \therefore The missing number = 25 + 4 = 29
- 18. The difference between consecutive numbers increases by 1.

Thus, the sequence is +2, +3, +4, +5 ...

- \therefore The missing number = 16 + 6 = 22.
- 19. The sequence is +2, +4, ..., +10, +12

So, the missing number = 6 + 6 = 12.

20. The numbers are alternately multiplied by 2 and increased by 3.

So,
$$5 \times 2 = 10$$
; $10 + 3 = 13$; $13 \times 2 = 26$; $26 + 3 = 29$ and so on.

- \therefore The missing number = $61 \times 2 = 122$.
- **21.** The sequence is $1^3 + 1$, $2^3 + 1$, $3^3 + 1$, $4^3 + 1$, $5^3 + 1$, ...
 - \therefore The missing number = $6^3 + 1 = 216 + 1 = 217$.
- 22. The sum of two consecutive numbers of the series gives the next number.

Thus,
$$4 + 9 = 13$$
; $9 + 13 = 22$; $13 + 22 = 35$ and so on.

- \therefore The missing number = 22 + 35 = 57.
- **23.** The numbers are 1^3 , 2^3 , 3^3 , 4^3 , 5^3 , 6^3 and so on.
 - \therefore The missing number = $7^3 = 343$.

24. The numbers are alternately multiplied by 2 and 3/2.

Thus,
$$1 \times 2 = 2$$
; $2 \times \frac{3}{2} = 3$; $3 \times 2 = 6$; $6 \times \frac{3}{2} = 9$; and so on.

- \therefore The missing number = $18 \times \frac{3}{9} = 27$.
- 25. The series consists of prime numbers.
 - ... The missing number is the next prime number, which is 43.
- **26.** The sequence is +3, +6, +12, +24 ...
 - \therefore the missing number = 47 + 48 = 95.
- 27. The given sequence is a combination of two series 4, 5, 7, 8, 10 and 9, 12, 15, (...)
 - \therefore The missing term = 15 + 3 = 18.
- 28. The given sequence consists of two series: 10, 13, 16, 19 and 5, 10, 20, (...)
 - \therefore The missing number = $20 \times 2 = 40$.
- **29.** The sequence is 1×2 , 2×3 , 3×4 , 4×5 , 5×6 , 6×7 , 7×8 .
 - \therefore The missing term = $8 \times 9 = 72$.
- 30. The given series consists of prime numbers starting from 2. The prime number after 11 is 13. So, 13 is the missing number.

Case II: Finding the Wrong Term in the Given Series

EXAMPLE 4 Find the wrong number in the series.

(a) 15

(b) 24

(c) 34

(d) 48

(e) 63

Solution The difference between consecutive terms of the given series are respectively 5, 7, 9, 11 and 13.

So 34 is the wrong number.

Answer is (c).

EXAMPLE 5 Find the wrong number in the series.

(a) 26

(b) 74

(c) 218

(d) 654

(e) 1946

Solution Each term is obtained by multiplying the preceding term by 3 and then subtracting 4 from it.

So,
$$26 = (10 \times 3) - 4$$
; $74 = (26 \times 3) - 4$; $218 = (74 \times 3) - 4$ and so on.

So the wrong number is 654.

Answer is (d).

EXERCISE 2

Directions: In each of the following questions, one term in the number series is wrong. Find out the wrong term.

- 1. 8, 14, 26, 48, 98, 194, 386
 - (a) 194
- (b) 98
- (c) 14
- (d) 48
- (e) 386

2. 8, 13, 21, 32, 47, 63, 83

- [Bank PO]

- (a) 21
- (b) 13
- (c) 32
- (d) 83
- (e) 47

3. 3, 7, 15, 39, 63, 127, 255, 511 (a) 39 (b) 15 (c) 7 (d) 63 (e) 127 4. 445, 221, 109, 46, 25, 11, 4 (a) 221 (b) 109 (c) 46 (d) 25 (e) 11 **5.** 1, 2, 6, 15, 31, 56, 91 (a) 31 (b) 15 (c) 56 (d) 91 (e) 2 **6.** 2, 5, 10, 17, 26, 37, 50, 64 (a) 50 (b) 17 (c) 26 (d) 37 (e) 64 **7.** 46080, 3840, 384, 48, 24, 2, 1 (a) 1 (b) 2 (c) 24(d) 48 (e) 384 8. 52, 51, 48, 43, 34, 27, 16 (a) 51 (b) 48 (c) 34 (d) 27 (e) 16 **9.** 325, 259, 202, 160, 127, 105, 94 (c) 105 (d) 202 (a) 94 (b) 127 (e) 259 **10.** 125, 126, 124, 127, 123, 129 (a) 123 (b) 124 (c) 126 (d) 127 (e) 129 11. 3, 4, 10, 32, 136, 685, 4116 (a) 136 (b) 10 (c) 4116 (d) 685 (e) 32 **12.** 25, 36, 49, 81, 121, 169, 225 (a) 36 (b) 49 (c) 121 (d) 169 (e) 225 **13.** 56, 72, 90, 110, 132, 150 (a) 72 (b) 90 (c) 110 (d) 132 (e) 150 **14.** 6, 13, 18, 25, 30, 37, 40 (a) 25 (c) 37 (d) 40 (b) 30 (e) 13 **15.** 10, 14, 28, 32, 64, 68, 132 (c) 64 (d) 132 (a) 32 (b) 68 (e) 28

ANSWERS

1. (d) 2. (e) 3. (a) 4. (c) 5. (d) 6. (e) 7. (c) 8. (c) 9. (d) 10. (e) 11. (e) 12. (a) 13. (e) 14. (d) 15. (d)

- 1. Each term in the series is less than twice the preceding term by 2. So, 48 is wrong, correct term = (26)2 2 = 52 2 = 50.
- 2. The sequence is +5, +8, +11, +14, and so on. Here 47 is wrong. Correct term =32+14=46.
- 3. Each number in the series is multiplied by 2 and the result increased by 1 to obtain the next number.
 - \therefore 39 is wrong. The correct term = $(15 \times 2) + 1 = 31$.
- 4. Each number is obtained by dividing the preceding number by 2 after subtracting 3 from it.

i.e.
$$221 = \left(\frac{445 - 3}{2}\right)$$
; $109 = \left(\frac{221 - 3}{2}\right)$ and so on.

$$\therefore$$
 46 is wrong. The correct term is $\left(\frac{109-3}{2}\right) = 53$.

- 5. The sequence is $+1^2$, $+2^2$, $+3^2+4^2$, $+5^2$, $+6^2$
 - \therefore 91 is wrong. The correct number is $56 + 6^2 = 56 + 36 = 92$.
- **6.** The numbers are $1^2 + 1$, $2^2 + 1$, $3^2 + 1$ and so on.
 - \therefore 64 is wrong. The correct term is $8^2 + 1 = 65$.
- 7. The terms are successively divided by 12, 10, 8, 6 etc. So, 24 is wrong. Correct term = 48/6 = 8.
- 8. The sequence is -1, -3, -5, -7, -9, -11, etc.
 - So, 34 is wrong. Correct term = 43 7 = 36.
- **9.** The sequence is -66, -55, -44, -33, -22, -11.
 - So, 202 is wrong. Correct term = 259 55 = 204.
- 10. Sequence is +1, -2, +3, -4, +5.
 - So, 129 is wrong.
 - Correct term = 123 + 5 = 128.
- 11. Sequence is obtained by adding 1 to preceding term and then multiplying successively by 1, 2, 3, and so on.
 - So, 32 is wrong. Correct term = (10 + 1)3 = 33.
- 12. The correct sequence is a set of squares of consecutive odd numbers.
 - i.e. 5^2 , 7^2 , 9^2 , 11^2 , 13^2 , 15^2 .
 - So, 36 is wrong in the series.
- 13. The sequence is +16, +18, +20, +22, +24.
 - So, 150 is wrong. Correct term = 132 + 24 = 156.
- 14. This is an alternate series.
 - i.e. +7, +5, +7, +5 and so on.
 - So, 40 is wrong. Correct term = 37 + 5 = 42.
- 15. Sequence is as follows:
 - 2nd term = 1st term + 4 = 10 + 4 = 14;
 - 3rd term = 2nd term \times 2 = 14 \times 2 = 28;
 - 4th term = 3rd term + 4 = 28 + 4 = 32;
 - 5th term = 4th term \times 2 = 32 \times 2 = 64 and so on.
 - So. 132 is wrong. Correct term = $68 \times 2 = 136$.

TYPE 2: ALPHABET SERIES

EXAMPLE 6 What will be the next term in

- (a) IJX
- (b) IGX
- (c) JGW
- (d) IGU
- (e) JGU

Solution In each term, the first letter is moved forward by 2 letters the second letter backward by 1 letter and the third letter moved forward by 1 letter. So the missing term is JGW. So the answer is (c).

EXAMPLE 7 Choose the alternative that will complete the series:

(a) pzj

- (b) jtd
- (c) jue
- (d) kve
- (e) ukv

Solution The letters in each term in all the positions are moved forward by 5. So the missing term = pzj. So the answer is (a).

EXAMPLE 8 What terms will fill in the blank spaces?

Y W U S Q (...) (...)

(a) N, J

(b) M, L

(c) J, R

(d) L, M

(e) O, M

Solution The given series consists of alphabet in the reverse order moved by 2 in each step. So the missing terms are O and M.

The answer is (e).

EXERCISE 3

Directions: In each of the following questions, various terms of a letter series are given with one term missing as shown by (?). Choose the missing term out of the given alternative.

1.	AZ, BY, CX, ?							[Bank PO]
	(a) EF (b	o) GH	(c)	IJ	(d)	DE	(e)	DW
2.	DEF HIJ MNO?							[Bank PO]
	(a) STU (b	o) RST	(c)	RTV	(d)	SRQ	(e)	TUV
3.	BXJ ETL HPN	KLP?						[SBI PO]
	(a) NHR (b	o) MHQ	(c)	MIP	(d)	NIR	(e)	None of these
4.	AB DEF HIJK	? STUVWX						[Bank PO]
	(a) MNOPQ (b	o) LMNOP	(c)	LMNO	(d)	QRSTU	(e)	None of these
5.	A Z X B V T C R (?),	(?)						[RRB]
	(a) P, D (b	o) E, O	(c)	Q, E	(d)	O, Q	(e)	Q, O
6.	G, H, J, M, (?), V							[Bank PO]
	(a) T (b	o) S	(c)	R	(d)	U	(e)	Q
7.	P3C, R5F, T8I, V12I	L, ?						[SBI PO]
	(a) Y17O (b		(c)	X17O	(d)	X16O	(e)	None of these
8.	AYD BVF DRH							[AAO Exam]
	(a) FMI (b	o) GMJ	(c)	HLK	(d)	GLJ	(e)	None of these
9.	CX FU IR ? OL RI							
	(a) LO (b		(c)	NO	(d)	OP	(e)	OR
10.	DEB IJG NOL							
	(a) RSP (b	•	(c)	RSQ	(d)	STQ	(e)	STO
11.	A CD GHI ? U							
	(a) LMNO (b	•	(c)	NOPQ	(d)	NOP	(e)	MNOP
12 .	EJOT DHLP CFI							
	(a) BDFH (b	•	(c)	DEIJ	(d)	DGKL	(e)	HFDB
13.	OTE PUF QVG							
	(a) SYJ (b		(c)	SXJ	(d)	SXI	(e)	TYJ
14.	DCXW FEVU HO							
	(a) LKPO (b	o) ABYZ	(c)	JIRQ	(d)	LMRS	(e)	JRIQ
15.	R, M, (?), F, D, (?)	\ T TT		D. II	<i>(</i> 1)	TT 0		T G
	(a) C, B (b		(c)	В, Н	(d)	H, C	(e)	I, C
16.	C, B, A, E, D, Z, G, F			**	/ 3 \	**		-
	(a) X (b)	D) Y	(c)	V	(d)	Н	(e)	า

CMW, HRB, (?), F	RBL, WGQ, BLV						
(a) MWG	(b) LVF	(c)	LWG	(d)	MXG	(e)	$\mathbf{W}\mathbf{M}\mathbf{X}$
HS, JQ, LO, NM,	(?)						
(a) PK	(b) RH	(c)	PL	(d)	TG	(e)	RT
QPO, SRQ, UTS,	WVU, (?)						
(a) XVZ	(b) ZYA	(c)	YXW	(d)	VWX	(e)	AZY
ABP,CDQ,EFR,	(?)						
(a) GHS	(b) GHT	(c)	HGS	(d)	GHR	(e)	GSH
	(a) MWGHS, JQ, LO, NM,(a) PKQPO, SRQ, UTS,(a) XVZABP, CDQ, EFR,	HS, JQ, LO, NM, (?) (a) PK (b) RH QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA ABP, CDQ, EFR, (?)	(a) MWG (b) LVF (c) HS, JQ, LO, NM, (?) (a) PK (b) RH (c) QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA (c) ABP, CDQ, EFR, (?)	(a) MWG (b) LVF (c) LWG HS, JQ, LO, NM, (?) (a) PK (b) RH (c) PL QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA (c) YXW ABP, CDQ, EFR, (?)	(a) MWG (b) LVF (c) LWG (d) HS, JQ, LO, NM, (?) (a) PK (b) RH (c) PL (d) QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA (c) YXW (d) ABP, CDQ, EFR, (?)	(a) MWG (b) LVF (c) LWG (d) MXG HS, JQ, LO, NM, (?) (a) PK (b) RH (c) PL (d) TG QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA (c) YXW (d) VWX ABP, CDQ, EFR, (?)	(a) MWG (b) LVF (c) LWG (d) MXG (e) HS, JQ, LO, NM, (?) (a) PK (b) RH (c) PL (d) TG (e) QPO, SRQ, UTS, WVU, (?) (a) XVZ (b) ZYA (c) YXW (d) VWX (e) ABP, CDQ, EFR, (?)

	ANSWERS									
1. (e)	2. (a)	3. (a)	4. (a)	5. (a)	6. (e)	7. (c)	8. (b)	9. (a)	10. (d)	
11. (e)	12. (a)	13. (d)	14. (c)	15. (e)	16. (b)	17. (a)	18. (a)	19. (c)	20. (a)	

- 1. In each term, the first letter is moved 1 step forward and the second letter is moved 1 step backward to obtain the corresponding letters of the successive terms.
- 2. The letters in each term are consecutive. There is a gap of one letter between the last letter of the first term and the first letter of the second term. In the successive terms the gap increases by one. So the last term is STU, which is having a gap of 3 letters from the last letter of the previous term.
- 3. The first, second and third letters of each term are moved three steps forward, four steps backward and two steps forward respectively to obtain the corresponding letters of the successive terms.
- 4. The number of letters in the terms increases by one at every step. Each term consists of letters in the alphabetical order. There is a gap of one letter between the last letter of a term and first letter of next term.
- 5. The first, fourth and seventh letters are in alphabetical order. So the tenth letter would be the letter after C. i.e., D.
 - The second and third are alternate and in the reverse order and so are the fifth and sixth and eighth and ninth. So 9th letter = R 2 = P.
 - So the answer is (a) P, D.
- 6. The first, second, third, fourth and fifth terms are moved one, two, three, four and five steps respectively forward to obtain the successive terms so, M + 4 = Q.
- 7. The first letters of the terms are alternate. The last letters of the terms are three steps ahead of last letter of the preceding term. The middle letter of the term follows the pattern +2, +3, +4, +5. So the next term = X170.
- 8. The first letters of the term are moved one, two, three and four steps forward respectively to obtain the first letter of the successive terms. The second letters of the term are moved three, four, five, and six steps backward respectively to obtain the second letters of the successive terms. The last letters of the terms are alternate.
- 9. The first letter of each term is moved three steps forward and the second letter is moved three steps backward to obtain the corresponding letters of the next term.
- 10. The letters in each term are moved five steps forward to obtain the letters of the next term.
- 11. The first term consists of one letter, the second term consists of two letters and the third letter consists of three letters. So, the required term consists of four letters. The last letter of first term and the first letter of the second term differ by two letters. Similarly, a gap of two

letters is there between the last letter of second term and the first letter of the third term. So the first letter of required term would be four steps ahead from the last letter of third term.

- 12. The letters of each term are moved one step backward to obtain the corresponding letters of the next term.
- 13. The first letters of the terms are in alphabetical order, and so are the second and third letters.
- 14. The first two letters of the term are in the reverse order. Similarly, the third and fourth letters of the term are also in the reverse order. Besides, the second letter of each term is the letter next to the first letter of the preceding term.
- 15. The letters are in the reverse order in which letters are moved 1, 2, 3, 4 and 5 steps from the last.
- 16. The third, sixth and ninth letters of the series are in reverse alphabetical order.
- 17. All the letters of each term are moved five steps forward to obtain the corresponding letters of the next term.
- 18. The first letter of each term is moved two steps forward and the second letter is moved two steps backward to obtain the corresponding letters of the next term.
- 19. Each term in the series consists of three consecutive letters in the reverse order. The first letter of each term and the last letter of the next term are the same.
- 20. The first and second letters of each term are moved two steps forward to obtain the corresponding letters of the next term. The third letter of the terms are consecutive terms in the alphabet.

TYPE 3: LETTER SERIES

This type of question usually consists of small letters which follow a certain pattern. Some letters are missing from the series. The missing letters are given in a proper sequence as one of the alternatives. You have to choose the correct alternative.

EXAMPLE 9	abca bcaab	ca bbc _	a
(a) ccaa	(b) bbaa	(c) abac	(d) abba

[AAO Exam]

Solution Proceed step by step as follows:

- 1. The first blank space should be filled using 'a'; so that the first alphabet doubles in the first step after 'abc'.
- 2. The second blank space should be filled using 'b' so that in the second step both 'a' and 'b' doubles.
- 3. The third blank space should be filled by 'a' and the fourth blank space should be filled by 'c' so that in the third stage, all characters 'a', 'b' and 'c' doubles.

 Thus, the answer is abac.

So, the answer is (c).

EXERCISE 4

Directions: In each of the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

Que	stions 1 to 5				[CBI Exa	m
1.	abca	bcaab aa	caa c			
	(a) bbac	(b) bbaa	(c) acbb		(d) acac	
2.	b b _	bb	bbb bb	b		
	(a) bbbbba	(b) bbaaab	(c) ababab		(d) aabaab	

4. cc (a 5 (a Questi 6. ba	bab ca	(b) (b) (b) (c) (b) (d) (d) (d) (e) (e) (e) (f) (f) (f) (f) (f) (f) (f) (f) (f) (f	bccc bbbabbca bcc abac bbaacc bcaa cbba	a acb c _ baaa bbcc	(c) (c) (c) (d) (d) (e) (e) (e) (f) (f)	bbab ababaa ac ccba abab bb babb		(d) (d) (d) (d) (d)	cbab babc aaaabb cbac aabc baab bcab	[SSC]
(a 5 (a Questi 6. ba (a 7. cc (a 8. aa (a 9. ac) babb abb) bababa ons 6 to 10 a ba) aacb ccbc) acac aa bb) abab cc bc _) abab ab bca ons 11 to 15	(b) (b) (b) (c) (b) (d) (d) (e) (e) (e) (f) (f) (f) (f) (h) (f) (h) (h) (h)	bbba bb bbca bcc bbaa cc bcaa cbba c	a acb c _ baaa bbcc	(c) (c) (d) (d) (e) (e) (e) (f) (f)	bbab ababaa cc ccba abab bb babb	ba	(d) (d) (d) (d)	aaaabb cbac aabc baab	[SSC]
5	abb abb) bababa ons 6 to 10 a ba) aacb ccbc.) acac aa bb) abab cc bc) abab ab bc) abab ab bba bba) bcca ons 11 to 15	(b) bac(b) acc(b) aal(b) a(b)	bbca bbca bbca bbaa bbaa bbaa cobbaa	acb acb c _ _ baaa cbbcc	(c) cba (c) (c) (c) (c) (c)	bbab ababaa cc ccba abab bb babb	ba	(d) (d) (d) (d)	aaaabb cbac aabc baab	[SSC
(a Questi 6. ba (a 7. cc (a 8. aa (a 9. ac) bababa ons 6 to 10 a ba) aacb ccbc aa bb) abab cc bc _) abab ab bba) bcca ons 11 to 15	(b) bac(b) acc(b) aal(b) a(b)	bbca bbca bbca bbaa bbaa bbaa cobbaa	acb acb c _ _ baaa cbbcc	(c) cba (c) (c) (c) (c) (c)	ababaa ccba abab bb babb		(d) (d) (d) (d)	aaaabb cbac aabc baab	[SSC]
(a Questi 6. ba (a 7. cc (a 8. aa (a 9. ac) bababa ons 6 to 10 a ba) aacb ccbc aa bb) abab cc bc _) abab ab bba) bcca ons 11 to 15	(b) bac _ (b) acc (b) aal (b) a (b)	bbabbb bbca bbca abac bbaa bbaa bcaa cbba	acb c _ c _ _ baaa cbbcc	(c)cbab(c)(c)(c)	ababaa ccba abab bb babb		(d) (d) (d)	cbac aabc baab	[SSC]
Questi 6. ba (a 7. cc (a 8. aa (a 9. ac	ons 6 to 10 a ba) aacb c ccbc aa bb) abab cc bc _) abab ab bba bca ons 11 to 15	bac (b) acc (b) aal (b) a (b) aa (b) aa (b)	bbca bcc abac bbaa bcaa bcaa cbba	acb c _ _ baaa :bbcc	(c) (c) (c)	ac ccba abab bb babb		(d) (d) (d)	cbac aabc baab	[SSC]
6. ba (a 7. cc (a 8. aa (a 9. ac	a ba) aacb : ccbc .) acac aa bb .) abab cc bc _) abab ab bba) bcca ons 11 to 15	(b) acc (b) aak (b) aak (b) (b) aaa	bbca bcc abac bbaa bcaa cbba	c _ _ baaa cbbcc	(c) (c) (c) (c)	ccba abab bb babb aabc		(d) (d)	aabc baab	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
(a 7. cc) aacb	(b) acc (b) aak (b) aak (b) (b) aaa	bbca bcc abac bbaa bcaa cbba	c _ _ baaa cbbcc	(c) (c) (c) (c)	ccba abab bb babb aabc		(d) (d)	aabc baab	
7. cc (a 8. aa (a 9. ac	ccbc . acac aa bb . abab cc bc _ abab ab bba bca bca coss 11 to 15	acc (b) aal (b) a (b)	abac bbaa bcaa cbba cbba	c _ _ baaa ebbee	(c) (c)	abab bb babb aabc		(d) (d)	aabc baab	
(a 8. aa (a 9. ac) acac aa bb .) abab ac bc _) abab ab bba) bcca ons 11 to 15	(b) ————————————————————————————————————	abac bbaa co bcaa cbba	_ baaa cbbcc	(c) (c) (c)	bb babb aabc		(d)	baab	
8. aa (a 9. ac	haa bb .) abab cc bc _) abab ab bba) bcca ons 11 to 15	aal (b) a (b)	bbaa co bcaa cbba	ebbcc	(c)	bb babb aabc		(d)	baab	
(a 9. ac) abab cc bc _) abab ab bba) bcca ons 11 to 15	(b) aa (b)	bbaa co bcaa cbba	ebbcc	(c) (c)	babb aabc		, ,		
9. ac	cc bc _) abab ab bba) bcca ons 11 to 15	(b)	bcaa cbba	8	(c)	aabc		, ,		
) abab ab bba) bcca ons 11 to 15	(b)	bcaa cbba	8	(c)			(d)	heah	
	ab bba) bcca ons 11 to 15	iaa	cbba	8				(4)		
•) bcca ons 11 to 15					ha			Doas	
	ons 11 to 15	(5)	CDUD		(c)	cbba		(d)	aabc	
•					(6)	сына		(4)		O Exam
		cahe		deba	he	, b	0		[AA]	O Exam
		cabe_ (b)					_ a	(d)	cbdaa	
•	, abuca cdaab						aaddd	(u)	Couaa	
			bddca				_ ccuuu	(4)	hhdaa	
,) bdbda					dbbca	ha	(a)	bbdac	
	abbb _			aac			ва	(4)	-11	
•) abcda	(b)				abdcb	1.1	(a)	abcad	
	bcdbc _						ba	(1)	11111	
,	,		ccccc			bbbbb		(a)	ddddd	
	lb ac _						cbda			
,) bccba	(b)	cbbaa		(c)	ccbba			bbcad	
-	ons 16 to 20				_		_	[He	otel Mana	igement]
	aaba						_ b			
) aabab					baaba		(d)	bbaba	
17. ak	bbc _	c	a	b	ab	b				
,	<i>'</i>	(b)			(c)			(d)	bccab	
	bca									
(a) aaaaa	(b)	bbbab		(c)	aabaa		(d)	bbabb	
19. b	ac	cc	cb		_ ab	ac				
(a) cbaba	(b)	bbaac		(c)	abbbc		(d)	aabba	
20. c.	ac	aa	aa		_ bc	bcc				
(a) cabba	(b)	ccbbb		(c)	bbbbb		(d)	cbacb	
				ANS	SWERS					
				1314	**********					
1. (c)	2. (c)	3. (d)	4. (a)	5. (b	6. ((c) 7.	(a) 8	3. (c)	9. (b)	10. (b)

- 1. The series is a / bcaa / bcaa / bcaa / bcaa / bc
 - Thus, the pattern bcaa is repeated.
- 2. The series is babb/bbab/bbba/bbbb
 - Thus, in each sequence 'a' moves one step forward and 'b' takes its place and finally to the fourth sequence, where 'a' is eliminated.
- 3. The series is aa / \underline{bcab} / \underline{bcab} / \underline{ccaa} / \underline{bcab} / \underline{bc}
 - Thus, the pattern ccaa followed by bcab repeated twice, makes up the series.
- 4. The series is ccba / bbca / aabc / ccba / b.
 - Thus, the pattern consists of first two letters moved 1 step backward and the third letter moved 1 step forward and the last letter remains the same in the first stage.
 - In the second stage, i.e. from second to third term, first two letters moved 1 step backward and last two letters moved 1 step backward.
 - In the third stage, first two letters moved 1 step backward, third letter unchanged and fourth letter moved 1 step forward.
 - In the fourth stage, first letter moved 1 step backward.
- 5. The series is babb / babb / babb / babb / ba. Thus pattern babb is repeated.
- 6. The series is bac / bac / bac / bac / bac / bac. Thus, the pattern bac is repeated.
- 7. The series is ccaccb / ccaccb / ccaccb. Thus, the pattern ccaccb is repeated.
- 8. The series is aaa / $\underline{b}bb$ / $\underline{a}aa$ / $\underline{b}\underline{b}b$ / $\underline{a}aa$ / $\underline{b}bb$.
- 9. The series is accb / bcca / accb /bcca. Thus, the pattern accb /bcca is repeated.
- 10. The series is aabcbba / aabcbba / aabcbba. Thus, the pattern aabcbba is repeated.
- 11. The series is <u>aababcabcddcbacbabaa</u>. Thus, the letters equidistant from the beginning and the end of the series are the same.
- 12. The series is a <u>b</u>cd / aab<u>b</u> cc<u>d</u>d / aa<u>a</u> bbb <u>c</u>cc ddd. Thus, each letter of first sequence is repeated two times in the second sequence and three times in the third sequence.
- 13. The series is $a\underline{a}a / bbb\underline{b} / cccc / d\underline{d}dd / ccc\underline{c} / bb\underline{b}b / a$.
- 14. The series is <u>a</u>bcd / bc<u>a</u>d / cabd / <u>a</u>bcd / bc<u>a</u>d / c<u>a</u>bd. Thus, the pattern abcd / bcad / cabd is repeated twice.
- 15. The series is adbcacbdabcddcbadbcacbda. The letters equidistant from the beginning and end of the series are the same.
- 16. The series is aaab / aabb / abbb / aaab / aabb.
- 17. The series is abc/b/bca/c/cab/a/abc/b.
- 18. The series is bbca / bcca / bcaa / bbc.
- 19. The series is baac / accb / cbba / baac.
- 20. The series is ccacc / aabaa / bbcbb / cc.

Coding and Decoding

Coding is a method of transmitting a message between the sender and the receiver without a third person knowing it.

The coding and decoding test is set up to judge the candidate's ability to decipher the law that codes a particular message and break the code to reveal the message. In these questions, the candidate is required to find out the original words from code values assigned to the word.

TYPE I: LETTER CODING: In these types of questions, code values are assigned to a word in terms of the alphabet and the requirement may be.

Case I: Coding: To form the code for the given word.

EXAMPLE 1 If in a certain language, TAP is coded as SZO, then how will FREEZE be coded?

- (a) ESDFYF
- (b) GQFDYF
- (c) EQDFYG
- (d) EQDDYD

Solution Clearly, each letter in the code is the alphabet before the corresponding letter in the word.

Thus, in FREEZE, F is coded as E, R as Q, E as D and Z as Y. So, the answer is (d).

EXAMPLE 2 If in a certain language GAMBLE is coded as FBLCKF, how will FLOWER be coded in that code?

(a) GKPVFQ (b) EMNXDS (c) GMPVDS (d) HNQYGT (e) EKNVDQ *Solution* The letters preceding the letters at odd places of the word and those succeeding the even places of the word in the alphabets form the code. Hence the answer is (b).

EXAMPLE 3 If in a certain language PENSION is coded as NEISNOP, how will FOLIAGE be coded in that code?

(a) OFILGAE (b) EOAILGF (c) FGLIAOE (d) EGAILOF (e) FILOGAE **Solution** In the code, first and last letters are reversed, second and second last letters are the same, third and third last letters are reversed and the middle letter remains the same. Hence the answer is (b).

EXERCISE 1 1. If PLANE is coded as OKZMD in a certain language, how will TRAIN be coded? (a) SQZHM (b) UQBHO (c) SQZJM (d) USBJM (e) USBJO 2. If in a certain language: SPACE is coded as TQBDF, how will PURSE be coded in that code? (a) QTSRF (b) OVQTD (c) QVSTF (d) ESRUP (e) OTQRD 3. If in a certain language NATURE is coded as MASUQE, how is FAMINE coded in that code? (a) FBMJND (e) FZNJME (b) FZMHND (c) GANIOE (d) EALIME 4. If in a certain language SECURE is coded as ERUCES, how is SALINE coded in that code? (b) EALINS (c) ENILAS (d) ERUCES (e) SLANIE (a) SALIQE 5. If in a certain language MECHANICS is coded as HCEMASCIN, how is POSTER coded in that code? (a) OPTSRE (b) SOPRET (c) RETSOP (d) TERPOS (e) POTSER 6. If in a certain language DISPEL is coded as IDPSLE, how is EFFECT coded in that language? (a) FEEFTC (b) CTFEEF (c) EFFFTC (d) ECTEFF (e) EEFFCT 7. If BATCH is coded as ABSDG, how is FORSAKE coded in that code? (a) ABDGS (b) EPQTZLD (d) GDSBA (c) EQPZLTD (e) None of these 8. In a certain code, GOODNESS is coded as HNPCODTR. How is GREATNESS coded in that code? [SBI PO] (a) HQFZUODTR (b) HQFZUMFRT (c) HQFZSMFRT (d) FSDBSODTR (e) HQFZUFRT 9. If FOUGHT is coded as EQRKCZ, how will MALE be coded? (b) NZMD (c) KCMI (d) NBIF (e) LBID 10. If in a certain language BLEMISH is coded as AODPHVG, how will CHAPTER be coded in that code? (a) DEBOVTDR (b) BKZSSHQ (c) CAHTPRE (d) BGAQMFP (e) ADGIQFS ANSWERS 1. (a) **2.** (c) **3.** (d) **4.** (c) **5.** (b) **6.** (a) **7.** (b) 8. (b) **9.** (a) **10.** (b)

- 1. The letter preceding each letter of the given word in the alphabet is taken as the subsequent letter of its code.
- 2. Each letter of the given word is moved one step ahead to obtain the subsequent letter of its code.
- 3. The letters preceding the letters at odd places in the given word are taken as the corresponding letters of the code while those at even places remain the same.

- 4. The word is wholly reversed in the code.
- 5. In the code the first four and the last four letters are reversed in order.
- **6.** In the code, every two letters are reversed in order.
- 7. In the code, the letters at odd places are one place before and those at even place are one place after the corresponding letter in the word.
- 8. In the code the letters at odd places are one alphabet ahead and those at even places are one alphabet before the corresponding letter in the word.
- 9. In the code the first letter is one place before, the second letter is two places ahead the third letter is three places before, the fourth letter is four places ahead, and so on.
- 10. In the code, the letters at odd places are one place before and those at even places are three places ahead of the corresponding letters in the word.

Case II: Decoding: To find the word from the given code. In decoding, instead of finding a code for a given word, you have to find the word from the given code.

EXAMPLE 4 If in a certain code, ALMIRAH is written as BNPMWGO, which word would be written as DNRWLUA?

(a) COSGOLT (b) TOGSOLC (c) TOGCLOS (d) CLOSGOT (e) COLSTOG **Solution** In the code, the first letter is one place ahead, the second letter is two places ahead and so on than the corresponding letter in the word.

So apply the same in the reverse direction to the given code so as to find the word. Therefore the answer is (d).

EXAMPLE 5 If in a certain code, SWITCH is written as TVJSDG, which word would be written as CQFZE?

(a) BARED (b) BRAED (c) BREAD (d) BRADE (e) BRDAE

Solution Each letter in the code is one place ahead in odd places and one place before in even places from the corresponding letters of the word. So the answer is (c).

EXAMPLE 6 In a certain code language, REFRIGERATOR is coded as ROTAREGIRFER. Which word would be coded as NOITINUMMA?

- (a) ANMOMIUTNI
- (b) AMNTOMUIIN
- (c) AMMUNITION

- (d) NMMUNITIOA
- (e) None of these

Solution The order of letters of the word is reversed in the code. So, reverse the letters in the code to get the word. So the answer is (c).

EXERCISE 2

- If in a certain language, UTENSIL is coded as WVGPUKN, which word would be coded as DMSFXG?
 - (a) BKQEVE
- (b) BKQDWE
- (c) BKQDWF
- (d) BKQDVF
- (e) BKQDVE
- 2. If in a certain code language, REMOTE is coded as ROTEME, which word would be coded as PNIICC?
 - (a) NPIICC
- (b) PICCIN
- (c) PINCIC
- (d) PICNIC
- (e) PICINC
- 3. If in a certain language, SHIFT is coded as RFFBO, which word would be coded as LKUMB?
 - (a) MMXQG
- (b) MLVNC
- (c) KJVLA
- (d) MJVLC
- (e) KJTLA

DWZLOKD?

	(a)	EXAMPLE	(b)	FIGMENT	(c)	DISM	ISS	(d)	DISJOIN	(e)	None	${ m of\ these}$
5.	5. If in a certain language GRASP is coded as BMVNK, which word would be coded as CRANE?											
	(a)	FUDQH	(b)	HWFSJ	(c)	GVEI	RI	(d)	XMVIZ	(e)	BQZM	ID
6.		n a certain lar ΓMFNB?	igua	ge, MACHIN	IE is	coded	as LBBI	HOI), which word	l wo	uld be	coded as
	(a)	RKSLEMA	(b)	TKULGMC	(c)	RMSI	NEOA	(d)	TMUNGOC	(e)	TMU	NGMC
7.		n a certain la: MOUSPP?	ngua	age PORCEI	LAIN	is code	ed as Q	OSC	FLBJO, whic	h w	ord is	coded as
	(a)	(a) ALTOLROPY (b)				ALLOTROPY			(c) ALOTR	OLPY		
	(d)	ATLOROPLY		(e)	None	e of the	ese					
8.	8. If in a certain language LBAEHC is the code for the word BLEACH, then which of the following is coded as NBOLZKMH?											
	(a)	OBNKLHM	(b)	LOBNHM	(c)	OCPI	<i>I</i> ALNI	(d)	MANKYJLG	(e)	BNLC	KZHM
9.	9. If in a certain language POPULAR is coded as QPQVMBS, which word would be coded as GBNPVT?											
	(a)	FAMOSU	(b)	FAMOUS	(c)	FASC	UM	(d)	FOSAUM	(e)	FAMS	SUO
10. If in a certain language, CRICKET is coded as FULFNHW, then EULGH is the code for which word?												
	(a)	PRIDE	(b)	BRIDE	(c)	BLAI	Œ	(d)	BLIND	(e)	None	of these
ANSWERS												
1.	(e)	2. (d) 3	3. (a)	4. (a)	5.	(b)	6. (b)	7.	(b) 8. (e)	9). (b)	10. (b)
Explanations												
1. Each letter of the word is two place behind the corresponding letters of the code. So the answer is (e).												
2.	2. The groups of second and third letters and fourth and fifth letters in the word interchange											

4. If in a certain language, TRIANGLE is coded as SQHZMFKD, which word would be coded as

- 2. The groups of second and third letters and fourth and fifth letters in the word interchange places in the code.
- 3. The first, second, third, fourth and fifth letters of the word are respectively one, two, three, four and five places ahead of the corresponding letters of the code.
- 4. Each letter of word is one place ahead of corresponding letter of the code.
- 5. Each letter of the word is five place ahead of the corresponding letter of the code.
- 6. In the code, we have alternatively one letter one place behind and the other one place ahead of the corresponding letter in the word.
- 7. In the code, we have one letter one place ahead of the corresponding letter in the word and the other letter coming in the even places remain unchanged.
- 8. The word is formed into pairs of letters and letters in each pair is reversed.
- 9. Each letter of the word is one place behind the corresponding letter of the code.
- 10. Each letter of the word is three places behind the corresponding letter of the code.

TYPE II: NUMBER CODING: In these type of questions, either numerical code values are assigned to a word or alphabetical code values are assigned to numbers. You have to analyse the code as per the directions.

Case I: When numerical values are assigned to words.

EXAMPLE 7 If in a certain code language SISTER is coded as 535301, UNCLE as 84670 and BOY as 129, how will SON be coded in that code language?

(a) 524

- (b) 643
- (c) 353
- (d) 846
- (e) None of these

Solution: Clearly, in the given code, the alphabets are coded as:

So, S is coded as 5, O is coded as 2 and N is coded as 4. Hence correct code is 524. Therefore, the answer is (a).

EXAMPLE 8 If GIVE is coded as 5137 and BAT is coded as 924, how is GATE coded?

- (a) 5427
- (b) 5724
- (c) 5247
- (d) 2547
- (e) None of these

Solution: Here the alphabet are coded as follows:

So G is coded as 5, A is coded as 2, T is coded as 4 and E is coded as 7. Hence correct code is 5247. Therefore the answer is (c).

EXERCISE 3

- 1. In a certain code, PALE is written as 2134, EARTH is written as 41590, how will PEARL be written in that code?
 - (a) 29530

- (c) 25413
- (d) 25430
- (e) None of these
- 2. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How will RIFFLE be written in that code?
 - (a) 968812
- (b) 869912

(b) 24153

- (c) 619982
- (d) 269981
- (e) None of these
- 3. If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?
 - (a) 246173
- (b) 214673
- (c) 214763
- (d) 216473
- (e) None of these
- 4. If in a certain code language, TWENTY is written as 863985 and ELEVEN is written as 323039, how will TWELVE be written in that code?
 - (a) 863203
- (b) 863584
- (c) 863903
- (d) 863063
- (e) None of these
- 5. If A is coded as 1, B is coded as 2, and so on, how is HIGH coded in that code?
 - (a) 9879
- (b) 7897
- (c) 8978
- (d) 8798
- (e) None of these

Directions (Questions 6 to 8): If in a certain language ENTRY is coded as 12345 and STEADY is coded as 931785, then state which is the correct code for each of the given words.

- 6. NEATNESS
 - (a) 2956169
- (b) 21732199
- (c) 21362199
- (d) 21823698
- (e) None of these

- 7. ARREST
 - (a) 744589
- (b) 744193 (c)
- (c) 166479
- (d) 745194
- (e) 188924

- 8. ENDEAR
 - (a) 524519
- (b) 174189
- (c) 128174
- (d) 124179
- (e) 164983

Directions (Questions 9 to 11): If in a certain language CHARCOAL is coded as 45164913 and MORALE is coded as 296137, how are the following words coded in that language? 9. REAL (a) 8519 (b) 6713 (c) 6513 (d) 6719 (e) None of these 10. COACH (a) 38137 (b) 49148 (c) 48246 (d) 49145 (e) None of these 11. COLLAR (a) 397758 (b) 497758 (c) 483359 (d) 493316 (e) None of these Directions (Questions 12 to 15): If MISTAKE is coded as 9765412 and NAKED is coded as 84123, how are the following words coded? 12. DISTANT (b) 4798165 (e) None of these (a) 3765485 (c) 3697185 (d) 4768296 13. ASSIST (a) 166762 (b) 466765 (c) 488976 (d) 435985 (e) 166872 14. INTIMATE (e) None of these (a) 89786145 (b) 79438163 (c) 78579452 (d) 78698365 15. STAIN (a) 98175 (b) 89483 (c) 68194 (d) 65478 (e) None of these 16. If ROPE is coded as 6821 and CHAIR is coded as 73456 then what will be the code for CRAPE? (a) 73456 (b) 76421 (c) 77246 (d) 77123 (e) None of these 17. If PLAY is coded as 8123 and RHYME is coded as 49367, then how is MALE coded? (a) 6217 (b) 6198 (c) 6395 (d) 6285 (e) None of these 18. If in a certain language PRIVATE is coded as 1234567 and RIST is coded as 2396, then how is RIVETS coded in that language? (b) 234769 (a) 687543 (c) 496321 (d) 246598 (e) None of these 19. In a certain code language 24685 is written as 33776. How is 35791 written in that code? (a) 44882 (b) 44880 (c) 46682 (d) 44682 (e) None of these 20. In a certain code language 35796 is written as 44887. How is 46823 written in that code? (e) None of these (a) 57914 (b) 55914 (c) 55934 (d) 55714 ANSWERS **4.** (a) **6.** (b) **1.** (b) **2.** (c) **3.** (b) **5.** (c) **7.** (b) 8. (c) **9.** (b) **10.** (d) 11. (d) 15. (d) 16. (b) **20.** (b) **12.** (a) **13.** (b) **14.** (c) 17. (a) **18.** (b) **19.** (a)

Explanations

1. The alphabets in the word are coded as shown below:

So PEARL is coded as 24153.

2. The alphabets in the word are coded as shown below:

So RIFFLE is coded as 619982.

3. The alphabets in the word are coded as shown below:

So SEARCH is coded as 214673.

4. The alphabets in the word are coded as shown below:

So TWELVE is written as 863203.

5. The alphabets in the word are coded as shown below:

So HIGH is coded as 8978.

Questions 6 to 8: The alphabets in the word are coded as follows:

- 6. So NEATNESS is coded as 21732199.
- 7. So ARREST is coded as 744193.
- 8. So ENDEAR is coded as 128174.

Questions 9 to 11: The alphabets in the word are coded as follows:

- 9. So REAL is coded as 6713.
- 10. So COACH is coded as 49145.
- 11. So COLLAR is coded as 493316.

Questions 12 to 15: The alphabets in the word are coded as shown below:

- 12. So DISTANT is coded as 3765485.
- 13. So ASSIST is coded as 466765.
- 14. So INTIMATE is coded as 78579452.
- 15. So STAIN is coded as 65478.
- 16. The alphabets in the word are coded as shown below:

So CRAPE is coded as 76421.

17. The alphabets are coded as shown below:

So MALE is coded as 6217.

18. The alphabets are coded as follows:

So RIVETS is coded as 234769.

19. Here, in the code, the letters at odd places are one place ahead and those at even places are one place before the corresponding letter in the word.

So 3 is coded as 4, 5 as 4, 7 as 8, 9 as 8 and 1 as 2. So 35791 is coded as 44882.

20. Here code follows same pattern as in Q. 19.

So 46823 is coded as 55914.

Blood Relations

TYPE 1

In these tests, the success of a candidate depends upon the knowledge of the blood relations.

Important Relations

Mother's or father's son : Brother
Mother's or father's daughter : Sister
Mother's or father's brother : Uncle
Mother's or father's sister : Aunt

Mother's or father's mother : Grandmother

Mother's or father's father : Grandfather

Son's wife : Daughter-in-law

Daughter's husband:Son-in-lawHusband's or wife's sister:Sister-in-lawHusband's or wife's brother:Brother-in-law

Brother's son : Nephew
Brother's daughter : Niece
Uncle or aunt's son or daughter : Cousin

Sister's husband : Brother-in-law Brother's wife : Sister-in-law

EXAMPLE 1 Pointing to a photograph, a man tells his friend, 'She is the daughter of the only son of my father's wife'. How is the girl in the photograph related to the man?

(a) Daughter (b) Cousin (c) Mother (d) Sister (e) Niece

Solution The relation may be represented as follows:

Father's wife—mother; mother's only son—himself. So the girl is man's daughter and the answer is (a).

	_		s the daughter of the woman, who in the relation of the lady to Suresh?	is
	(a) Aunt	(b) Granddaughter	(c) Daughter	
	(d) Sister	(e) Sister-in-law		
Moth		ther's mother—Grand	ows: Imother; Grandmother's daughter– Suresh's aunt and the answer is (a	
_	MPLE 3 A girl introduce. The boy is girl's	ced a boy as the son c	of the daughter of the father of he	r
	(a) Brother (b) Son	(c) Uncle	(d) Son-in-law (e) Nephew	
Fath	ation The relation may be ser of uncle—Grandfather other—Brother. So the an	; Daughter of grandfa swer is (a).	ws: ther—Uncle's sister or mother; So	n
		EXERCISE 1		K
1.	Introducing a man to her higrandfather. How is the wor		is brother's father is the only son of m? [Bank PC	-
	(a) Mother (b) Aunt	(c) Sister	(d) Daughter (e) Grandmother	r
2.	Pointing to a lady a girl said only son'. How is the lady re		n-law of the grandmother of my father	's
	(a) Sister-in-law	(b) Mother	(c) Aunt	
	(d) Mother-in-law	(e) Cousin		
3.	Introducing a man a woma woman related to the man?	n said, He is the only s	on of my mother's mother. How is th [Officer's Grade	
	(a) Mother (b) Aunt	` '	(d) Niece (e) None of these	
4.	Introducing a man, a woman man related to the woman?	n said, 'His wife is the on	ly daughter of my father'. How was tha [AO Exan	_
	(a) Brother	(b) Father-in-law	(c) Maternal uncle	
_	(d) Husband	(e) None of these		
5.	How is Ashok related to tha	t lady?	daughter of my grandfather's only son Bank PC)]
	(a) Brother (b) Cous	` '	(d) Uncle (e) None of these	
6.	grandfather'. How is the wo	man related to the man)]
	(a) Mother (b) Aunt	, ,	(d) Daughter (e) Grandmother	
7.	Pointing to a person a man s How was the woman related		ther is the only daughter of your father	٠,٠
	(a) Aunt (b) Moth	` '	(d) Daughter (e) None of these	е
8.		-	ny mother'. Who is Arun to the girl?	
	(a) Father	(b) Grandfather	(c) Husband	
_	(d) Father-in-law	(e) None of these		
9.	the lady related to the man'	?	rother is the brother of my wife'. How	is
	(a) Mother's sister	(b) Grandmother	(c) Mother-in-law	
	(d) Sister of father-in-law	(e) Maternal aunt		

- 10. Pointing to a lady on the platform, Anju said, 'She is the sister of the father of my mother's son'. Who is the lady to Anju?
 - (a) Mother
- (b) Sister
- (c) Aunt
- (d) Niece

(e) None of these

ANSWERS

1. (c) **2.** (b)

(b) **3.** (d)

4. (d)

5. (a)

6. (c)

7. (a)

8. (d)

9. (d)

10. (c)

Explanations

- 1. Only son of her grandfather—her father; man's brother's father man's father; So man's father is her father. i.e. She is man's sister.
- 2. My father's only son—My brother; grandmother of my brother—My grandmother; Daughter-in-law of my grandmother—My mother. So the lady is girl's mother.
- 3. My mother's mother—my grandmother; My grandmother's only son—my maternal uncle. So, the woman is man's niece.
- 4. Only daughter of my father—myself; So, man is woman's husband.
- 5. Grandfather's only son—father; Daughter of father—sister. So Ashok is lady's brother.
- 6. Only son of woman's grandfather—woman's father; Man's brother's father—Man's father. So, the woman is man's sister.
- 7. Only daughter of your father—your sister; Person's mother—woman's sister. So the woman is person's aunt.
- 8. Mother's grandson—son; Son's wife—daughter-in-law. So, Arun is the father-in-law of the girl.
- 9. Brother of my wife—My brother-in-law; Son of lady's brother is brother-in-law of man. So lady's brother is man's father-in-law. i.e. Lady is sister of man's father-in-law.
- 10. Mother's son—brother; My brother's father—father; my father's sister—aunt; So the lady is Anju's aunt.

TYPE 2

In this type mutual blood relations or other information of more than two persons are mentioned and information about any two is questioned.

EXAMPLE 4 A and B are married couple. X and Y are brothers. X is the brother of A. How is Y related to B?

- (a) Brother-in-law
- (b) Brother

(c) Son-in-law

(d) Cousin

(e) None of these

Solution A and B are husband and wife. Since X and Y are brothers, and X is the brother of A, Y is also the brother of A. Thus Y is the brother-in-law of B. Hence the answer is (a).

EXAMPLE 5 A and B are brothers, C and D are sisters. A's son is D's brother. How is B related to C?

(a) Father

(b) Brother

(c) Grandfather

(d) Uncle

(e) None of these

Solution Clearly, the relations can be as follows. B is the brother of A. A's son is D's brother. i.e. D is the daughter of A and hence C is also the daughter of A. So, B is the uncle of C. Hence the answer is (d).

EXAMPLES 6-11

Read the following information carefully and answer the questions given below.

All the six members of a family A, B, C, D, E, and F are travelling together. B is the son of C but C is not the mother of B. A and C are a married couple. E is the brother of C. D is the daughter of A. F is the brother of B.

e aau	gnter of A. F is	tne	brotner o	IВ.					
6.	How many ma	le m	embers a	re the	re in the fan	ily?			
	(a) 1	(b)	3	(c)	2	(d)	4	(e)	5
7.	Who is the mo	ther	of B?						
	(a) D	(b)	F	(c)	E	(d)	A	(e)	None of these
8.	How many chil	ldre	n does A	have?					
	(a) One	(b)	Two	(c)	Three	(d)	Four	(e)	None of these
9.	Who is the wife	e of	E?						
	(a) A			(b) F	י		(c) B		
	(d) Can't be de	terr	nined	(e) N	None of these	9			
10.	Which of the fo	llow	ing is a p	oair of	females?				
	(a) A, E	(b)	B, D	(c)	D, F	(d)	A, D	(e)	None of these
11.	How is E relate	ed to	D?						
	(a) Father			(b) E	Brother		(c) Uncle		

Solution B is the son of C but C is not the mother of B means C is the father of B. A is married to C means A is the mother of B. F is brother of B means F is the son of A and C. E is the brother of C. D is the daughter of A.

(e) None of these

i.e. D is the daughter of A and C.

(d) Can't be determined

Therefore, E is the uncle of D.

- 6. A is the mother and hence female. B is the son and hence male. C is the husband and hence male. D is the daughter and hence female. E is the brother and hence male. F is the son and hence male. So there are four males. The answer is (d).
- 7. Clearly A is the mother of B. The answer is (d).
- 8. A has son B, son F and daughter D. i.e. three children. The answer is (c).
- 9. Clearly, from the data available, the wife of E cannot be determined. The answer is (d).
- 10. Clearly, the females are only the mother A and the daughter D. The answer is (d).
- 11. E is the brother of C who is the father of D. So, E is the uncle of D. The answer is (c).

EXERCISE 2

Read the following information and answer the questions that follow (Q. 1-4).

A is the son of B; C, B's sister has a son D and a daughter E; F is the maternal uncle of D.

1. How is A related to D?

(a) Cousin (b) N

(b) Nephew

(c) Uncle

(d) Brother

(e) None of these

2. How is E related to F?

(a) Sister

(b) Daughter

(c) Niece

(d) Wife

(e) None of these

3.	How many neph	news does F h	ave?					
	(a) Nil	(b) One		(c) Two	(d) T	hree	(e)	None of these
4.	How is F relate	d to E?						
	(a) Uncle		(b)	Brother	((c) Matern	al ur	icle
	(d) Nephew		(e)	None of these				
5.	P is the son of Q son of T, which			the sisters to on atement is corre		T is the mo	other	of R. If S is the
	(a) T is the brot	ther of Q		(b) S	is the cou	ısin of P		
	(c) Q and S are	sisters		(d) S	is the ma	aternal unc	le of	P
	(e) R is the gran	ndfather of P						
6.	P is the brother cannot be define	-	is R's	mother. T is fatl	ner of P. W	hich of the	follov	wing statement
	(a) T is father	of Q	(b)	S is mother of H	•	(c) P is son	of S	
	(d) T is husban	nd of S	(e)	Q is son of T				
7.	A and B both a daughter of C, h			f C is the mother utually related?	r of A, A is	s the son o	f C b	ut B is not the
	(a) A is the bro	ther of B	(b)	A is the sister of	f B	(c) A is the	cou	sin of B
	(d) A is the nep	hew of B	(e)	None of these				
8.	E is the son of A E?	A. D is the son	of B.	E is married to	C. C is dau	ghter of B.	How	is D related to
	(a) Brother		(b)	Uncle	((c) Father	in-la	w
	(d) Brother-in-	law	(e)	None of these				
				another woman	and on hoi	ng asked ah	out h	er relationshir
9.	A woman walking with the boy, silbrothers'. How is	he says, 'My	mate	rnal uncle and l		-		_
9.	with the boy, sl	he says, 'My	mate	rnal uncle and l	nis materr	-	mate	_
	with the boy, sibrothers'. How	he says, 'My is the boy rela (b) Son of D. X is the	mate ated t sister	rnal uncle and loo the woman? (c) Grandson of P. A is the brown.	nis materr (d) H	nal uncle's Iusband	mate (e)	ernal uncle are Brother-in-law
	with the boy, so brothers'. How is (a) Nephew P is the brother	he says, 'My is the boy rela (b) Son of D. X is the	mate ated t sister	rnal uncle and loo the woman? (c) Grandson of P. A is the brown.	nis materr (d) H	nal uncle's Iusband F is the dat	mate (e) ughte	ernal uncle are Brother-in-law
10.	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P	mate ated t sister of A?	rnal uncle and le the woman? (c) Grandson of P. A is the brown (c) F	d) Hother of F.	nal uncle's [usband F is the day	(e) ughte	Brother-in-law er of D. M is the None of these
10. Reac	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X d the following is persons namely	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an	mate ated t sister of A? care d Z ar	rnal uncle and le the woman? (c) Grandson of P. A is the broce. (c) F fully and answer esitting on a pa	(d) Hother of F. (d) Mer the que	nal uncle's Iusband F is the dav I Stions tha	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these ow (Q. 11-14).
10. Read Five Z. Y	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X d the following is persons namely is the brother of	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the	mate ated t sister of A? care d Z ar	rnal uncle and le the woman? (c) Grandson of P. A is the broken (c) F fully and answers sitting on a pa	(d) Hother of F. (d) Mer the que	nal uncle's Iusband F is the dav I Stions tha	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these ow (Q. 11-14).
10. Read Five Z. Y	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X I the following is persons namely is the brother of How is P related	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the	mate ated to sister of A?	rnal uncle and less the woman? (c) Grandson of P. A is the brown of Co. (c) F fully and answers sitting on a passband of P.	d) Hother of F. (d) More the que rk. P is the	nal uncle's (usband F is the dau I stions tha e mother of	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these ow (Q. 11-14).
10. Read Five Z. Y	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X I the following is persons namely is the brother of How is P related (a) Mother	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z?	mate ated to sister of A? care d Z are e hus	rnal uncle and less the woman? (c) Grandson of P. A is the brown of Co. (c) F fully and answers sitting on a particular of P. Aunt	d) Hother of F. (d) More the que rk. P is the	nal uncle's Iusband F is the dav I Stions tha	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these ow (Q. 11-14).
10. Read Five Z. Y: 11.	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X d the following is persons namely is the brother of How is P related (a) Mother (d) Mother-in-lated	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z?	mate ated to sister of A? care d Z are e hus	rnal uncle and less the woman? (c) Grandson of P. A is the brown of Co. (c) F fully and answers sitting on a passband of P.	d) Hother of F. (d) More the que rk. P is the	nal uncle's (usband F is the dau I stions tha e mother of	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these ow (Q. 11-14).
10. Read Five Z. Y: 11.	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X defined the following is the brother of How is P related (a) Mother (d) Mother-in-left How is Y related the following is the brother of How is P related (a) Mother (d) Mother-in-left How is Y related the following is Y related to the following is Y re	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z?	mated to sister of A? care d Z are e hus (b) (e)	rnal uncle and less the woman? (c) Grandson of P. A is the brown of these	(d) Hother of F. (d) Merthe que	nal uncle's (usband F is the day I stions tha e mother of (c) Sister	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these low (Q. 11–14).
10. Read Five Z. Y: 11.	with the boy, sibrothers'. How is a Nephew P is the brother father of X. Who (a) X definition that the following is the brother of How is P related (a) Mother (d) Mother-in-left How is Y related (a) Brother	he says, 'My is the boy rela (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z?	mated to sister of A? care d Z are e hus (b) (e)	rnal uncle and le to the woman? (c) Grandson of P. A is the broke of P. A is the broke of the second of P. Aunt None of these Brother-in-law	(d) Hother of F. (d) Merthe que	nal uncle's (usband F is the dau I stions tha e mother of	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these low (Q. 11–14).
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10. Reac Five Z. Y: 11.	with the boy, sibrothers'. How is (a) Nephew P is the brother father of X. Who (a) X d the following is persons namely is the brother of How is P related (a) Mother (d) Mother-in-lated in Brother (d) Uncle How is X related in Daughter	he says, 'My is the boy reli (b) Son of D. X is the to is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z? aw d to Q?	mated to sister of A? care d Z are huse (b) (e) (b) (e)	rnal uncle and le the woman? (c) Grandson of P. A is the brown of the woman? (c) F fully and answer of the second of P. Aunt None of these Brother-in-law Son Daughter-in-law	(d) Hother of F. (d) Merthe querk. P is the	nal uncle's (usband F is the day I stions tha e mother of (c) Sister	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these low (Q. 11–14).
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10. Read Five Z. Y: 11.	with the boy, sibrothers'. How is Y relate (a) Brother (d) Uncle How is Q relate (d) Aunt How is Q relate (d) Aunt How is Q relate (d) Aunt How is Q relate (d) Property (d) Property (d) Aunt How is Q relate (d) Property (d) Property (d) Aunt How is Q relate (e) Property (d) Property (d) Aunt How is Q relate (e) Property (d) Prop	he says, 'My is the boy relication (b) Son of D. X is the to is the uncle (b) P information P, Q, X, Y, an P and Q is the d to Z? d to Q?	mated to sister of A? care d Z are huse (b) (e) (b) (e)	rnal uncle and le the woman? (c) Grandson of P. A is the brown of the woman? (c) F fully and answer of the second of P. Aunt None of these Brother-in-law Son Daughter-in-law	(d) Hother of F. (d) Merthe querk. P is the	nal uncle's [usband] F is the day [stions thate mother of the continuous co	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these low (Q. 11–14).
10. Read Five Z. Y: 11.	with the boy, sibrothers'. How is Y relate (a) Brother (d) Uncle How is X relate (a) Daughter (d) Aunt	he says, 'My is the boy reli (b) Son of D. X is the o is the uncle (b) P information P, Q, X, Y, an P and Q is th d to Z? aw d to Q? d to Q?	mated to sister of A? care d Z are huse (b) (e) (b) (e) (b) (e) (b) (f) (f) (f)	rnal uncle and le the woman? (c) Grandson of P. A is the brown of the woman? (c) F fully and answer of the second of P. Aunt None of these Brother-in-law Son Daughter-in-law	(d) Hother of F. (d) More the quench rk. P is the	nal uncle's [usband] F is the day [stions thate mother of the continuous co	(e) ughte (e) t follow	Brother-in-lawer of D. M is the None of these low (Q. 11–14).

Study the following information and answer the questions (15-19).

There is a family of six persons A, B, C, D, E, and F. They are lawyer, doctor, teacher, salesman, engineer and accountant. There are two married couples in the family. D, the salesman is married to the lady teacher. The doctor is married to the lawyer. F, the accountant is the son of B and brother of E. C, the lawyer is the daughter-in-law of A. E, is the unmarried engineer. A, is the grandmother of F.

Hov	w is E related t	to F?	•							
(a)	Brother			(b)	Siste	\mathbf{r}		(c) Cousin		
(d)	Can't be deter	rmin	ed	(e)	None	of these				
Wh	at is the profes	ssior	of B?							
(a)	Teacher			(b)	Docto	\mathbf{r}		(c) Lawyer	•	
(d)	Can't be deter	rmin	ed	(e)	None	of these				
Wh	ich is the profe	essio	n of A?							
(a)	Lawyer			(b)	Teac	her		(c) Doctor		
(d)	Can't be deter	rmin	ed	(e)	None	of these				
Wh	ich of the follo	wing	s is one o	f th	e coup	oles?				
(a)	F and D	(b)	D and E	3	(c)	E and A	(d)	A and C	(e)	None of these
Hov	w is D related t	to F	?							
(a)	Grand father	(b)	Father		(c)	Uncle	(d)	Brother	(e)	None of these
l the	e following in	forr	nation c	are	fully	and answ	er the	questions (2	0-25).
nily	consists of six	mei	nbers P,	Q,	R, X,	Y, and Z. (Q is the	son of R but	R is	not the mother
P aı	nd R are a mar	ried	couple.	Y is	the b	rother of R	R. X is th	ie daughter o	f P. 2	Z is the brother
				•						
				.?		**	(3)	**		3.7
` '		` ′			(c)	Y	(d)	X	(e)	None of these
		of Q	?		_			-		
` '			_	` '				(c) R		
					None	of these				
				ve?		_		_		
` /		. ,		_	` '		` '	One	(e)	None of these
				e th				_		
` '		` '			(c)	Three	(d)	Four	(e)	Five
` '		` '		•	(c)	Father	(d)	Husband	(e)	None of these
	=									
(a)	R and Y	(h)	Q and X		(c)	P and Z	(b)	P and X	(e)	None of these
` '			-							
d the	e following in	forr	nation o	are	fully	and answ	er the	questions (2	6–30	
	(a) (d) Wh (a) (d) Wh (a) (d) Wh (a) Hor (a) Hor (a) Hor (a) Hor (a) Wh (a) Wh (a) Hor (a) Wh	(a) Brother (d) Can't be deter What is the profes (a) Teacher (d) Can't be deter Which is the profes (a) Lawyer (d) Can't be deter Which of the follor (a) F and D How is D related to (a) Grandfather d the following in mily consists of six P and R are a mar Who is the brothe (a) P Who is the father (a) P (d) Can't be deter How many childre (a) Four How many female (a) One How is Q related to (a) Uncle Which is a pair of	(a) Brother (d) Can't be determine What is the profession (a) Teacher (d) Can't be determine Which is the profession (a) Lawyer (d) Can't be determine Which of the following (a) F and D (b) How is D related to Final (a) Grandfather (b) (b) the following informally consists of six mendal P (b) Who is the brother-informal P (b) Who is the father of Q (a) P (d) Can't be determine How many children do (a) Four (b) How many female mendal (a) One (b) How is Q related to Xinal (a) Uncle (b) Which is a pair of brother informal pair of brother informal (a) One (b) How is Q related to Xinal (a) Uncle (b) Which is a pair of brother informal (b) Which is a pair of brother informal (c) One (b) How is Q related to Xinal (c) Uncle (c) Which is a pair of brother informal (c) One (d) Which is a pair of brother informal (c) One (d) How is Q related to Xinal (d) Uncle (d) Which is a pair of brother informal (d) One (d) How is Q related to Xinal (d) Uncle (d) Which is a pair of brother informal (d) One (d) How is Q related to Xinal (d) Uncle (d) Which is a pair of brother informal (d) One (d) How is Q related to Xinal (d) Uncle (d) Which is a pair of brother informal (d) One (d) How is Q related to Xinal (d) Uncle (d) Which is a pair of brother informal (d) One	(a) Brother (d) Can't be determined What is the profession of B? (a) Teacher (d) Can't be determined Which is the profession of A? (a) Lawyer (d) Can't be determined Which of the following is one of (a) F and D (b) D and B How is D related to F? (a) Grandfather (b) Father d the following information of mily consists of six members P, P and R are a married couple. Who is the brother-in-law of R (a) P (b) Z Who is the father of Q? (a) P (d) Can't be determined How many children does P hav (a) Four (b) Three How many female members an (a) One (b) Two How is Q related to X? (a) Uncle (b) Brother Which is a pair of brothers?	(a) Brother (b) (d) Can't be determined (e) What is the profession of B? (a) Teacher (b) (d) Can't be determined (e) Which is the profession of A? (a) Lawyer (b) (d) Can't be determined (e) Which of the following is one of the (a) F and D (b) D and B How is D related to F? (a) Grandfather (b) Father d the following information care mily consists of six members P, Q, P and R are a married couple. Y is Who is the brother-in-law of R? 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(a) Uncle (b) Brother (c) Which is a pair of brothers?	(a) Brother (b) Sister (d) Can't be determined (e) None of these What is the profession of B? (a) Teacher (b) Doctor (d) Can't be determined (e) None of these Which is the profession of A? (a) Lawyer (b) Teacher (d) Can't be determined (e) None of these Which of the following is one of the couples? (a) F and D (b) D and B (c) E and A How is D related to F? (a) Grandfather (b) Father (c) Uncle d the following information carefully and answering consists of six members P, Q, R, X, Y, and Z. Q P and R are a married couple. Y is the brother of F Who is the brother-in-law of R? (a) P (b) Z (c) Y Who is the father of Q? (a) P (b) Z (d) Can't be determined (e) None of these How many children does P have? (a) Four (b) Three (c) Two How many female members are there in the family (a) One (b) Two (c) Three How is Q related to X? (a) Uncle (b) Brother (c) Father Which is a pair of brothers?	(a) Brother (b) Sister (d) Can't be determined (e) None of these What is the profession of B? (a) Teacher (b) Doctor (d) Can't be determined (e) None of these Which is the profession of A? (a) Lawyer (b) Teacher (d) Can't be determined (e) None of these Which of the following is one of the couples? (a) F and D (b) D and B (c) E and A (d) How is D related to F? (a) Grandfather (b) Father (c) Uncle (d) d the following information carefully and answer the of the following information carefully and answer the family consists of six members P, Q, R, X, Y, and Z. Q is the P and R are a married couple. Y is the brother of R. X is the Who is the brother-in-law of R? (a) P (b) Z (c) Y (d) Who is the father of Q? (a) P (b) Z (d) Can't be determined (e) None of these How many children does P have? (a) Four (b) Three (c) Two (d) How many female members are there in the family? (a) One (b) Two (c) Three (d) How is Q related to X? (a) Uncle (b) Brother (c) Father (d) Which is a pair of brothers?	(a) Brother (b) Sister (c) Cousin (d) Can't be determined (e) None of these What is the profession of B? (a) Teacher (b) Doctor (c) Lawyer (d) Can't be determined (e) None of these Which is the profession of A? (a) Lawyer (b) Teacher (c) Doctor (d) Can't be determined (e) None of these Which of the following is one of the couples? (a) F and D (b) D and B (c) E and A (d) A and C How is D related to F? (a) Grandfather (b) Father (c) Uncle (d) Brother definition of the following information carefully and answer the questions (2) and R are a married couple. Y is the brother of R. X is the daughter of the father of Q? (a) P (b) Z (c) Y (d) X Who is the brother-in-law of R? (a) P (b) Z (c) Y (d) X Who is the father of Q? (a) P (b) Z (c) Two (d) One How many children does P have? (a) Four (b) Three (c) Two (d) One How many female members are there in the family? (a) One (b) Two (c) Three (d) Four How is Q related to X? (a) Uncle (b) Brother (c) Father (d) Husband Which is a pair of brothers?	(a) Brother (b) Sister (c) Cousin (d) Can't be determined (e) None of these What is the profession of B? (a) Teacher (b) Doctor (c) Lawyer (d) Can't be determined (e) None of these Which is the profession of A? (a) Lawyer (b) Teacher (c) Doctor (d) Can't be determined (e) None of these Which of the following is one of the couples? (a) F and D (b) D and B (c) E and A (d) A and C (e) How is D related to F? (a) Grandfather (b) Father (c) Uncle (d) Brother (e) d the following information carefully and answer the questions (20-25 mily consists of six members P, Q, R, X, Y, and Z. Q is the son of R but R is P and R are a married couple. Y is the brother of R. X is the daughter of P. Z Who is the brother-in-law of R? (a) P (b) Z (c) Y (d) X (e) Who is the father of Q? (a) P (b) Z (c) Y (d) C (c) R (d) Can't be determined (e) None of these How many children does P have? (a) Four (b) Three (c) Two (d) One (e) How many female members are there in the family? (a) One (b) Two (c) Three (d) Four (e) How is Q related to X? (a) Uncle (b) Brother (c) Father (d) Husband (e) Which is a pair of brothers?

and the father of T. U is the grandfather of R and is a contractor. S is grandmother of T and is a housewife. There is one doctor, one contractor, one nurse, one housewife and two students in the family.

9	ß	Who	ie	the	hije	han	А	of '	\mathbf{p}_{2})
4	u.	VV IIO	- 15	LILE	HUS	บลบ	u			

(a) R

(b) U

(c) Q

(d) S

(e) T

- **27.** Who is the sister of T?
 - (a) R

(b) U

(c) T

- (d) Can't be determined
- (e) None of these
- 28. What is the profession of P?
 - (a) Doctor

(b) Nurse

(c) Doctor or Nurse

(d) Housewife

- (e) None of these
- **29.** Which of the following are two married couples?
 - (a) US, QT
- (b) US, QP
- (c) TS, RU
- (d) US, RP
- (e) None of these
- **30.** Which of the following is definitely a group of male members?
 - (a) QU
- (b) QUT
- (c) QUP
- (d) UT
- (e) None of these

ANSWERS

1. (a)	2. (c)	3. (c)	4. (a)	5. (d)	6. (e)	7. (a)	8. (d)	9. (b)	10. (b)
11. (d)	12. (b)	13. (a)	14. (a)	15. (d)	16. (b)	17. (b)	18. (e)	19. (a)	20. (b)

21. (c) 22. (c) 23. (b) 24. (b) 25. (a) 26. (c) 27. (a) 28. (b) 29. (b) 30. (a)

Explanations

- 1. A is the son of B. D is the son of B's sister. So A is the cousin of D.
- 2. E is the daughter of C and D is the son of C. So, F, who is the maternal uncle of D, is also maternal uncle of E. Thus, E is the niece of F.
- 3. Clearly, F is the maternal uncle of D means F is the brother of D's mother, i.e. F is the brother of C. C is the sister of B. So, F is the brother of B, who is mother of A. Thus, F is the maternal uncle of A. So, A and D are the nephews of F. i.e. F has two nephews.
- 4. From Q. 2, E is the niece of F, who is a male. So, F is the uncle of E.
- 5. Q and R are sisters. So, T is the mother of R means T is the mother of Q and R. S is the son of T means S is the brother of Q. Thus, P is the son of Q means S is the maternal uncle of P.
- 6. P, Q and R are children of the same parents. So, S who is mother of R and T, who is father of P will be mother and father of all three. However, it is not mentioned whether Q is male or female. So (e) can't be definitely true.
- 7. B is the child of C, but not daughter means B is the son of C. Also, A is the son of C. So, A is the brother of B.
- 8. C is B's daughter and D is B's son. So, D is the brother of C. E is a male married to C. So, E is the husband of C, whose brother is D. So, D is the brother-in-law of E.
- 9. Boy's maternal uncle will be brother of boy's mother. Maternal uncle of mother's brother and maternal uncle of lady are brothers means lady is sister to mother's brother. i.e., lady is the mother of the boy. So, the boy is woman's son.
- 10. A is the brother of F who is the daughter of D. So, A is the son of D. P is the brother of D. So, P is the uncle of A.
- 11. P is the mother of X who is the wife of Z. So, P is the mother-in-law of Z.
- 12. Q is the husband of P means P is the wife of Q. So, Y is the brother of P who is the wife of Q. Thus, Y is the brother-in-law of Q.
- 13. P is the mother of X who is a female. So, X is the daughter of P. Q is the husband of P. Thus, X is the daughter of Q.
- 14. P is the mother-in-law of Z. So, Q, the husband of P will be father-in-law of Z.

Questions (15 to 19)

Given E is the unmarried engineer. C is the daughter-in-law of A who is the grandmother of F means C is the mother of F. But F is the son of B. So, B is the husband of C. But C, the lawyer is married to the doctor. Thus C, the lawyer is the wife of B who is a doctor. F, the accountant will be the son of B and C. So, the other married couple can be that of grandmother of F. i.e. A and D. But D, the salesman is married to the lady teacher. So D, the salesman is the grandfather of F, father of B and the husband of A, the lady teacher.

- 15. Clearly, from the given data the relation between E and F cannot be determined.
- 16. Clearly, B is a doctor.
- 17. A is the lady teacher.
- 18. The two couples are C and B; and D and A which is none among the choices.
- 19. D is the grandfather of F.

Questions (20 to 25)

Q is the son of R but R is not the mother. So, R is the father of Q. P is married to R. So, P is the wife of R and the mother of Q. X is the daughter of P and hence of R and so it is the sister of Q. Y is the brother of R and Z is the brother of P.

- 20. R is the husband of P and Z is the brother of P. So, Z is the brother-in-law of R.
- 21. R is the father of Q.
- 22. Clearly, Q is the son of P and X is the daughter of P. So P has two children.
- 23. There are two females only—mother P and daughter X.
- 24. X is the sister of Q who is a male. So, Q is the brother of X.
- 25. Clearly, Y is the brother of R who is a male. So, Y and R are a pair of brothers.

Questions (26 to 30)

- Q, the doctor, is the father of T. S, the housewife, is the grandmother of T and hence the mother of Q. Since there are only two married couples one being that of Q, the grandfather of R, i.e. U must be married to S. Thus, R and T will be both children of Q and these must be the students. So, P, who remains, shall be the wife of Q and she alone can be the nurse. Thus, U must be the contractor.
- 26. The husband of P will be Q.
- 27. Clearly, R and T are children of the same parents. So, R will be the sister of T.
- 28. P is the nurse.
- 29. The two married couples are Q, P and U, S.
- **30.** Clearly, the males are Q, the father, and U, the grandfather.

TYPE 3

Coded Relationships

In such questions, the relationships are represented by certain codes or symbols such as +, \times , -, \div etc. Then relationships between certain persons, given in the form of these codes, are to be analysed.

EXAMPLES (12–15)

Read the following information carefully and answer the questions that follow:

A + B means A is the son of B; A - B means A is the wife of B; $A \times B$ means A is the brother of B; $A \div B$ means A is the mother of B and A = B means A is the sister of B.

- 12. What does P + R Q means?
 - (a) Q is the father of P

(b) Q is the son of P

(c) Q is the uncle of P

(d) Q is the brother of P

- 13. What does $P \times R \div Q$ mean?
 - (a) P is the brother of R
 - (c) P is the uncle of Q
- 14. What does P = R + Q mean?
 - (a) P is the aunt of Q
- (c) P is the niece of Q 15. What does $P = R \div Q$ mean?
 - (a) P is the aunt of Q

 - (c) Q is the niece of P

- (b) P is the father of Q
- (d) P is the nephew of Q
- (b) P is the daughter of Q
- (d) P is the sister of Q
- (b) P is the sister of Q
- (d) Q is the daughter of P

Solution

- 12. P + R Q means P is the son of R who is the wife of Q. i.e. Q is the father of P. So, the answer is (a).
- 13. $P \times R \div Q$ means P is the brother of R who is the mother of Q. i.e., P is the uncle of Q. So the answer is (c).
- 14. P = R + Q means P is the sister of R who is the son of Q. i.e. P is the daughter of Q. So, answer is (b).
- 15. $P = R \div Q$ means P is the sister of R who is the mother of Q. i.e., P is the aunt of Q. So the answer is (a).

EXERCISE 3

Read the following information carefully and answer the questions that follow: (Q. 1-8).

 $A \div B$ means A is the father of B, A - B means A is the wife of B; $A \times B$ means A is the brother of B, A ÷ B means A is the daughter of B.

- 1. If $P \div R + S + Q$, which of the following is true?
 - (a) P is the daughter of Q
- (b) Q is the aunt of P
- (c) P is the aunt of Q

- (d) P is the mother of Q
- (e) None of these
- 2. If P R + Q, which of the following statement is true?
 - (a) P is the mother of Q
- (b) Q is the daughter of P
- (c) P is the aunt of Q

- (d) P is the sister of Q
- (e) P is the niece of Q
- 3. If $P \times R \div Q$, which of the following statement is true? (a) P is the uncle of Q
 - (b) P is the father of Q
- (c) P is the brother of Q

- (d) P is the son of Q
- (e) None of these
- **4.** If $P \times R Q$, which of the following is true?
 - (a) P is brother-in-law of Q
- (b) P is the brother of Q
- (c) P is the uncle of Q

- (d) P is the father of Q
- (e) None of these
- 5. If $P + R \div Q$, which of the following is true?
 - (a) P is the brother of Q
- (b) P is the son of Q
- (c) P is the husband of Q

- (d) P is the father of Q
- (e) P is the uncle of Q
- **6.** If $P \div R + Q$, which of the following is true?
 - (a) P is the father of Q
- (b) P is the brother of Q
- (c) P is the mother of Q

- (d) P is the sister of Q
- (e) None of these

7.	If P×	$R \div Q$	which	of the	following	is true?
----	-------	------------	-------	--------	-----------	----------

- (a) P is the uncle of Q
- (b) P is the father of Q
- (c) P is brother-in-law of Q

- (d) P is grandfather of Q
- (e) P is son-in-law of Q

8. If $P - R \times Q$, which of the following is true?

- (a) P is the sister of Q
- (b) Q is the husband of P
- (c) P is the sister-in-law of Q

- (d) Q is the sum of P
- (e) None of these
- 9. If A + B means A is the mother of B; $A \div B$ means A is the brother of B; $A \times B$ means A is the son of B and A B means A is the daughter of B, which of the following means C is the niece of D?
 - (a) D-C
- (b) $D \times P C$
- (c) $C P \div D$
- (d) $P + D \div C$
- (e) $D P \div C$
- 10. If A + B means A is the sister of B, A B means A is the brother of B, $A \times B$ means A is the daughter of B, which of the following shows the relation that E is the maternal uncle of D?
 - (a) $D + F \times E$
- (b) $D F \times E$
- (c) $D \times F + E$
- (d) $D \times F E$
- (e) None of these

ANSWERS

1. (c) 2. (a) 3. (d) 4. (a) 5. (c) 6. (d) 7. (a) 8. (c) 9. (c) 10. (c)

Explanations

- 1. $P \div R + S + Q$ means P is the daughter of R who is the father of S who is the father of Q i.e. P is the sister of S who is the father of Q i.e. P is the aunt of Q.
- 2. P R + Q means P is the wife of R who is the father of Q, i.e. P is the mother of Q.
- 3. $P \times R \div Q$ means P is the brother of R who is the daughter of Q, i.e. P is the son of Q.
- 4. $P \times R Q$ means P is the brother of R who is the wife of Q, i.e. P is the brother-in-law of Q.
- 5. $P + R \div Q$ means P is the father of R who is the daughter of Q, i.e. P is the father of R and Q is the mother of R, i.e. P is the husband of Q.
- 6. $P \div R + Q$ means P is the daughter of R who is the father of Q, i.e., P is the sister of Q.
- 7. $P \times R + Q$ means P is the brother of R who is the father of Q, i.e. P is the uncle of Q.
- 8. $P R \times Q$ means P is the wife of R who is the brother of Q, i.e. P is the sister-in-law of Q.
- 9. C is the niece of D means C is the daughter of the brother (say P) of D, i.e. $C P \div D$.
- 10. E is the maternal uncle of D means D is the daughter of the sister (say F) of E, i.e. $D \times F + E$.

Puzzle Test

This chapter includes questions in the form of puzzles involving certain number of items, which may be persons or things. You are required to analyse the given information, condense it in a suitable form and answer the questions asked.

EXAMPLE 1 Read the following information carefully and answer the questions given below:

- 1. There are five friends—Sonal, Ratheesh, Manoj, Ashok and Gireesh.
- 2. Sonal is shorter than Ratheesh but taller than Gireesh.
- 3. Manoj is the tallest.
- 4. Ashok is a little shorter than Ratheesh and little taller than Sonal.

Questions:

1.	w no is the shor	test	:						
	(a) Gireesh	(b)	Sonal	(c)	Ashok	(d)	Ratheesh	(e)	None of these
2.	If they stand in	the	order of t	heir	heights, v	who v	will be in the n	nidd	le?
	(a) Ratheesh	(b)	Gireesh	(c)	Sonal	(d)	Ashok	(e)	None of these
3.	Who is the seco	nd t	allest?						
	(a) Sonal	(b)	Ratheesh	(c)	Ashok	(d)	Gireesh	(e)	None of these
4.	Who is taller th	an A	Ashok but	sho	rter than I	Mano	oj?		
	(a) Manoj	(b)	Ratheesh	(c)	$\operatorname{Gireesh}$	(d)	Ashok	(e)	Sonal
5 .	If they stand in	the	order of i	ncre	easing heig	ghts,	who will be th	e se	cond?
	(a) Ashok	(b)	Sonal	(c)	Gireesh	(d)	Ratheesh	(e)	None of these

Solution Let the friends be denoted by the first letter of each name, namely S, R, M, A

and G. Then G < S < R S < A < R

Manoj is the tallest.

$$\therefore \qquad \qquad G < S < A < R < M$$

- 1. (a) Gireesh is the shortest.
- 2. (d) Ashok is in the middle.
- 3. (b) Ratheesh is the second tallest.
- 4. (b) Ratheesh is taller than Ashok but shorter than Manoj.
- 5. (b) In the order of increasing heights, Sonal is in the second position.

EXERCISE 1

Directions (Q. 1 to 5): Read the following information carefully and answer the questions given below:

- 1. There is a group of six persons A, B, C, D, E and F from a family. They are Professor, Manager, Lawyer, Jeweller, Doctor and Engineer.
- 2. The Doctor is the grandfather of F who is a Professor.
- 3. The Manager D is married to A.
- 4. C, the Jeweller is married to the Lawyer.
- 5. B is the mother of F and E.
- 6. There are two married couples in the family.

[Bank PO]

Questions:

1. What is the profession of I

- (a) Doctor
- (b) Jeweller
- (c) Manager
- (d) Professor
- (e) None of these

- 2. How is A related to E?
 - (a) Brother
- (b) Uncle
- (c) Father
- (d) Grandfather (e) None of these
- 3. How many male members are there in the family?

(b) Lawver

(a) One

(b) Three

(c) Four

- (d) Data inadequate
- (e) Can't be determined
- 4. What is the profession of A?
 - (a) Doctor

- (c) Jeweller
- (d) Manager
- (e) None of these
- 5. Which of the following is one of the pairs of couples in the family?
 - (a) AB

(b) AC

(c) AD

- (d) Can't be determined
- (e) None of these

Directions (Q. 6 to 10): Study the following information carefully and answer the questions given below it:

Five friends A, B, C, D and E are sitting on a bench.

- 1. A is sitting next to B.
- 2. C is sitting next to D.
- 3. D is not sitting with E.
- 4. E is on the left and of the bench.
- 5. C is on second position from the right.
- 6. A is on the right side of B and to the right side of E.
- 7. A and C are sitting together.

Questions:

- **6.** Where is A sitting?
- (a) Between B and D
- (b) Between D and C
- (c) Between E and D

- (d) Between C and E
- (e) Between B and C

7	Who is sitting in	the centre?							
٠.	(a) A	(b) B		(c)	C	(d)	D	(۵)) E
Q	C is sitting between	() —		(0)	O	(u)	D	(6)	, 1
0.	(a) B and D	een	(h)	A and	d E		(c)	D and E	
	(d) A and D		` ′	A an			(0)	D and D	
9	What is the posit	tion of D?	(0)	n an	ав				
υ.	(a) Extreme left		(h)	Extra	eme right		(c)	Third from	left
	(d) Second from				of these		(0)	imia iiom	icit
10.	What is the posit		(0)	110110	of these				
200	(a) Second from		(b)	Cent	re		(c)	Extreme le	ft
	(d) Second from				of these		(0)		
Diro	ctions (Q. 11 to					an ear	cofull	v and anew	or the augetion
	below it:	10). Study	die ic)110 W 1	ing information	on car	cruii	y and answ	[Bank PO]
-	nining college has	s to conduct	a refi	eshe	r course for t	teach	ers o	f seven diffe	
Mech	nanics, Psychology to 29th July.								
1.	The course shoul	ld start with	n Psych	ology	у.				
2.	23rd July, being	Sunday, she	ould be	e holi	day.				
3.	Science subject s	hould be on	the pr	eviou	us day of the	e ngin	eerin	g subject.	
4.	The course shoul	ld end with	Mecha	nics.					
5.	Philosophy shoul	ld be immed	liately	after	the holiday.				
6.	There should be	a gap of one	day b	ewee	n Economics	and E	Ingin	eering.	
Ques	stions:								
11.	The refresher cou	urse will sta	rt witl	h whi	ch one of the	follov	ving	subjects?	
	(a) Psychology	(b) Mech	anics	(c)	Philosophy	(d)	Ecor	nomics (e)	None of these
12.	Which subject wi	ill be on Tue	esday?						
	(a) Mechanics	(b) Engir	neering	(c)	Economics	(d)	Psyc	chology (e)	None of these
13.	Which subject pr								
	(a) Economics	_			Philosophy	` '	•	chology (e)	None of these
14.	How many day's		e betwe						
	(a) One	(b) Two			Three	(d)	No g	gap (e)	None of these
15 .	Which subject is					. = .	_		
	(a) Engineering	(b) Psych	ology	(c)	Philosophy	(d)	Ecor	nomics (e)	None of these
	ctions (Q. 16 to below it.	18): Read	the fo	llowi	ng informatio	n cai	efull	y and answ	er the questions
1.	Six friends A, B,	C, D, E and	l F are	sittii	ng in a closed	circle	e faci	ng the centr	·e.
2.	E is to the left of	D.							
3.	C is between A a	nd B.							
4.	F is between E a	nd A.							
Ques	stions:								
16.	Who is to the left	t of B?							
	(a) A	(b) C		(c)	D	(d)	\mathbf{E}	(e)	None of these
17.	Who is to the rig								
	(a) A	(b) B		(c)		(d)	\mathbf{E}	(e)	F
18.	Which of the abo		itemen						
	(a) 1	(h) 2		(c)	3	(d)	1	(۵)	None of these

ANSWERS

- 1. (e) 2. (d) 3. (e) 4. (a) 5. (c) 6. (e) 7. (a) 8. (d) 9. (b) 10. (d)
- 11. (a) 12. (c) 13. (e) 14. (a) 15. (d) 16. (c) 17. (a) 18. (e)

Explanations

Given F is a professor, B is the mother of F and E implies that E is a brother or sister of F.

There are only two married couples in the family.

Since D is married to A, C, the jeweller who is married to a lawyer who will be B.

Manager D is married to A means A is the doctor and grandfather of F and E. So E must be an engineer.

1. Here E is an engineer.

So the answer is 'None of these'

- 2. A is the grandfather of F and E is the brother or sister of F. So A is the grandfather of E.
- 3. Since nothing is mentioned about E and F, the number of males cannot be determined.
- 4. A, who is the grandfather of F is doctor.
- 5. D is the manager married to A. So AD is one of the couples in the family.

Questions 6 to 10

- 6. A is sitting between B and C.
- 7. A is sitting in the centre.
- 8. C is sitting between A and D.
- 9. D is on the extreme right.
- 10. B is second from left.

veen A and D. 5 4 3 2 1 eme right.

Questions 11 to 15

22	23	24	25	26	27	28	29
Psychology	Sunday	Philosophy	Economics	Science	Engineering	Sociology	Mechanics

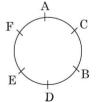
- 11. The course will start with Psychology.
- 12. Economics will be on Tuesday.
- 13. Sociology precedes Mechanics.
- 14. There is only one day's gap between Science and Philosophy.
- 15. Economics is followed by Science.

Questions 16 to 18

Seating arrangement is as shown:

- 16. D is to the left of B.
- 17. A is to the right of C.
- 18. Here all statements are necessary to determine the arrangement.

 Therefore none of these is superfluous.



EXERCISE 2

Examine the following relationships among members of a family of 6 persons A, B, C, D, E, and F.

- 1. The number of males equals that of females.
- 2. A and E are sons of F.

[IAS]

3.	D is the mother of two, one boy and one girl	
4.	B is the son of A.	
5.	There is only one married couple in the fam	aily at present.
1.	Which one of the following references can b	e drawn from the above?
	(a) A, B and C are all females	(b) A is the husband of D
	(c) E and F are the children of D	(d) D is the granddaughter of F

2. A, B, C, D, E, and F not necessarily in that order are sitting in 6 chairs regularly placed around a round table. It is observed that

A is between D and F

C is opposite to D

D and E are not on neighbouring chairs.

Which one of the following must be true?

[IAS]

(a) A is the opposite to B

(b) D is the opposite to E

(c) C and B are neighbours

- (d) B and E are neighbours
- 3. A, B, C, D, E and F, not necessarily in that order are sitting on six chairs regularly placed around a round table. It is observed that

A is between D and F

C is opposite to D and

D and E are not on neighbouring chairs.

Which one of the following pairs must be sitting on neighbouring chairs?

[IAS]

(a) A and E

(b) C and E

(c) B and F

(d) A and C

4. A, B, C, D, E, F, and G are members of a family consisting of 4 adults and 3 children, two of whom, F and G are girls. A and D are brothers and A is a doctor. E is an engineer married to one of the brothers and has two children. B is married to D and G is their child. Who is C?

[IAS]

- (a) G's brother
- (b) F's father
- (c) E's daughter
- (d) A's son.
- 5. Seven persons P, Q, R, S, T, U, and V participate in and finish all the events of a series of swimming races. There are no ties at the finish of any of the events. V always finishes somewhere ahead of P. P always finishes somewhere ahead of Q. Either R finishes first and T finishes last or S finishes first and U or Q finishes last. If in a particular race V finished fifth, then which one of the following would be true?
 - (a) S finishes first (b) R finishes second (c) T finishes third (d) R finishes fourth
- **6.** Consider the following:
 - 1. Saxena, David, Jain and Kumar were District Collectors at places P, Q, R and S respectively in 1970.
 - In 1972 they were transferred, Saxena and Jain interchanged places. Kumar and David also interchanged places.
 - 3. One year later in 1973 they were again transferred such that David and Jain interchanged places and Saxena and Kumar were also interchanged.

What should be the next round of transfers so that all the four persons could have been posted at all the four places? [IAS]

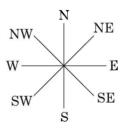
- (a) Interchange Saxena and David as well as Jain and Kumar.
- (b) Interchange Saxena and Kumar as well as David and Jain.
- (c) Interchange David and Kumar as well as Saxena and Jain
- (d) It is not possible for all the four persons to have been posted at all the four places.

7.	violinists. Girija and V how to play the violin. and Thanuja are all vo	a group of six women, there are four dancers, four vocal musicians, one actress and three clinists. Girija and Vanaja are among the violinists while Jalaja and Shailaja do not know to play the violin. Shailaja and Thanuja are among the dancers, Jalaja, Vanaja, Shailaja d Thanuja are all vocal musicians and two of them are also violinists. If Pooja is an actress to among the following is both a dancer and a violinist?						
0	(a) Jalaja	(b) Shailaja	(c) Thanuja	(d) Pooja				
8.	one postgraduate in c the bank employees v among the table tenni economics of whom to	ommerce and three bar while Amala and Koma is players. Amala, Kam wo are bank employees ring is both a table tenr	nk employees. Vimala ala are unemployed. K ala, Komala and Nirma s. If Shyamala is a pos	graduates in economics, and Kamala are among omala and Nirmala are ala are postgraduates in stgraduate in commerce employee? [IAS]				
_	(a) Nirmala	(b) Vimala	(c) Amala	(d) Komala				
9.	A is seated between D opposite to B?	and F at a round table	e. C and not E is seated	l opposite to D. Who sits [IAS]				
	(a) A	(b) D	(c) C	(d) F				
10.	are not terminal stops after A. D is the first s the downward journey	s. C comes twice as man stop in the downward jo y	y stops before D in upv urney. Give the correct	he middle stop. A and E vard journey as B comes sequence of the stops in [IAS]				
	(a) DEACB	(b) DAECB	(c) DACEB	(d) DCBAE				
11.		ınd R are industrious. A		honest, L, M, and N are dustrious nor ambitious [IAS]				
	(a) K alone	(b) L and R	(c) M and N	(d) None in the group				
12.	Six roads lead to a couthere is storm, Y is bl	ntry. They may be indicocked. When there are	cated by letters X, Y, Z, floods X, 1 and 2 will b	and digits 1, 2, 3. When be affected. When road 1 storm also blows, which [IAS]				
	(a) Z and 2	(b) Only Z	(c) Only 3	(d) Only Y				
13.		motor cycle have equal after travelling a dista	_	qual stopping forces are ectively, then				
				[IAS]				
	(a) $X > Y > Z$	(b) $X < Y < Z$	(c) $X = Y = Z$	(d) $X = 4 Y = 8Z$				
14.	a cap of a different col see in front of him gre	our like violet, indigo, ken and blue, but not viours other than orange.	olue, green, yellow, ora olet. E can see violet ar	der. Each one is wearing nge and red, D is able to nd yellow, but not red. G go coloured cap, then the [IAS]				
	(a) Blue	(b) Violet	(c) Red	(d) Orange				
D are	e ladies who are unma and. B is the brother o		. Of the married coup	nan and an artist. A and le in the group E is the it. E's wife is an artist.				
15.	Who is the professor?	(1) D	()	(I) D				
10	(a) A	(b) B	(c) C	(d) D				
16.	Who is the artist?	(L) D	(-) C	(J) T				
	(a) A	(b) B	(c) C	(d) E				

17.	Who is the wi	fe of E?							
	(a) A	((b) B		(c) C		(d)	D	
18.	Which of the	following a	groups inc	lude all th	ne men?				
	(a) BE	((b) ABC		(c) BC	D	(d)	None of t	$_{ m hese}$
righ	C, D, and E a t of D who is trician. In bet	an advoc	ate. B is	a scienti	st and is	standing	g to the le	eft of E w	ho is an
19.	Who is in the	middle?							
	(a) A	((b) B		(c) C		(d)	D	
20.	Who is to the	extreme l	eft?						
	(a) A	((b) B		(c) C		(d)	D	
21.	Who is to the	extreme r	right?						
	(a) A	((b) B		(c) C		(d)	D	
22 .	How many pe	rsons are	there to tl	he right of	E?				
	(a) 4	((b) 3		(c) 2		(d)	1	
23.	Six persons A E and C. F is						tween D a	ınd C. A is	s between
	(a) B	((b) C		(c) D		(d)	E	
24.	300 persons a Indians. Out of and 35 are we the meeting?	of the Indi omen judg	ians, there es. There	e are 110 i	men who eign judg	are not ju	dges, 160 any India	are men o in women	or judges
	(a) 45	`	(b) 55		(c) 35		(d)		
25.	In an examine examinees are list B. Furthe (i) No exam (ii) Answers	e requeste r it was gi inee has g	d to match ven that given the c	each iter correct ans	n under li swer.	st A with i	its corresp	oonding it	em under
	the test		exammee	s were rue	entical. F	mu the nu	imber or e	xammees	attenue
	(a) 24	((b) 26		(c) 119	9	(d)	129	
				ANSW	ERS				
1.	(b) 2. (d)	3. (b)	4. (d)	5. (a)	6. (c)	7. (c)	8. (a)	9. (a)	10. (a)
11.	(d) 12. (c)	13. (a)	14. (c)	15. (b)	16. (d)	17. (c)	18. (a)	19. (d)	20. (b)
21.	(c) 22. (b)	23. (d)	24 . (b)	25. (a)					
				. ,					

Direction Sense Test

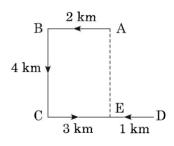
In this test, questions consist of a sort of direction puzzle. A successive follow-up of directions is formulated and the candidate is required to ascertain the final direction or the distance between two points. The test is meant to judge the candidate's ability to trace and follow correctly and sense the direction correctly.



EXAMPLE 1 Ravi walked 2 km west of his house and then turned south covering 4 km. Finally, he moved 3 km towards east and then again 1 km west. How far is he from his initial position?

- (a) 2 km
- (b) 4 km
- (c) 9 km
- (d) 10 km

Solution Ravi started from his house at A, moves 2 km west up to B, 4 km towards south up to C, 3 km east up to D and finally 1 km west up to E. Thus, his distance from the initial position A = AE = BC = 4 km. Hence the answer is (b).



EXAMPLE 2 A man walks 6 km to the east and then turns to the south and walks 5 km. Again he turns to the east and walks 6 km. Next, he turns towards north and walks 10 km. How far is he now from his starting point?

- (a) 5 km
- (b) 12 km
- (c) 13 km
- (d) 17 km
- (e) 11 km

Solution The man starts from the point A and walks 6 km, 5 km, 6 km and 10 km respectively towards east, south, east and north respectively.

E

Draw BO | DE

Now BO = CD = 6 km

$$\therefore$$
 AO = AB + BO = 6 + 6 = 12 km
OE = DE - DO = 10 - 5 = 5 km

... Man's distance from his starting point

$$AE = \sqrt{AO^2 + OE^2} + \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ km}$$

Hence the answer is (c).

EXAMPLE 3 Arun moved a distance of 75 metres towards north. He then turned to the left and walked for about 25 metres, turned left again and walked 80 metres. Finally he turned to the right at an angle of 45°. In which direction was he moving finally?

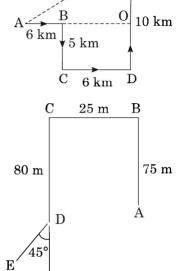
- (a) North-east
- (b) North-west
- (c) South
- (d) South-east (e) South-west

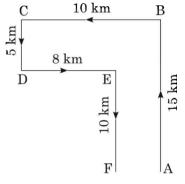
Solution Arun started from A, moved 75 m towards north up to B, turned left and walked 25 m up to C. He then turned left again and moved 80 m up to D. Turning to the right at an angle of 45°, she was finally moving in the direction DE, i.e. south-west. Hence, the answer is (e).

EXAMPLE 4 Satheesh left for his office in his car. He drove 15 km towards north and then 10 km towards west. He then turned to the south and covered 5 km. Further, he turned to the east and moved 8 km. Finally, he turned right and drove 10 km. How far and in which direction is he from his starting point?

- (a) 2 km west (b) 5 km east (c) 3 km north
- (d) 6 km south (e) None of these

Solution Satheesh started from A towards B and moved a distance of 15 km and 10 km from B to C towards west. He then moved from C to D, 5 km towards south and





8 km towards E. Finally he turned right and moved a distance of 10 km up to F.

Now F and A are in the same straight line and F lies to the west of A.

:. Distance from starting point A

$$= AF = BC - DE = 10 - 8 = 2 \text{ km}$$

Hence, the answer is (a).

EXERCISE 1

- 1. Kishore walks 10 km towards north. Then he walked 6 km towards south. Then he walks 3 km towards east. How far and in which direction is he with reference to his starting point?
 - (a) 7 km east
- (b) 5 km west
- (c) 5 km north east (d) 7 km west

(a) South-east

2. (b)

3. (a)

1. (c)

(b) North-east

4. (b)

2.	A man leaves for of 20 m, he turns further 5 m towar distance in metre	s towards south rds north. He the	and walks 10 en turns toward	m. Then he wa ds east and walk	lks 35 m tow	ards west and	
	(a) 0	(b)	5	(c)	10		
	(d) None of these	e (e)	Can't be dete	ermined			
3.	James started from towards right and his left again and his left. In which	d walked 20 m. H l walked 40 m. H direction is he v	Ie then turned Ie now turns to walking now?	left and moved a the left and wa	a distance of 1 lks 5 m. Final	.0 m, turned to lly, he turns to	
	(a) North	(b) South	(c) East	(d) Sout		West	
4.	Sudheesh walked moved to his right and moved a dist point?	ht. Then he turi	ned right and	walked 20 metr	es. Finally h	e turned right	
	(a) 10 m north	(b) 20 m soutl	n (c) 20 m n	orth (d) 10 m	ı south (e)	None of these	
5.	A man walked 30 to his left, he wal initial position?						
	(a) 30 m	(b) 20 m	(c) 80 m	(d) 60 m	n (e)	None of these	
6.	Gopal started wa and walked 5 km. and walked 9 km	. Again, he turne	ed to east and v	valked 2 km. Fir			
	(a) 7 km	(b) 5 km	(c) 4 km	(d) 3 km	n (e)	None of these	
7.	Sumesh went 15 Then he turned so direction is he fro	outh and covered					
	(a) East	(b) West	(c) North	(d) Sout	:h		
8.	After walking a distance of 50 m to the south of her house, Lakshmi turns left and walks another 20 m. Then she turns north and walks 30 m and then starts walking to her house. In which direction is she walking now?						
	(a) North-west	(b) North	(c) South-	east (d) East	;		
9.	Dinesh walks 20 walks 35 metres. Finally, turning loriginal position?	Now, turning left, he again wal	t, he walks 15	m. Again, he tur	ns left and mo	oves 15 metres.	
	(a) 15 m east	(b) 45 m east	(c) 15 m w	` ′	` '	None of these	
10.	Anil's house faces turns to the righ walking 25 m. No	nt and walks 50	m again. Fin	ally, he turns to	owards left a		

(ANSWERS)

5. (e)

6. (b)

(c) South-west (d) North-west (e) None of these

8. (a)

9. (e)

10. (d)

7. (c)

Explanations

1. The movements of Kishore are shown in the adjoining figure.

$$(A \to B \to C \to D)$$

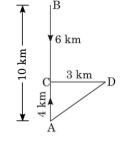
$$AC = AB - BC = 10 - 6 = 4 \text{ km}$$

 $CD = 3 \text{ km}$

:. AD =
$$\sqrt{AC^2 + CD^2} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

So, Kishore is 5 km north-east.

2. Movements of man shown in the adjoining figure.

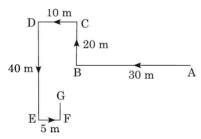


Here, DC = AB + EF

.. F is in a line with A.

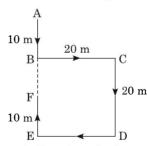
$$AF = BC - ED = 10 - 5 = 5 m$$

- .. The man is 5 metres away from his initial position.
- 3. The movements of James are shown in the adjoining figure from A to G.



James is walking in the direction FG, i.e. towards north.

4. The movement of Sudheesh are shown in the adjoining figure.



Distance from initial position of Sudheesh to final position

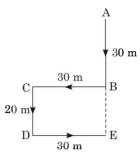
$$AF = AB + BF$$

= $AB + (BE - FE)$
= $10 + (20 - 10)$
= 20 m

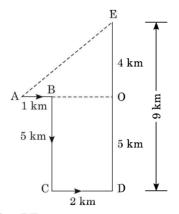
F lies in the south of A.

: 20 m south is the final position of Sudheesh from the starting point.

5. The movements of man are shown in the adjoining figure.



- :. Man's distance from the initial position = AE = AB + BE= AB + CD = 30 + 20 = 50 m.
- 6. The movements of Gopal are shown in the adjoining figure.



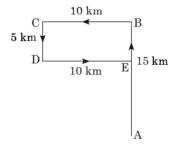
$$BO \perp DE$$

 $BO = CD = 2 \text{ km}$
 $AO = AB + BO = 1 + 2 = 3 \text{ km}$
 $OE = DE - DO = DE - BC$
 $= 9 - 5 = 4 \text{ km}$

:. Distance of Gopal from starting point A.

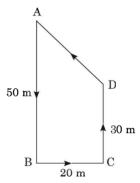
$$= AE = \sqrt{AO^2 + OE^2} = \sqrt{3^2 + 4^2} = 5 \text{ km}$$

7. The movements of Sumesh are shown in the adjoining figure.



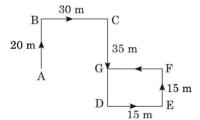
The final position E is north of Sumesh's house at A.

8. The movements of Lakshmi are shown in the adjoining figure.



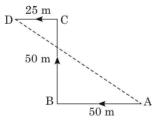
She is moving in the direction DA towards her house. i.e. north-west.

9. The movements of Dinesh are shown in the adjoining figure.



He is moving from A to G.

- :. Distance of Dinesh from initial position = AG = BC = 30 m east.
- 10. The movements of Anil are shown in the adjoining figure.



He starts walking from back of his house, i.e. towards west.

Now, the final position is D, which is to the north west of his starting point A.

EXERCISE 2

- Binoj's school bus is facing north when it reaches his school. After starting from Binoj's house, it turns right twice and then left before reaching the school. What direction was the bus facing when it left the bus stop in front of Binoj's house. [Bank PO]
 - (a) South
- (b) North
- (c) East
- (d) West
- (e) None of these
- 2. A, B, C, and D are playing cards. A and B are partners. D faces towards north. If A faces west, then who faces south?

 [AO Exam]
 - (a) C

(b) B

(c) D

- (d) Data inadequate
- (e) None of these

1. (d)

2. (a)

3. (c) **4.** (b)

5. (d)

6. (d)

7. (c)

8. (c)

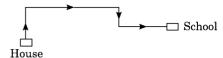
9. (c)

10. (d)

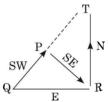
3.	There are four persons P , Q , R and T . Q is to the south-west of P ; R is to the east of Q and south-east of P , and T is to the north of R in line with QP . In which direction of P is T located?
	(a) South-east (b) North (c) North-east (d) East
4.	A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now?
	(a) 50 m (b) 40 m (c) 30 m (d) 20 m
5.	Two ladies and two men are playing cards and are seated at north, east, south and west of a table. No lady is facing east. The persons sitting opposite to each other are not of the same sex. One man is facing south. Which directions are the ladies facing?
	[RBI Officer's Exam]
	(a) East and west (b) South and east (c) North and east
	(d) North and west (e) None of these
6.	One morning after sunrise, Vishakh and Satheesh were standing in a lawn with their backs towards each other. Vishakh's shadow fell exactly towards left hand side. Which direction was Satheesh facing? [BSRB Exam]
	(a) East (b) West (c) North (d) South
7.	The post office is to the east of the school while my house is to the south of the school. The market is to the north of the post office. If the distance of the market from the post office is equal to the distance of my house from the school, in which direction is the market with respect to my school? [AAO Exam]
	(a) North (b) East (c) North-east (d) South-west
8.	From the positions in original figure, A and C move diagonally to opposite corners and then one side each clockwise and anticlockwise respectively. D and B move two sides each clockwise and anticlockwise respectively. Where is A now?
	(a) At the north-west corner
	(b) At he north-east corner
	(c) At the south-east corner
	(d) At the south-west corner
	(e) Midway between original position of B and D.
9.	From the original position given in the above figure, A and B move one arm length clockwise and then cross over to the corner diagonally opposite, C and D move one arm length anticlockwise and cross over the corner diagonally opposite. The original configuration ADBC has now changed to
	(a) CBDA (b) BDAC (c) DACB (d) ACBD (e) BCAD
10.	Facing the east, Rajesh turned left and walked 10 m, then turned to his left again and walked 10 m. He then turned 45° towards his right and went straight to cover 25 m. In which direction from his starting point is he?
	(a) South-west (b) South-east (c) North-west (d) North-east (e) East
	ANSWERS
_	

Explanations

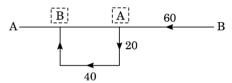
1. In the given diagram, the route of the bus from Binoj's house to the school is shown. As the bus faces north on reaching the school, from the diagram, it faces west in front of Binoj's house.



- 2. As per the data, D faces north. A faces towards west. So, its partner B will face towards A and hence towards east. So, C who will face D will face south.
- 3. Clearly, the arrangement according to the given directions is as shown. So, comparing the direction diagram, T is north east of P.



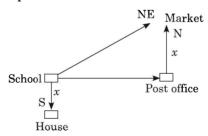
4. If both A and B move with the same speed, they will travel the same distance = 60 + 20 + 40 + 20 = 140 m in the same time. So, A will travel 140 m in the horizontal direction. B travels 100 m horizontally. So distance between A and B will now be the difference = 140 - 100 = 40 m.



5. No lady is facing east means a man faces east. The persons opposite are not of same sex. So a woman will be facing west. Again a man faces south. So a woman will be facing north.

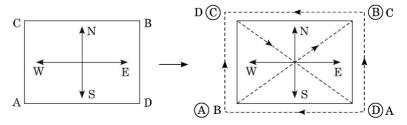


- 6. Since Vishakh's shadow falls towards left, therefore, Vishakh is facing north. So Satheesh with his back towards Vishakh, will be facing south.
- 7. Distance of house from school = x
 - = Distance of market from post office

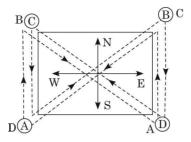


So, direction of market school with respect to school = North-east

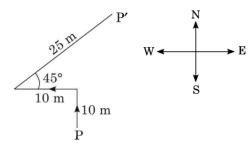
8. A is now at south-east corner.



9. ADBC is changed to DACB.



10. P' is North-east of P



Logical Venn Diagrams

This chapter deals with questions which aim at analysing a candidate's ability to relate a certain given group of items and illustrate it diagrammatically.

Certain types of Venn diagrams are given with its implication.

- 1. If *three* items are given belonging to three different groups, the same can be represented using Venn diagram as shown in Figure 7.1.
 - For example: Doctors, engineers, lawyers.
 - The above three bear no relationship with each other. So they are represented by 3 disjoint figures.
- 2. If one item belongs to the class of second and the second belongs to the class of third, then these items can be represented as three concentric circles as shown in Figure 7.2.
 - For example: Seconds, minutes, hours.
 - Here, seconds are part of minute and minutes are part of hours. So the Venn diagram would be as shown in Figure 7.2 with circle A representing seconds, B representing minutes and C representing hours.
- 3. If *two* separate items belong to the class of the third, they are represented by two disjoint circles inside a bigger circle as shown in Figure 7.3.
 - For example: Table, chair, furniture.
 - Here, table and chair are separate items both belonging to the class of furniture. So they would be represented as two disjoint circles A and B respectively representing table and chair within a circle C representing furniture.



Figure 7.1

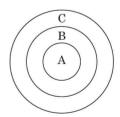


Figure 7.2

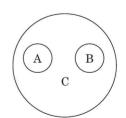


Figure 7.3

4. If two items belong to the class of third such that some items of each of these two groups are common in relationship, then they are represented by two intersecting circles enclosed within a bigger circle as shown in Figure 7.4.

For example: Males, Fathers, Brothers.

Here, some fathers may be brothers. So, fathers and brothers would be represented by two intersecting circles A and B. Also, both fathers and brothers are males. The bigger circle C represents males.

- 5. If two items are partly related to the third, and are themselves independent of each other they are represented by three intersecting circles in a line as shown in Figure 7.5. For example: Dogs, Pets, Cats.
 - Here, some dogs and some cats are pets. But all the pets are not dogs or cats. Dogs and cats are not related to each other. So the given three items can be represented using three circles A, B, and C representing dogs, pets and cats respectively as shown in Figure 7.5.
- 6. If *three* items are partly related to each other, they are represented as shown in Figure 7.6.

For example: Clerks, Government employees, Educated persons.

Here, some clerks may be government employees and some may be educated. Similarly, some government employees may be clerks and some may be educated. Also, some educated persons may be clerks and some may be government employees.

7. If *one* item belongs to the class of second while the third item is entirely different from the two, then they may be represented by Figure 7.7.

For example: Engineers, Human beings, Rats.

Here, all engineers are human beings. This would be represented by two concentric circles. But the class of rats is entirely different from these two. In Figure 7.7, circle A represents engineers, circle B represents human beings and circle C represents rats.

8. If *one* item belongs to the class of second and the third item is partly related to these two, they can be represented as in Figure 7.8.

For example: Females, Mothers, Doctors.

All mothers are females. This would be represented by two concentric circle. But some females and some mothers can be doctors. So the circle representing doctors would intersect the two concentric circles. Thus is Figure 7.8, circle A represents mothers, circle B represents females and circle C represents doctors.

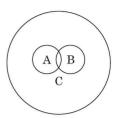


Figure 7.4

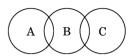


Figure 7.5



Figure 7.6

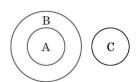


Figure 7.7

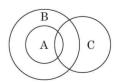


Figure 7.8

9. If one item belongs to the class of second and the third item in partly related to the second, they can be represented as in Figure 7.9.

For example: Grass-eating animals, cows, flesh eating

animals. Here, cows are grass-eating animals. So, they would be

represented by two concentric circles. But some grass-eating

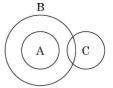


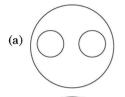
Figure 7.9

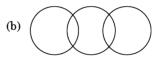
animals are flesh-eating also. Thus the Venn diagram can be represented as in Figure 7.9 with circle A representing cows, circle B representing grass-eating animals and circle C representing flesh eating animals.

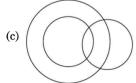
EXERCISES

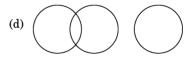
1. Which of the following diagrams correctly represents elephants, wolves, animals?

[IAS]

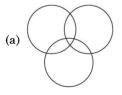


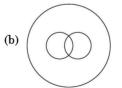


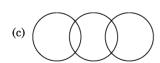


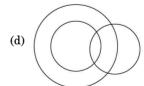


2. Which one of the following diagrams correctly represents the relationship among tennis fans, cricket players and students? [IAS]



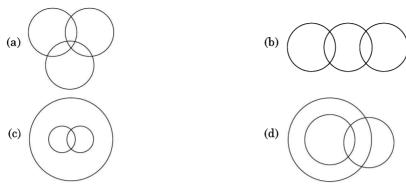




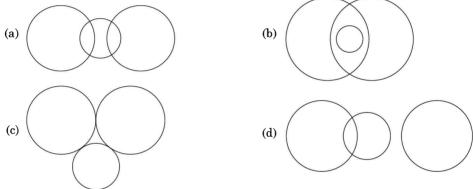


3. Which is the most suitable Venn diagram among the following, which represents interrelationship among anti-social elements, pickpockets and blackmailers?

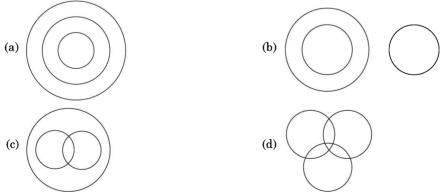
[IAS]



4. If animals that live on land and the animals that live in water are represented by two big circles and animals that live in water and on land are represented by small circle, the combination of these three can be represented as [IAS]

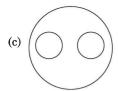


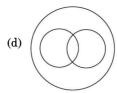
5. Which of the following gives the proper relation of tall men, black haired people, Indians? [IAS]



6. Choose from the four diagrams given below, the one that illustrates the relationship among languages, French, German.

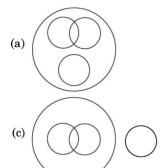


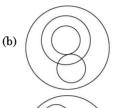




7. In a dinner party both fish and meat were served. Some took only fish and some only meat. There were some vegetarians who did not accept either. The rest accepted both fish and meat. Which one of the following logic diagrams correctly reflects the above situation?

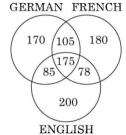
[IAS]







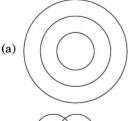
8. A survey was conducted on a sample of 1,000 persons with reference to their knowledge of English, French and German. The results of the survey are presented in the given Venn diagram. The ratio of the number of the persons who do not know any of the three languages to those who know all the three languages is [IAS]

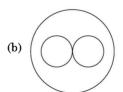


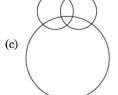
- (a) 1/27
- (c) 1/550

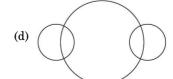
- (b) 1/25
- (d) 175/1000

9. Which one of the following Venn diagrams correctly illustrates the relationship among the classes: carrot, food, vegetable? [IAS]

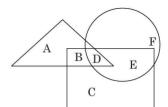








10. In the given figure, the triangle represents girls, the square represents sports persons and the circle represents coaches. The portion in the figure which represents girls who are sports persons but not coaches is the one labelled.
[IAS]

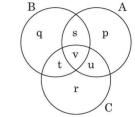


(a) A

(b) B

(c) D

- (d) E
- 11. In the given diagram, circle A represents teachers who can teach Physics, circle B represents teachers who can teach Chemistry and circle C represents those who can teach Mathematics. Among the regions marked p, q, r ... the one which represents teachers who can teach Physics and Mathematics but not Chemistry is [IAS]

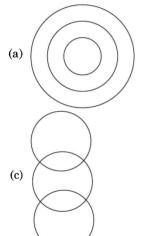


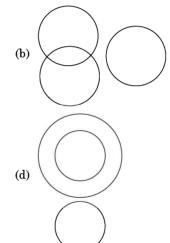
(a) v

(b) u

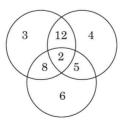
(c) s

- (d) t
- 12. Which one of the following four logical diagrams represent correctly the relationship between Musicians, Instrumentalists, Violinists? [IAS]





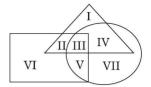
13. Consider the Venn diagram given below.



The numbers in the Venn diagram indicated are persons reading the newspapers. The diagram is drawn after surveying 50 persons. In a population of 10,000 how many can be expected to read at least two newspapers? Choose the correct answer from the following: [IAS]

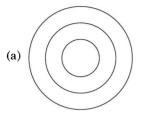
- (a) 5000
- (b) 6250
- (c) 6000
- (d) 5400

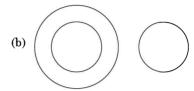
14. The triangle, square and circle shown below respectively represent the urban, hard working and educated people. Which one of the areas marked I-VII is represented by the urban educated who are not hardworking? [IAS]

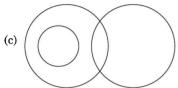


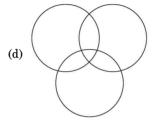
(a) II(c) IV

- (b) I (d) III
- 15. Which one of the following Venn diagrams best illustrates the three classes: rhombus, quadrilaterals and polygons?

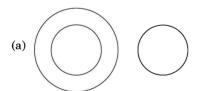


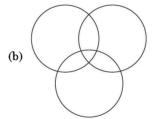


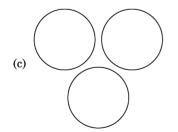


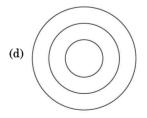


16. Select from the four alternative diagrams, the one that best illustrates the relationship among the three classes: pigeons, birds, dogs.

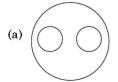






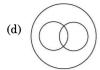


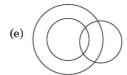
In each of the following questions, choose the Venn diagram which best illustrates the relationship among three given circles.











- 17. Diseases, leprosy, scurvy
- 18. Hockey, cricket, games
- 19. Yak, zebra, bear
- 20. Sun, moon, stars
- 21. Animals, men, plants
- 22. Mercury, mars, planets

Choose the Venn diagram which best illustrates the three given classes in each question. Mark your answer as (a), (b), (c) or (d).

- 23. Protons, electrons, atoms
- 24. Sun, planets, earth
- 25. Dog, animal, pet
- 26. Science, physics, chemistry
- 27. Atmosphere, hydrogen, oxygen
- 28. Wheat, grains, maize

[CBI]

[Railways]

- 29. Machine, lathe, mathematics
- 30. Biology, botany, zoology(a) Indicates that one class is completely contained in the other but not the third.



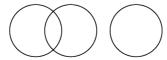
(b) Indicates that two classes are completely contained in the third.



(c) Indicates that neither class is completely contained in the other but the two have common members, forming one entity.



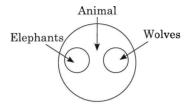
(d) Indicates that two classes are inter-related and third one is not.



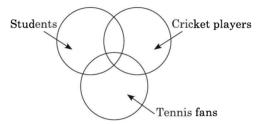
(ANSWERS)									
1. (a)	2. (a)	3. (c)	4. (a)	5. (d)	6. (c)	7. (c)	8. (b)	9. (a)	10. (b)
11. (b)	12. (d)	13. (d)	14. (c)	15. (a)	16. (a)	17. (a)	18. (a)	19. (b)	20. (c)
21. (c)	22. (a)	23. (b)	24. (a)	25. (c)	26. (b)	27. (b)	28. (b)	29. (a)	30. (b)

Explanations

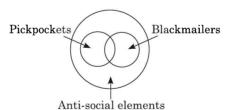
1. Elephants and wolves bear no relationship to each other. But, both of them are animals.



2. Some students can be cricket players. Some cricket players can be tennis fans. Some students can be tennis fans. So, the given items are partly related to each other.

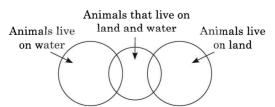


3. Both pickpockets and blackmailers are antisocial elements. But, some pickpockets can be blackmailers and vice versa.

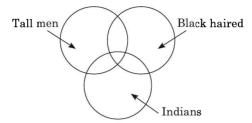


4. Here three classes are given. Two classes are not having any common characteristics. One class is having some common characteristic with the other two.

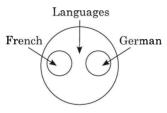
That is, 'Animals live on both land and water' is having common characteristic with the classes 'Animals live on land' and 'Animals live on water'.



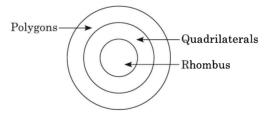
5. Some tall men can be black haired. Some black haired persons can be Indians. Some tall men can be Indians. So, all the three are partly interrelated.



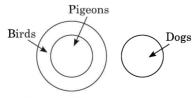
6. Both French and German are languages. But both of them are different from each other.



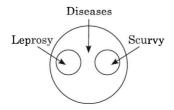
- 8. Required ratio = $\frac{7}{175} = \frac{1}{25}$
- 12. In the given three classes Musicians, Instrumentalists and Violinists,—Violinists come under the class of Instrumentalists. Musicians are different from two classes. Sometimes a musician may also be an Instrumentalist. But from the available choices only (d) is the most appropriate one.
- 15. All rhombus are quadrilaterals. All quadrilaterals are polygons.



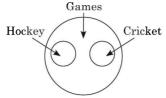
16. All pigeons are birds. But, dogs are entirely different.



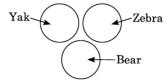
17. Both leprosy and scurvy are diseases. But, both are entirely different from each other.



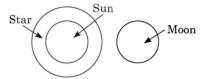
18. Both hockey and cricket are games. But, both are entirely different from each other.



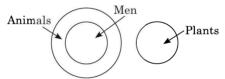
19. Yak, zebra and bear are all different from each other.



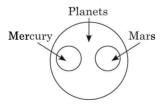
20. Sun is a star, moon is entirely different from the two.



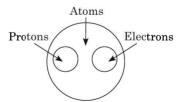
21. Men belongs to the class of animals. Plants are entirely different from the two.



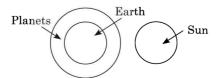
22. Mercury and mars are entirely different from each other. But both are planets.



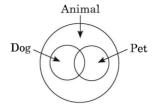
23. Protons and electrons are entirely different from each other. But, both are parts of atoms.



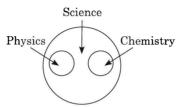
24. The earth belongs to the class of planets. But, sun is entirely different from the two.



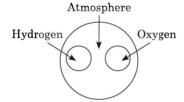
25. Some dogs are pets and some pets are dogs. Both dog and pet are animals.



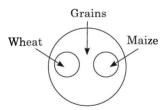
26. Physics and chemistry are entirely different from each other. But, both belong to the class of science.



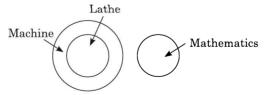
27. Hydrogen and oxygen are entirely different from each other. But, both are parts of atmosphere.



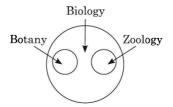
28. Wheat and maize are two different items. But, both belong to the class of grains.



29. Lathe is a type of machine. But, Mathematics is entirely different from the two.



30. Botany and zoology are entirely different from each other. But, both are branches of biology.



Number Ranking and Time Sequence Test

NUMBER TEST: In the number test, you are given a series of numbers. You have to find out how many times a number satisfying the conditions specified in the question occurs.

EXAMPLE 1 How many 7s immediately preceded by 6 but not immediately followed by 4 are there in the following series? [AO Exam]

7 4 2 7 6 4 3 6 7 5 3 5 7 8 4 3 7 6 7 2 4 0 6 7 4 3

(a) 1 (b) 2 (c) 4 (d) 6 (e) None of these **Solution** Clearly, the numbers satisfying the given conditions can be shown as follows: 7 4 2 7 6 4 3 6 7 5 3 5 7 8 4 3 7 6 7 2 4 0 6 7 4 3

Here there are two such 7s in the number series which is preceded by 6 and not followed by 4. Therefore the answer is (b).

EXAMPLE 2 How many 1s are there in the following sequence which are immediately preceded by 9 but not immediately followed by 7? [Bank PO]

 $7\ 1\ 9\ 1\ 1\ 7\ 1\ 8\ 9\ 1\ 7\ 1\ 2\ 1\ 3\ 1\ 4\ 5\ 7\ 1\ 3\ 9\ 1\ 7$

(a) One (b) Two (c) Three (d) Four (e) None of these **Solution** The answer is (a).

EXAMPLE 3 In the following series of numbers, how many times have the figures 9, 1 and 8 appeared together, 1 being in the middle and 9 and 8 being on either side of 1?

[RRB Exam]

2 1 9 8 1 9 8 3 71 9 7 8 1 2 9 1 9 8 1 8 2 1 2

(a) One (b) Six (c) Three (d) Four (e) None of these **Solution** There is only one digit between 9 and 8. Therefore, the answer is (a).

RANKING TEST: In the ranking test, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. However, sometimes the question is put in the form of a puzzle of interchanging seats by two persons.

EXAMPLE 4 Sajith ranked thirteenth from the top and twenty sixth from the bottom among those who have passed in the annual examination in a class. If six students have failed in the annual examination, what was the total number of students in that class?

[Bank PO]

(a) 38

(b) 44

(c) 45

(d) 50

(e) None of these

Solution

Number of students passed = (12 + 1 + 25) = 38

Number of students failed = 6

 \therefore Total number of students = 38 + 6 = 44

EXAMPLE 5 Sanal's position in a row is 12th from the front side and 7th from the back side. How many persons are standing in that row? [Railway Recruitment]

(a) 17

(b) 18

(c) 19

(d) 20

(e) 21

Solution Number of persons in the row = 11 + 6 + 1 = 18

TIME SEQUENCE TEST: In the time sequence test, you will be asked to find a day which falls between two pair of dates. From that you have to locate the exact day.

EXAMPLE 6 The Managing Director entered the conference room ten minutes before 12.30 hours for interviewing. He came 20 minutes before the chairman who was 30 minutes late. At what time, the interviews were scheduled? [AO Exam]

(a) 12.50

(b) 12.40

(c) 12.20

(d) 12.10

(e) 12.00 Noon

Solution The Managing Director came at 12.20. Thus chairman came at 12.40. Since the chairman was late by 30 minutes, interviews were scheduled to be held at 12.10. The answer is (c).

EXERCISES

1. How many 8s are there in the following number sequence which are preceded by 7 but not immediately followed by 4? [SBI PO]

2 3 8 2 5 7 8 3 7 8 4 6 9 8 4 3 2 7 8 9 5 7 8 1 5 2 9

(a) One

(b) Two

(c) Three

(d) Four

(e) None of these

2. How many 3s are there in the following number sequence which are immediately preceded by 6 but not immediately followed by 7? [Bank PO]

2 3 7 4 3 5 6 3 7 4 6 3 8 9 6 3 5 1 8 3 7 2 4 2 8 6 3 9

(a) One

(b) Two

(c) Three

(d) Four

(e) More than four

3. How many numbers from 11 to 50 are there which are exactly divisible by 7 but not by 3?

[Bank PO]

(a) Two

(b) Four

(c) Five

(d) Six

(e) Seven

4. How many numbers from 1 to 100 are there each of which is not only exactly divisible by 4 but also has 4 as a digit? [SBI PO]

(a) 7

(b) 10

(c) 20

(d) 21

(e) More than 21

5.	Abhishek ranks	thirteenth in a	a class of thirty one.	What is his rank	from the last?
					[RBI Exam]
	(a) 15th	(b) 17th	(c) 19th	(d) 20th	(e) None of these
6.	_				s nineth from the right w many girls are there [Bank PO]
	(a) 16	(b) 18	(c) 19	(d) 22	(e) None of these
7.	How many days	will there be fro	om 26th January, 198	88 to 15th May, 198	88 (both days included)?
				[Ra	ilway Recruitment]
	(a) 110	(b) 111	(c) 112	(d) 113	(e) None of these
8.		s that Lakshma			nd November, whereas 14th November. Which
	(a) 20th Novem	aber	(b) 21st November	(c) 22nd	November
	(d) 23rd Noven	aber	(e) None of these		
9.			ınt each 5 which is w many such 5s are		preceded by 3 but is
	1 5	7 3 5 7 4 7	3 7 2 5 6 5 8 5 7	4 5 6 5 5 7 1	5 7 7 5 5
	(a) 1	(b) 2	(c) 3	(d) 4	(e) 5
10.	Below is given a	a number series	5		
		1 8 5 7	2 9 8 4 3 6 2 7 5	1 8 9 4 3 6 5	9
	How many insta	ances are there	in which an even nu	umber is followed	by two odd numbers?
	(a) Nil	(b) One	(c) Two	(d) Three	(e) None of these
11.	A number is green number is	eater than 3 bu ——.	t less than 8. Also, i	t is greater than 6	but less than 10. The
	(a) 5	(b) 6	(c) 7	(d) 8	(e) 9
12.	$16 \times 85 = 8651$,	what is 73×42	??		
	(a) 4372	(b) 3723	(c) 4327	(d) 4732	(e) 2734
13.			od was shifted by tw s earlier position fro		the left, he became 7th f the row?
	(a) 7th	(b) 8th	(c) 9th	(d) 10th	(e) None of these
14.	a passenger tha	at the bus had		utes ago and the	nquiring, the clerk told next bus will leave at enger?
	(a) 9.10 am	(b) 8.55 am	(c) 9.08 am	(d) 9.05 am	(e) 9.15 am
15.			ta was shifted by for as her earlier positi		the right, she became end of the row?
	(a) 9th	(b) 10th	(c) 11th	(d) 12th	(e) 14th
			ANSWERS		
1	(c) 9 (c)	3 (b) 4 (c	5 (c) 6 (c)	7. (b) 8. ((b) 9 (d) 10 (d)
т.	(c) 2. (c)	3. (b) 4. (a)	5. (c) 6. (e)	7. (b) 8. ((b) 9. (d) 10. (d)

11. (c)

13. (b)

14. (e)

15. (e)

12. (c)

Explanations

- 1. There are three such 8s which are preceded by 7 but not immediately followed by 4.
- 2. There are three such 3s which are immediately preceded by 6 but not immediately followed by 7.
- 3. No. of numbers which are divisible by 7 from 11 to $50 = \frac{49-14}{7} + 1 = 6$. But out of these

which are divisible by 3 = 21 and 42.

- \therefore Required numbers = 14, 28, 35, 49.
- \therefore Required no. of numbers = 4.
- 4. Numbers from 1 to 100 which are divisible by 4 and has 4 as a digit are 4, 24, 40, 44, 48, 64, 84.
 - \therefore Required number of numbers = 7.
- 5. Rank of Abhishek from last = 31 13 + 1 = 19.
- 6. Savitha's present position = 15th from left = Anita's previous position = 9th from right
 - \therefore No. of girls in the row = 15 + 9 1 = 23.
- 7. Number of days = 6 + 29 + 31 + 30 + 15 = 111
 - (: Number of days in February in the year 1988 is 29).
- 8. According to Ram, Lakshman's birthday is on one of the days among 20th and 21st November. According to Anil, Lakshman's birthday is on one of the days among 21st, 22nd and 23rd November.

The day common in both is 21st November. Therefore, Lakshman's birthday is on 21st November.

- 9. There are four such 5s in the given series.
- 10. There are three such instances in which an even number is followed by two odd numbers.
- 11. As per the first condition, if x is the number, 8 > x > 3. As per second condition 10 > x > 6 Taking two conditions together, 8 > x > 6.
 - ... The number is 7.
- 12. Given, $16 \times 85 = 8651$

Similarly $73 \times 42 = 4327$

i.e. by placing from unit's place of product, digits in ten's place of first number, unit's place of second number, unit's place of first number and ten's place of second number.

13. 1 2 3 4 5 6 P 8 9 10 11 12 13 14 15 16

P denotes Pramod's new position.

- \therefore His earlier position from right end of row = 16 9 + 1 = 8
- 14. Bus leaves to Trivandrum every 30 minutes

The last bus left 10 minutes before

The next bus will leave at 9.35 am

- \therefore Time at which information got = 9.35 0.30 + 0.10 = 9.15 am
- 15. Anita's earlier position = 12 4 = 8th from left end.
 - \therefore Position from right = 21 8 + 1 = 14th

Decision Making

In this type of test, a vacancy is declared. The necessary qualifications required in the candidates coming up to fill the vacancy are provided and the merits of the candidates mentioned. The decision about each candidate has to be made from among the five choices named (a), (b), (c), (d), and (e) which state the courses of action to be taken as per the candidate's potentials.

EXAMPLE (1-5): Study the following information carefully and answer the questions given below it.

Following are the qualifications necessary for the recruitment of a librarian in a state university.

The candidate must:

- 1. have a master's degree in Library Science with at least 55% marks or the equivalent grade and a consistently good academic record.
- 2. have one year specialisation in an area of Information Technology/Archives and manuscript keeping master's degree in an area of thrust in the institution.
- 3. have at least ten year's experience as a deputy librarian in a university.
- 4. bear an evidence of innovative library service and organisation of published work. In the case of a candidate who:
- 5. has a 15 year's experience as a college librarian, the case may be referred to the Vice Chancellor.
- 6. having obtained less than 55% marks in library science but has 13 years' experience as a Deputy Librarian in a University, the case may be referred to the Registrar of the University for his consideration.
- 7. having an M.Phil./Ph.D. degree in Library Science/ Information Science/ Documentation/Archives and manuscript keeping but has only ten year's experience as a college librarian, the condition at (1) may be waived.

EXERCISE 1

Based on the above conditions and the information provided against each of the candidates in the questions given below, decide which of the following courses of action should be taken against each candidate.

Mark the answer as (a) if the candidate is to be selected; (b) if the candidate is not to be selected; (c) if the data are inadequate; (d) if the case is to be referred to the Registrar and (e) if the case is to be referred to the Vice Chancellor.

- Ashok having Master's degree in Library Science with 70% marks and with one year specialisation in an area of Information Technology joined as a Librarian in the Indian College on 22nd February 1985. He also holds a certificate of innovative library science in the college.
- 2. Satheesh, an M.Phil in Library Science has been a Deputy Librarian in the MG University since 27th September 1988. He has also obtained master's degree in Archives and manuscript keeping. He holds the evidence of innovative organisation of published work of the college students doing Ph.D.
- 3. Gopala Krishnan has been a Deputy Librarian in the MS University since 1981. He holds an evidence of contributing library service in the same institution. He has a master's degree in Library Science with 53% marks.
- 4. Roy holding a Ph.D. degree in Library Science has one year specialisation in the Archives and manuscript keeping. He has been a Deputy Librarian in the Agriculture University since 11 May 1990. He also has a certificate of innovative Library service in a public library for three years.
- 5. Suresh Kumar has been a college librarian since 15 January, 1983. He holds an M.Phil Degree in Library Science.

ANSWERS

1. (e) 2. (a) 3. (b) 4. (a) 5. (c)

Explanations

- 1. Ashok fulfills conditions (1), (2) and (4). Being a college librarian for 16 years, he satisfies condition (5). So, the answer is (e).
- 2. Satheesh, being an M.Phil degree holder, satisfies condition (7) so that condition (1) may be waived. He satisfies conditions (2), (3) and (4). So he is selected and the answer is (a).
- 3. Gopala Krishnan satisfies condition (4). Being a Deputy Librarian for last 20 years, he satisfies condition (3). As the marks scored by him are less than 55%, he violates condition (1). So the answer is (b).
- 4. Roy, being a Ph.D. holder, satisfies condition (7) so that (1) is waived. He satisfies condition (2) and (4) and being a Deputy Librarian for eleven years, he satisfies condition (3). So the answer is (a).
- 5. As the candidate Suresh Kumar, being an M.Phil, satisfies condition (7) and so condition (1) is waived. He being a college librarian for 18 years satisfied condition (3). As there is no information regarding condition (2) and (4) are not mentioned, answer is (c).

EXERCISE 2

Study the following information carefully and answer the questions given below it.

[Bank PO]

The following are the criteria for the admission to medical courses in a college.

The student must:

- 1. have passed XII standard examination in science with at least 60 % marks.
- 2. be at least 18 years old as on 23.3.93.
- 3. have obtained at least 71% marks in the entrance examination.
- 4. be able to pay monthly tuition fee of $\stackrel{?}{\sim}$ 560.
- 5. be able to pay one time deposit of $\overline{\varsigma}$ 18000.

In the case of a student who:

- 6. satisfies all other conditions except (5) and can pay one time deposit only upto ₹ 12,000, the case may be referred to the Director of the Institute.
- 7. satisfies all criteria except that at (1), the case may be referred to the chairman.
- 8. satisfies all criteria but has not yet received the result of the final examination of XII std. may be provisionally admitted.

On the basis of the above conditions and the information provided against each student in the questions given below, decide which of the following courses of action should be taken against each candidate.

Mark answer as (a) if the student is to be admitted; (b) if the student is not to be admitted; (c) if the case is to be referred to the Director; (d) if the case is to be referred to the Chairman; (e) if the student is to be provisionally admitted.

- 1. Nitin Sharma has passed XII standard examination with 62% marks and was 20 years old on 9th January, 1993. He has secured 75% marks in the entrance test. He can pay a tuition fee of ₹ 560 and a one time deposit of only ₹ 12000.
- 2. Rajendra Kumar has passed XII standard examination in Science with 55% marks. He was born on 26th March, 1974. He has secured 83% marks in the entrance examination. He can pay one time deposit of ₹ 18000 and monthly tuition fee of ₹ 560.
- 3. Prakash Mehra secured 60% and 72% marks in the XII standard and entrance examination respectively. He was 19 years old on 2nd December, 1992. He is able to pay the stipulated one time deposit and monthly tuition fee.
- 4. Mukesh Maheshwari was born on 15th February, 1974. He attained 51% marks in XII standard examination and 75% marks in entrance test. He is able to pay monthly tuition fee of ₹ 560 and one time deposit of ₹ 18000.
- 5. Peeyush Yadav has appeared for the final exam of XII standard in Science stream. He was 19 years old on 25.7.92. He has secured 75 % in the entrance examination. He is able to pay one time deposit of ₹ 18000 and monthly tuition fee of ₹ 560.
- 6. Deepak Gupta passed XII standard in Science with 63% marks and was born on 1.6.73. He has secured 74% marks in the entrance examination and can pay monthly tuition fees of ₹ 560. He can pay one time deposit of ₹ 9000 only.
- 7. Harpal Singh, born on 8th September, 1974, passed XII standard examination in Science with 68% marks. He attained 69% marks in the Entrance Test and is able to pay the stipulated monthly tuition fee and one time deposit.
- 8. Rakesh Sharma, who attained 59% marks in XII std. examination and 73.5% marks in the entrance test, is able to pay the monthly tuition fee of ₹ 560 and one time deposit of ₹ 18000. He is 22 years of age.

- 9. Nisha Gupta, a 21 year old girl, passed the XII standard examination with 64% marks and the entrance test with 77% marks. She is able to pay the tuition fee of ₹ 560 and a one time deposit of ₹ 18000.
- 10. Jatin Narang was born on 7.9.74. He secured 72% marks in the entrance test. He has appeared for XII standard examination and can pay the stipulated tuition fee and one time deposit.

ANSWERS

1. (c) 2. (d) 3. (a) 4. (d) 5. (e) 6. (b) 7. (b) 8. (d) 9. (a) 10. (e)

Explanations

- 1. The candidate satisfies condition (6) instead of condition (5).
- 2. The candidate satisfies condition (7) instead of condition (1).
- **3.** All conditions of eligibility are satisfied.
- 4. The candidate satisfies condition (7) instead of condition (1).
- 5. The candidate satisfies condition (8).
- 6. Condition (5) is not fulfilled.
- 7. Condition (3) is not fulfilled.
- 8. Condition (7) is not satisfied instead of condition (1).
- 9. All conditions of eligibility are satisfied.
- 10. As the candidate has not yet received the result of the final examination of XII standard, he satisfies condition (8).

Assertion and Reason

In this type of questions two statements will be given: Assertion (A) and Reason (R). You have to go through these statements and check whether the statement given as (A) and statement given as (R) are as stated in the following manner and you have to answer either (a), (b), (c) or (d).

- (a) Both (A) and (R) are individually true and (R) is the correct explanation of (A).
- (b) Both (A) and (R) are individually true, but (R) is not the correct explanation of (A)
- (c) (A) is true but (R) is false.
- (d) (A) is false but (R) is true.

EXERCISES

- 1. Assertion (A): The same face of the Moon is always presented to the Earth.
 - Reason (R): The moon rotates about its own axis in $23\frac{1}{2}$ days which is about the same time that it takes to orbit the earth. [IAS]
- 2. Assertion (A): Existence of human life on Venus is highly improbable.
 - Reason (R): Venus has extremely high level of CO_2 in its atmosphere [IAS]
- 3. Assertion (A): All the proteins in our food are digested in small intestine only.
 - Reason (R): The protein-digesting enzymes from pancreas are released into small intestine. [IAS]
- 4. Assertion (A): Amoeba reproduces by fission.
 - Reason (R): All unicellular organisms reproduce by asexual methods. [IAS]
- 5. Assertion (A): Bangalore receives much higher annual rainfall than that of Mangalore.
 - Reason (R): Bangalore has the benefit of receiving rainfall both from south-west and north-east monsoons. [IAS]
- 6. Assertion (A): The Central Rural Sanitation Programme was launched in 1986 to improve the quality of life of rural people in India.
 - Reason (R): The rural sanitation is a subject in the concurrent list in the constitution of India. [IAS]

- 7. Assertion (A): The west flowing rivers of peninsular India have no deltas.
 - Reason (R): These rivers do not carry any alluvial sediments.
- 8. Assertion (A): The thickness of the atmosphere is maximum over the equator.
 - Reason (R): High insulation and strong convection currents occur over the equator.

[IAS]

[IAS]

- 9. Assertion (A): In our houses, the current in AC electricity line changes directions 50 times per second.
 - Reason (R): The frequency of alternating voltage supplied is 50 Hz.
- 10. Assertion (A): Fatty acids should be a part of the balanced human diet.
 - Reason (R): The cells of the human body can't synthesize any fatty acids.
- 11. Assertion (A): India does not export natural rubber.
 - Reason (R): About 97% of India's demand for natural rubber is met from domestic production.
- 12. Assertion (A): For the first time, India had no trade deficit in the year 2002-03.
 - Reason (R): For the first time, India's exports crossed \$50 billion in the year 2002-03.

[IAS]

- 13. Assertion (A): To dilute sulphuric acid, acid is added to water and not water to acid.
 - Reason (R): Specific heat of water is quite large. [IAS]
- 14. Assertion (A): Devaluation of currency may promote export.
 - Reason (R): The price of the country's product in the international market may fall due to devaluation.
- 15. Assertion (A): The fiscal deficit is greater than the budgetary deficit.
 - Reason (R): The fiscal deficit is the borrowing from RBI plus other liabilities of government to meet its expenditure.
- 16. Assertion (A): According to statistics, more female children are born each year than male children in India.
 - Reason (R): In India, the death rate of male child is higher than that of the female child.
- 17. Assertion (A): Insect resistant transgenic cotton has been produced by inserting Bt gene.
 - Reason (R): The Bt gene is derived from a bacterium.
- 18. Assertion (A): Information technology is fast becoming a very important field of activity in India.
 - Reason (R): Software is one of the major exports of the country and India has a very strong base in hardware.
- 19. Assertion (A): Chile continues to be an important producer of copper in the world.
 - Reason (R): Chile is endowed with the world's largest deposit of porphyry copper.
- 20. Assertion (A): Dolly was the first cloned mammal.
 - Reason (R): Dolly was produced by *in-vitro* fertilization.
- 21. Assertion (A): During the time of Akbar, for every ten cavalrymen, the mansabdars had to maintain twenty horses.
 - Reason (R): Horses had to be rested while on march and replacements were necessary in times of war.
- 22. Assertion (A): Lord Linlithgow described the August Movement of 1942 as the most serious rebellion since the Sepoy Mutiny.
 - Reason (R): There was massive upsurge of the peasantry in certain areas. [IAS]

- 23. Assertion (A): The Gaudham School of Art bears the mark of Hellanistic influence.
 - Reason (R): Hinayana form was influenced by that art.

[IAS]

- 24. Assertion (A): Formic acid is a stronger acid than acetic acid.
 - Reason (R): Formic acid is an organic acid.

[IAS]

- 25. Assertion (A): At first the Turkish administration in India was essentially military.
 - Reason (R): The country was parcelled out as Iqtas among leading military leaders.

[IAS]

- **26.** Assertion (A): According to Asoka's edicts social harmony among the people was more important than religious devotion.
 - Reason (R): He spreaded ideas of equity instead of promotion of religion. [IAS]
- 27. Assertion (A): The temperature of a metal wire rises when an electric current is passed through it.
 - Reason (R): Collission of metal atoms with each other releases heat energy.
- 28. Assertion (A): The Khilafat movement did bring the urban Muslims into the fold of the National Movement.
 - Reason (R): There was a predominant elements of anti-imperialism in both the National and Khilafat Movement.
- 29. Assertion (A): Phenyl is used as a household germicide.
 - Reason (R): Phenyl is a phenol derivative and phenol is an effective germicide.
- 30. Assertion (A): Partition of Bengal in 1905 brought to an end the Moderate's role in the Indian Freedom Movement.
 - Reason (R): The Surat session of Indian National Congress separated the Extremists from the Moderates.
- 31. Assertion (A): The first ever bill to make primary education compulsory in India was rejected in 1911.
 - Reason (R): Discontent would have increased if everybody could read.
- 32. Assertion (A): Sodium metal is stored under kerosene.
 - Reason (R): Metallic sodium melts when exposed to air.
- **33.** Assertion (A): The congress rejected the Cripps proposals.
 - Reason (R): The Cripps Mission consisted solely of the whites.
- 34. Assertion (A): The United States of America has threatened to ask the World Trade Organisation (WTO) to apply sanctions against the developing countries for the non-obsergance of ILO conventions.
 - Reason (R): The United States of America itself has adopted and implemented those ILO conventions.
- **35.** Assertion (A): During the reign of Shahjahan, Dara Sikh was sent on an expedition to Balkha, Badakhshan and Qandahar.
 - Reason (R): The expedition sent by Shahjahan to the Middle-east was a marvellous success.
- **36.** Assertion (A): Gandhi stopped the Non-Co-operation movement in 1922.
 - Reason (R): Violence at Chauri Chaura led him to stop the movement. [IAS]
- 37. Assertion (A): The reservation of thirty-three per cent of seats for women in Parliament and State Legislatures does not require constitutional amendment.
 - Reason (R): Political parties contesting elections can allocate thirty-three per cent of seats they contest to women candidates without any constitutional amendment. [IAS]

Reason (R):

Reason (R):

38. Assertion (A):

39. Assertion (A):

40.	Assertion (A):	A diamond sparkles more than a glass imitation cut to the same shape.
	Reason (R):	The refractive index of diamond is less than that of glass. [IAS]
41.	Assertion (A):	The monsoonal rainfall decreases as one goes towards the west and northwest in the Ganga plain.
	Reason (R):	The moisture bearing monsoonal winds go higher up as one moves up in the Ganga plain. [IAS]
42.	Assertion (A):	A lock of Einstein's hair, if scientists could locate it and extract its DNA, could help in producing another Einstein, by cloning.
	Reason (R):	The DNA extracted from the cell of an embryo at an early stage of development can be transferred to individual eggs which in turn can be implanted into the uterus of a surrogate mother to give birth to an identical offspring.
43.	Assertion (A):	The USA re-emerged as India's single largest import source in the early nineties.
	Reason (R):	With swift political developments in the erstwhile Soviet Union, India gradually began to rely on the USA for its defence requirement.
		[IAS]
44.	Assertion (A):	In India, the political parties which formed the governments represented the majority of seats secured in the elections to the House of the people at the centre and the Legislative Assemblies in the states but not the majority of votes.
	Reason (R):	The elections based on the majority-vote-system decides the result on the basis of relative majority of votes secured. [IAS]
45 .	Assertion (A):	A mixture of salt and ice gives temperature below 0°C.
	Reason (R):	The salt raises the freezing point of ice. [IAS]
46.	Assertion (A):	Hong Kong is to revert to China from British control in a few years.
	Reason (R):	The people of Hong Kong have opted for it in a referendum. [IAS]
47.	Assertion (A):	Balan wrote his memoirs in Turki.
	Reason (R):	Turki was the official language of the Mughal Court. [IAS]
48.	Assertion (A):	Minimum wages in India are fixed in accordance with the levels of living and the labour participation ratios.
	Reason (R):	All workers covered by the Minimum Wages Acts are above the poverty line. [IAS]
49.	Assertion (A):	Italy, Switzerland, Sweden and Norway have abundant power resources.
	Reason (R):	They have the largest coal deposits in Europe. [IAS]
50.	Assertion (A):	The Quit India movement marked the culmination of Indian national movement.
	Reason (R):	After the Quit India movement it was matter of time to find a suitable mechanisms for transfer of power. [IAS]

Wilful disobedience or non-compliance of court orders and use of derogatory

Judicial activism can't be practised without aiming the judiciary with

The emergence of economic globalisation does not imply the decline of

[IAS]

[IAS]

language about judicial behaviour amount to contempt of court.

The ideology of socialism believes universalism and globalism.

punitive powers to punish contemptuous behaviour.

socialist ideology.

51.	Assertion (A):	The form of gove	ernment i	n Rigvedio	c period w	as monaro	ehy.	
	Reason (R):	Priest enjoyed bo	Priest enjoyed both social and political status and influenced adminis					istration.
				_				[IAS]
52 .	Assertion (A):	Rainfall is scant	y on east	of wester:	n ghats.			
	Reason (R):	The east of west	ern ghats	is on the	lee side.			[IAS]
53 .	Assertion (A):	Insects are not a	affected by	pesticide	es.			
	Reason (R):	Insects are kille	d by pesti	cides.				[IAS]
54.	Assertion (A):	The finance comstates.	ımission a	nims at sa	ıfeguardin	g the fisca	al autono	my of the
	Reason (R):	The finance com	mission is	s constitu	ted every i	fifth year.		[IAS]
55.	Assertion (A):	Soap removes oi	l and dirt					
	Reason (R):	Soap increases t	he surface	e tension	of water.			[IAS]
56.	Assertion (A):	Red phosphorus	is used in	n matchsti	icks.			
	Reason (R):	Red phosphorus	is less da	ngerous co	mpared to	the white	e one.	[IAS]
57 .	Assertion (A):	Earthworm is a	friend to	the man.				
	Reason (R):	It decreases the	soil erosie	on.				[IAS]
58.	Assertion (A):	Dry battery can	t be recha	rged.				
	Reason (R):	The chemical re	action is r	eversible.				[IAS]
59.	Assertion (A):	India adopted U	K's Parlia	mentary	system.			
	Reason (R):	The Upper Hous	se has jud	icial powe	r.			[IAS]
60.	Assertion (A):	Colonialism has	started in	n India du	ring the 1	9th centu	ry.	
	Reason (R):	Industrial revol	ution dem	anded ma	rket place	s.		[IAS]
			ANSV	VERS)				
1.	(c) 2. (b)	3. (b) 4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (a)	10. (c)
11.	(a) 12. (a)	13. (a) 14. (a)	15. (a)	16. (c)	17. (a)	18. (a)	19. (a)	20. (b)
21.	(d) 22. (a)	23. (c) 24. (b)	25. (b)	26. (b)	27. (b)	28. (d)	29. (c)	30. (d)
31.	(b) 32. (a)	33. (b) 34. (a)	35. (c)	36. (a)	37. (d)	38. (b)	39. (b)	40. (c)

ANSWERS									
1. (c)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (a)	10. (c)
11. (a)	12. (a)	13. (a)	14. (a)	15. (a)	16. (c)	17. (a)	18. (a)	19. (a)	20. (b)
21. (d)	22. (a)	23. (c)	24. (b)	25. (b)	26. (b)	27. (b)	28. (d)	29. (c)	30. (d)
31. (b)	32. (a)	33. (b)	34. (a)	35. (c)	36. (a)	37. (d)	38. (b)	39. (b)	40. (c)
41. (a)	42. (c)	43. (b)	44. (b)	45. (a)	46. (c)	47. (c)	48. (c)	49. (c)	50. (d)
51. (b)	52. (a)	53. (d)	54. (b)	55. (a)	56. (a)	57. (c)	58. (c)	59. (c)	60. (a)

Situation Reaction Test

In this test, a certain situation is described and the candidate is required to choose the most suitable reaction to the given situation from among the alternatives given. The test judges the reasoning power of the candidate and his ability to get correctly and promptly to a situation that may arise in emergency.

EXAMPLE 1 You are visiting a place for first time and are travelling in a bus. Suddenly you realise that the driver is taking the bus to a lonely place with no right intentions. You would:

- (a) with the help of some other passengers, try to baffle the driver and take over the bus.
- (b) sit and wait to face repercussions.
- (c) jump out of the running bus.
- (d) console the worried passengers.

Solution When you expect wrong doing of the driver, the immediate action to prevent it is the need. So the answer is (a).

EXAMPLE 2 While playing cricket in the school, suddenly when you hit the ball, it strikes your classmate on the forehead and blood starts oozing out. You would:

- (a) run away from the field.
- (b) start fighting with the boy why he came in the way.
- (c) blame somebody else for the accident.
- (d) take the boy to the first-aid room.

Solution In the above situation, the urgent need is to provide first aid to the boy so that bleeding may stop. So the answer is (d).

EXAMPLE 3 While you board a train at the station, you find a suitcase beneath your seat. You would:

- (a) report the matter to the police.
- (b) open the suitcase to look through its contents.

- (c) try to find out the address of the owner from the papers, etc. in the suitcase.
- (d) finding no one to claim it, take it into your own possession.

Solution As the case is a loss of a valuable article for the concerned owner, necessary steps have to be taken to hand over the article, by reporting the matter to the police. So the answer is (a).

EXERCISES

To each of the following questions, four probable answers have been given. Select the most appropriate alternative as the answer.

- 1. You are living in a college hostel. The *dal* served to you in the mess has a lot of stones. What would you do?
 - (a) leave eating the dal altogether.
 - (b) bring the matter to the notice of mess incharge.
 - (c) speak to the cook about changing the dal.
 - (d) buy your own dal and cook it in your room.
- 2. You are entrusted with the job of taking care of a child. If the child insists on doing something which you would not allow, you would:
 - (a) beat the child.
 - (b) threaten the child and make him quiet.
 - (c) try to make him understand why you will not allow the action.
 - (d) let the child cry.
- 3. You are driving your car on the road when you hit against a fruit vendor's cart. You would:
 - (a) escape from the site by driving away.
 - (b) abuse the fruit vendor for putting his cart on the way.
 - (c) pay the fruit vendor for the damage done to him.
 - (d) insist that it was not your fault.
- 4. While travelling in a train, you notice a man from the coach behind yours fall off the train. You would:
 - (a) pull the alarm chain so that the train may stop and the man may be helped.
 - (b) shout at the falling man asking him to get up quickly and entrain.
 - (c) jump off the train to assist the falling man.
 - (d) wait till the train stops at the next station and inform the railway authorities there.
- 5. You find that the person whom you call your friend has been cheating you. What would you do?
 - (a) break relations with him.
 - (b) give him tit for tat.
 - (c) make him realise his mistake.
 - (d) tell other friends about him.
- 6. You are alone in the house and there is quite a danger of thieves around. Just then, you hear a knock at the door. You would:
 - (a) open the door to see who is there.
 - (b) first peep out from the window to confirm whether you know the person.
 - (c) not open the door.
 - (d) ask the servant to see who is there.

- 7. You are returning from school. On the way, you find a sealed envelope in a street, fully addressed with unused stamps on it. You would:
 - (a) leave it there as it was and walk away.
 - (b) remove the stamps and destroy the envelope.
 - (c) open the envelope, find out who has dropped it by mistake, and send it to him if possible.
 - (d) post it at the nearest letter box.
- 8. While firing crackers, a child gets severe burns on the hand. What would you do?
 - (a) dip the child's hands in cold water till there is no more burning sensation.
 - (b) wash the hands with dettol.
 - (c) send some one to call the doctor.
 - (d) apply some ointment on the affected area.
- 9. If in the examination hall, you find that the question paper is too tough to be answered satisfactorily by you, the best thing to do for you is to:
 - (a) tell the examiner that the questions are out of course.
 - (b) provoke the candidates to walk out of the examination hall.
 - (c) try to know something from your neighbour.
 - (d) try to solve the questions as much as you know with a cool head.
- 10. On reaching the railway station, you find that the train you wanted to catch is just to start and there is hardly any time for purchasing the ticket. The best thing for you is to:
 - (a) rush to the train rather than miss it and inform the TTI at the next stoppage about your inability to purchase the ticket.
 - (b) rush to the train and perform your journey quietly.
 - (c) first purchase the ticket and then catch the train if it is there.
 - (d) miss the train rather than take the risk of boarding the moving train.
- 11. Your bathroom tap is leaking and is a constant source of irritating noise. You would:
 - (a) sleep with pillows upon your ears.
 - (b) put a bucket underneath.
 - (c) try to put up a cork upon the mouth of the tap.
 - (d) call a plumber to repair the tap.
- 12. While attending your friend's partly, you see your friend's muffler catching fire from the candle on the table behind him. You would:
 - (a) ask your friend to see behind him.
 - (b) rush to call friend's mother.
 - (c) rush and take out the muffler from his neck, drop it and pour water on it.
 - (d) take out the muffler and throw it away.

ANSWERS

1. (b) 2. (c) 3. (c) 4. (a) 5. (c) 6. (b) 7. (d) 8. (a) 9. (d) 10. (a)

11. (d) **12.** (c)

Mathematical Operations

This chapter deals with the questions on simple mathematical operations. Here, the four fundamental operations: plus (+), minus (-), multiplication (×), and division (÷) and also, statement such as 'less than', 'greater than', 'equal to', 'not equal to', etc. are represented by symbols, different from the usual ones. The questions involving these operations are set using artificial symbols. The candidate has to substitute the real signs and solve the questions accordingly, to get the answer. Here are a few examples.

EXAMPLE 1 If '+' means '+', '-' means 'x', '+' means '+' and 'x' means '-', then

$$36 \times 12 + 4 \div 6 + 2 - 3 = ?$$

(a) 2

(d)
$$6\frac{1}{2}$$

(e) None of these

Solution Putting the proper sign in the given expression, we get

$$36 - 12 \div 4 + 6 \div 2 \times 3 = 36 - 3 + 3 \times 3$$

= $36 - 3 + 9 = 42$

So, the answer is (c).

EXAMPLE 2 If '+' means 'minus', '-' means 'multiplied by', '÷' means 'plus' and '×' means 'divided by', then

$$10 \times 5 \div 3 - 2 + 3 = ?$$

(a) 5

(b) 53/3

(c) 18

(d) 21

(e) None of these

Solution Using the proper signs, we get

$$10 \div 5 + 3 \times 2 - 3 = 2 + 3 \times 2 - 3$$

= $2 + 6 - 3 = 5$

So, the answer is (a).

EXAMPLE 3 If 'P' denotes '+', 'Q' denotes '-', 'R' denotes '÷' and 'S' denotes '×', then

(a) 115

(b) 55

(c) 648/13

(d) 25

(e) None of these

Solution Using proper signs, we get

$$18 \times 36 \div 12 - 6 + 7 = 18 \times 3 - 6 + 7$$

= $54 - 6 + 7 = 55$

So, the answer is (b).

EXAMPLE 4 If '>' denotes '+', '<' denotes '-', '+' denotes '÷', '-' denotes '=', '=' denotes 'less than' and 'x' denotes 'greater than', find which of the following statement is correct.

(a) 3+2>4=9+3<2

(b) 3 > 2 > 4 = 18 + 3 < 1

(c) $3 > 2 < 4 \times 8 + 4 < 2$

(d) $3+2 < 4 \times 9 + 3 < 3$

Solution Using proper notations, we have

- (a) $3 \div 2 + 4 < 9 \div 3 2$ or $\frac{11}{2} < 1$, which is false.
- (b) $3 + 2 + 4 < 18 \div 3 1$ or 9 < 5, which is false.
- (c) $3+2-4>8 \div 4-2$ or 1>0, which is true.
- (d) $3 \div 2 4 > 9 \div 3 3$ or $\frac{-5}{2} > 0$, which is false.

So, the statement (c) is true.

EXERCISES

1.	If '-' means 'x', 'x' means '+', '+' means '\ddots' and '\ddots	' means '-',	, then what will be	the value of
	$40 \times 12 + 3 - 6 \div 60$?			[Bank PO]

- (a) 44
- (b) 7.95
- (c) 16
- (d) 479.95
- (e) None of these
- 2. If '+' means '+', '+' means '-', '-' means 'x' and 'x' means '+', what will be the value of the following expression $8 + 4 \div 3 \times 5 - 9$? [Bank PO]
 - (a) 44
- (b) $5\frac{2}{3}$ (c) $6\frac{1}{3}$
- (d) 46
- (e) None of these
- 3. If '+' means ' \div ', ' \times ' means '-', ' \div ' means ' \times ' and '-' means '+', then $9 + 3 \div 4 8 \times 2 = ?$

[Bank PO]

- (a) $-6\frac{1}{4}$ (b) $6\frac{3}{4}$ (c) $-1\frac{3}{4}$ (d) 18

- (e) None of these
- 4. If '+' means '÷', '-' means 'x', '÷' means '-' and 'x' means '+', what will be the value of $8 + 6 \div 4 - 7 \times 3$? [SBI PO]
 - (a) $-\frac{23}{2}$ (b) 14
- (c) $-\frac{71}{3}$
- (d) 12
- (e) None of these
- 5. If '+' means ' \times ', '-' means '+' and ' \times ' means ' \div ', find the value of $5+4-18\times3$.

[Bank PO]

- (a) -34
- (b) 6
- (c) 26
- (d) 14
- (e) None of these
- 6. If '+' means '+' '-' means '+', 'x' means '-' and '+' means 'x', then $8 \div 4 6 + 3 \times 4 = ?$

[AAO Exam]

- (b) 46 (a) 8
- (c) 4
- (d) 13
- (e) None of these

7. If '÷' means '+', '-', means '÷', '×' means '-' and '+' means '×', then
$$\frac{(36 \times 4) - 8 \times 4}{4 + 8 \times 2 + 16 \div 1} = ?$$

[Railway Recruitment]

- (a) 16
- (b) 12
- (c) 8
- (d) 0

- 8. If '+' stands for 'division', '-' stands for 'equal to', '×' stands for 'addition', '÷' stands for 'greater than', '=' stands for 'less than', '>' stands for 'multiplication' and '<' stands for 'subtraction', then which of the following alternatives is correct?
 - (a) $5 + 2 \times 1 = 3 + 4 > 1$

(b) $5 > 2 \times 1 - 3 > 4 < 1$

(c) $5 \times 2 < 1 - 3 < 4 \times 1$

- (d) $5 < 2 \times 1 \div 3 > 4 \times 1$
- 9. If 'L' denotes 'x', 'M' denotes '÷', 'P' denotes '+' and 'Q' denotes '-', then

16P24M8Q6M2L3=?

- (a) 13/6
- (b) $-\frac{1}{6}$
- (c) $14\frac{1}{2}$
- (d) 10
- (e) None of these
- 10. If 'P' denotes '+', 'Q' denotes 'x', 'R' denotes '+', 'S' denotes '-' and '=' denotes greater than, then which of the following statement is true?
 - (a) 4 P 8 R 2 S 1 Q 6 = 1

(b) 3 S 1 Q 8 P 6 R 2 = 0

(c) 8 R 2 S 3 Q 4 P 6 = 2

(d) 9 P 2 Q 6 S 4 R 2 = 21

ANSWERS

- 1. (e) 2. (a)
- a) 3. (d)
- **4.** (c)
- **5.** (c)

6. (e)

- 7. (d)
- 8. (b)
- **9.** (d) **10.** (c)

Explanations

1. Using the correct symbols, we have:

Given expression = $40 + 12 \div 3 \times 6 - 60 = 40 + 4 \times 6 - 60 = 40 + 24 - 60 = 4$

2. Using correct symbols, we have:

$$8 \div 4 - 3 + 5 \times 9 = 2 - 3 + 5 \times 9 = 2 - 3 + 45 = 44$$

3. Using correct symbols, we have:

$$9 \div 3 \times 4 + 8 - 2 = 3 \times 4 + 8 - 2 = 12 + 8 - 2 = 18$$

4. Using correct symbols, we get:

$$8 \div 6 - 4 \times 7 + 3 = \frac{8}{6} - 28 + 3 = \frac{-71}{3}$$

5. Using correct symbols, we have:

$$5 \times 4 + 18 \div 3 = 20 + 6 = 26$$

6. Using correct symbols, we have:

$$8 \times 4 + 6 \div 3 - 4 = 32 + 2 - 4 = 30$$

7. Using correct symbols, we have

$$\frac{(36-4) \div 8-4}{4 \times 8-2 \times 16+1} = \frac{32 \div 8-4}{32-32+1} = \frac{4-4}{1} = 0$$

8. Using correct symbols in (b), we get

$$5 \times 2 + 1 = 3 \times 4 - 1$$
 or $11 = 11$, which is true.

9. Using correct symbols, we get:

$$16 + 24 \div 8 - 6 \div 2 \times 3 = 16 + 3 - 3 \times 3 = 16 + 3 - 9 = 10$$

- 10. Using correct symbols we get:
 - (a) $4+8 \div 2-1 \times 6=1$ or 4+4-6=1 or 2=1, which is false.
 - (b) $3-1\times 8+6 \div 2=0$ or 3-8+3=0 or -2=0, which is false.
 - (c) $8 \div 2 3 \times 4 + 6 = -2$ or 4 12 + 6 = -2 or -2 = -2, which is true.
 - (d) $9 + 2 \times 6 4 \div 2 = 21$ or 9 + 12 2 = 21 or + 19 = 21, which is false.

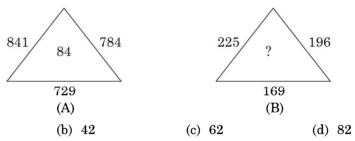
So, the answer is (c).

Inserting the Missing One

In this type of problems, a figure is given which is subdivided into a certain number of parts and each one bears a number except one subdivision. The numbers in the figure follow a particular pattern. You have to analyse the given figure and then fill in the blank space with the most suitable alternative.

Directions: Find the missing number from among the given alternatives.

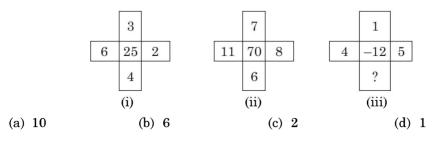
EXAMPLE 1



Solution In figure (A), $27^2 = 729$, $28^2 = 784$, $29^2 = 841$ and 27 + 28 + 29 = 84. Similarly, in figure (B), $13^2 = 169$, $14^2 = 196$, $15^2 = 225$ and 13 + 14 + 15 = 42. So, the answer is **(b)**.

EXAMPLE 2

(a) 32

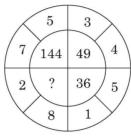


Solution In figure (i),
$$(3^2 + 6^2) - (2^2 + 4^2) = (9 + 36) - (4 + 16) = 45 - 20 = 25$$

In figure (ii), $(7^2 + 11^2) - (6^2 + 8^2) = (49 + 121) - (36 + 64) = 170 - 100 = 70$
In figure (iii), $(1^2 + 4^2) - (5^2 + x^2) = -12$
or $(1 + 16) - (25 + x^2) = -12$
or $x^2 = 17 - 25 + 12 = 4$
∴ $x = 2$

So, the answer is (c).

EXAMPLE 3



(d) 121

Solution Here, $(7+5)^2 = 144$; $(3+4)^2 = 49$; $(5+1)^2 = 36$;

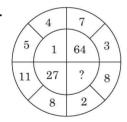
Similarly $(2 + 8)^2 = 100$

: Missing number is 100.

EXERCISES

Directions: Find the missing number in each of the following questions:

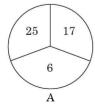
1.





(c) 0

2.

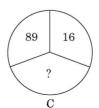


(a) 19



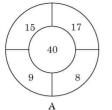


(b) 17

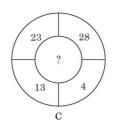


(c) 15

3.

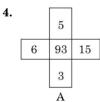


(b) 8



(c) 12

(a) 1



28

В

12

(a) 27

(b) 19

(c) 89

(d) 5

(d) 27

5.	7B	5C	6B
	3C	9B	19A
	15A	17A	?

(a) 10C

(b) 12C

(c) 14C

(d) 16C

6. 9 A 12 B 10 ? 8 C 11

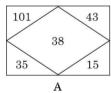
(a)
$$A = 13$$
, $B = 11$, $C = 9$

(b)
$$A = 13$$
, $B = 9$, $C = 11$

(c)
$$A = 9$$
, $B = 11$, $C = 13$

(d)
$$A = 9$$
, $B = 13$, $C = 11$

7.



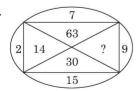
(a) 198

(b) 158

(c) 142

(d) 127

8.



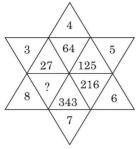
(a) 33

(b) 145

(c) 135

(d) 18

9.



(a) 64

- (b) 512
- (c) 16
- (d) 24

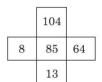
10.	3	8	10	2	?	1
	6	56	90	2	20	0
((a) 0		(b)	3		

(c) 5

(d) 7

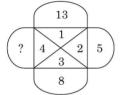
[Railways]

Directions: In each of the following questions, numbers have been arranged according to the pattern shown in the sample figure given below. Find the most correct alternative to fill in the space provided by question mark.



Sample figure

11.

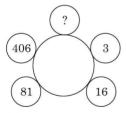


(a) 10

(b) 11

- (c) 12
- (d) 13

12.



- (a) 2031
- (b) 731
- (c) 1625
- (d) 1

13.

	154	
14	221	196
	?	

(a) 11

- (b) 13
- (c) 15
- (d) 17

14. \lceil

BD_3	CE_5	DF_{15}
EG_2	FH_4	GI_8
HJ_{4}	IK_6	?

- (a) JL_{24}
- (b) IJ_{18}
- (c) JK_{18}
- (d) JL_{12}

15.



(a) 33

(b) 81

(c) 243

(d) 42

ANSWERS

1. (d) 2. (c)

. (c) 3. (a)

4. (c)

(c) **5.** (d) **6.** (d)

7. (c)

8. (c)

9. (b) **10.** (c)

11. (c) 12. (a)

13. (a)

14. (a)

15. (b)

Explanations

1. $(5-4)^3 = 1$; $(7-3)^3 = 64$; $(11-8)^3 = 27$; $(8-2)^3 = 216$.

.. The missing number is 216.

2. The sum of the numbers in the upper two parts of each circle is seven times the number in the third part.

In figure A, $(25 + 17) = 42 = 7 \times (6)$

In figure B, $(38 + 18) = 56 = 7 \times (8)$

In figure C, $(89 + 16) = 105 = 7 \times (15)$

.. The missing number is 15.

3. The difference of the product of the numbers in two lower parts and the sum of the numbers in two upper parts is the number inside the smaller circle.

In figure A, $(9 \times 8) - (15 + 17) = 72 - 32 = 40$

In figure B, $(3 \times 12) - (6 + 2) = 36 - 8 = 28$

Similarly, in figure C, $(13 \times 4) - (23 + 28) = 52 - 51 = 1$

.. The missing number is 1.

4. In figure A, $(6 \times 3) + (5 \times 15) = 18 + 75 = 93$

In figure C, $(4 \times 8) + (18 \times 1) = 32 + 18 = 50$

Similarly, in figure B, $(9 \times 6) + (7 \times 5) = 54 + 35 = 89$

.. The missing number is 89.

5. In each column, out of the letters A, B and C, each of these must appear once. Along the diagonal, the sum of two numbers is equal to the third number.

 \therefore The missing number will be 7 + 9 = 16 and the letter will be C as A and B already appears once in that column.

6. The total of numbers in each row and each column is 30.

7. In figure (A), (101 + 15) - (35 + 43) = 116 - 78 = 38.

The same pattern would be followed in figure (B).

 \therefore The missing number = (48 + 184) - (56 + 34) = 232 - 90 = 142

8. We have $15 \times 2 = 30$; $2 \times 7 = 14$; $7 \times 9 = 63$;

 \therefore Missing number = $9 \times 15 = 135$.

9. We have, $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$; $7^3 = 343$.

 \therefore Missing number = $8^3 = 512$.

10. Let X denotes the number in the first row and Y that in the second row.

∴
$$Y = X(X - 1)$$

∴ $6 = 3(3 - 1); 56 = 8(8 - 1);$
 $90 = 10(10 - 1); 2 = 2(2 - 1);$
 $0 = 1(1 - 1);$
So, $X(X - 1) = 20$
∴ $X = 5$

11. Arrangement is as follows:

Taking clockwise;

$$5 + 3 = 8$$
; $8 + 4 = 12$; $12 + 1 = 13$;

So, the missing number is 12.

12. Here taking the arrangement clockwise,

$$(3 \times 5) + 1 = 16; (16 \times 5) + 1 = 81; (81 \times 5) + 1 = 406;$$

So $(406 \times 5) + 1 = 2031$.

- ... The missing number is 2031.
- 13. Let the missing number be x

From the given sample figure, $8 \times 13 = 104$; $8^2 = 64$; 8 + 13 + 64 = 85.

Similarly, here $14^2 = 196$;

$$14 + x + 196 = 221$$

$$x = 11$$

$$14 \times 11 = 154$$

So, the missing number is 11.

- 14. The number in the third column in each row is obtained by multiplying the numbers in the first and second column in the same row.
 - \therefore In first row, $3 \times 5 = 15$ and in second row, $2 \times 4 = 8$.
 - \therefore Number to be filled in blank space = $4 \times 6 = 24$.
 - :. From the given alternatives only (a) contains suffix 24.

So, the answer is (a).

15. Here, 3 + 27 + 9 = 39

$$? = 3 \times 27 = 81 \text{ i.e., missing number} = 3 \times 27 = 81.$$

So the answer is (b).

Logical Sequence of Words

In these type of questions, certain inter-related words are given and numbered, followed by various sequences of numbers denoting them, as alternatives. The candidate is required to arrange these words in a logical sequence based on a common property and then choose the correctly graded sequence from the given alternatives.

Some common sequences have been mentioned as follows:

I. Sequence of occurrence of events or various stages in a process:

EXAMPLE 1 Arrange the following in a logical order.

1. Birth 2. Marriage 3. Death 4. Funeral 5. Education

(a) 1, 4, 2, 5, 3

(b) 1, 5, 2, 3, 4

(c) 3, 4, 2, 5, 1

(d) 2, 5, 4, 1, 3

Solution The given words when arranged in the order of various events occur in a man's life, form the sequence: Birth, Education, Marriage, Death, Funeral.

Therefore, the correct order becomes 1, 5, 2, 3, 4.

Hence, the answer is (b).

EXAMPLE 2 Arrange the following in a meaningful sequence:

1. Doctor 2. Fever 3. Prescribe 4. Diagnose 5. Medicine (a) 1, 4, 3, 2, 5

(b) 2, 1, 3, 4, 5

(c) 2, 1, 4, 3, 5

(d) 2, 4, 3, 5, 1

[CBI]

Solution Here fever occurs first. One then goes to a doctor. After diagnosing he prescribes medicine. Therefore, the correct order is: Fever, Doctor, Diagnose, Prescribe and Medicine. Hence, the answer is (c).

II. Sequence of objects in a class or group, from part to the whole:

EXAMPLE 3 Arrange the following in a logical order.

1. Leaf 2. Fruit 3. Stem 4. Root 5. Flower

(a)	3,	4	5	1	2
(a)	υ,	ъ,	υ,	т,	_

(b) 4, 1, 3, 5, 2

(d) 4, 3, 1, 5, 2

[SSC]

Solution The given words are part of a plant. Arranging them in order from bottom to top in the order of their occurrence, we get Root, Stem, Leaf, Flower and Fruit.

Hence the sequence is 4, 3, 1, 5, 2.

Hence, the correct answer is (d).

EXAMPLE 4 Arrange the following in a meaningful order, from particular to general.

1. District 2. Village 3. Country 4. Town 5. State

(b) 2, 1, 4, 5, 3

(d) 2, 5, 3, 4, 1

Solution Here the village is the smallest area. A number of villages are combined to form a town. A number of towns are combined to form a district. So many districts are combined to form a state. So many states are clubbed to form a country. Hence the sequence is Village, Town, District, State, Country.

Thus, the correct order is 2, 4, 1, 5, 3.

Hence, the answer is (a).

III. Sequence of increasing/decreasing size, value, intensity, etc.

EXAMPLE 5 Arrange the following in a logical order.

1. Gold 2. Iron 3. Silver 4. Platinum 5. Diamond

(a) 3, 4, 2, 5, 1

(b) 2, 3, 1, 5, 4

(c) 4, 5, 1, 2, 3

(d) 5, 4, 2, 3, 1

Solution The given names when arranged in the order of increasing values, (i.e. from cheapest to costly) form the sequence:

Iron, Silver, Gold, Diamond, Platinum.

Hence, the answer is (b).

EXAMPLE 6 Arrange the following in a logical sequence from small to big.

1. Elephant 2. Cat 3. Mosquito 4. Tiger 5. Whale

(b) 2, 5, 1, 4,3

(a) 1, 3, 5, 4, 2(c) 3, 2, 4, 1, 5

(d) 5, 3, 1, 2, 4

[CBI]

Solution Here the given names can be arranged (from small to big) in a sequence as:

Mosquito, Cat, Tiger, Elephant, Whale.

Hence, the answer is (c).

IV. Sequence in which a chain of given objects is formed:

EXAMPLE 7 Arrange the following in a logical order.

1. Book 2. Pulp 3. Timber 4. Jungle 5. Paper

(a) 2, 5, 1, 4, 3

(b) 3, 2, 5, 1, 4

(c) 4, 3, 2, 5, 1

(d) 5, 4, 3, 1, 2

[SSC]

Solution Here the names can be arranged in the order in which they are used for the manufacture of a book. It can be written as a sequence:

Jungle, Timber, Pulp, Paper, Book

Hence the sequence is 4, 3, 2, 5, 1

Hence, the answer is (c).

EXAMPLE 8 Arrange the following in a meaningful sequence:

- 1. Yarn 2. Plant 3. Saree 4. Cotton 5. Cloth
 - (a) 2, 4, 1, 5, 3

(b) 2, 4, 3, 5, 1

(c) 2, 4, 5, 1, 3

(d) 2, 4, 5, 3, 1

[CBI]

Solution Here the names given are the stages in the manufacture of saree. Therefore the sequence can be formed as: Plant, Cotton, Yarn, Cloth, Saree.

Hence the sequence is 2, 4, 1, 5, 3

Hence, the answer is (a).

EXERCISES

Directions (Q.1 to 10): In each of the following questions, arrange the given words in a meaningful sequence and then choose the most appropriate sequence from amongst the alternatives provided below each question:

- 1. 1. Sentence 2. Chapter 3. Letter 4. Book 5. Word 6. Paragraph
 - (a) 4, 2, 1, 6, 5, 3

(b) 4, 2, 6, 1, 5, 3

(c) 4, 6, 1, 2, 3, 5

(d) 4, 6, 2, 5, 1, 3

[CBI]

- 2. 1. Police 2. Punishment 3. Crime 4. Justice 5. Judgement
 - (a) 1, 2, 3, 4, 5

(b) 3, 1, 2, 4, 5

(c) 3, 1, 4, 5, 2

(d) 5, 4, 3, 2, 1

[SSC]

- 3. 1. College 2. Child 3. Salary 4. School 5. Employment
 - (a) 1, 2, 4, 3, 5

(b) 2, 4, 1, 5, 3

(c) 4, 1, 3, 5, 2

(d) 5, 3, 2, 1, 4

[RRB]

- 4. 1. Mother 2. Child 3. Milk 4. Cry 5. Smile
 - (a) 1, 5, 2, 4, 3

(b) 2, 4, 1, 3, 5

(c) 2, 4, 3, 1, 5

- (d) 3, 2, 1, 5, 4
- 5. 1. Atomic Age 2. Metallic Age 3. Stone Age 4. Alloy Age
 - (a) 1, 3, 4, 2

(b) 2, 3, 1, 4

(c) 3, 2, 4, 1

(d) 4, 3, 2, 1

[MAT]

- 6. 1. Heel 2. Shoulder 3. Skull 4. Neck 5. Knee 6. Chest 7. Thigh 8. Stomach 9. Face 10. Hand
 - (a) 2, 4, 7, 10, 1,5,8,9,6,3

(b) 3, 4, 7, 9, 2, 5, 8, 10, 6, 1

(c) 4, 7, 10, 1, 9, 6, 3, 2, 5, 8

(d) 3, 9, 4, 2, 10, 6, 8, 7, 5, 1

[SSC]

- 7. 1. Rain 2. Monsoon 3. Rescue 4. Flood 5. Shelter 6. Relief
 - (a) 1, 2, 3, 4, 5, 6

(b) 1, 2, 4, 5, 3, 6

(c) 2, 1, 4, 3, 5, 6

(d) 4, 1, 2, 3, 5, 6

[SSC]

- 8. 1. Never 2. Sometimes 3. Generally 4. Seldom 5. Always
 - (a) 5, 2, 1, 3, 4

(b) 5, 2, 4, 3, 1

(c) 5, 3, 1, 2, 4

(d) 5, 3, 2, 4, 1

[SSC]

9. 1. Butterfly 2. Cocoon 3. Egg 4. Worm

(a) 1, 3, 4, 2

(b) 2, 4, 1, 3

(c) 1, 4, 3, 2

(d) 3, 4, 2, 1

[CBI]

10. 1. Site 2. Plan 3. Rent 4. Money 5. Building 6. Construction

(a) 1, 2, 3, 6, 5, 4

(b) 2, 3, 6, 5, 1, 4

(c) 3, 4, 2, 6, 5, 1

(d) 4, 1, 2, 6, 5, 3

[Hotel Management]

ANSWERS

1. (b) **2.** (c)

3. (b) **4.** (b)

5. (c)

6. (d)

7. (c)

8. (d)

9. (d)

10. (d)

Solutions and Hints

In this words are arranged such that each successive term constitute part of first word.
 i.e., Book → Chapter → Paragraph → Sentence → Word → Letter.

Chapter constitutes part of a book.

Paragraph constitutes part of a chapter etc.

2. In this words are arranged according to the order of occurrence.

Here *crime* was done first. Interference of *Police* occurs as second. After the criminal was caught by the police he will be brought to the count before the *Justice*. After the hearing is over, *judgement* will be issued. Based on that *punishment* will be given to the criminal. Therefore crime, police, justice, judgement and punishment is the required order.

3. The sequence of words here represents different stages from birth of a person to his final stage in his carrer. Here Child → School → College → Employment → Salary the sequence mention that first stage of a person is child. After some time that child will go to school, then college. After completing his study in the college, he will appear for various competitive examinations for a job. Then on getting success in the examination, he will get an employment. After getting employed he will get salary. Therefore the sequence is 2, 4, 1, 5, 3.

Hence answer is (b).

4. The sequence here represents various expression of a child in different situations.

Here *child* first *cries*. While crying *mother* will give *milk* to the child. Then the child will smile on getting milk.

Hence sequence is Child \rightarrow Cry \rightarrow Mother \rightarrow Milk \rightarrow Smile.

Hence answer is (b).

The sequence here represents different periods in order. i.e., Stone age → Metallic age →
Alloy age → Atomic age.

Hence answer is (c).

- 6. This sequence consists of body parts from head to toe in order. Therefore sequence is $Skull \rightarrow Face \rightarrow Neck \rightarrow Shoulder \rightarrow Hand \rightarrow Chest \rightarrow Stomach \rightarrow Thigh \rightarrow Knee \rightarrow Heel.$ Hence answer is (d).
- 7. This sequence consists of climatic changes from monsoon and subsequent processes. Therefore sequence in Monsson → Rain → Flood → Rescue → Shelter → Relief. Hence answer is (c).
- 8. This sequence contains the same word with different intensity in the decreasing order of intensity. i.e., Always → Generally → Sometimes → Seldom → Never. Hence answer is (d).

- 9. This sequence consists of different stages of formation of Butterfly from its egg. i.e., $Egg \to Worm \to Cocoon \to Butterfly$. Hence answer is (d).
- 10. This sequence consists of different names related to construction of a building in the order of execution/priority. i.e., Money → Site → Plan → Construction → Building → Rent. i.e., First priority is given for money. After arranging money, place or land is bought for construction which is the site. After having the site, plan is prepared for construction of building. After construction is over building got ready for occupying. Then the building is given for rent.

Hence answer is (d).

SECTION B LOGICAL DEDUCTION

Chapter 16

Logic

Logic is the science of thought as expressed in language. Questions based on logic are to be solved considering the given statements as they are true without considering the formal validity or the truth of the statements. i.e., the conclusion should follow directly from the statements given.

LOGICAL REASONING: In logic, a statement of certain relation between two or more terms is analogous to a sentence in grammar.

The proposition consists of three parts, namely subject, predicate and copula.

- 1. Subject: The subject is about which something is said.
- 2. Predicate: The predicate is the part of the proposition denoting which is affirmed or denied about the subject.
- **3. Copula:** The copula is that part of the proposition which denotes the relation between the subject and the predicate.

EXAMPLE 1 Consider the proposition 'Man is intelligent'. Here the information is given about the man. So 'Man' is the subject.

'Intelligent' is the quality affirmed for this subject. So it is the predicate. 'Is' denotes the relation between the subject and the predicate. So, it is the copula.

CLASSIFICATION OF PROPOSITIONS: 'Propositions' can be classified into four types:

1. Universal Affirmative Proposition (denoted by A): This distributes only the subject i.e. the predicate is not interchangeable with the subject while maintaining the validity of the proposition.

EXAMPLE 2 All books are pens.

This is proposition A since we cannot say 'All pens are books'.

- 2. Universal Negative Proposition (denoted by E): This distributes both the subject and the predicate, i.e. an entire class of predicate term is denied to the entire class of the subject term, as in the proposition. For example, No pen is pencil.
- 3. Particular Affirmative Proposition (denoted by I): This distributes neither the subject nor the predicate. For example,

 Some pupil are brilliant.

Here subject term 'some pupil' is used not for all but only for some students and similarly the predicate term 'brilliant' is affirmed for a part of subject class. So both are undistributed.

4. Particular Negative Proposition (denoted by O): This distributes only the predicate. For example,

Some cats are not dogs.

Here, the subject term 'some cats' is used only for a part of its class and hence is undistributed while the predicate term 'dogs' is denied in entity to the subject term and hence is distributed.

The above four types of proposition can be summarised as follows:

Proposition	Type
(a) (A) distributes subject only	All S is P
(b) (E) distributes both subject and predicate	No S is P
(c) (I) distributes neither	Some S is P
(d) (O) distributes predicate only	Some S is not P

Statement Form	Type of Proposition	Distributes
(A) All S is P	Universal affirmative	S only
(B) No S is P	Universal negative	Both S and P
(I) Some S is P	Particular affirmative	Neither S nor P
(O) Some S is not P	Particular negative	P only

LOGICAL DEDUCTION: The phenomenon of deriving a conclusion from a single proposition or a set of given proposition is known as *logical deduction*. The given proposition are also referred to as the premise.

There are two inferential processes of deduction.

Immediate Deductive Inference: Here the conclusion is deduced from one of the given propositions, by any of the three ways—conversion, obversion and contraposition.

Conversion: The conversion proceeds with interchanging the subject term and predicate term, i.e. the subject term of the premise becomes the predicate term of the conclusion and the predicate term of the premise becomes the subject of the conclusion.

The given proposition is called *convertend*, whereas the conclusion drawn from it is called its *converse*.

Valid conversions

Convertend	Converse
A: All S is P	I: Some P is S
Example: All books are pens.	Some pens are books.
E: No S is P	E: No P is S
Example: No cat is dog.	No dog is cat.
I: Some S is P	I: Some P is S
Example: Some boys are foolish.	Some fools are boys.
O: Some S is not P	No valid conversion

In a conversion, quality remains the same and the quantity may change.

Obversion: In obversion, we change the quality of the proposition and replace the predicate term by its complement.

Valid obversions

Obvertend	Obverse
A: All birds are mammals.	E: No birds are non-mammals.
E: No poets are singers.	A: All poets are non-singers
I: Some men are engineers.	O: Some men are not non-engineers.
O: Some politicians are not honest.	I: Some politicians are non-honest.

Contraposition: In contraposition, we first replace the subject and predicate terms in the proposition and then exchange both these terms with their complements.

Valid contrapositions

Proposition	Contrapositive
A: All humans are animals.	A: All non-animals are non-humans.
I: Some humans are animals.	I: Some non-animals are non-humans.

MEDIATE DEDUCTIVE INFERENCE (SYLLOGISM): A syllogism is a deductive argument in which the conclusion has to be drawn from two propositions referred to as the premises.

Example: 1. All s

- 1. All girls are students.
- 2. All students are brilliant.
- 3. All girls are brilliant.

Here, the propositions 1 and 2 are the premises and the proposition 3, which follows from the first two propositions, is called the *conclusion*.

In logic, a *term* is a word or a combination of words, which by itself can be used as a subject or predicate of a proposition.

Syllogism is concerned with three terms:

- 1. **Major Term:** This is the predicate of the conclusion and is denoted by **P** (first letter of 'Predicate').
- 2. Minor Term: This is the subject of the conclusion and is denoted by S (first letter of 'Subject').
- 3. Middle Term: This is the term common to both the premises and is denoted by M (first letter of 'Middle').

Example: Premises: 1. All cats are animals.

2. All tigers are cats.

Conclusion: All tigers are animals.

Here, 'Animals' is the predicate of the conclusion and so it is the major term P.

'Tigers' is the subject of the conclusion and so, it is the minor term, S.

'Cats' is the term common to both the premises and so it is the middle term, M.

Major and Minor Premises: Of the two premises, the major premise is that in which the middle term is the subject and the minor premise is that in which the middle term is the predicate.

RULES FOR DERIVING THE CONCLUSION FROM TWO GIVEN PREMISES

1. The conclusion does not contain the middle term.

EXAMPLE 1

Statements: 1. All desks are tables.

2. Some tables are chairs.

Conclusions: 1. All tables are desks.

2. Some tables are not chairs.

Since both the conclusions 1 and 2 contain the middle term 'tables', neither of them can follow.

2. No term can be distributed in the conclusion unless it is distributed in the premises.

EXAMPLE 2

Statements: 1. Some students are boys.

2. All boys are fools.

Conclusions: 1. All fools are boys.

2. Some students are fools.

Statement 1 is an I-type proposition which distributes neither the subject nor the predicate. Statement 2 is an A-type proposition which distributes the subject i.e., 'boys' only.

Conclusion 1 is an A-type proposition which distributes the subject 'fools' only.

Since the term 'fools' is distributed in conclusion 1 without being distributed in the premise, so conclusion 1 cannot follow.

3. The middle term (M) should be distributed at least once in the premise. Otherwise, the conclusion cannot follow.

For the middle term to be distributed in a premise,

- (i) M must be the subject if premise is an A type proposition.
- (ii) M must be subject or predicate if premise is an E type proposition.
- (iii) M must be predicate if premise is an O type proposition.

In an I type proposition, which distributes neither the subject nor the predicate, the middle term cannot be distributed.

EXAMPLE 3

Statements: 1. All clocks are watches.

2. Some watches are red.

Conclusions: 1. All watches are clocks.

2. Some clocks are red.

In the premises, the middle term is 'watches'. Clearly, it is not distributed in the first premise, which is an A type proposition as it does not form its subject. It is not distributed in the second premise which is an I type proposition.

Since the middle term is not distributed even in both the premises, so no conclusion follows:

4. No conclusion follows:

(a) if both the premises are particular.

EXAMPLE 4

Statements: 1. Some pens are pencils.

2. Some pencils are rulers.

Conclusions: 1. All pens are rulers.

2. Some rulers are pens.

Since both the premises are particular, no definite conclusion follows:

(b) if both the premises are negative.

EXAMPLE 5

Statements: 1. No mango is pineapple.

2. No pineapple is coconut.

Conclusions: 1. No mango is coconut.

2. Some coconuts are pineapples.

Since both the premises are negative, neither conclusion follows:

(c) if the major premise is particular and the minor premise is negative.

EXAMPLE 6

Statements: 1. Some cows are bulls.

2. No cats are cows.

Conclusions: 1. No cows are cats.

2. Some bulls are cats.

Here, the first premise containing the middle term 'cows' as the subject is the major premise and the second premise containing the middle term 'cows' as the predicate in the minor premise. Since the major premise is particular and the minor premise is negative, no conclusion follows.

5. If the middle term is distributed twice, the conclusion cannot be universal.

EXAMPLE 7

Statements: 1. All tables are cots.

2. No chairs are tables.

Conclusions: 1. No chairs are cots.

2. Some chairs are cots.

Here, the first premise is an A type proposition and so, the middle term 'tables' forming the subject is distributed. The second premise is an E type proposition and so the middle term 'tables' forming the predicate is distributed.

Since the middle term is distributed twice, the conclusion cannot be universal.

6. If one premise is negative, the conclusion must be negative.

EXAMPLE 8

Statements: 1. All pens are rubbers.

2. No rubber is pencil.

Conclusions: 1. No pens are pencils.

2. Some pencils are pens.

Since one premise is negative, the conclusion must be negative. So conclusion 2 cannot follow.

7. If one premise is particular, the conclusion must be particular.

EXAMPLE 9

Statements: 1. Some men are thieves.

2. All thieves are intelligent persons.

Conclusions: 1. Some men are intelligent persons.

2. All intelligent persons are men.

Since, one premise is particular, the conclusion must be particular. so, conclusion 2 cannot follow.

8. If both the premises are affirmative, the conclusion must be affirmative.

EXAMPLE 10

Statements: 1. All cows are cats.

2. All cats are bulls.

Conclusions: 1. All cows are bulls.

2. Some cows are not bulls.

Since both the premises are affirmative, the conclusion must be affirmative. So, conclusion 2 cannot follow.

9. If both the premises are universal, the conclusion must be universal.

Complementary pair: A pair of contradictory statements, i.e. a pair of statements such that if one is true, the other is false and when no definite conclusion can be drawn, either of them is bound to follow, is called a *complementary pair*.

E and I type propositions together form a complementary pair and usually either of them follows, in a case where we cannot arrive at a definite conclusion, using the rules of syllogism.

Study the various possible cases and draw all possible inferences in each case, along with verification through Venn diagrams.

Case 1: All men are boys. All boys are players.

Immediate Deductive Inferences: The converse of first premise 'Some boys are men' and the converse of second premise 'Some players are boys' both hold good.

Mediate Deductive Inferences: Since both the premises are universal and affirmative, the conclusion must be universal and affirmative. Also, the conclusion should not contain the middle term. Therefore, it follows that 'All men are players'. The converse of this conclusion 'Some players are men' also holds good.

Venn Diagram	Inferences
Players Boys Men	 Some boys are men. Some players are boys. All men are players. Some players are men.

Case 2: All birds are mammals.

All animals are mammals.

Immediate Deductive Inferences: The converse of first premise, 'Some mammals are birds' and the converse of second premise 'Some mammals are animals' both hold good.

Mediate Deductive Inferences: Both, being A-type propositions, distribute subject only.

Thus, the middle term 'mammals' is not distributed even once in the premises. Therefore, no definite conclusion follows.

Venn Diagram	Inferences			
Mammals or Birds Animals Birds Animals	 Some mammals are birds. Some mammals are animals. Either 'No bird is animal' or 'Some birds are animals' as E and I-type propositions form a complementary pair. 			

Case 3: All men are employees.

Some employees are players.

Immediate Deductive Inferences: The converse of first premise 'Some employees are men' and the converse of the second premise 'Some players are employees', both hold good.

Mediate Deductive Inferences: The first premise is an A-type proposition which distributes the subject only while the second premise is an I-type proposition which distributes neither subject nor predicate. Since the middle term 'boys', is not distributed even once in the premises, no definite conclusion can be drawn.

Venn l	Diagram	Inferences			
Employees Men Players	Employees or Players	 Some employees are men. Some players are employees. Either 'No men is player' or 'Some men are players' as E and I-type proposition form a complementary pair. 			

Case 4: Some singers are writers.

All writers are artists.

Immediate Deductive Inferences: The converse of the first premise 'Some writers are singers' and the converse of the second premise 'Some artists are writers', both hold good.

Mediate Deductive Inferences: Since one premise is particular the conclusion must be particular and should not contain the middle term. so it follows that 'Some singers are artists.' The converse of this conclusion 'Some artists are singers' also holds good.

Venn Diagram	Inferences
Artists Singers Writers	 Some writers are singers. Some artists are writers. Some singers are artists. Some artists are singers.

Case 5: All boys are players. Some boys are writers.

Immediate Deductive Inferences: The converse of the first premise 'Some players are boys' and the converse of the second premise 'Some writers are boys' both hold good.

Mediate Deductive Inferences: Since the premise is particular, the conclusion must be particular and should not contain the middle term. So, it follows that 'Some players are writers' and the converse of this conclusion 'Some writers are players' also holds good.

Venn Diagram	Inferences
Players Boys Writers	 Some players are boys. Some writers are boys Some players are writers. Some writers are players.

Case 6: All boys are students. Some men are students.

Immediate Deductive Inferences: The converse of the first premise 'Some students are boys' and the converse the second premise 'Some students are men' both hold good.

MEDIATE DEDUCTIVE INFERENCES: The first premise is an A-type proposition, which distributes subject only. The second premise is an I-type proposition, which distributes neither subject not predicate. So the middle term 'students' is not distributed even once in the premises. Hence, no definite conclusion can be drawn.

Venn Diagram	Inferences
Students Boys Men	 Some students are boys. Some students are men. Either 'No boy is man' or 'some boys are men' follows.

Case 7: Some balls are sticks. Some sticks are erasers.

Immediate Deductive Inferences: The converse of the first premise 'Some sticks are balls' and the converse of the second premise 'Some erasers are sticks' both hold good.

Mediate Deductive Inferences: Since both premises are particular, no definite conclusion follows.

Venn Diagram	Inferences
Balls Sticks Balls Sticks Erasers or Erasers	 Some sticks are balls. Some erasers are sticks. Either 'Some balls are erasers' or 'No ball is eraser' follows as I and E-type propositions form a complementary pair.

Case 8: All books are pencils. No pencil is eraser.

Immediate Deductive Inferences: The converse of the first premise 'Some pencils are books' and the converse of the second premise 'No eraser is pencil', both hold good.

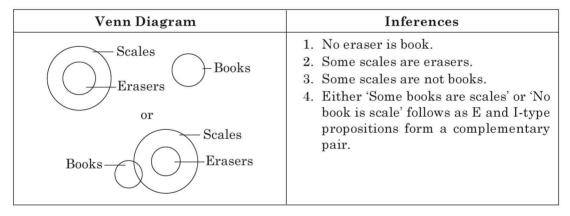
Mediate Deductive Inferences: Since both premises are universal, the conclusion must be universal. Since one premise is negative, the conclusion must be negative. So 'No book is eraser' follows. The converse of the conclusion 'No eraser is book' also holds good.

Venn Diagram	Inferences		
Pencils Books —Erasers	 Some pencils are books. No eraser is pencil. No book is eraser. No eraser is book. 		

Case 9: No book is eraser. All erasers are scales.

Immediate Deductive Inferences: The converse of the first premise 'No eraser is book' and the converse of the second premise 'Some scales are erasers', both hold good.

Mediate Deductive Inferences: The first premise, an E-type proposition, distributes both the subject and the predicate. Second premise, an A-type proposition, distributes the subject. Thus, the middle term 'eraser' is distributed twice in the premises. So conclusion cannot be universal. One premise in negative. So the conclusion must be negative. Thus the conclusion must be particular negative (i.e., O-type). Therefore 'Some scales are not books' follows.



Case 10: Some fans are heaters. No heater is refrigerator.

Immediate Deductive Inferences: The converse of the first premise 'Some heaters are fans' and the converse of the second premise 'No refrigerator is heater', both hold good.

Mediate Deductive Inferences: Since the first premise is particular and the second premise negative, the conclusion must be particular and negative or O-type. Therefore, it follows that 'Some fans are not refrigerators.'

Venn Diagram	Inferences			
Fans Heaters Refrigerator or Fans Heaters Refrigerator	 Some heaters are fans. No refrigerator is heater. Some fans are not refrigerators. Either 'Some fans are refrigerators' or 'No fan is refrigerator' follows as E and I-type propositions form a complementary pair. 			

Note: While deriving logical conclusions, always remember that the following conclusions hold:

- (i) The converse of each of the given premises;
- (ii) The conclusion that directly follows from the given premises in accordance with the rules of syllogism; and
- (iii) The converse of the derived conclusions.

TYPE I: TWO-PREMISE ARGUMENTS (a) WITH TWO CONCLUSIONS

Directions (1 to 10): In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from the commonly known facts and then decide which of the given conclusions logically follows from the given two statements, disregarding commonly known facts.

Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows: and (e) if both I and II follow.

1. Statements: All cakes are ice creams.

All ice creams are toffees.

Conclusions: I. All cakes are toffees.

II. All toffees are ice creams. [RRB Exam]

2. Statements: Some pearls are gems.

Some gems are ornaments.

Conclusions: I. Some gems are pearls.

II. Some ornaments are gems. [Bank PO]

3. Statements: All lights are trucks.

Some trucks are jeeps.

Conclusions: I. All jeeps are lights.

II. Some lights are jeeps. [Bank PO]

4. Statements: Lawyers married only fair girls.

Sobha is very fair.

Conclusions: I. Sobha was married to a lawyer.

II. Sobha was not married to a lawyer.

[Railway Recruitment]

5. Statements: All pencils are bricks.

All bricks are bottles.

Conclusions: I. All pencils are bottles.

II. All bricks are pencils. [RBI Exam]

6. Statements: Some books are pencils.

Some pencils are pens.

Conclusions: I. Some books are pens.

II. Some pens are books. [SBI PO]

7. Statements: All trays are erasers.

All pens are erasers.

Conclusions: I. All trays are pens.

II. Some pens are trays. [BSRB Exam]

8. Statements: All birds are trees.

Some trees are hens.

Conclusions: I. Some birds are hens.

II. Some hens are trees.

9. Statements: Some cooks are lazy.

All boys are lazy.

Conclusions: I. Some boys are cooks.

II. Some cooks are boys. [Bank PO]

10. Statements: Sohan is a good sportsmen.

Sportsmen are healthy.

Conclusions: I. All healthy person are sportsmen.

II. Sohan is healthy. [Bank PO]

ANSWERS

1. (a) 2. (d) 3. (d) 4. (c) 5. (a) 6. (d) 7. (d) 8. (a) 9. (d) 10. (b)

Solutions and Hints

- 1. Since both the statements are affirmative, the conclusion must be affirmative. Conclusion II cannot follow as it contains the middle term. So, only conclusion I follows.
- 2. Since both the premises are particular, no conclusion follows.
- 3. The first premise, being an A type proposition distributes subject only. So middle term 'trucks' forming the predicate is not distributed.

The second premise, being an I type proposition distributes neither the subject nor the predicate. So, the middle term 'trucks' forming the subject is not distributed.

- 4. The data does not mention whether all fair girls were married to lawyers. So, either of the two conclusions may follow.
- 5. Since both the premises are affirmative, the conclusion must be affirmative and only conclusion I follows.
- 6. Since, both the premises are particular, no conclusion follows.
- 7. Both the premises are A type propositions. So, in either, the middle term 'erasers' forming the predicate is not distributed.

Since the middle term is not distributed at least once in the premises, no conclusion follows.

- 8. Since one premise is particular, the conclusion must be particular. Conclusion II cannot follow as it contains the middle term. So only conclusion I follows.
- 9. The first premise is an I type proposition and distributes neither the subject nor the predicate. So the middle term 'lazy' forming the predicate is not distributed.

The second premise is an A type proposition and distribute subject only. So, the middle term 'lazy' forming the predicate is not distributed. Since the middle term is not distributed at least once in the premises, no conclusion follows.

10. Conclusion I cannot follow as it contains the middle term. So only conclusion II follows.

TWO-PREMISE ARGUMENTS (b) WITH MORE THAN TWO CONCLUSIONS

In these type of questions, two statements called *premises* are given followed by four *conclusions*. The candidate has to find out which of the conclusions follow from the given premises. First the given statements are analysed. If the middle term is not distributed at least once, no conclusion follows. No conclusion follows when

- (i) both the premises are particular.
- (ii) both the premises are negative.

EXAMPLE 1

Statements: Some mangoes are apples.

Some apples are lemons.

Conclusions: I. No mango is lemon.

II. All apples are lemons.

III. Some lemons are mangoes

IV. No apple is mango.

Solution Since both the premises are particular, (i.e. of type 'Some S is P'), no conclusion follows.

EXAMPLE 2

Statements: All birds are tigers.

All tigers are animals.

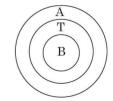
Conclusions: I. All birds are animals.

II. Some tigers are birds.

III. Some animals are birds.

IV. Some animals are tigers.

Solution Region for birds 'B' lies entirely within the region for animals 'A'. So conclusion I follows. Since T has a common area with B, A has a common area with B and A has a common area with T, conclusions II, III and IV follow. Thus all the conclusions follow.



EXAMPLE 3

Statements: All books are papers.

Some papers are pencils.

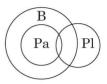
Conclusions: I. All pencils are papers.

II. All pencils are books.

- III. Some books are pencils.
- IV. No book is pencil.

Solution Clearly, the middle term is 'Papers' denoted by 'Pa'.

The first premise is an A type proposition which distributes subject only. So the middle term 'Papers', forming the predicate is not distributed. The second premise is an I type proposition and distributes neither the subject nor the predicate. So, the middle term 'Papers', forming the subject is not distributed. Since the middle term is not distributed at least once in the premises, no conclusion follows.



[Bank PO]

[AFO Exam]

EXERCISES

Directions (Q.1 to 5): In each question below are given two statements followed by four conclusions numbered I, II, III and IV. You have to consider the given statements to be true even if they seem to be at variance from the commonly known facts. Then decide which of the given conclusion logically follow from the given two statements, disregarding the commonly known facts.

1. Statements: Some teachers are students.

All students are girls.

Conclusions:

- I. All teachers are girls.
- II. Some girls are teachers.
- III. Some girls are students.
- IV. All students are teachers.
- (a) Only I follows

(b) Only I. II and III follow

(c) All follow

(d) Only II and III follow

(e) None follows

2. Statements: All soaps are clean.

All clean are wet.

Conclusions:

- I. Some clean are soaps.
- II. No clean is soap.
- III. Some wet are soaps.
- IV. All wet are soaps.
- (a) Only I follows

(b) Either I or II follows

(c) Either III or IV follows

(e) None follows

(d) Only I and III follow

3. Statements: All typists are stenographers.

Some stenographers are boys.

Conclusions:

- I. All boys are stenographers.
- II. All boys are teachers.
- III. Some teachers are boys.
- IV. No teacher as boy.
- (a) Only I follows

(b) Only II and III follow

(c) Either II or III follows

(d) Either I or IV follows

(e) None follows

- [Bank PO]
- 4. Statements: Some green are blue.

No blue is white.

II. Some white are green.

III. Some green are not white.

IV. All white are green.

(a) Only I follows

(b) Only II and III follow

(c) Only I and II follow

(d) Only I and III follow

(e) Either III or IV follows

[AFO Exam]

5. Statements: All scientists are fools.

All fools are illiterate.

Conclusions: I. All scientists are illiterate.

II. All illiterates are scientists.

III. All illiterates are fools.

IV. Some illiterates would be scientists.

(a) Only I and IV follow

(b) Only II follows

(c) Only III and IV follow

(d) Only IV follows

(e) None follows

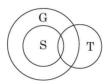
[UTI Exam]

ANSWERS

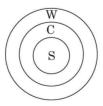
- 1. (d) 2. (d)
- **3.** (e)
- **4.** (d)
- **5.** (a)

Solutions and Hints

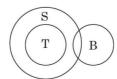
1. From the Venn diagram, it is clear that some girls are teachers and some girls are students, i.e. conclusions II and III follow. But conclusions I and IV cannot follow.



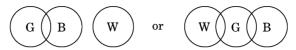
2. From the Venn diagram, C and S have a common area. So some clean are soaps. Similarly W and S have a common area. So some wet are soaps. Therefore, only conclusions I and III follow.



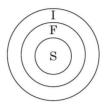
3. From the Venn diagram, none of the conclusions follow.



4. From the Venn diagram, conclusion I and III follow, i.e. 'Some blue are green' and 'Some green are not white'



5. From the Venn diagram, area of scientists 'S' lie entirely within the area of illiterates 'I'. So conclusion I follows. The area 'I' does not lie completely within 'S' or 'F'. Therefore, conclusion II and III cannot follow. But illiterates and scientists have a common area. So conclusion IV follows. Therefore, only conclusion I and IV follow.



TYPE 2: THREE-PREMISE ARGUMENTS: Here, you are given three premises, containing four terms in all, followed by three or four conclusions. In such type of questions, first check for the converse of each of the premises amongst the given conclusions. Then apply the rules of syllogism to the given premises taking two interrelated premises at a time. Then use the conclusion so derived as a premise together with another premise to infer yet another conclusion.

Directions: In each of the following questions, three statements are given followed by four conclusions numbered I, II, III and IV. You have to take the given statements to be true even if they seem to be at variance with commonly known facts and then decide which of the given conclusions logically follow from the given statements disregarding the commonly known facts.

EXAMPLE 1

Statements: Some flowers are rods.

Some rods are doors. Some doors are houses.

Conclusions: I. Some houses are flowers.

II. Some doors are flowers.

III. Some flowers are doors.

IV. No house is flower.

(a) Either I or II follows

(b) Either I or IV follows

(c) Only II and III follow

(d) Only I and IV follow

(e) None of these

[SBI PO]

Solution Each combination of premises shall contain two particular premises and as such no definite conclusion follows.

Here, I and IV are contradictory statements involving only the extreme terms and none of the middle term 'rods' or 'doors'. Thus they form a complementary pair. So either I or IV follows.

Hence, the answer is (b).

EXAMPLE 2

Statements: All buildings are rains. All papers are buildings. All dogs are papers.

Conclusions:

- I. All dogs are rains.
- II. Some papers are rains.III. Some rains are buildings
- IV. Some rains are papers.
- (a) Only I and II follow

(b) Only II and III follow

(c) Only I, II and III follow

(d) All follow

(e) None of these

[Bank PO]

Solution Conclusion III is the converse of first premise and so it holds.

All papers are buildings. All buildings are rains.

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All papers are rains' and IV is the converse of this conclusion and so it holds.

All dogs are papers. All papers are buildings.

Clearly it follows that 'All dogs are buildings'.

All dogs are buildings. All buildings are rains.

Clearly it follows that 'All dogs are rains' and so conclusion I follows. Hence I, III and IV follow from the given premises.

Hence, the answer is (e).

EXAMPLE 3

Statements: Some fruits are flowers.

No flower is a boat.

All boats are rivers.

Conclusions:

- I. Some fruits are rivers.
- II. Some rivers are boats.
- III. Some rivers are fruits.
- IV. Some flowers are fruits.
- (a) Only I and III follow

(b) Only II and III follow

(c) Only II and IV follow

(d) All follow

(e) None of these

[SBI PO]

Solution IV is the converse of first premise and so it holds.

II is the converse of third premise and so it holds.

Some fruits are flowers. No flower is a boat

Since one premise is particular and one negative, conclusion must be particular and negative and should not contain the middle term. So it follows that 'Some fruits are not boats'.

No flower is a boat. All boats are rivers.

Since the middle term is distributed twice, the conclusion cannot be universal. Again, since one premise is negative, the conclusion must be negative and shouldn't contain the middle term. Therefore, it follows that 'Some flowers are not rivers'.

Therefore, only II and IV follow.

Hence, the answer is (c).

EXERCISES

Directions (Q.1 to 5): In each of the following questions, three statements are followed by three conclusions numbered I, II and III. You have to take the given statements to be true even if they seem to be at variance from the commonly known facts. You have to decide which of the given conclusions logically follow from the given statements disregarding the commonly known facts.

1. Statements: Some blades are hammers.

Some hammers are knives.

Some knives are axes.

Conclusions: I. Some axes are hammers.

II. Some knives are blades.

III. Some axes are blades.

(a) Only I follows

(b) Only II follows

(c) Only III follows

(d) None follows

(e) None of these

2. Statements: All dolls are windows.

All bottles are windows.

All cars are bottles.

Conclusions: I. All

I. All cars are windows.

II. Some cars are dolls.

III. Some windows are cars.

(a) Only I and II follow

(b) Only II and III follow

(c) Only I and III follow

(d) All follow

(e) None of these

[Bank PO]

3. Statements: Some hills are rivers.

Some rivers are deserts.

All deserts are roads.

Conclusions:

I. Some roads are rivers.

II. Some roads are hills.

III. Some deserts are hills

(a) Only I follows

(b) Only I and II follow

(c) Only II and III follow

(d) All follow

(e) None follow

10.00

4. Statements: All fruits are vegetables.

All pens are vegetables. All vegetables are rains.

Conclusions:

I. All fruits are rains.

II. All pens are rains.

III. Some rains are vegetables.

(a) Only I and II follow

(b) Only II and III follow

(c) Only I and III follow

(d) All follow

(e) None follows

[Bank PO]

[Bank PO]

[Bank PO]

5. Statements: All dogs are fruits.

No chair is fruit.

Some chairs are clowns.

147 Logic Conclusions: I. Some clowns are dogs. II. Some chairs are dogs. III. No chair is dog. IV. No dog is clown. (a) Either I or IV follows (b) Either I or IV and III follow (c) Either I or IV and II follow (d) Either II or III follows and either I or IV follows. ISBI POI Directions (Q. 6): In the following question, select one alternative in which the third statement is implied by the first two statements. 6. (a) All dogs are mad. All sick persons are mad. So, all sick persons are dogs. (b) All oranges are black. All figs are oranges. So, all figs are black. (c) All windows are dogs. Some doors are dogs. So, all windows are doors. (d) No man can fly. No kite can fly. So, all men are kites. **Directions (Q. 7 to 10):** Each question given below has a set of three or four arguments. Each set of statements is further divided into three segments. Choose the alternative when the third segment in the statement can be logically deduced using both the preceding two, but not just from one of them. 7. (A) All good people are knights. All warriors are good people. All knights are warriors. (B) No footballers are ministers. All footballers are tough. Some ministers are players. (C) All pizzas are snacks. Some meals are pizzas. Some meals are snacks. (D) Some students are boys. All students are girls. Some girls are boys. (a) A only (b) B and C (c) C only (d) C and D [CAT] 8. (A) All beautiful things are sad. She is beautiful. She is sad. (B) All nice things are flat. TVs are flat. TVs are nice things. (C) Potatoes are stems. All stems are fruits. Potatoes are fruits. (a) A only (b) A and B (c) C only (d) A and C [MAT] 9. (A) P is an artist. Some artists are pretty. P is pretty. (B) All actors are brave. Some men are actors. Some men are brave. (C) All actors are pretty. P is not an actor. P is not pretty. (D) Some men are employees. All employees are efficient. Some efficient people are employees. (a) A only (b) B and D (c) B only (d) C only 10. (A) Ravens are black. Ravens are evil. All evil are black.

(B) Horses are faster than eagles. All eagles are hawks. Horses are faster than hawks.

(C) No priest is a saint. Peter is a priest. Peter is a saint.

(a) A only

(b) B only

(c) C only

(d) None of these.

[MAT]

ANSWERS

1. (d) 3. (a) **4.** (d) **5.** (b) **6.** (b) 7. (c) 8. (c) 9. (c) 10. (b) **2.** (c)

Solutions and Hints

1. Clearly each combination of premises shall contain two particular premises and as such no definite conclusion follows. Here,

Some blades are hammers. Some hammers are knives.

Since both the premises are particular, conclusion must be particular and should not contain the middle term 'hammers' So, it follows that 'Some blades are knives'.

Some blades are knives. Some knives are axes

Since both the premises are particular, conclusion must be particular and should not contain the middle term 'knives'. So, it follows that 'Some blades are axes'. From the given conclusions, none of them follows.

Hence, the answer is (d).

2. All cars are bottles. All bottles are windows.

Clearly it follows that 'All cars are windows'. Therefore conclusion I follows.

Conclusion III is the converse of I. So III follows.

Thus only I and III follows.

Hence, the answer is (c).

3. Some hills are rivers. Some rivers are deserts

Since both the premises are particular, no definite conclusion follows.

Some rivers are deserts. All deserts are roads

Since one premise is particular, conclusion should be particular and should not contain the middle term. So it follows that 'Some rivers are roads'. I is the converse of this conclusion. So I follows.

Hence, the answer is (a).

4. All fruits are vegetables. All vegetables are rains

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All fruits are rains'. So I follows.

All pens are vegetables. All vegetables are rains.

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All pens are rains'. So II follows. Conclusion III is the converse of the third premise and so it holds. So III follows.

Hence, the answer is (d).

5. All dogs are fruits. No chair is fruit.

Since one premise is negative, conclusion must be negative. Since both the premises one universal, the conclusion must be universal. Therefore, it follows that 'No dog is chair', which is the converse of this conclusion and so it holds. Hence III follows.

No dog is chair. Some chairs are clowns.

Since one premise is negative and one particular, conclusion must be particular and negative. Therefore it follows that 'Some dogs are not clowns'. I and IV involves only the extreme terms and not the middle term 'chairs' and form complementary pair. So either I or IV follows.

Hence, the answer is (b).

6. Only the statement (b) in which third statement is implied by first two statements. Hence, answer is (b).

7. Only the statement (c) in which third segment can be logically deduced from the first two segment. i.e. All pizzas are snacks. Some meals are Pizzas.

Hence, the answer is (c).

8. Only the statement (c) in which third segment can be logically deduced from the first two segment.

i.e., Potatoes are stems.

All stems are fruits.

Potatoes are fruits.

Hence, answer is (c).

9. Only statement (c) in which third segment can be logically deduced from the first two segment. i.e., All actors are pretty.

P is not an actor.

P is not pretty.

Hence answer is (c).

 $\textbf{10.} \ \ \textbf{Only statement (b) in which third segment can be logically deduced from the first two segment.}$

i.e., Horses are faster than eagles.

All eagles are hawks.

Horses are faster than hawks.

Hence, answer is (b).

Statements and Arguments

In this type of questions, a statement is given and two arguments for or against or both follow. Analyse the statement first and then decide which of the given arguments is strong with respect to the statement.

If argument I is strong, then answer (a); if argument II is strong, then answer (b); if either of the arguments I or II is strong then answer (c); if none of the arguments I or II is strong, then answer (d) and if both the arguments I and II are strong, then answer (e).

EXAMPLE 1

Statement: Should colleges be given the status of a university in India?

Arguments:

- I. Yes. Colleges are in a better position to assess the students' performance and therefore the degrees will be more valid.
- II. No. It is utopian to think that there will not be nepotism and corruption in awarding degrees by colleges.

Solution At the college level, all the students are assessed according to their performance in the university exams and not on the basis of any criteria of a more intimate dealing with the students. So, argument I is vague. At this level, the awarding of degrees is impartial and simply based on his performance. So, argument II also does not hold. So, the answer is (d).

EXAMPLE 2

Statement: Should government jobs in rural areas have more incentives?

Arguments:

- I. Yes. Incentives are essential for attracting government servants there.
- II. No. Rural areas are already cheaper, healthier and less complex than urban areas. So, why offer extra incentives?

Solution Government jobs in rural areas are having several difficulties as mentioned in the statement. In lieu of these, extra incentives are needed. So only argument I is strong. So the answer is (a).

EXAMPLE 3

Statement: Should computers be used in all possible sectors in India?

Arguments: I. Yes. It will bring efficiency and accuracy in the work.

II. No. It will be an injustice to the monumental human resources which are at present underutilised.

Solution It is clear that underutilised human resources have to be put together for better use. Computers with better and speedy efficiency can accomplish this. So, only argument I is strong. So the answer is (a).

EXERCISES

1. Statement: Should all news be controlled by government in a democracy?

[Bank PO]

Arguments: I. Yes. A variety of news only confuses people.

II. No. The controlled news loses credibility.

2. Statement: Should higher education be completely stopped for sometime?

[Bank PO]

Arguments: I. No. It will hamper the country's future progress.

II. Yes. It will reduce the educated unemployment.

3. Statement: Should workers be allowed to participate in the management of factories in India? [Bank PO]

Arguments: I. Yes. It is the present management theory.

II. No. Many workers are illiterate and so their contributions will not be of any value.

4. Statement: Should the private sector be permitted to operate telephone services?

[Bank PO]

Arguments: I. Yes. They are operated in advanced western countries.

II. No. It is risky to put them in private hands.

5. Statement: Should new big industries be started in Bombay? [Bank PO]

Arguments: I. Yes. It will create new job opportunities.

II. No. It will further add to the pollution of the city.

6. Statement: Should India manufacture atom bombs?

[Bank PO]

Arguments: I. Yes. It is imperative to protect the sovereignty and integrity of the country.

II. No. This will create imbalance in the power of nations in this region.

7. Statement: Should non-vegetarian food be totally banned in our country?

[Bank PO]

Arguments: I. Yes. It is expensive and therefore it is beyond the means of most people in our country.

II. No. Nothing should be banned in a democratic country like ours.

8. Statement: Should luxury hotels be banned in India?

[Bank PO]

Arguments: I. Yes. They are places from where international criminals operate.

II. No. Affluent foreign tourists will have no place to stay.

9. Statement: Should adult education programme be given priority over compulsory education programmes? [Bank PO]

Arguments: I. No. It will also help in success of compulsory education programme. II. Yes. It will help to eliminate the adult illiteracy. 10. Statement: Should India go in for computerisation in industry? [Bank PO] Arguments: I. I. No. Computerisation demands a lot of money. We should not waste money on it. II. Yes. When advanced countries are introducing computers in various areas, how can India afford to lag behind? [UTI Examl 11. Statement: Should telecasting feature films be stopped? Arguments: I. Yes. Young children are misguided by the feature films. II. No. This is the only way to educate the masses. 12. Statement: Should all the remote parts of a country be connected by road? [RBI Exam] Arguments: I. No. It will disturb peaceful simple life of the villagers. II. Yes. It must be done immediately. 13. Statement: Should there be only one university throughout India? [UTI Exam] I. Yes. This is the only way to bring about uniformity in the educational Arguments: standards. II. No. This is administratively impossible. 14. Statement: Should election expenses to central and state legislatures be met by the [AAO Exam] government. Arguments: I. Yes. It will put an end to political corruption. II. No. It is not good in any country. 15. Statement: Should fashionable dresses be banned? [AAO Exam] I. Yes. Fashions keep changing and hence consumption of cloth increases. Arguments: II. No. Fashionable clothes are a persons's self expression and therefore his/her fundamental right. 16. Statement: Should political parties be banned? [AAO Exam]

Arguments: I. Yes. It is necessary to teach a lesson to the politicians.

II. No. It will lead to an end of democracy.

17. Statement: Should a total ban be put on trapping wild animals? [AAO Exam]

Arguments: I. Yes. Trappers are making a lot of money.

II. No. Bans on hunting and trapping are not effective.

18. Statement: Should the illiterate be debarred from voting?

I. Yes. They are easily misguided. Arguments:

II. No. It is their constitutional right.

19. Statement: Should religion be taught in our schools?

Arguments: I. No. Ours is a secular state.

II. Yes. Teaching religion helps inculcate moral values among children.

20. Statement: Should there be no place of interview in selection?

Arguments: I. Yes. It is very subjective inassessment.

II. No. It is the only instrument to judge candidates motives and personality.

21. Statement: Should articles of only deserving authors be allowed to be published?

[Bank PO]

	Arguments:	I. Yes. It will save a lot of paper which is in short supply.
		II. No. It is not possible to draw a line between the deserving and the undeserving.
22.	Statement:	Should India encourage exports, when most things are insufficient for internal use itself? [Bank PO]
	Arguments:	I. Yes. We have to earn foreign exchange to pay for our imports.
		II. No. Even selective encouragement would lead to shortages.
23.	Statement:	Should income tax be evaded by people? [Bank PO]
	Arguments:	I. Yes. Taxes are excessively high.
		II. No. It is anti-national to do so.
24.	Statement:	Should personal tax be abolished in India? [Bank PO]
	Arguments:	I. Yes. It will motivate people to earn more.
		II. No. Individuals must learn to share their wealth with other people.
25 .	Statement:	Should India have no military force at all? [Bank PO]
	Arguments:	I. No. Other countries in the world do not believe in non-violence.
		II. Yes. Many Indians believe in non-violence.
26 .	Statement:	Should selection tests be of the objective rather than of the descriptive type?
		[Bank PO]
	Arguments:	I. Yes. The assessment of answers to objective type questions is fair and impartial.
		II. No. The descriptive type test is certainly a better tool than the objective type test.
27.	Statement:	Should those who receive dowry, despite the law prohibiting it, be punished? [Bank PO]
	Arguments:	I. Yes. Those who violate the law, must be punished.
		II. No. Dowry system is firmly rooted in the society since time immemorial.
28.	Statement:	Should smoking be prohibited? [BSRB Exam]
	Arguments:	I. Yes. It is wrong to smoke away millions of money.
		II. No. It will throw thousands of workers in the tobacco industry out of employment.
29.	Statement:	Should taxes on colour television be further increased? [AAO Exam]
	Arguments:	I. Yes. Colour television is a luxury item and only rich people buy them.
		II. No. Televisions are bought by the poor too.
30.	Statement:	Should loyalty be the only criterion for promotion in any organisation?
		[Officer's Grade]
	Arguments:	I. Yes. Without loyal men, no organisation can function.

	ANSWERS									
1. (b)	2. (a)	3. (b)	4. (c)	5. (e)	6. (a)	7. (b)	8. (b)	9. (b)	10. (d)	
11. (d)	12. (b)	13. (b)	14. (a)	15. (b)	16. (d)	17. (d)	18. (b)	19. (b)	20. (a)	
21. (b)	22. (a)	23. (b)	24. (d)	25. (d)	26. (a)	27. (a)	28. (d)	29. (d)	30 . (d)	

II. No. It leads to hypocrisy and partiality.

Explanations

- 1. A variety of news helps people to develop their own news. So argument I is strong. The controlled news shall be partial. So it loses credibility. Thus argument II is strong.
- 2. Higher education is not the cause of unemployment. In fact, it has created greater job opportunities. So argument II is vague. Besides higher education promotes the country's development. So argument I is strong.
- 3. The argument I in support does not provide a valid reason for the pursuance of the policy. So it is vague. Argument II provides a valid reason that illiterate workers will create wastefulness in management. So only argument II is strong.
- 4. In a developing country, a policy cannot be followed merely on the pretext that it is pursued in the western countries. So argument I does not hold good. Putting telephone services in private hands may even bring out greater efficiency. So, argument II is also vague.
- 5. With big industries, pollution is always a problem. But it is having some advantage as there are more openings in the field of employment. So, both the arguments are strong.
- 6. For attaining nuclear power, manufacture of atom bombs is an inevitability to protect the country from the threat of nuclear power. So, argument I is strong. Argument II against the statement is, however, weak.
- 7. Restriction on the diet of people will be denying them from their basic human rights. So, only argument II holds.
- 8. Luxury hotels are a mark of country's standard and a place for staying the affluent foreign tourists. So, argument II is strong. Ban on hotels is not a way to end the success of international criminals. So argument I is weak.
- 9. Argument I gives a reason in support of the statement and so it is not strong. The adult education programme needs to be given priority because it shall eliminate adult illiteracy and thus help in further spread of education. So argument II is strong.
- 10. Development in a new field is not a matter of merely following other countries. So, argument II is not strong. Computerisation is a much beneficial project and investment in it is not at all a waste. So argument I is also not strong enough.
- 11. Argument I is not strong because films also educate masses. Similarly, argument II against the statement is weak because it is not the only way to educate the masses.
- 12. Connecting remote parts by roads will only help the people there. So, argument II alone holds strong.
- 13. To bring uniformity in educational standards, we can have many universities all following same curricular and policies under one board. But one university throughout the country makes impossible the management of education. So, argument II is strong.
- 14. The policy will end political corruption to extract these amounts. The second argument is vague. So only argument I is strong.
- 15. Imposing ban on fashionable dresses will be a restriction on the personal choice and hence the right to freedom of an individual. So, only argument II is strong.
- 16. The ban on political parties, candidates can independently contest elections. So, it will not end democracy. So, argument II is not strong. Argument I does not give a strong reason.
- 17. Ban is necessary to protect our natural environment. So, none of the arguments is strong enough.
- 18. Argument I is not strong as no one can be debarred from their constitutional right even if they cannot practise it. In the same context argument II is strong.
- 19. Ours is a secular state does not mean that religion and religious values are to be eradicated. These inculcate moral values. So, argument I is vague and only argument II is strong.

- **20.** Besides interview, there can be written examination to judge candidate's motives. So, the second argument is not strong enough. However, interview is a subjective assessment. So argument I is strong.
- 21. The first argument is not a strong reason in support of the statement. It is not possible to analyse the really deserving and not deserving. So argument II is strong.
- 22. India can export only the surplus and those which can be saved from the luxury needs to pay for its import. Encouragement to export cannot lead to shortages as it shall provide the resources for imports. So, only argument I is strong.
- 23. Income tax taken from the people is utilised for the welfare of the people. So it will be antinational on their part to evade taxes. Thus, only argument II is strong.
- 24. Abolishing the tax will increase the people's income and make them passive. So, argument I is vague. The personal tax is no way of sharing wealth with other people. So, argument II also does not hold.
- 25. India needs to have military force to defend itself against the threat of other military powers in the world. So, both the arguments are weak.
- 26. Judgement being subjective tests depends on the individual who judges while that in objective tests is fair and impartial. So, argument I is strong and argument II is weak.
- 27. Laws are made to ensure that no person pursues the practice. So, person, who violate the laws, is to be punished. So, argument I is strong. Even though wrong practice is firmly rooted, it should be ended. So, argument II is not strong.
- 28. Smoking is to be abolished as it is injurious to health and not only to save money. So argument I is vague. To provide employment one cannot continue a hazardous task. So, argument II is also vague.
- 29. The colour television that has now become more of a means of entertainment and television is bought by all those who can afford it and not only by rich. So, argument I is vague. Poor people cannot buy televisions. So, argument II is not strong.
- **30.** The argument in support of the statement is vague. When loyalty is considered, hypocrisy does not matter much as the fact that efficiency is neglected. So, the arguments are not strong enough.

Statements and Assumptions

An assumption is something that can be supposed on considering a given statement.

Generally, a statement is given, followed by two assumptions. You have to assess the given statement and then decide which of the given assumptions is implicit in the statement and choose the correct alternative provided.

Directions: In the questions of the type, a statement is given followed by two assumptions numbered I and II. You have to consider the statement and decide which of the given assumptions is implicit.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II is implicit and (e) if both I and II are implicit.

EXAMPLE 1

Statement : A good book, even if costly is sold.

Assumptions: I. Some books are better than others.

II. Most of the books are costly.

Solution Statement mentions a 'good' book. This means some books may not be good. So assumption I is implicit. The words 'if costly' show that most books are not costly. So assumptions II is not implicit. Hence the answer is (a).

EXAMPLE 2

Statement : "You must learn to refer the dictionary if you want to become a good

writer"—A advises B.

Assumptions: I. Only writers refer to the dictionary.

II. All writers good or bad refer to the dictionary.

Solution It does not follow from the statement that only writers and nobody else refer to the dictionary. Nothing is mentioned about the bad writers. So, both the assumptions I and II are not implicit and therefore the answer is (d).

EXAMPLE 3

Statement: "All are cordially invited to attend the entertainment programme. It is

free"—An announcement in a newspaper.

Assumptions: I. A negligible number of readers read announcements in a newspaper.

II. People generally do not go to entertainment programme which are free.

Solution Nothing is mentioned in the statement about the number of people who read announcements or whether people do or do not go to free entertainment. Hence, the answer is (d).

EXERCISES

Directions: In the following question given a statement is followed by two assumptions numbered I and II. An assumption is something supposed or taken for granted. You have to consider the statement and the following assumptions and decide which of the assumption is implicit in the statement.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II implicit and (e) if both I and II are implicit.

1. Statement : The boy is too honest to speak the truth.

Assumptions: I. Very honest boys also tell lies.

II. Dishonest boys also speak the truth.

2. Statement : Of all the radio sets manufactured in India, the 'X' brand has the largest

sale.

Assumptions: I. The sale of all the radio sets manufactured in India is known.

II. The manufacturing of no other radio set in India is as large as 'X' brand radio.

3. Statement : Warning: Cigarette smoking is injurious to health. [Bank PO]

Assumptions: I. Non-smoking promotes health.

II. Really, this warning is not necessary.

4. Statement : The next meeting of the Governing Board of the Institute will be held after

one year.

Assumptions: I. The Institute will remain in function after one year.

II. The Governing Board will be dissolved after one year.

5. Statement : Let us change the eligibility criteria to reduce the number of applicants.

[RBI Exam]

[Bank PO]

Assumptions: I. The present eligibility criteria is very low.

II. A large number of applications are not desirable.

6. Statement : The integrated steel plants in India would no longer have to depend on

[Bank PO]

Assumptions: I. Continuous casting refractories are needed by India.

imports for continuous casting refractories.

II. Continuous casting refractories are in demand.

7. Statement : A sentence in the letter to the candidates called for written examination.

"You have to bear your expenses on travel etc."

[SBI PO]

Assumptions: I. If not clarified all the candidates may claim reimbursement of expenses.

II. Many organisation7s reimburse expenses on travel to candidate called for written examination. 8. Statement : It is through participative management policy alone that indiscipline in

our industries can be contained and a quality of life ensured to the worker.

[SBI PO]

[Bank PO]

Assumptions: I. Quality of life in our industry is better.

II. Indiscipline results in poor quality of life.

9. Statement : "Blue tie would help us identify our staff from others"—A suggestion in a

company.

Assumptions: I. The company needs to identity its staff.

II. Blue tie is the latest fashion.

10. Statement : "A car is required on rent"—An advertisement. [Bank PO]

Assumptions: I. All types of vehicles are available on rent.

II. People will respond to the advertisement.

11. Statement : Read this notice before entering the club.

Assumptions: I. People are literate.

II. No blind person comes to the club.

12. Statement : Postal rates have been increased to meet the deficit.

Assumptions: I. The present rates are very low.

II. If the rates are not increased the deficit cannot be met.

13. Statement : "Present day education is in shambles and the country is going to the dogs".

[AEO]

Assumptions: I. A good education system is essential for the well being of a nation.

II. A good education alone is sufficient for the well being of a nation.

14. Statement : "Computer education should start at schools itself".

Assumptions: I. Learning computers is easy.

II. Computer education fetches jobs easily.

15. Statement : The new education policy envisages major modifications in the education

system.

Assumptions: I. The present education system is inconsistent with national needs.

II. The present education system needs change.

16. Statement : "Do not lean out of the door of the bus"—A warning in a school bus.

Assumptions: I. Leaning out of a running bus is dangerous.

II. Children do not pay any heed to such warnings.

17. Statement : Be humble even after being victorious.

Assumptions: I. Many people are humble after being victorious.

II. Generally, people are not humble.

18. Statement : Opening a library in Rambli will be a wastage.

Assumptions: I. Inhabitants of Rambli are illiterate.

II. Inhabitants of Rambli are not interested in reading.

19. Statement : Films have become indispensable for the entertainment of people.

Assumptions: I. Films are the only media of entertainment.

II. People enjoy films.

20. Statement : Children are influenced more by their teachers now-a-days.

Assumptions: I. The children consider teachers as their models.

II. A large amount of children's time is spent in school.

21. Statement : "In order to bring punctuality in our office, we must provide conveyance allowance to our employees"—Incharge of a company tells Personnel

Manager.

[Bank PO]

Assumptions: I. Conveyance allowance will not help in bringing punctuality.

II. Discipline and reward should always go hand in hand.

22. Statement : A warning in a train compartment—"To stop train, pull chain. Penalty for improper use ₹ 500".

Assumptions: I. Some people misuse the alarm chain.

II. On certain occasions, people may want to stop a running train.

23. Statement : "You should not grant him leave in this week due to exigency of work"—A supervisor advises the administrative officer. [Officer's Grade]

Assumptions: I. Request for leave can be turned down also.

II. The supervisor has reviewed the work required to be done during the said period.

24. Statement : "We offer best training in the field of computers"—An advertisement.

[Bank PO]

Assumptions: I. People are interested in getting training in computers.

II. People want best training.

25. Statement: "Use aluminium—The versatile metal for packaging"—An advertisement.

[Bank PO]

Assumptions: I. Aluminium is the only versatile metal.

II. Some companies use metallic packing.

26. Statement : The president assured the people that elections will be held here after every

five years.

[SBI PO]

Assumptions: I. People are afraid that the elections may not be held at all.

II. People are afraid that elections may not be held after five years.

27. Statement : If it is easy to become an engineer, I don't want to be an engineer.

[SBI PO]

Assumptions: I. An individual aspires to be professional.

II. One desires to achieve a thing which is hard earned.

28. Statement : You can hereby appointed as a programmer with a probation period of one

year and your performance will be reviewed at the end of the period for confirmation"—A line in an appointment letter. [Bank PO]

Assumptions: I. The performance of an individual generally is not known at the time of appointment offer.

II. Generally an individual tries to prove his worth in the probation period.

29. Statement : Lock your valuables in a cupboard and call everybody gentleman.

[SBI PO]

Assumptions: I. Valuables locked in cupboard cannot be stolen.

II. Stealing is a crime.

30. Statement : "Banking services are fine-tuned to meet growing business needs"—An

advertisement. [SBI PO]

Assumptions: I. Banking is a part of business activity.

II. Industrialists prefer better banking services.

31. Statement The successful man has the ability to judge himself correctly.

[AAO Exam]

Assumptions: I. To judge others is of no use to a successful man.

II. The successful man cannot make a wrong judgement.

32. Statement Television X-the neighbour's envy, the owner's pride—A TV advertisement.

[RBI Exam]

I. People are envious of their neighbours superior possessions. Assumptions:

II. People want to be envied by their neighbours.

33. Statement "The function will start at 3 PM. You are requested to take your seats before

3 PM"—Last sentence in an invitation card.

I. If the invitee is not in his seat before 3 PM, the function will not start. Assumptions:

II. Function will start as scheduled.

34. Statement : If you have any problems, bring them to me.

Assumptions: I. You have some problems.

II. I can solve any problem.

Vitamin E tablets keep your complexion in a glowing condition by improving 35. Statement

circulation.

Assumptions: I. People like a glowing complexion.

II. Complexion becomes dull in the absence of circulation.

36. Statement "Ensure a good night's sleep for your family with safe and effective X

mosquito coil"—An advertisement.

[UTI Exam]

Assumptions: I. X mosquito coil is better than any other mosquito coil.

II. A good night's sleep is desirable.

37. Statement All existing inequalities can be reduced, if not utterly eradicated by action

> of government or by revolutionary change of government. [LIC]

Assumptions: I. Inequality is a man-made phenomenon.

II. No person would voluntarily part with what he possesses.

38. Statement The taste of food contributes to the intake of nourishment which is essential

> for the survival of human beings. [Management Trainee's Exam]

Assumptions: I. Human beings take food for the enjoyment of its taste.

II. Human beings experience the taste of food.

39. Statement Every year doctors, scientists and engineers migrate from India to greener

> pastures. [Income Tax and Central Excise]

Assumptions: I. Brain drain has affected India adversely.

II. Better scales and better standards of living act as a bait to lure them.

40. Statement The present examination system needs overhauling thoroughly.

[Investigator's Exam]

Assumptions: I. The present examination system is obsolete.

II. Overhauling results in improvement.

ANSWERS

1. (d)	2. (a)	3. (d)	4. (e)	5. (d)	6. (e)	7. (e)	8. (b)	9. (a)	10. (b)
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- 16. (a) 18. (c) 11. (e) **12.** (b) **13.** (e) **14.** (a) **15.** (e) 17. (d) **19**. (b) **20**. (a)
- **21.** (b) **22.** (e) **23.** (e) **24.** (e) **25**. (e) **26.** (e) **27.** (b) 28. (e) **29**. (a) **30.** (e)
- **31.** (e) **32.** (a) **33.** (b) **34.** (b) **35.** (e) **36.** (e) **37.** (a) **38.** (b) **39.** (d) **40.** (e)

Explanations

- 1. The statement means that boy is so honest that he will definitely speak the truth. i.e. very honest boy will not tell lies. So assumption I is not implicit. The statement mentions nothing about dishonest boys. So, assumption II is not implicit.
- 2. Comparison could not be made without knowing the sale of all the radio sets. So assumption I is implicit. The statement mentions only that the sale is largest and nothing is mentioned about the manufacture. So, assumption II is not implicit.
- 3. The statement confirms that smoking affects health adversely and so non-smoking will keep the health as it is. Thus, assumption I is not implicit. The warning is necessary to tell the people that injury would be caused by smoking. So, assumption II is not implicit.
- 4. The meeting of the Governing Board will be held after one year means the institute will start functioning at that time. So, assumption I is implicit. The board cannot be dissolved at the time when its meeting starts. So, assumption II is implicit.
- 5. The statement shows that the eligibility criteria needs to be raised but not because they are very low but only to reduce the number of applicants. So, assumption I is not implicit. The statement does not mention anything about the worth of applicants. So, assumption II is also not implicit.
- 6. The statement mentions the self-sufficiency of India in continuous casting refractories. This means that they are needed in the country. So, assumption I is implicit. Since continuous casting refractories are needed in integrated steel plants, it means that they are in demand. So assumption II is implicit.
- 7. Not mentioning the condition may provoke all the candidates to demand their claim. So, assumption I is implicit. The condition is mentioned because some companies do reimburse the travel expenses. So, assumption II is also implicit.
- 8. The statement mentions that participative management policy will provide quality life to the workers. So, assumption I is not implicit. The statement mentions that participative management will contain the indiscipline and ensure quality life to workers. So, assumption II is implicit.
- 9. Here, the suggestion is given for providing a method of identification. This means that the staff needs to be identified. So assumption I is implicit. The statement does not mention anything about the fashion. So, assumption II is not implicit.
- 10. The advertisement clearly mentions that a car is required on rent. So, assumption I is not implicit. An advertisement is given for the people to respond it. So, assumption II is implicit.
- 11. The statement is meant for the people to read. So, it is assumed that the people are literate and assumption I is implicit. Since the notice is to be read by everyone entering the club, so it is assumed that no blind person comes to the club. Thus, assumption II is also implicit.
- 12. The statement shows that the present postal rates are not sufficient to meet the deficit but they may still be high. So, assumption I is not implicit. The rates are increased to meet the deficit. So, assumption II is implicit.
- 13. The statement mentions the degradation of the country with the disruption of the education system. So, assumption I is implicit. Clearly, from the statement it does not follow whether education alone is required or whether other things go with it for the well being of the nation. So assumption II is also implicit.
- 14. The computer education can be started at the school level only if it is easy. So, assumption I is implicit. In the statement nothing is mentioned about the link between the job and the computer education. So, assumption II is not implicit.

- 15. Modifications are made in the present education system finding that it was inconsistent with the needs and required to be changed. So, both the assumptions I and II are implicit.
- 16. The warning against leaning out of running bus is made to heed against the dangers involved. So, assumption I is implicit. The warning is meant to be heeded but nothing is mentioned whether children pay heed to it or not. So, assumption II is not implicit.
- 17. Nothing is mentioned about the nature of the people in the statement. It simply gives advice. So, both the assumptions I and II are not implicit.
- 18. A library will be a wastage only when the people of the place cannot read or they are not interested in reading. So, either of the assumptions I or II is implicit.
- 19. 'Films are indispensable' doesn't mean that they are the only means of entertainment. So, assumption I is not implicit. Assumption II follows the statement. So assumption II is implicit.
- 20. Children consider teachers as their model as they are more influenced by them. So, assumption I is implicit. It is not necessary that the children are influenced by teachers because they spend much time in school. So, assumption II is not implicit.
- 21. Assumption I is against the statement. So it is not implicit. The allowance will serve as a reward to the employees and shall provoke them to come on time. So, assumption II is implicit.
- 22. Penalty is imposed to prevent people from misusing the alarm chain. This means that some people misuse it. So, assumption I is implicit. The alarm chain is provided to stop the running train in case of emergency. So, assumption II is also implicit.
- 23. The advice is given to turn down the request for leave. So assumption I is implicit. The mention of the exigency of work makes assumption II implicit.
- 24. The advertisement is meant to cater to the people's demand of computer training. So, assumption I is implicit. The offer of 'best training' makes assumption II implicit.
- 25. 'Versatility' is mentioned as the special quality of aluminium. So, assumption I is implicit. The advertisement is meant for those who use metallic packing. So, assumption II is also implicit.
- 26. The statement is made to eliminate the fear of the people that the elections may not be held at all or they may not be held after five years. So, both assumptions I and II are implicit.
- 27. Nothing is mentioned about the professional nature of the job. So, assumption I is not implicit. From the statement, it is clear that one rejects a thing that is easy to achieve. So, assumption II is implicit.
- 28. The performance of the individual has to be tested over a span of time as the statement mentions. So, assumption I is implicit. The statement mentions that the individual's worth shall be reviewed before confirmation. So assumption II is also implicit.
- 29. The statement points out that a person who keeps his things locked away shall feel that every person is good because he has no danger for his things. So, assumption I is implicit. The statement mentions nothing about the lawful nature of the act of stealing. So, assumption II is not implicit.
- **30.** According to the statement, banking is connected to business activity. So, assumption I is implicit. Banking is adjusted in a way to promote business needs. This means that business is promoted by better banking. So, assumption II is also implicit.
- 31. The basic quality of a successful man is that he can judge himself. This means that he need not judge others. So, assumption I is implicit. The word 'correctly' makes assumption II implicit.
- 32. The advertisement attacks the particular quality mentioned in the assumption. So, assumption I is implicit. The statement mentions that the thing shall be coveted because it shall be his superior possession. So, assumption II is not implicit.
- 33. In the statement, it is mentioned that the function will start at 3 PM and not that the invitees will be waited for. So, assumption I is not implicit and only assumption II is implicit.

- 34. The statement doesn't show that you have problems. So, assumption I is not implicit. The statement states that problems will be solved by 'me'. So, assumption II is implicit.
- 35. The advice is given for people who like glowing complexion. So, assumption I is implicit. Since complexion glows if circulation is improved. So, assumption II is also implicit.
- 36. The advertisement mentions the coil safe and effective pointing to the best qualities. So, assumption I is implicit. The advertisement is meant to enable people to have a good night's sleep. So, assumption II is also implicit.
- 37. Since inequality can be reduced, it means that it is not natural but created. So, assumption I is implicit. Nothing is mentioned about the people's response. So, assumption II is not implicit.
- 38. It is mentioned that nourishment is essential for survival. So, this is the basic cause of intake of food. Hence, assumption I is not implicit. Since taste of food affects the intake of nourishment, it means that human beings are affected by taste. So, assumption II is implicit.
- **39.** Nothing is mentioned about the effect of migration on the country. So, assumption I is not implicit. Nothing is mentioned about the reason for the migration. So, assumption II is also not implicit.
- **40.** 'Thorough' overhauling is needed only in case of an obsolete system. So, assumption I is implicit. Overhauling is done for improvement. So, assumption II is also implicit.

Statements and Course of Action

In such questions, a statement mentioning a particular problem is given and is followed by two courses of action numbered I and II. You have to analyse the statement and then decide which of the courses of action should be a step or decision taken for improvement, follow-up or further action in regard to the given statement. Everything in the statement is assumed to be true and logical courses of action should follow from it.

EXAMPLE 1

Statement : Many cases of cholera were reported from a nearby village.

Courses of action : I. The question should be raised in the Legislative Assembly.

II. A team of doctors should be rushed to the village.

Solution The disease has to be eradicated. For this proper and immediate medication and preventive measures by doctors are necessary. So only course II follows.

EXAMPLE 2

Statement : People residing in some remote tribal areas are far from education.

Courses of action: I. Schools for children and adults should be opened there.

II. Social workers should be entrusted with the job of educating them.

Solution Clearly, to make permanent arrangements for education in remote tribal areas, schools have to be opened in those areas. Education by social workers shall be a temporary remedy. So only course of action I follows.

EXAMPLE 3

Statement : The librarian finds some cases in which the pages from certain books

issued from the library, are torn.

Courses of action: I. The librarian should keep a record of books issued to each student, and if the pages are found torn, strict measures should be taken against the student who had been issued that book.

II. Some funds should be collected from the students collectively to renovate the library.

Solution Clearly, precaution should be taken to catch the guilty person and punish him for the act. This alone will help curb the wrong practice. So, only course I will follow.

EXERCISES

Directions: In the following question a statement is followed by two courses of action numbered I and II. A course of action is a step or decision to be taken for improvement, follow-up, or further action in regard to the problem policy, etc. on the basis of the information given in the statement. You have to assume everything in the statement to be true, then decide which of the two given courses of actions logically follows for pursuing.

Give answer (a) if only I follows: (b) if only II follows; (c) if either I or II follows; (d) if neither I nor II follows; and (e) if both I and II follow.

1. Statement : A leading US multinational engineering and construction firm is keen to invest in India in a variety of sectors ranging from power to land management. [Bank PO]

Courses of action: I. Such multinational companies should not be allowed to operate in India.

II. India should encourage multinational companion from other developed countries to invest in the power sector to bring in competitive climate.

2. Statement : A shopkeeper was reported to be selling adulterated grains.

Courses of action: I. He should be fined and his shop sealed.

II. He should be asked to leave the town and open a shop elsewhere.

3. Statement : A train has derailed near a station while moving over a bridge and fell into the river.

Courses of action: I. The railway authorities should clarify the reason of the accident to the government.

II. The government should allocate funds to compensate the destruction caused.

4. Statement : The Central Bureau of Investigation receives the complaint of an officer taking bribe to do the duty he is supposed to.

Courses of action: I. The CBI should try to catch the officer red handed and then take a strict action against him.

II. The CBI should wait for some more complaints about the officer to be sure about the matter.

5. Statement : Most of the children in India are not able to get education, because they get employed to earn livelihood in their childhood only.

Courses of action: I. Education should be made compulsory for all children up to the age

II. Employment of children below the age of 14 years, should be banned.

6. Statement : The Finance Minister submits his resignation a month before the new budget is to be presented in the Parliament.

Courses of action: I. The resignation should be accepted and another person should be appointed as the Finance Minister.

II. The resignation should not be accepted.

 $\textbf{7. Statement} \hspace{1.5cm} : \hspace{1.5cm} \textbf{Courts take too long in deciding important disputes of various departments}.$

Courses of action: I. Courts should be ordered to speed up matters.

II. Special powers should be granted to officers to settle disputes concerning their department.

8. Statement : A group of school students was reported to be enjoying at a picnic spot during school hours.

Courses of action : I. The principal should contact the parents of those students and tell them about the incident with a real warning for future.

II. Some disciplinary action must be taken against those students and all other students should be made aware of it.

9. Statement : Footpaths of a busy road are crowded with vendors selling cheap items.Courses of action : I. The help of police should be sought to drive them away.

II. Some space should be provided to them where they can earn their

bread without blocking the footpaths.

10. Statement : The sale of a particular product has gone down considerably causing great concern to the company.

Courses of action: I. The company should make a proper study of rival products in the market.

II. The price of the product should be reduced and quality improved.

11. Statement : A recent study shows that children below five die in the cities of the developing countries mainly from diarrhoea and parasitic intestinal worms.

[Bank PO]

Courses of action: I. The government of developing countries should take adequate measures to improve the hygienic conditions in the cities.

II. Children below five years in the cities of the developing countries need to be kept under constant medication.

12. Statement : The Doordarshan is concerned about the quality of its programmes particularly in view of stiff competition it is facing from Star and other satellite TV channels and is contemplating various measures to attract talent for its programmes.

[Bank PO]

Courses of action: I. In an effort to attract talent, the Doordarshan has decided to revise its fee structure for the artists.

II. The fee structure should not be revised until other electronic media also revise it.

13. Statement : The Indian electronic component industry venturing into the West European markets faces tough competition from the Japanese.

Courses of action: I. India should search for other international markets for its products.

II. India should improve the quality of the electronic components to compete with the Japanese in capturing these markets.

14. Statement : Exporters in the capital are alleging that commercial banks are violating the Reserve Bank of India directive to operate a post shipment export credit denominated in foreign currency at internatioal interest rates from January this year.

[Bank PO]

Courses of action: I. The officers concerned in the commercial banks are to be suspended.

II. The RBI should be asked to stop giving such directives to commercial banks.

15. Statement : One of the problems facing the food processing industry is the irregular supply of raw material. The producers of raw material are not getting a reasonable price.

[Bank PO]

Courses of action: I. The government should regulate the supply of raw materials to other industries also.

II. The government should announce an attractive package to ensure regular supply of raw materials for food processing industry.

Statement : India's performance in the recent Olympic Games was very poor. Not even a single medal could be bagged by the players. The government has spent ₹ 5 crores in training and deputing a team of players to participate in the Olympic Games.

Courses of action: I. India should stop sending players to the future Olympic Games.

- II. Government should immediately set up an enquiry commission to find out the reason for India's dismal performance.
- 17. Statement : The government will slap legally enforceable penalties on coal companies defaulting on quality and quantity of coal supplies to bulk consumers, especially to the thermal power station. [Bank PO]
 - Courses of action: I. The requirement of coal for thermal power stations should be assessed realistically.
 - II. The coal companies should introduce welfare measures for their employees.
- Statement : Some serious blunders were detected in the accounts section of a factory.
 Courses of action : I. An efficient team of auditors should be appointed to check the accounts.
 - II. A show cause notice should be issued to all the employees involved in the irregularity.
- 19. Statement : The police department has come under a cloud with recent revealations that at least two senior police officials are suspected to have been involved in the illegal sale of a large quantity of weapons from the state police armoury.
 - Courses of action: I. A thorough investigation should be ordered by the state government to bring out all those who are involved into the illegal sale of arms.
 - State police armoury should be kept under central government's control.
- 20. Statement : Orissa and Andhra Pradesh have agreed in principle to set up a joint control board for better control, management and productivity of several interstate multipurpose projects. [Bank PO]
 - Courses of action: I. Other neighbouring states should setup such control boards.
 - II. The proposed control board should not be allowed to function as such joint boards are always ineffective.
- 21. Statement : The committee has criticised the institute for the failure to implement a dozen of regular programmes despite an increase in the staff strength and not drawing up a firm action plan for studies and research.

[SBI PO]

Courses of action: I. The broad objectives of the institute should be redefined to implement a practical action plan.

II. The institute should give a report on reasons for not having implemented the planned programmes.

ANSWERS

- 1. (b) 2. (a) 3. (d) 4. (a) 5. (e) 6. (b) 7. (e) 8. (e) 9. (e) 10. (a)
- 11. (e) 12. (a) 13. (b) 14. (d) 15. (b) 16. (b) 17. (d) 18. (a) 19. (a) 20. (a)
- **21.** (e)

- 1. Clearly, financing is the major problem in starting any project. The investment by multinational companies shall, therefore, be a way to development. So only II follows.
- 2. It is clear that if shopkeeper is allowed to continue without being punished, he would create the problem elsewhere. So only I follows.
- 3. It is necessary to take preventive measures so as to protect the passengers and pay them adequate compensation. So none of the two courses follows.
- 4. One complaint is enough for a wrong doing. This should be confirmed by catching the guilty one and then strict action is to be taken against him. So only course I follows.
- 5. To educate all children, enforcement of education is necessary. Also, the reason is that they are employed. So ban on such employment is also needed. Thus, both the courses follow.
- 6. The present finance minister shall know better all the plans and resources of the government and he alone can present a suitable budget. So course II should be followed.
- 7. For quick disposal of cases, either the matters in the court should be speeded up or the matters should be cleared up in their respective departments to prevent the delay. So both the courses follow
- 8. Both warning and future prevention are necessary. So both the courses follow.
- 9. The footpaths are meant for an entirely different purpose. So, they need to be kept empty. For this, police help has to be sought. Also, the vendors cannot be deprived of a living. So, both the courses follow.
- 10. A study of rival products in the market will help assess the cause for the lowering down of the prices and then a suitable action will be taken. So only I follow.
- 11. The two diseases mentioned are caused by unhygienic conditions. So, improving the hygienic conditions is a step towards their eradication. Also, constant medication will help timely detection of the disease and hence a proper treatment. So both I and II follow.
- 12. The decision to revise its fee structure for artists is taken by Doordarshan as a remedy to the challenging problem that had arisen before it. It cannot wait till other media take action. So only course I follows.
- 13. An escapist attitude does not help much. The need is to compete and emerge successful. So, only course II follows.
- 14. The statement mentions that the commercial banks violate a directive issued by the RBI. The remedy is only to make the banks implement the act. So none of the courses follows.
- 15. In order to solve the problem of food industry, a regular supply of raw materials should be ensured. So course II shall follow.
- 16. To compete against a challenge, the first step must be to find out where the lackening is. So, only course II follows.
- 17. None of the courses of action is a suitable follow up of the government's act against defaulters. So, none of the courses of action follows.
- 18. Detect the blunder and its improvement is the urgent need. So, only course I follows.
- 19. Here the first course of action is suitable follow up of the problem.
- 20. The effectiveness of such control boards is established by the fact that Orissa and A.P. have agreed to it for better control of its multipurpose projects. So only course of action I follows.
- 21. The problem is that despite an increase in staff strength, the institute has failed in its objective of implementing its plan. So, either there should be reasons for the lack or the plans are a failure and must be revised for practical implementation. Thus, both the courses follow.

Statements and Conclusions

In this type of questions, one or more statements are given. You have to analyse the given statements, understand their indirect implications and then decide which of the given conclusion follows.

Directions: In each of the following questions, a statement is given followed by two conclusions I and II. Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow.

EXAMPLE 1

Statement: The distance of 900 km by road between Bombay and Jafra will be reduced to 280 km by sea. This will lead to a saving of ₹ 7.92 crores per annum on fuel.

- Conclusions: I. Transportation by sea is cheaper than that by road.
 - II. Fuel must be saved to the greatest extent.

Solution According to the statement, the sea transport is cheaper than the road transport in the case of route from Bombay to Jafra, not in all the cases. So, conclusion I does not follow. The statement stresses on the saving of fuel. So, conclusion II follows. Hence, the answer is (b).

EXAMPLE 2

Statement: Sealed tenders are invited from competent contractors experienced in executing construction jobs.

Conclusions: I. Tenders are invited only from experienced contractors.

II. It is difficult to find competent tenderers in construction jobs.

Solution According to the statement, tenders are invited from contractors experienced in executing construction jobs. So, conclusion I follows. The availability (c) of competent tenderers in construction is not mentioned. So, conclusion II does not follow. Hence the answer is (a).

EXAMPLE 3

Statement : Any young man who makes dowry as a condition for marriage discredits

himself and dishonours womanhood.

Conclusions: I. Those who take dowry should be condemned by society.

II. Those who do not take dowry respect womanhood.

Solution Clearly, the statement declares dowry as an evil practice and reflects its demerits. Thus, conclusion I follows. Also, it is given that those who take dowry, dishonour womanhood. This implies that those who do not take dowry respect womanhood. So, conclusion II follows. Hence the answer is (e).

EXERCISES

Directions: In each question below are given certain statements followed by some conclusions. Choose the conclusion which follows from the given statements.

1. Statements:

- 1. None but the rich can afford air-travel.
- 2. Some of those who travel by air become sick.
- 3. Some of those who become sick require treatment.

Conclusions:

- (a) All the rich persons travel by air.
- (b) Those who travel by air become sick.
- (c) All the rich persons become sick.
- (d) All those who travel by air are rich.

2. Statements:

- 1. Only students can participate in the race.
- 2. Some participants in the race are females.
- 3. All female participants in the race are invited for coaching.

Conclusions:

- (a) All participants in the race are invited for coaching.
- (b) All participants in the race are males.
- (c) All students are invited for coaching.
- (d) All participants in the race are students.

3. Statements:

- 1. A forest has as many sandal trees as it has Ashoka trees.
- 2. Three-fourths of the trees are old one and half of the trees are at the flowering stage.

Conclusions:

- (a) All Ashoka trees are at the flowering stage.
- (b) All sandal trees are at the flowering stage.
- (c) At least one-half of the Ashoka trees are old.
- (d) One-half of the sandal trees are at the flowering stage.
- (e) None of these.

4. Statements:

- 1. Shyam is not the father of Hari.
- 2. Hari is the son of Suresh.
- 3. Suresh has three sons.

Conclusions:

- (a) Shyam is son of Suresh.
- (b) Hari is the brother of Shyam.
- (c) Suresh is the father of Hari.
- (d) Shyam has no children.

[Railways]

Directions: In each of the following questions, a statement is followed by two conclusions. Give answer (a) if the conclusion I follows; (b) if conclusion II follows; (c) if both I and II follow; (d) if either I or II follows; and (e) if neither I nor II follows.

- 5. Statement : Of all the television sets manufactured in India, is the 'solar' brand has the largest sale.

 [Bank PO]
 - Conclusions: I. The volume of sales of all the brands of television sets manufactured in India, is known.
 - II. The production of no other television set in India is as large as that of 'solar'.
- 6. Statement: Parents are prepared to pay any price for an elite education to their children.
 - Conclusions: I. All parents these days are very well-off.
 - II. Parents have an obsessive passion for a perfect development of their children through good schooling.
- 7. Statements: 1. The TV programmes, telecast specially for women are packed with a variety of recipes and household hints.
 - A major portion of magazines for women also contains the items mentioned above.
 - Conclusions: I. Women are not interested in other things.
 - II. An average woman's primary interest lies in home and specially in the kitchen.
- 8. Statement: Patients with minor ailments usually do not go to eminent doctors.
 - Conclusions: I. Eminent doctors remain too busy with patients suffering from serious complications.
 - II. Their charges are rather high.
- 9. Statement : From all available cultural records, it is evident that even in ancient India, both the masters and disciples valued not the quantity but the quality of knowledge.
 - Conclusions: I. Giving importance to quantity of knowledge is meaningless.
 - II. There was an identity of educational values between teachers and students in ancient India.
- Statement : The doctors nursed only those bleeding wounds, which were caused by bullets.A patient X was bleeding profusely.
 - Conclusions: I. X was nursed by the doctors.
 - II. The doctors did not nurse X.
- 11. Statement : In a one day cricket match, the total runs scored by a team were 200. Out of these 160 runs were made by spinners.
 - Conclusions: I. 80% of the team consists of spinners.
 - II. The opening batsman was spinner.
- 12. Statement : Players who break various records in a fair way get special prizes. Player X broke the word record but was found to be under the influence of a prohibited drug.

Conclusions: I. X will get the special prize.

II. X will not get the special prize.

13. Statement : National Aluminium Company has moved India from a position of shortage

to self sufficiency in the metal.

Conclusions: I. Previously, India had to import aluminium.

II. With this speed, it can soon become a foreign exchange earner.

14. Statement : The domestic demand has been increasing faster than the production of

indegeneous crude oil.

Conclusions: I. Crude oil must be imported.

II. The domestic demand should be reduced.

15. Statement: He stressed the need to step the present examination system and its

replacement by other methods which would measure the real merit of the

students.

Conclusions: I. Examinations should be abolished.

II. The present examination system does not measure the real merit of the students.

Directions: In each of the following questions, a statement has been given followed by two conclusions. Give answer (a) if conclusion I is true and II is wrong; (b) if both the conclusions are true; (c) if both the conclusions are wrong and (d) if conclusion II is true and I is wrong.

16. Statement: The percentage of the national income shared by the top 10 per cent of households in India is 35.

Conclusions: I. When an economy grows fast, concentration of wealth in certain packets of population takes place.

II. The national income is unevenly distributed in India.

 $\textbf{17. Statement} \quad : \ The \ use \ of \ non-conventional \ sources \ of \ energy \ will \ eliminate \ the \ energy \ crisis$

in the world.

Conclusions: I. Modern technology is gradually replacing the conventional sources of

energy.

II. The excessive exploitation of environment has led to depletion of

conventional sources of energy.

18. Statement: In India, the fruits of development have not been equitably distributed between

our rural and urban sectors.

Conclusions: I. Rural poverty is substantially higher than urban poverty.

 A large family size in India may be a consequence rather than a cause of poverty.

Directions: In each of the following questions, a statement is given followed by two conclusions I and II. Mark answer (a) if both the conclusions can be drawn from the statement; (b) if only I can be drawn, (c) if only II can be drawn, (d) if neither I nor II can be drawn and (e) if either I or II can be drawn.

19. Statement: The TV staff deserves an applaud for showing booth capturing.

Conclusions: I. TV always aims at showing things in their true perspective.

II. People involved in booth-capturing have been recognized and are being tried by law.

20. Statement : Smoking is one of those human weakness which tends to test the will power of the smoker to the edge.

of the smoker to the eage.

Conclusions: I. It is very difficult for the smokers to give up smoking even if they want to do so.

II. Human beings have other weaknesses as well.

21. Statement: Video libraries are flourishing very much these days.

Conclusions: I. People in general have got a video craze.

II. It is much cheaper to see as many movies as one likes on videos rather than going to the cinema hall.

22. Statement : All those political prisoners were released on bail who had gone to jail for reasons other than political dharnas. Bail was not granted to persons involved

in murders.

Conclusions: I. No political prisoner had committed murder.

II. Some politicians were not arrested.

23. Statement : No country is absolutely self-dependent these days.

Conclusions: I. It is impossible to grow and produce all that a country needs.

II. Countrymen in general have become lazy.

Directions: In each question below is given a statement followed by two conclusions I and II. Give answer (a) if conclusion I is implicit; (b) if conclusion II is implicit; (c) if both I and II are implicit; (d) if either I or II is implicit and (e) if neither I nor II is implicit.

24. Statement : The best way to escape from a problem is to solve it.

Conclusions: I. Your life will be dull, if you don't face a problem.

II. To escape from problems, you should always have some solutions with you.

25. Statement: Good is the enemy of the best.

Conclusions: I. You should be good to your best enemy.

II. The best do not like the good.

ANSWERS									
1. (d)	2. (d)	3. (e)	4. (c)	5. (c)	6. (b)	7. (b)	8. (c)	9. (b)	10. (d)
11. (e)	12. (b)	13. (c)	14. (d)	15. (b)	16. (d)	17. (b)	18. (a)	19. (b)	20. (a)
21. (a)	22. (b)	23. (b)	24. (e)	25. (a)					

Deriving Conclusions from Passages

This section comprises some questions on logical deduction. Here, each question consists of a passage followed by five inferences based on it. Readers are required to analyse the passage and grasp desirable facts from it. Then, he has to consider each inference in context of the given passage, decides its degree of truth or falsity and then choose the best alternative provided in the question.

Directions: Read the following passage and examine each inference given below it in the context of this passage. Mark answer (a) if the inference is definitely true; (b) if the inference is probably true; (c) if the data is inadequate; (d) if the inference is probably false; and (e) if the inference is definitely false.

EXAMPLE 1 The bauxite deposits in India are widely distributed. Recently, deposits in Orissa have been developed and the largest plant of its kind in Asia has been set up to produce alumina and aluminium. Its annual capacity is 800,000 tonnes of alumina and 2,25,000 tonnes of aluminium. It uses the latest French technology. The ore is exported to Japan and European countries. In 1987, the output of bauxite was 2.8 million tonnes. The country's reserves are estimated at 270 million tonnes, of which 73 million tonnes are of high quality.

- (i) The plant set up in Orissa is the largest in India.
- (ii) Aluminium is exported to Japan and European countries.
- (iii) Orissa is the largest producer of bauxite in India.
- (iv) The plant in Orissa was set up by financial aid from France.
- (v) About 27% of total reserves of bauxite are of high quality.

Solution

- (i) (a) Since, the plant set up in Orissa is the largest in Asia, it is evident that it is also the largest in India.
- (ii) (e) Not aluminium, but its ore is exported to Japan and European countries.
- (iii) (b) Since the largest bauxite producing plant is in Orissa, it is much possible that Orissa is the largest producer of bauxite.

- (iv) (e) The plant set up in Orissa uses French technology; it was not set up by financial aid from France.
- (v) (a) Total reserves of bauxite = 270 million tonnes. High quality reserves = 73 million tonnes. Percentage of high quality bauxite reserves = $\left(\frac{73}{270} \times 100\right)\% = 27\%$.

EXAMPLE 2 According to trade estimates, the black pepper crop will be in the range of 75,000 to 95,000 tonnes during the pepper season from November 1989 to October 1990. The crop will be thus the double of that for the 1988–89 season. Besides good monsoon, the record production is attributed to the crash programme launched in Kerala by the State Government last June. Although 80% of the country's total black pepper production is grown in Kerala, the yield per hectare is low at 300 kg as compared to 550 kg in Indonesia and 1200 kg in Brazil.

- (i) India exports black pepper more than that of Brazil.
- (ii) In no other state in the country black pepper yield is more than 300 kg/hectare.
- (iii) Good water supply is needed for cultivating black pepper.
- (iv) The black pepper crop during 1988-89 was about 30,000 tonnes.
- (v) The Central Government helped the Kerala Government in implementing the crash programme. [RBI Exam]

Solution

- (i) (c) Nothing is mentioned about the exports of India and Brazil in the passage.
- (ii) (b) As mentioned in the passage, Kerala is the largest pepper producing state and the yield per hectare is 300 kg. But other states show a low yield and so a less production of black pepper.
- (iii) (a) The good monsoon has led to a high production of black pepper as mentioned in the passage.
- (iv) (e) The total produce during 1989–90 was estimated to be 75,000 to 95,000 tonnes and the produce during 1988–89 was half of this, which is more than 30,000 tonnes.
- (v) (c) It is mentioned in the passage that crash programme was launched by Kerala Government. But there is no mention of the efforts or aid of the Central Government to the programme.

EXERCISES

Directions: In each question below is given a passage followed by several inferences. You have to examine each inference separately in the context of the passage and decide upon its degree of truth or falsity.

Mark answer (a) if the inference is 'definitely true'; (b) if the inference is 'probably true' though not definitely true in the light of facts given; (c) if the data are inadequate; (i.e. from facts given you cannot say whether the inference is likely to be true or false); (d) if the inference is 'probably false' though not definitely false in the light of the facts given; and (e) if the inference is 'definitely false'.

1. In the forties, nationalisation was considered the panacea for all socio-economic ills. Today, privatisation has become the buzzword that has been sweeping both the developed and the developing world for more than a decade now. Even in India, the idea has been gathering momentum. But before the idea is transplanted in India, there are several aspects of privatisation that need to be understood. It would be worthwhile, in this context, to look at the experiences of other countries.

- Other countries which have adopted privatisation are considering now to change over to industrialisation.
- (ii) Privatisation has been practiced in the USA since long.
- (iii) India is also thinking of privatisation.
- (iv) Nationalisation has failed to improve substantially the socio-economic situation of some countries.
- (v) India is the first country in Asian subcontinent to adopt privatisation.

[Bank PO]

- 2. The Ministry of Environment and Forest has granted environmental clearance to the Karkatla open-cast expansion project of the Central Coal fields Ltd. in Bihar that envisages exploitation of non-cocking coal reserves. The present production level of 0.8 million tonnes is proposed to be expanded to 1.5 million tonnes per annum at an estimated cost of 67.82 crores under the project. The total land area requirement for the proposed mining activities is about 651 hectares which includes about one sixth of it as forest land.
 - (i) The expansion plan would require about 100 hectares of forest land.
 - (ii) Karkatla open-cast mine is the only one of non-cocking coal in the country.
 - (iii) There is no demand for non-cocking coal.
 - (iv) The production cost of one tonne of non-cocking coal from Karkatla mine will be about ₹ 450
 - (v) Environmental concerns get less priority over the need of the coal.

[Bank PO]

- 3. There is more bad news on food front. It now appears certain that there will be a shortfall of about 9 million tonnes in the food production achieved in the last *kharif* season. However, rice procurement may only be partially affected since West Bengal and Andhra Pradesh have had sufficient rainfall while Punjab, the major contributor to the central pool is less dependent on rainfall. Still, the overall availability of rice may go down by more than four million tonnes. There may be worst news ahead.
 - (i) There is no canal water facility in West Bengal and Andhra Pradesh.
 - (ii) The procurement price of rice will increase this year.
 - (iii) Rice is mainly produced in the kharif season.
 - (iv) In the last year, there was a deficit production of rice by five million tonnes.
 - (v) It is likely that production of rice will be below the normal level in the next year.

[Bank PO]

(ANSWERS)

1. (i) (e) (ii) (c) (iii) (a) (iv) (a) (v) (e) 2. (i) (a) (ii) (c) (iii) (a) (iv) (a) (v) (c)

3. (i) (a) (ii) (b) (iii) (a) (iv) (e) (v) (b)

- (i) As mentioned in the passage, the idea of privatisation has been sweeping both the developed and the developing world for more than a decade.
 - (ii) In the passage, nothing has been mentioned about the USA.
 - (iii) It has been clearly mentioned in the passage that the idea of privatisation has been gathering momentum in India.

- (iv) It is mentioned in the passage that earlier nationalisation was considered the remedy for socio-economic ills whereas at present, the idea of privatisation is dominant. This implies that nationalisation failed to improve the socio-economic situations of some countries.
- (v) As mentioned in the passage, the idea of privatisation is being promoted all over the world and has not been successfully transplanted in India.
- 2. (i) According to the passage, land required for expansion plan = 651 hectares.

Forest land =
$$\left(\frac{1}{6}\text{ of total land}\right) = \frac{1}{6} \times 651$$

= 108.5 = 100 (approximately)

- (ii) It is mentioned only that Karkatla mine deals with exploitation of non-cocking coal reserves. But, it is not given that it is the only such mine.
- (iii) The granting of environmental clearance to Karkatla mine shows that there is a demand for non-cocking coal.
- (iv) Total estimated production = 1.5 million tonnes = (1.5×10^6) tonnes.

Total estimated cost = ₹ 67.82 crores = ₹ (67.82×10^7)

Cost per tonne of coal =
$$\mathbf{\xi} \left[\frac{67.82 \times 10^7}{1.5 \times 10^6} \right]$$

= $\mathbf{\xi} 452.13 = \mathbf{\xi} 450$ (approximately)

- (v) The given fact is neither mentioned nor can be derived from the passage.
- 3. (i) As mentioned in the passage, the rice production in West Bengal and Andhra Pradesh would not be affected since they had sufficient rainfall. This implies that the farming there is dependent mainly on rain and no other irrigation facilities are available.
 - (ii) As mentioned in the passage, the rice production has gone down which may lead to a rise in procurement price of rice.
 - (iii) The given fact is clearly evident from the given passage.
 - (iv) As mentioned in the passage, there is a deficit production of rice by five million tonnes in the present year.
 - (v) It is mentioned in the passage that 'There may be worst news ahead'. There is no surety about the given fact, but it appears to be true according to the present trends.

SECTION C NONVERBAL REASONING

Chapter 22

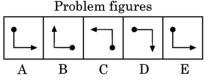
Series

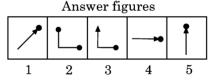
In this chapter, four types of series of figures are given. There will be a set of answer figure also. You have to identity the most appropriate figure from the set of answer figures.

TYPE 1: FIVE FIGURE SERIES

This type of problems on series consists of five figures numbered A, B, C, D and E forming the problem set, followed by five other figures numbered 1, 2, 3, 4 and 5 forming the answer set. The five consecutive problem figures form a definite sequence and it is required to choose one of the figures from the answer set which will continue the same sequence.

EXAMPLE 1

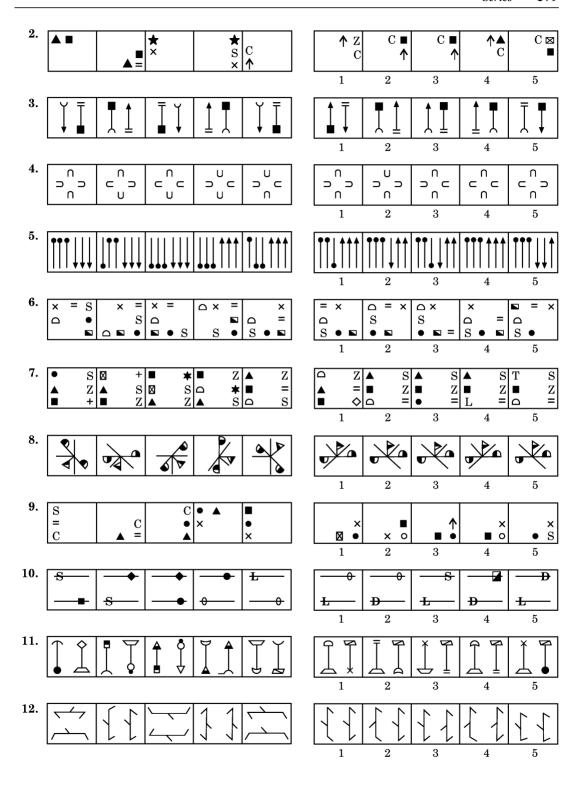


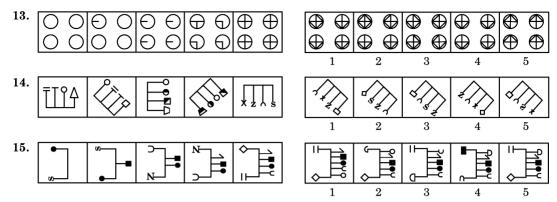


Solution In this case, the pin rotates 90° clockwise and the arrow rotates 90° anticlockwise in each step. Hence, the answer is figure (3).

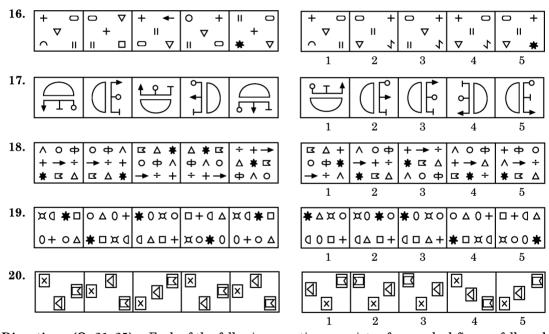
EXERCISE 1

Directions (Q. 1-15): In each of the questions given below which one of the five answer figures on the right should come after the problem figures on the left, if the sequence were continuted?

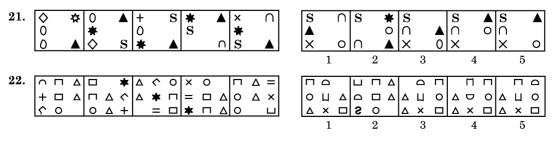


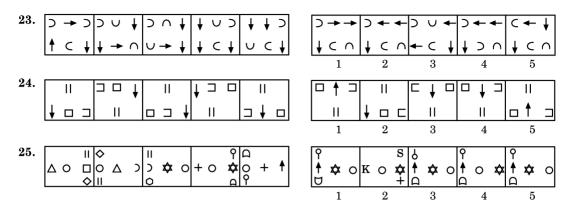


Directions (Q. 16–20): Each of the following questions consists of unmarked figures followed by figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.

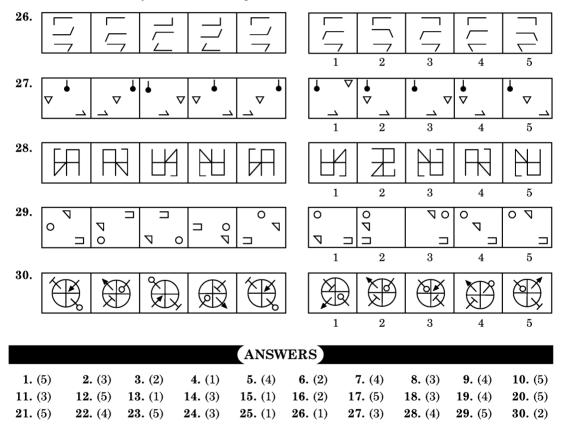


Directions (Q. 21–25): Each of the following questions consists of unmarked figures followed by five figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.





Directions (Q. 26-30): Each of the following questions consists of unmarked figures followed by five figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.



- 1. In subsequent steps one, two three one line segments are added.
- 2. In subsequent steps, the ACW-end element shifts one-and-a-half side ACW. The CW-end element shifts one side CW while a new element appears in the middle.

- 3. In each step the elements get inverted while the arrows and the end-elements inter-change places in alternate steps.
- 4. Two end one arcs get inverted in alternate steps.
- 5. One, two, three, elements get inverted in subsequent steps in succession.
- 6. In subsequent steps 5, 4, 3, 5, ... elements from CW and shift half-a-side CW while others shift half-a-side ACW.
- 7. In alternate steps the elements of LHS and RHS columns shift one step downward while one of the elements in the other column gets replaced by a new one.
- 8. In each step whole figure rotates by 45° ACW. The triangular element gets inverted and its shading is changed while the semi-circular elements change their side and shading in alternate steps.
- 9. In alternate steps the elements shift one-and-a-half sides and half a side ACW while one element is replaced by a new one in each step. The replacement happens in succession.
- 10. In each step one element is replaced by a new one, alternately on upper and lower parts, while the other interchanges place in one step and gets inverted in the next step.
- 11. In subsequent steps the upper part of LHS element becomes lower without getting inverted while the lower part of RHS becomes the upper after getting inverted.
- 12. The whole figure rotates by 90° ACW. All the three dashes on lower figure change direction while the two dashes on upper figure change direction in each step.
- 13. In subsequent steps 1, 3, 5, 7, 9 lines are added beginning from the upper-left.
- 14. The whole figure rotates by 45° CW in each step. In alternate steps three elements shift one step in cyclic order while one end element is replaced by a new one.
- 15. In alternate steps the LHS elements interchange position while a new element is added alternately at upper and lower ends of RHS.
- 16. The central element and two other elements move one step ACW in cyclic order. The other two interchange positions and one of them is replaced by a new one. The cyclic movement rotates one step in ACW order.
- 17. The whole figure rotates by 90° ACW. The middle tip becomes lowest tip, lowest becomes upper and upper becomes middle in alternate steps.
- 18. In the first step elements move one step from right to left, the left most becoming the rightmost. In the next step the row of elements moves one step downwards. This goes on alternately.
- 19. In the first step the pairs of elements interchange diagonally. In the next step the elements on the outer side move one side CW while those on the inner side move one side ACW. This goes on alternately.
- 20. In each step elements move half a side downwards. The lower most moves to upper most.
- 21. In the first step the upper-and-lower left elements interchange places. The lower-right moves to upper-right, the upper-right moves to middle left while a new element appears at lower right. In the next step the upper and lower-right elements interchange place. The elements in the left column move one step downwards while a new element appears at upper-left position.
- 22. Two elements of the middle column interchange places sequentially while the upper two and lower two elements of the first column interchange places and the third is replaced by a new one. The columns of elements move one step towards left while the left most element moves to the rightmost position. The column with only two elements moves up and down in subsequent steps.
- 23. In the first step the rightmost element of each row moves to the left most while other two rotate by 90° CW and move one step towards right. In the next step the end elements of the

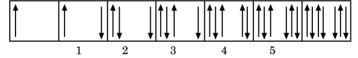
lower row and the middle element of the upper row move one step ACW in cyclic order while others remain unchanged.

- 24. In every step middle two elements get inverted and two end elements moved CW by two sides in first step and half-a side and two-and-a half side each respectively CW in the next step. Then the process repeats.
- 25. In the first step the odd-numbered elements from top rotate by 90° CW. In the next step the even-numbered elements from top rotate by 90° ACW.
- 26. The middle and side elements rotate by 180° alternately.
- 27. The elements move half-a-side in rows from left to right in each step. After reaching the rightmost position they shift to the left most position.
- 28. The rectangle moves one step ACW while the dash moves one step CW in each step.
- 29. Elements move one side and half-a-side ACW alternately in each step.
- 30. The elements move one step ACW and change their direction in each step.

TYPE 2: FIND OUT THE WRONG FIGURE IN THE SERIES

This type of questions begins with an un-numbered figure followed by five figures numbered from 1 to 5 and then again an un-numbered figure on the extreme right. These seven figures together form a series which starts at the first (un-numbered) figure and ends at the last (un-numbered) figure. However, only one of these figures does not fit into the series. The number of that figure is the answer.

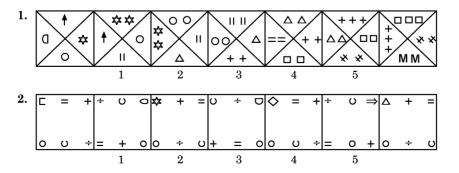
EXAMPLE 2

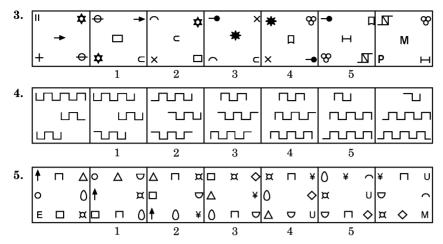


Solution In the above set of figures, the arrows are added to the right and left sides alternately. But in the third figure the arrow which was to be added to the right side, has been added to the left side. Hence figure (3) is the answer.

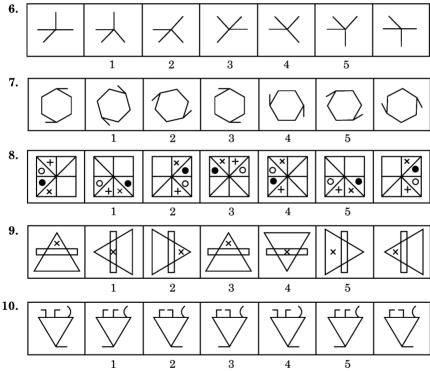
EXERCISE 2

Directions (Q. 1-5): In each of the following problems, a related pair of figures is given followed by five numbered pairs of figures. Four out of five pairs have similar relationship as that of the original pair. Select a pair that does not have a relationship similar to that in the original pair.

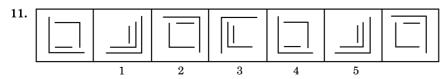


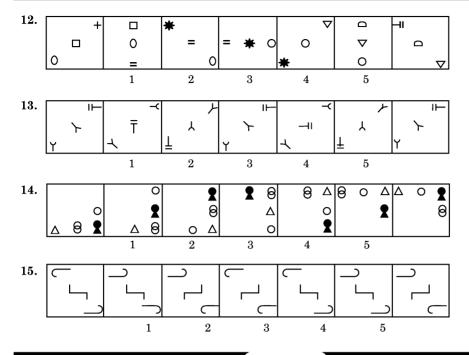


Directions (Q. 6–10): Each of the following series consists of seven figures, two of which at the ends are unnumbered. One of the five numbered figures does not fit into the series. Find-out the figure.



Directions (Q. 11–15): Each of the following series consists of seven figures, two of which at the ends are unnumbered. One of the five numbered figures does not fit into the series. Find out the figure.





ANSWERS

- 1. (4)
 2. (5)
 3. (2)
 4. (3)
 5. (1)
 6. (3)
 7. (2)
 8. (4)
 9. (2)
 10. (3)
- **11.** (3) **12.** (2) **13.** (4) **14.** (5) **15.** (1)

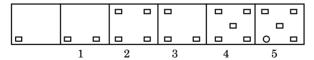
- 1. The elements move one step ACW. The element reaching the upper part increases by one while that reaching lower part is replaced by a new one.
- 2. In alternate steps the upper-right and the upper-left pair move downwards diagonally. While a new element appears alternately on upper-right and upper-left. The circle in the lower row moves from one end to the other in each step. The other two elements in the lower row interchange places and move upwards diagonally.
- 3. In each step three elements move diagonally while other two interchange places diagonally. Two elements—one at centre and one at a corner—are replaced by new ones. This movement rotates by 90° CW.
- 4. In each step one and two line segments alternately from left and right side of the upper element are lost while two line segments-one each in the middle and the lower element are added alternately on right and left side.
- 5. In the first step elements at positions 1 and 4 interchange places. So do elements at positions 6 and 9. 3 moves to 2 → 8 → 7 while a new element appears at position 3. In the next step 3 and 6 and 4 and 7 interchange their places. 9 moves to 8 → 2 → 1 while a new element appears at 9.
- 6. In each step one arm gets rotated by 45° CW. This movement proceeds in ACW direction.
- 7. The whole figure rotates by 45° CW and 90° ACW alternately while end elements get inverted in each step.

- 8. The whole figure rotates through 90° ACW in each step and symbols inside a square gets interchanged within it in every step.
- 9. The whole figure rotates by 90° and 180° ACW alternately. 'x' moves from vertex to base.
- 10. The lower dash changes direction in each step while the upper elements change direction alternately.
- 11. The outermot and innermost sides rotate by 90° ACW while the middle one rotates by 90° CW.
- 12. The elements shift one step in cyclic order and one element is replaced by a new one. The line of orientation rotates by 45° ACW.
- 13. The uppermost element rotates 90° CW and moves to the middle. The middle element rotates by 180° and moves to lower position while the lower element rotates by 90° CW and moves to the upper position.
- 14. The elements move one step ACW in one step while in the next the element on extreme ACW position moves to CW and others move one step ACW.
- 15. The upper and the lower element rotate by 180° while the middle one gets inverted vertically in alternate steps.

TYPE 3: DETECTING THE INCORRECT ORDER IN A SERIES

The third type of questions on series consists of an un-numbered figure followed by five other figures numbered as 1, 2, 3, 4 and 5. All the six figures together form a series. The un-numbered figure marks the beginning of the series and so its position is fixed. However, the positions of two figures in the series are incorrect and the series would be complete if these figures are interchanged. The earlier of the two numbered figures whose positions are interchanged is the answer. In case, the position of no two figures is to be interchanged, then the answer is 5.

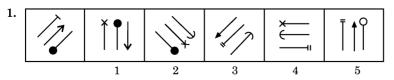
EXAMPLE 3

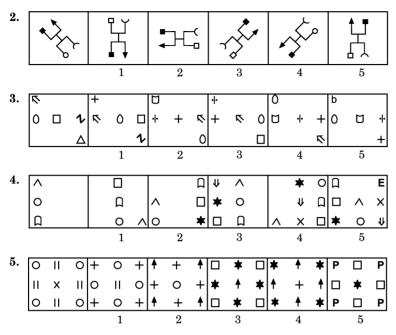


Solution Here, the number of squares increases in step by one and then these squares change into circles stepwise. But this series will be established only if figure (2) and figure (3) are interchanged. So, the answer is (2).

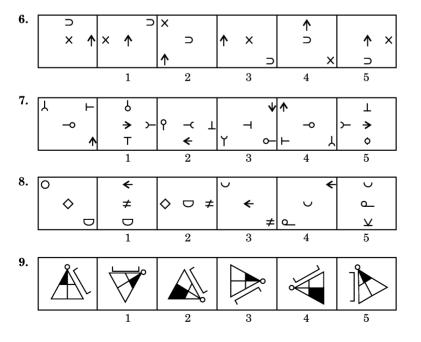
EXERCISE 3

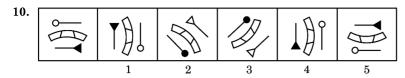
Directions (Q. 1-5): Each of the following questions consists of six figures, the first of which is unnumbered and marks the beginning of the series continued in the successiove figures numbered from 1 to 5. However, the series will be established only if the positions of two of the numbered figures are interchanged. The number of the first of the two figures is the answer. If no figure need to be interchanged then the answer is 5.





Directions (Q. 6-10): Each of the following questions consists of six figures, the first of which is unnumbered and marks the beginning of the series continued in the successive figures numbered from 1 to 5. However, the series will be established only if the positions of two of the numbered figures are interchanged. The number of the first of the two figures is the answer. If no figure need to be interchanged then the answer is 5.





(ANSWERS) 1. (3) 2. (1) 3. (2) 4. (5) 5. (3) 6. (3) 7. (5) 8. (4) 9. (5) 10. (1)

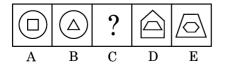
Explanations

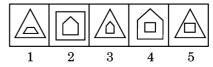
- 1. The central element interchanges place alternately with the element on either side and gets inverted while the third one is replaced by a new one. The whole figure rotates by 45° ACW in each step.
- 2. The whole figure rotates by 45°, 90°, 135°, 180° ACW in subsequent steps while the end elements interchange their places alternately on either end in subsequent steps.
- 3. In each step the elements move one step downwards and towards right. A new element appears at the top-left position while the lower one is lost.
- 4. The column of elements moves half-a-side from left to right. The central element interchanges alternately with upper and lower elements and the third element is placed at the lower end of the next column.
- 5. In each step the similar corner elements move to the middle of the sides while one of the middle elements moves to the centre. New elements appear at the corners.
- 6. The horizontal arrow rotates by 90° ACW and is placed on LHS. The vertical arrow on the RHS rotates by 90° ACW and its head gets inverted while the LHS arrow rotates by 180° and its head gets inverted.
- 7. The outermost and innermost elements interchange places and sizes. The second element from outside rotates by 180° while the second from inside rotates by 90° CW.
- 8. The upper-right and lower-left elements get vertically inverted and move to middle-right and centre respectively. The central element gets laterally inverted and moves to the lower right.
- 9. The LHS and RHS elements get enlarged. The RHS element becomes the outer one while the LHS becomes the inner one. The number of sides of LHS element increases by one. The middle element gets reduced in size rotates by 135° CW and becomes the innermost.
- 10. The elements move two sides. The end elements get reversed while the middle one remains unchanged.

TYPE 4: CHOOSING THE MISSING FIGURE IN A SERIES

In this type of questions, you are given a set of four of five figures (labelled A, B, C, D and E) following a certain sequence and hence forming a series. However, one of figures in the problem set will be missing. You have to choose this figure from the five alternatives given in the Answer set.

EXAMPLE 4

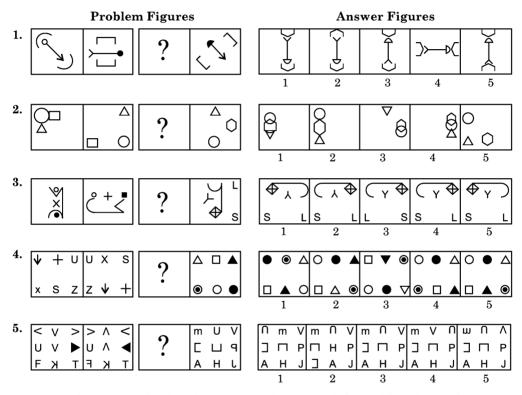




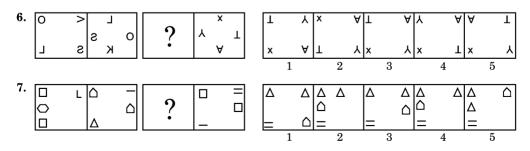
Solution Here, the inner small figure becomes the outer large figure and a new small figure appears inside it in every step. Hence, the answer is (3).

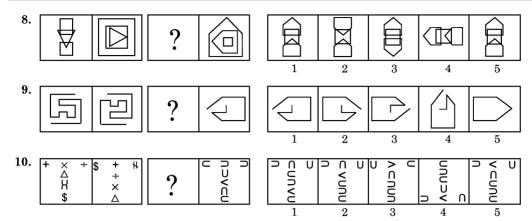
EXERCISE 4

Directions (Q. 1-5): The second figure in the first unit of the problem figures bears a certain relationship to the first figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.

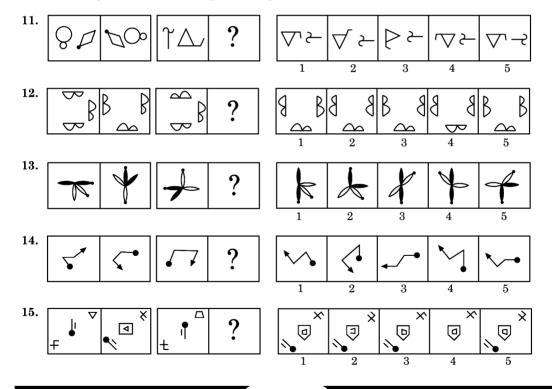


Directions (Q. 6–10): The first figure in the first unit of the problem figures bears a certain relationship to the second figures. Similarly, one of the figures in the answer figures bears the same relationship to the first figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.





Directions (Q. 11-15): The first figure in the first unit of the problem figures bears a certain relationship to the second figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.



ANSWERS

1. (2) 2. (2) 3. (4) 4. (5) 5. (3) 6. (3) 7. (4) 8. (1) 9. (2) 10. (5) 11. (3) 12. (1) 13. (3) 14. (4) 15. (3)

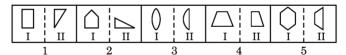
- 1. From figure II to I: The inner element rotates by 135° ACW, the arrowhead gets inverted, while the other end changes its shading. The outer element rotates by 45° ACW and assumes new form.
- 2. From figure II to I: The middle element moves to the opposite side and gets enlarged. The CW element moves $1\frac{1}{2}$ sides ACW while the ACW element moves $1\frac{1}{2}$ sides CW.
- 3. From figure II to I: The main figure rotates by 90° ACW and the curve attached to it gets inverted. The middle element rotates by 45° while the other two elements move one side ACW.
- 4. From figure II to I: The elements at positions 1, 3 and 8 move one step CW while elements at positions 2, 7 and 9 move one step ACW.
- 5. From figure II to I: The first and third columns have their mirror image while the middle column has its water image in the LHS figure.
- 6. From figure II to I: Each element moves one side and a half ACW without getting inverted.
- 7. From figure II to I: One side increases in each element except in the upper-left, in which one side is lost. The middle row element changes side.
- 8. From figure II to I: The middle figure rotates by 90°CW and is placed on the upper and the lower part while the innermost is placed in the middle.
- 9. From figure II to I: The first is laterally inverted form of the second.
- 10. From figure II to I: The upper three elements in the column go two steps downward. The right element in the row goes down in third row of the column and left element in the lower. The lowermost goes to the second row and the second row to right of the upper row.
- 11. From figure I to II: The elements interchange positions. The LHS rotates by 90° ACW while the RHS rotates by 90° CW.
- 12. From figure I to II: The RHS remains constant, the lower element gets vertically inverted while the upper one rotates by 90° ACW and shifts to left.
- 13. From figure I to II: The shaded petal with the dot rotates by 90° ACW. The unshaded petal with dot rotates by 45° CW while the other two rotate by 135° CW.
- 14. From figure I to II: The arrow rotates by 90° CW, the pin rotates by 45° ACW while the middle bar rotates by 45° ACW.
- 15. From figure I to II: The central element rotates by 135° CW gets inverted and shifts to the lower-left. The lower-left rotates by 45° CW gets inverted and shifts to the upper-right while the upper-right rotates by 90° CW and shifts to the centre.

Analogy

This chapter is dealt with identifying the odd pair of figures which are not having a particular relation between them in one way or another.

CHOOSING THE ODD RELATIONSHIP: In this type of questions, five pairs of figures are given. The two figures in four out of five pairs are related in a particular manner. You have to select the pair which does not show the relationship.

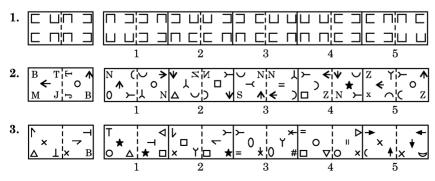
EXAMPLE 1

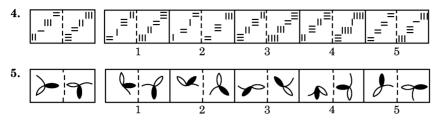


Solution In all the pairs except (2), the second element is half of the first element.

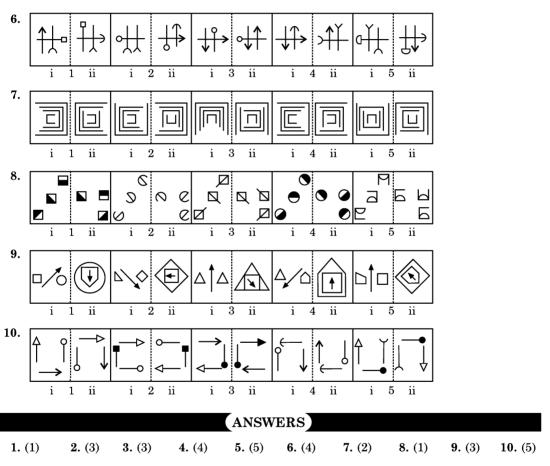
EXERCISE 1

Directions (Q. 1-5): In each of the following problems, a related pair of figures is given followed by five numbered pairs of figures. Four out of five pairs have similar relationship as that of the original pair. Select a pair that does not have a relationship similar to that in the original pair.





Directions (Q. 6–10): In each of the following questions in four out of the five figures, element I is related to element II in the same particular way. Find out the figures in which element I is not related to element II.



- 1. The upper-left and the lower-right elements rotate by 90° CW while the upper-right and the lower-left elements rotate by 90° ACW.
- 2. The upper-left element shifts to lower-right position. The upper-right and lower-right shift to upper-left and lower-left position respectively and rotate by 90° ACW. The central element shifts to the upper-right position and rotates by 90° CW. The lower-left element shifts to centre and changes its shape.

- 3. The upper-left element rotates by 90° CW and shifts to the centre. The central element moves to the lower-left. The lower-right element rotates by 90° ACW and shifts to the upper-right position while a new element appears at lower-right position.
- 4. The uppermost and the lowermost sets of dashes rotate by 90° and an additional dash is added to each of them. One dash is lost from second upper set of dashes while one dash is added to the second from the bottom.
- 5. The unshaded leaf rotates by 45° CW. The shaded leaf rotates by 90° CW. The half-leaf rotates by 135° CW and changes its direction.
- 6. One of the elements move half-a-side CW one by one while the other two move one step once in row and twice diagonally and again in row in subsequent steps.
- 7. The CW element shifts to the centre while the central element becomes the ACW element. Other two elements move one-and-a half sides CW. Elements rotate by 90° CW.
- 8. In each step the elements move from right to left and one of the elements is replaced by a new one. The line on which the elements are arranged rotates by 45° and 90° ACW.
- 9. The triangle rotates by 45°, 90°, 135°, 180° ... CW in subsequent steps. The shading moves one step CW. The bar outside moves one step CW. The bar outside moves from one side of the outer circle to the other in alternate steps.
- 10. In the first step the whole figure rotates by 90° CW while the elements on the ends further rotate by 180°. In the next step the whole figure rotates by 45° ACW, elements on the ends interchange positions and their shading also changes. The process goes on.

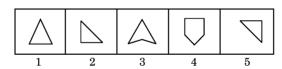
Classification

In this chapter, problem figures are having same characteristics. You have to identify the most appropriate figure from answer figures, which show some characteristics.

CHOOSING THE FIGURE WITH SAME PROPERTIES: This type of questions contain two figures forming the problem set followed by five other figures forming the answer set. The problem figures have sme common characteristics. You are required to select one of the figures from the answer set which also exhibits the same characteristics.

EXAMPLE 1

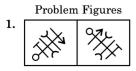


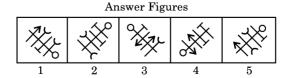


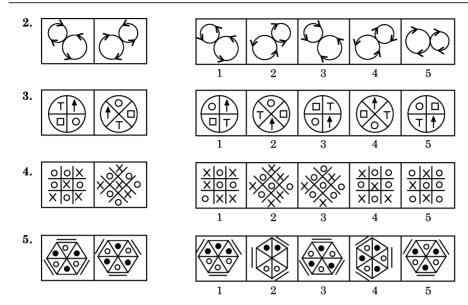
Solution Each one of the two figures on the left has been formed by four distinct lines. Similarly, figure (3) is made up of four lines. Hence figure (3) is the answer.

EXERCISE 1

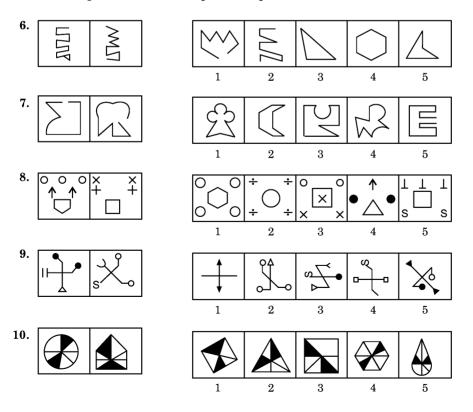
Directions (Q. 1-5): In each question below, two problem figures, followed by five answer figures numbered (1), (2), (3), (4) and (5) are given. The two problem figures have some common characteristics/features. You have to find out one figure out of the five answer figures which has the same features/characteristics. The number of that figure is the answer.







Directions (Q. 6-10): The first figure in the first unit of the problem figures bears a certain relationship to the second figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.



(ANSWERS)

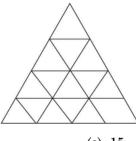
1. (5) **2.** (4) **3.** (3) **4.** (4) **5.** (4) **6.** (1) **7.** (5) **8.** (4) **9.** (2) **10.** (5)

- 1. Two identical elements are placed in the middle and are in the opposite direction.
- 2. Directions of arrows are CW in one and ACW in the other.
- 3. Positions of the elements are similar in respect of other elements.
- 4. Element 'x' is placed in the centre.
- 5. Shaded and unshaded circles are placed in a definite sequence and outer line segments lie adjacent to unshaded circles.
- 6. The figures have only one right angle.
- 7. The figures are open-ended.
- 8. The number of lines of the enclosed figure is equal to the number of smaller elements.
- 9. The titled lines have similar shapes on them.
- 10. The shaded parts have three unshaded parts on one side and two on the other.

Analytical Reasoning

The chapter on analytical reasoning involves the problems relating to the counting of geometrical figures in a given complex figure. You have to analyse the complex figure and determine the number of any particular type of figure as shown in the examples.

EXAMPLE 1 How many straight lines are there in the following figure?



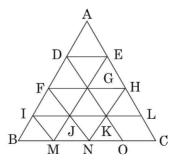
(a) 9

(b) 11

(c) 15

(d) 48

Solution The given figure is as shown below.

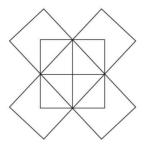


Horizontal lines are DE, FH, IL and BC. i.e., 4 nos.

Slanting lines are IM, FN, DO, AC, AB, EM and HN. i.e. 7 nos.

 \therefore Total number of lines = 4 + 7 = 11.

EXAMPLE 2 How many rectangles does the following figure have?



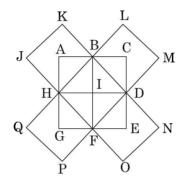
(a) 10

(b) 12

(c) 13

(d) 14

Solution The given figure can be labelled as shown below:

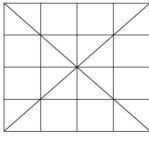


The above figure consists of:

- (i) rectangle JKNO composed of four components JKBH, DNOF, JKDF and BNOH i.e. altogether 5 in numbers;
- (ii) rectangle QLMP composed of four components QHFP, BLMD, QBDP and HLMF i.e. altogether 5 in numbers;
- (iii) square ACEG composed of four components ACDH, BCEF, DEGH and GABF i.e. altogether 4 in numbers.

Hence, total number of rectangles = 5 + 5 + 4 = 14.

EXAMPLE 3 How many triangles does the following figure have?



(a) 36

(b) 40

(c) 44

(d) 48

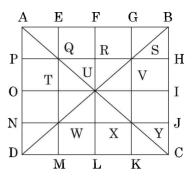
Solution The figure may be labelled as shown below.

The simplest triangles are APQ, QTU, UXY, YKC, AEQ, QRU, UVY, YJC, BGS, SRU, UTW, WND, BHS, SVU, UXW and WMD i.e. in all 16 numbers.

The triangles having two components each are QSU, SYU, YWU and WQU i.e. in all 4 numbers.

The triangles having three components each are AFU, FBU, BIU, CLU, LDU, DOU OAU i.e. in all 8 numbers.

The triangles having four components each are QSY, SYW, YWQ, WQS i.e. in all 4 numbers.



The triangles having six components each are ABU, BCU, CDU, DAU i.e. in all 4 numbers.

The triangles having seven components each are ANY, GAY, BEW, WJB, HQC, CQM, KSD and DPS i.e. in all 8 numbers.

 \therefore Total number of triangles = 16 + 4 + 8 + 4 + 4 + 8 = 44. Answer is (c)

EXERCISES

1. How many triangles are there in the figure given below?

[IAS]



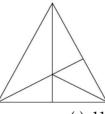
(a) 5

(b) 6

(d) 10

2. How many triangles are there in the following figure?

[IAS]



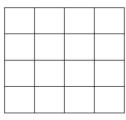
(a) 9

(b) 10

(c) 11

(d) 12

3. How many squares are there in the following figure?



(a) 16

(b) 17

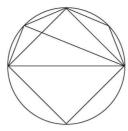
(c) 25

(d) 27

(e) 30

[SSC]

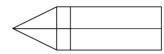
4. Count the number of triangles in the following figure.



(a) 8

(b) 10

- (d) 12
- 5. How many rectangles are there in the following figure?

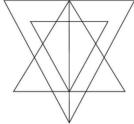


(a) 6

(b) 7

(c) 8

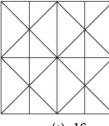
- (d) 9
- [Hotel Management]
- 6. Count the number of triangles in the following figure.



(a) 27

(b) 25

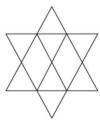
- (d) 21
- 7. What is the number of straight lines in the following figure?



(a) 11

(b) 14

- (d) 17
- 8. Determine the number of pentagons in the following figure?



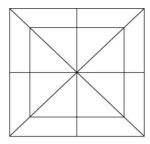
(a) 5

(b) 6

(c) 8

(d) 10

9. What is the number of triangles and squares in the following figure?

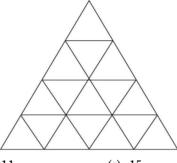


(a) 28 triangles, 10 squares

(b) 28 triangles, 8 squares

(c) 32 triangles, 10 squares

- (d) 32 triangles, 8 squares
- 10. How many straight lines are there in the following figure?

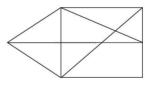


(a) 9

(b) 11

- (c) 15
- (d) 48
- 11. How many triangles are there in the following figure?

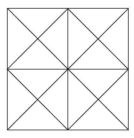
[SSC]



(a) 12

(b) 13

- (d) 15
- 12. How many triangles and squares are there in the given figure?
- [Railways]



(a) 44 triangles, 10 squares

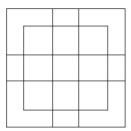
(b) 14 triangles, 16 squares

(c) 24 triangles, 6 squares

(d) 24 triangles, 9 squares

13. How many squares does the following figure contain?

[Hotel Management]

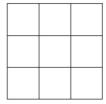


(a) 18

(b) 19

- (c) 25
- (d) 27
- 14. The maximum number of squares in the following figure is ____

[Railways]

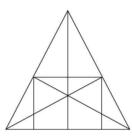


(a) 14

(b) 13

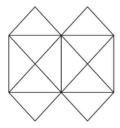
(c) 10

- (d) 9
- 15. What is the number of straight lines and the number of triangles in the given figure?



- (a) 10 straight lines and 34 triangles
- (b) 9 straight lines and 34 triangles
- (c) 9 straight lines and 36 triangles
- (d) 10 straight lines and 36 triangles.

Directions: Study the following figure and answer the questions (16 to 18).



- 16. What is the minimum number of straight lines that is needed to construct the figure?
 - (a) 11

(b) 13

- (c) 15
- (d) 21

- 17. Count the number of triangles in the figure
 - (a) 12

(b) 16

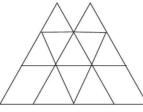
- (c) 20
- (d) 24

- 18. How many squares does the figure contain?
 - (a) 5

(b) 6

(c) 7

- (d) 8
- 19. Count the number of triangles and parallelogram in the figure given below.



- (a) 16, 22
- (b) 18, 23
- (c) 14, 20
- (d) 15, 21
- 20. In the following figure, if the centres of all the circles are joined by horizontal and vertical lines, then find the number of squares that can be formed.



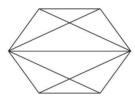
(a) 6

(b) 7

(c) 8

(d) 10

Directions: Analyse the following figure and answer the questions (21 and 22).



- 21. Find the number of quadrilaterals
 - (a) 5

(b) 7

(c) 9

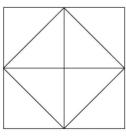
(d) 10

- 22. Find the number of pentagons
 - (a) 2

(b) 3

(c) 4

- (d) 6
- 23. Count the number of pentagons in the following figure

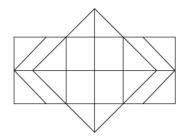


(a) 16

(b) 14

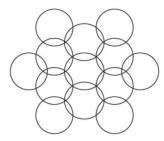
- (c) 12
- (d) 10

24. Determine the number of rectangles and hexagons in the following figure.



(a) 8 rectangles, 3 hexagons

- (b) 15 rectangles, 3 hexagons
- (c) 24 rectangles, 5 hexagons
- (d) 30 rectangles, 5 hexagons
- 25. How many circles are there in the figure given below?



(a) 11

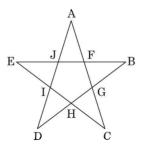
- (b) 12
- (c) 13
- (d) 14

ANSWERS

- 1. (d) 2. (d) 3. (e) 4. (b) 5. (d) 6. (a) 7. (b) 8. (d) 9. (c) 10. (b)
- 11. (d) 12. (a) 13. (d) 14. (a) 15. (c) 16. (b) 17. (c) 18. (c) 19. (b) 20. (c)
- 21. (c) 22. (d) 23. (c) 24. (d) 25. (c)

Explanations

1. The given figure can be represented as shown below.



The simplest triangles are AJF, BFG, CGH, DHI and EIJ i.e. in total 5 numbers.

The triangles having 3 components are AIC, ADG, BEH, BJD and CFE i.e. in total 5 numbers.

 \therefore Total number of triangles = 5 + 5 = 10.

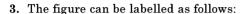
2. The given figure can be labelled as follows:

The simplest triangles are AFE, EFC, CFD, DFB and BFA i.e. 5 numbers.

The triangles having two components are AFC, CFB, ABD and EAB i.e. 4 numbers.

The triangles having three components are ADC and EBC i.e. 2 numbers.

The triangles having five components is ABC i.e. 1 number. So total number of triangles = 5 + 4 + 2 + 1 = 12



The simplest squares are ABGF, BCHG, DIHC, EJID, FGLK, GHML, GLKF, OTSN, NSRM, MRQL, LQPK, TYXS, SXWR, RWVQ and QVUP i.e. 16 numbers.

The squares composed of four simple squares are ACMK, DNLB, EOMC, HRPF, ISQG, JTRH, KMWU, LNXV and MOYW i.e. 9 numbers.

The squares composed of nine simple squares are ADSP, BETQ, FIXU and GJYV i.e. 4 numbers.

The square composed of sixteen simple squares is AEYU i.e. 1 number.

- \therefore Total number of squares = 16 + 9 + 4 + 1 = 30
- 4. The figure can be labelled as follows:

The simplest triangles are ABG, BCG, CDE, GCE, AGE and EFA i.e. 6 numbers.

The triangles composed of two triangles each are ABC, ABE, BCE and ACE i.e. 4 numbers.

- \therefore There are 6 + 4 = 10 triangles.
- 5. The given figure can be labelled as follows:

The simplest rectangles are BCJI, CDEJ, EFGJ and JGHI i.e. 4 numbers.

The rectangles having two components are BDEI, EFHI, CDFG and BCGH i.e. 4 numbers. The only rectangle composed of four components is BDFH.

- \therefore Total number of rectangles = 4 + 4 + 1 = 9
- **6.** The figure can be labelled as follows:

DOA, DAQ and HGI i.e. 5 numbers.

The simplest triangles are GKL, MNH, DLJ, DJM, QRE, OFP, PAI and RIA i.e. 8 numbers.

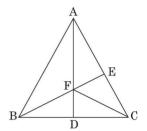
The triangles having two components each are BOD, CDQ, ENI, RPA, IKF, HJI, JGI, DLM, KPI and DKI i.e. 10 numbers. The triangles having four components each are DIE, DFI,

The triangles having six components each are DBA and CDA i.e. 2 numbers.

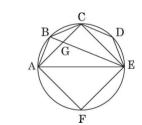
Only triangle having eight components is DEF.

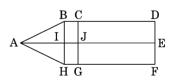
The only triangle having twelve components is ABC

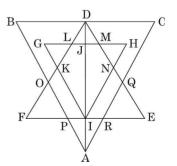
 \therefore Total number of triangles = 8 + 10 + 5 + 2 + 1 + 1 = 27



A	A 1	3 () E	1
_		G	Н	Ι	
F		L	M	N	J
K		Q	R	s	O
P		ષ	n		Т
U		V	W	X	Y







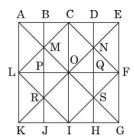
7. The given figure may be labelled as follows:

There are 3 horizontal lines, namely AE, LF and KG.

There are 5 vertical lines i.e. AK, BJ, CI, DH and EG.

There are 6 slanting lines i.e. AG, KE, LC, CF, LI and IF.

- \therefore Total number of straight lines = 3 + 5 + 6 = 14.
- .. The answer is (b)



M

8. The figure may be labelled as follows:

Here six pentagons are there having three triangles and two rhombuses as components: i.e. AJHFD, KHFDB, IFDBL, GDBLJ, EBLJH and CLJHF.

Four pentagons are formed which are composed of three triangles and one rhombus.

i.e. LCFHM, BEHML, KHFMB, and IFMBL

 \therefore Total number of pentagons = 6 + 4 = 10.

9. The figure may be labelled as follows:

Triangles

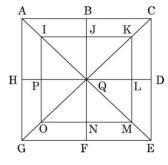
The simplest triangles are IJQ, JKQ, KLQ, LMQ, MNQ, NOQ, OPQ and PIQ i.e. 8 numbers.

Triangles composed of two components are ABQ, BCQ, CDQ, DEQ, EFQ, FGQ, GHQ, HAQ, IKQ, KMQ, MOQ and OIQ. i.e. 12 numbers.

The triangles composed of four components are ACQ, CEQ, EGQ, GAQ, MOI, OIK, IKM and KMO i.e. 8 numbers.

The triangles composed of eight components are ACE, CEG, EGA and GAC i.e. 4 numbers.

Therefore, total number of triangles = 8 + 12 + 8 + 4 = 32.



Sauares

The squares composed of two components are IJQP, JKLQ, LMNO and NOPQ i.e. 4 numbers. The squares composed of four components are ABQH, BCDQ, DEFQ and QFGH i.e. 4 numbers.

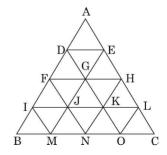
The only square composed of eight components is IKMO.

Only square composed of sixteen components is ACEG.

- \therefore Total number of squares = 4 + 4 + 1 + 1 = 10.
- 10. The figure may be labelled as follows:

The horizontal lines are DE, FH, IL and BC i.e., 4 numbers. The slanting lines are IM, FN, DO, AC, AB, ME and NH i.e. 7 numbers.

 \therefore Total number of lines is 4 + 7 = 11.



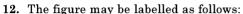
11. The figure may be labelled as follows:

The simplest triangles are AFE, ABF, BCI, CHI, GIH and FGE i.e. 6 numbers.

The triangles composed of two components are ABE, BHF, BIE, BCH, CHG and AGE i.e. 6 numbers.

The triangles composed of three components are ABH, BCE and CDE i.e. 3 numbers.

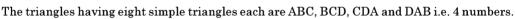
Hence total number of triangles = 6 + 6 + 3 = 15.



Triangles

The simplest triangles are AFI, FBJ, BGJ, JGO, FJO, OGK, GCK, KCH, OKH, OHL, LHD, ELD, EOL, IOE, FOI and AIE i.e. 16 numbers. The triangles having two simple triangles each are AFE, AFO, FOE, AOE, FBG, BGO, GOF, OFB, OGC, GCH, CHO, HOG, EOH, OHD, HDE and DEO i.e. 16 numbers.

The triangles having four simple triangles each are AOB, BOC, COD, DOA, GFE, FEH, EHG and GFH i.e. 8 numbers.



 \therefore Number of triangles in the figure = 16 + 16 + 8 + 4 = 44.

Squares

The squares containing two triangles each are FJOI, JGKO, KHLO and OLEI i.e. 4 numbers. The squares containing four triangles each are AFOE, FBGO, OGCH and OHDE i.e. 4 numbers. The square containing eight triangles is EFGH.

The square containing sixteen triangles is ABCD.

 \therefore Total number of squares = 4 + 4 + 1 + 1 = 10.

13. The figure may be labelled as follows:

The simplest squares are EFRQ, RNSZ, SGHT, QRZY, MQYX, A_1TOU , B_1A_1UV , VUIJ, WB_1VP , XYB_1W , YZA_1B_1 and $ZSTA_1$ i.e. 13 numbers.

The squares having two components each are AEYL, FBGZ, A₁HCI and KB₁JD i.e. 4 numbers.

The squares having four components each are XZUP, YSOV, QNTB₁, and MRA₁W i.e. 4 numbers.

The squares having seven components each are AFA₁K, EBHB₁, YGCJ and LYJD i.e. 4 numbers.

The square having nine components is MNOP.

The square having seventeen components is ABCD.

... Total number of squares = 13 + 4 + 4 + 4 + 1 + 1 = 27.

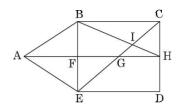
14. Given figure is labelled as follows:

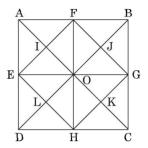
The simplest squares are ABFE, BCGF, CDHG, HLKG, GKJF, FJIE, LPOK, KONJ and JNMI i.e. 9 numbers.

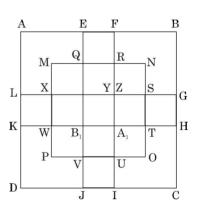
The squares composed of four components are ACKI, BDLJ, EGOM and FHPN i.e. 4 numbers.

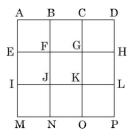
The square composed of nine components is ADPM.

 \therefore Total number of squares = 9 + 4 + 1 = 14









The horizontal lines are DF and BC i.e. 2 numbers.

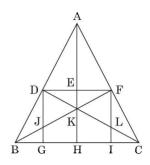
The vertical lines are DG, AH and FI i.e. 3 numbers.

The slanting lines are DC, BF, AB and AC i.e. 4 numbers.

 \therefore Total number of straight lines = 2 + 3 + 4 = 9 numbers.

The simplest triangles are AED, AFE, DEK, EFK, DKJ, KFL, DJB, FCL, BJG, and LCI i.e. 10 numbers.

The triangles composed of two components each are AFD, DFK, AKD, AFK, KFC, DKB, FCI, DGB, BKH and KCH i.e. 10 numbers.



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The triangles composed of three components each are DFJ and DFL i.e. 2 numbers.

The triangles composed of four components each are AKB, ACK, BKC, BDF, DFC, DGC and FIB i.e. 7 numbers.

The triangles composed of six components each are AHB, AHC, DAC, DFB, BDC and BFC i.e. 6 numbers.

The triangle composed of twelve components is ABC.

- \therefore Total number of triangles = 10 + 10 + 2 + 7 + 6 + 1 = 36
- 16. The figure may be labelled as follows:

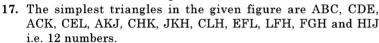
The straight lines are:

Horizontal: AE and JF i.e. 2 numbers.

Vertical: AJ, CH and EF i.e. 3 numbers.

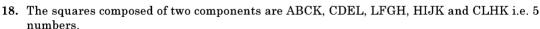
Slanting: JI, AG, BF, DE, AB, DJ, EI and FG i.e. 8 numbers.

i.e. Total number of straight lines = 2 + 3 + 8 = 13



The triangles composed of two components are AHJ, ACH, CFH, CEH, ACJ, CHJ EFH and CEF i.e. 8 numbers.

Total number of triangles in the given figure = 12 + 8 = 20



The squares composed of four components are ACHJ and CEFH i.e. 2 numbers.

- \therefore Number of squares = 5 + 2 = 7
- 19. The figure may be represented as follows:

Triangles

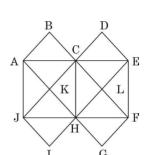
The simplest triangles are ABL, CDB, BML, DNB, KLM, MBN, NDE, IMH, MNH and NGH i.e. 10 numbers.

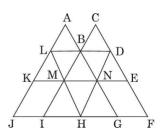
The triangles composed of three components are LHJ and DFH i.e. 2 numbers.

The triangles composed of four components are: ANK, CEM, BGI and HLD i.e. 4 numbers.

The triangles composed of eight components are AGJ and CFI i.e. 2 numbers.

 \therefore Total number of triangles = 10 + 2 + 4 + 2 = 18.





Parallelograms

The simplest parallelograms are KMIJ and NEFG i.e. 2 numbers.

The parallelograms composed of two components are BMKL, BNML, BDEN, BDNM, MNHI, MNGH, BNHM, ABML and CDNB i.e. 9 numbers.

The parallelograms composed of three components are KNHJ, MEFH, LBIJ and BDFG i.e. 4 numbers.

The parallelograms composed of four components are ANHL, CDHM, LBGH, BDHI, LDNK and LDEM i.e. 6 numbers.

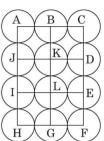
The parallelograms composed of seven components are LDFH and LDHJ i.e. 2 numbers.

- \therefore Total number of parallelogram = 2 + 9 + 4 + 6 + 2 = 23.
- 20. The centres of all the circles are joined and all the vertices are labelled as follows:

The simplest squares are ABKJ, BCDK, JKLI, KDEL, ILGH and LEFG i.e. 6 numbers.

The squares composed of four simple squares each are ACEI and ${
m JDFH}$ i.e. 2 numbers.

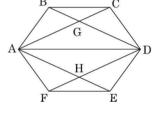
Thus, total number of squares formed = 6 + 2 = 8.

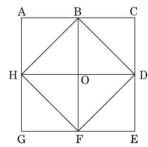


21. The figure may be labelled as follows:

The quadrilaterals in the figure are ABDE, ABDF, ACDF, ACDE, ACDH, AGDH, ABDH, AGDE and AGDF.

- : Total number of quadrilaterals = 9
- 22. The pentagons in the above figure are ABCDE, CDEFA, DEFAB, FABCD, AGDEF and ABCDH i.e. 6 numbers.
- 23. The figure may be labelled as follows:





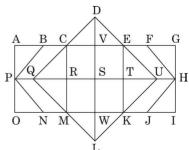
The pentagons in the above figure are ABDFH, BCDFH, DEFHB, FGHBD, ABDFG, GHBDE, ACDFH, BCEFH, GABDE, GACDF, HACEF and BCEGH i.e. 12 numbers.

24. The simplest rectangles are CVSR, VETS, STKW and RSWM i.e. 4 numbers.

The rectangles having two components each are: CVWM, VEKW, CETR and RTKM i.e. 4 numbers.

The rectangles having three components each are: ACRP, PRMO, EGHT and THIK i.e. 4 numbers.

The rectangles having four components each are: AVSP, PSWO, CEKM, VGHS and SHIW i.e. 5 numbers.



The rectangles having five components each are AETP, PTKO, CGHR and RHIM i.e. 4 numbers.

The rectangles having six components each are ACMO and EGIK i.e. 2 numbers.

The rectangles having eight components each are AVWO, VGIW, AGHP and PHIO i.e. 4 numbers.

The rectangles having ten components each are: AEKO and CGIM i.e. 2 numbers.

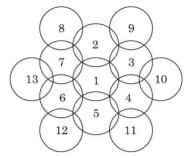
The rectangle having sixteen components each is AGIO.

 \therefore Total number of rectangles = 4 + 4 + 4 + 5 + 4 + 2 + 4 + 2 + 1 = 30.

The hexagons in the given figure are CEUKMQ, BFHJNP, CDEKLM, CFHJMQ, and BEUKNP i.e. 5 numbers.

Total number of hexagons = 5.

25. The given figure may be labelled as follows:



Total number of circles = 13.

Problems on Cubes and Dice

In this chapter you will be given with a dice marked from 1 to 6 in different faces placed in 4 or 5 positions. You have to identify the face coming opposite to a specified face or a face coming in between two different specified faces.

Or you will be given with a cube painted with different colours in different faces. If the cube is cut into smaller cubes. You have to find out the number of cubes, which are:

- (i) at least one face painted
- (ii) at least two faces painted
- (iii) at least three faces painted
- (iv) no faces painted, etc.

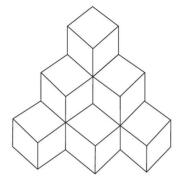
PROBLEMS ON DICE: In this test, you are given figures showing the same die in different positions. Study the figures carefully and you have to find the number opposite a given number on the die.

PROBLEMS ON CUBES: In this test, you have to study the given figure carefully and answer the questions that follow. You may have to find either of the following:

- (i) Count the number of cubes.
- (ii) Count the number of cubes that are painted with one or more particular colour.
 - (a) in one face only, (b) in more than one faces
- (iii) Find the colour of a face which is opposite to another face painted with a particular colour.
- (iv) Count the number of cubes with specific number of faces painted.
- (v) (a) no face painted, (b) one face painted, (c) two faces painted, (d) three faces painted, (e) four faces painted, (f) five faces painted, (g) all faces painted

EXAMPLE 1 Find the number of cubes in the given figure.

[IAS]



Solution There are three columns containing 1 cube each, two columns containing 2 cubes each and one column containing 3 cubes.

 \therefore Total number of cubes in the figure = $3 \times 1 + 2 \times 2 + 1 \times 3 = 3 + 4 + 3 = 10$.

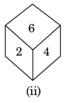
EXERCISES

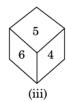
Observe the figures carefully and find the number opposite a given number on the dice.

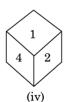
1. Consider the following pictures of a dice.

[IAS]









What is the number opposite to 3?

(a) 1

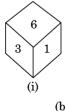
(b) 4

(c) 5

(d) Data insufficient

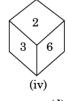
2. The number opposite to 3 is

[IAS]









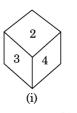
(a) 2

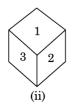
(b) 3

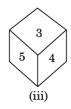
(c) 4

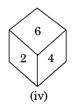
(d) 6

3. A die is thrown four times and its four different positions are given below. Find the number on the face opposite the face showing 2.









(a) 3

(b) 4

(c) 5

(d) 6

(a) 1

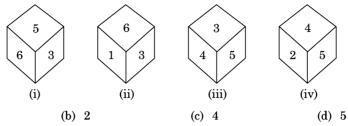
(a) 1

(a) 1

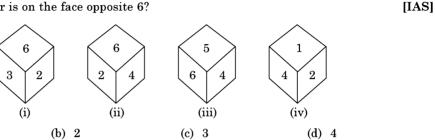
(a) 1

(a) 1

4. Shown below are four different positions of the same dice. Find the number on the face opposite the face showing 6.

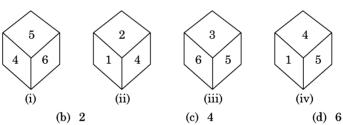


5. Which number is on the face opposite 6?

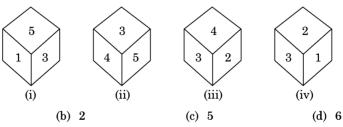


[IAS]

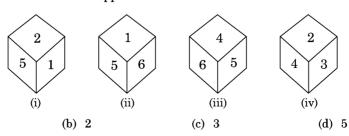
6. Which number is opposite to 3?



7. Which number is opposite to 4?



8. Which number is on the face opposite 4?



(d) 4

[IAS]

9. What should be the number opposite 3?

(a) 1

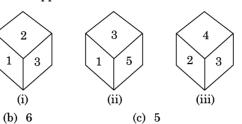
(a) 6, 6, 2

(a) 5

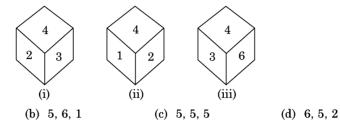
(c) 4

(a) 1

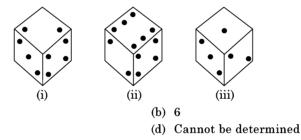
(a) 1



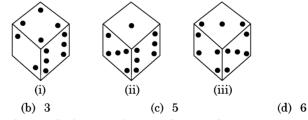
10. What numbers occur at the bottom face in the three positions of the same die?



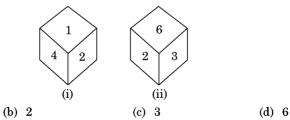
11. Find the number of dots on the face opposite the face bearing 3 dots.



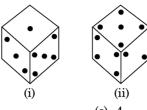
12. How many dots lie opposite 2 dots?



13. What will be the number at the bottom if 5 is at the top; the two positions of the dice being as given below.



14. What is the number of dots on the face opposite 2 dots?



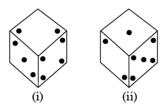
(a) 1

(b) 3

(c) 4

(d) 6

15. What is the number of dots at the bottom face of the left hand side dice?



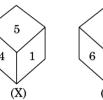
(a) 3

(b) 4

(c) 5

(d) 6

16.



(Y) (Z)

(i) Which number lies at the bottom face of the die X?

(b) 2

(d) 4

(ii) Which number lies at the bottom face of the die Y?

(a) 6

(b) 5

(c) 2

(d) 1

(iii) Which number lies opposite 6?

13. (b)

(a) 1

(b) 2

(c) 4

(d) 5

(iv) Which numbers are hidden behind the numbers 6 and 5 in the die Z?

(a) 1 and 4

(b) 1 and 3

(c) 4 and 3

(d) 1 and 2

(v) Which of the hidden numbers adjacent to 5 in die X are common to the hidden numbers adjacent to 5 in die Z?

(a) 1 and 4

(b) 2

(c) 6

(d) None of these

ANSWERS

1. (b) **3.** (c) **2.** (c) 11. (b) **12.** (c)

4. (c) **14.** (b)

5. (a) **15.** (d) **6.** (c)

16. (i) (c) (ii) (c)

7. (a)

8. (a) (iii) (a)

9. (b) (iv) (b) **10.** (c) (v) (d)

Explanations

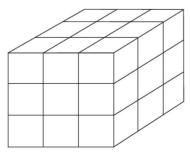
2. From figure (i), (iii) and (iv) it is clear that the numbers 6, 1, 5 and 2 lie adjacent to 3. So, 4 lies opposite 3.

- 3. From figure (i) and (iv) 3 is opposite to 6. From figure (i) and (ii) 1 is opposite to 4. So number opposite to 2 is 5.
- 4. From figure (i) and (iii), 4 is opposite to face showing 6.
- 5. From figure (ii) and (iv) 1 is opposite to face showing 6.
- 6. From figure (i) and (iii) number opposite 3 is 4.
- 7. From figure (iii) and (iv) face opposite 4 is 1.
- 8. From figure (ii) and (iii) face opposite 4 is 1.
- 9. From figure (i), (ii) and (iii), face opposite 3 is 6.
- 10. From figure (i), (ii) and (iii) 1, 2, 3 and 6 appears on sides of dice and 4 appears on top. So number at bottom on each dice is 5. i.e. 5, 5, 5.
- 11. From figure (i) and (iii) 1 dot appears opposite 4. From figure (ii) and figure (i), 6 appears opposite to 3.
- 12. From figure (i) and (ii), 5 lies opposite 2.
- 13. From figure (i) and (ii) number opposite to 2 is 5. i.e. if 5 is at the top, number at bottom will be 2.
- 14. From figure (i) and (ii) 3 dots lies opposite 2.
- 15. From figure (i) and (ii), 3, 4, 5 and 1 lies by the side of 2. So 6 lies opposite to 2 i.e. bottom of the left hand side dice.
- 16. (i) From figure (X) and (Z), 4, 1, 6 and 2 lies by the side of 5. So number lies at the bottom face of die X is 3.
 - (ii) From figure (X) and (Y), 1, 5, 6 and 3 lies by the side of 4. So number that lies at bottom face of Y is 2.
 - (iii) From figure (Y) and (Z), 2, 5, 4 and 3 lies by the side of 6. So number opposite to 6 is 1.
 - (iv) From figure (Y) and (Z), number 1 is opposite to 6. From figure (X) and (Z), number 3 is opposite to 5.
 - ∴ So 1 and 3 is the answer.
 - (v) From figure (X) and (Z), number adjacent to 5 are 4, 1, 2 and 6. No numbers are in common to dice X and Z which are hidden and adjacent to 5.

PAINTING A STACK OF CUBES

EXERCISES

Directions: A wooden cube is painted blue on all the four adjoining sides and green on two opposite sides, i.e. top and bottom. It is then cut at equal distances at right angles four times vertically (top and bottom) and two times horizontally (along the sides) as shown in the figure. Study the diagram and answer the following questions:



1.	How many cubes	will l	have one face	pair	nted only	in bl	ue?					
	(a) 1	(b)	2	(c)	3		(d)	4		(e)	5	
2.	How many cubes	will l	have one face	pair	nted only	in gr	een?	•				
	(a) 1	(b)	2	(c)	3		(d)	4		(e)	5	
3.	How many cubes	are fe	formed in all?									
	(a) 16	(b)	24	(c)	27		(d)	32		(e)	48	
4.	How many cubes	will l	have at least t	hre	e sides p	ainte	d?					
	(a) 8	(b)	6	(c)	3		(d)	2		(e)	1	
5 .	How many cubes	will l	have no face p	ain	ted at all	?						
	(a) 1	(b)	2	(c)	3		(d)	4		(e)	5	
	ctions (6-8): A ver the following q		_	ie o	n all face	s is c	ut in	ito 125 (cubes	of e	qual size. N [M]	
6.	How many cubes	are n	ot painted on	any	y face?							
	(a) 8	(b)	16	(c)	18		(d)	27		(e)	54	
7.	How many cubes	are p	ainted on one	fac	e only?							
	(a) 8	(b)	16	(c)	36		(d)	54		(e)	None of th	ese
8.	The minimum nu faces may have the			uire	ed to pain	t all t	he si	des of a	cube t	hat	no two adjao [M]	
	(a) 1	(b)	2	(c)	3		(d)	4		(e)	6	
Dire	ctions (9-13): A	cube	e painted red o	n t	wo adjace	nt fa	ces a	nd blac	k on t	he fa	ces opposit	e to
the r	ed faces and gree	n on t	the remaining	g fac	ces is cut	into	$64 \mathrm{s}$	maller c	ubes	of e	lual size.	
											[C	BI
9.	How many cubes	are t	here which ha	ve :	no faces p	ainte	ed?					
	(a) 0		(b) 4		(c)	8			(d)	16		
10.	How many cubes	have	only one face	pai	\mathbf{nted} ?							
	(a) 8		(b) 16		(c)	24			(d)	32		
11.	How many cubes	have	less than 3 fa	ces	painted?							
	(a) 8		(b) 24		(c)	28			(d)	48		
12 .	How many cubes	are t	here with thre	ee fa	aces pain	ted?						
	(a) 4		(b) 8		(c)	16			(d)	24		
13.	How many cubes	have	one face gree	n aı	nd one of	the a	djac	ent face	s blac	k or	red?	
	(a) 8		(b) 16		(c)	24			(d)	28		
	ctions (14–20): posite faces. It is t								d, blu	e and	d black on p	airs
14.	How many cubes	have	no faces pain	tedʻ	?							
	(a) 0		(b) 4		(c)	8			(d)	12		
15.	How many cubes	have	three faces pa	aint	ed with d	liffere	ent c	olours?				
	(a) 0		(b) 4		(c)	8			(d)	12		
16.	How many cubes	have	two faces pai	nte	d red and	blac	k and	d all oth	er fac	es u	npainted?	
	(a) 4		(b) 8		(c)	16			(d)	32		
17.	How many cubes	have	to faces paint	ted	black?							
	(a) 2		(b) 4		(c)	8			(d)	Nor	ne of these	

- 18. How many cubes have one face painted blue and one face painted red? (the other faces may be painted or not painted)
 - (a) 16

(b) 12

(c) 8

- (d) (
- 19. How many cubes have only one face painted red and all other faces unpainted?
 - (a) 4

(b) 8

- (c) 12
- (d) 16

- 20. How many cubes have three faces painted?
 - (a) 0

(b) 4

(c) 6

(d) 8

(ANSWERS)

- 1. (d) 2. (b) 3. (c) 4. (a) 5. (a) 6. (d) 7. (d) 8. (c) 9. (c) 10. (c)
- 11. (d) 12. (b) 13. (c) 14. (c) 15. (c) 16. (b) 17. (d) 18. (a) 19. (b) 20. (d)

Explanations

Note: The figure may be analyzed by dividing it into three horizontal layers.

- 1. There are four cubes in the middle layer, which have one face painted only in blue.
- 2. There is only one cube each at top and bottom layer which has one face painted only in green.
- 3. There are 9 cubes in each layer. Therefore, altogether in all the three layers there are 27 cubes.
- 4. Four cubes in the top layer and four cubes in the bottom layer have three sides painted. Hence there are 8 such cubes.
- 5. There is only one central cube in the middle layer, which has no faces painted at all.
- 6. There will be 5 cubes width for the given large cube as length, width and depth. So in the middle layers there will be $3 \times 3 = 9$ cubes/layer. So in all, there will be $9 \times 3 = 27$ cubes unpainted on all faces.
- 7. Number of cubes which are one side painted -9 cubes at top +9 cubes at bottom + (12 cubes/layer \times 3 for middle layers).
 - \therefore Total number of cubes with only one face painted = $9 + 12 \times 3 + 9 = 54$.
- 8. Adjacent faces are avoided for same colours, opposite faces may be provided with same colours. So as a cube is having 3 pair of faces, minimum number of colours required to paint all sides of cube = 3.
- 9. There are 8 cubes with no face painted.
- 10. There are 24 cubes having only one face painted.
- 11. There are 24 cubes having only one face painted and 24 cubes having only two faces painted. So the number of cubes having less than 3 faces painted = 24 + 24 = 48.
- 12. There are 8 cubes having three faces painted.
- 13. There are 8 + 4 + 4 + 8 = 24 such cubes which have one face green and one of the adjacent faces black or red.
- 14. Four central cubes each in 2 middle layers have no face painted, i.e. total number of such cubes = 8.
- 15. Four corners cubes in top and bottom layers have three faces painted with different colours. Thus, there are 8 such cubes.
- 16. There are four cubes in top layer and four cubes in bottom layer which have two faces painted red and black and all other faces unpainted. So, there are 8 such cubes.

- 17. There will be no cubes which have two of its faces both painted black. So number of cubes = 0. The answer is (d).
- 18. There are 8 cubes in top layer and 8 cubes in bottom layer which have one face painted blue and one face painted red. So, the total number of such cubes = 8 + 8 = 16.
- 19. There are 4 cubes in top layer and 4 cubes in bottom layer which have only one face painted red and all other faces unpainted.
 - \therefore Total number of such cubes = 4 + 4 = 8.
- 20. Four corner in top layer and four corner cubes in bottom layer have faces painted. So there are 8 such cubes.

Mirror Images

Image of an object, as seen in a mirror, is called the mirror image or mirror reflection.

The right side of an object appears on the left side in its image and vice versa. Phenomenon of getting a letter inverted in its mirror image is called the *lateral inversion*.

Note: Letters having identical mirror images are: A H I M O T U V W X and Y

EXAMPLE 1 Find out the mirror image of the word 'SECOND'.

Object | Image | SECOND | COND

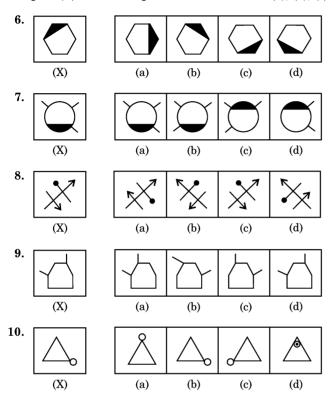
Image of word SECOND will be as seen on the right side after dotted line.

EXERCISES

Directions (Q. 1-5): In each of the following questions, you are given a combination of letters or/and numbers followed by four alternatives (a), (b), (c) and (d). Choose the alternative which most closely resembles the mirror-image of the given word.

1.	ABILITY			
	(a) YTILIBA	YTILIBA (d)	ABILITY (a)	(d) LITYABI
2.	97563			
	(a) 63975	(b) 36579	57936 (b)	97563 (b)
3.	mineral			
	(a) eralmin	mineral (d)	nimlare (a)	(d) larenim
4.	A1B2C3			
	(a) 3C2B1A	(b) 2C3A1B	A1B2C3 (a)	B1A3C2 (b)
5.	GANDHI1948			
	GANDHI1948 (a)		(b) 1948 GANDHI	
	(c) 8491IHDNAG		IHDNAG8491 (b)	

Directions (Q. 6–10): In each one of the following questions, choose the correct mirror-image of the figure (X) from among the four alternatives (a), (b), (c) and (d) given along with it.



(ANSWERS)

1. (c)

2. (d)

3. (b)

4. (c)

5. (a)

6. (b)

7. (b)

8. (b)

9. (c)

10. (c)

Water Images

The reflection of an object as seen in water is called its **water image**. It is the inverted image obtained by turning the object upside down.

Note: The letters whose water images remain unchanged are: C, D, E, H, I, K, O, and X.

EXAMPLE 1 Find out the water image of the word 'SERIES'.

- (a) SEIRES
- (p) SERIES
- SEIRES (2)
- IESSEK (b)

The answer is (b).

EXERCISES

Directions: In each one of the following questions, you are given a combination of letters or/and numbers followed by four alternatives (a), (b), (c), and (d). Choose the alternative which most clearly resembles the water image of the given combination.

- 1. NEW DELHI
 - (a) DETHI NEM
- DELHI NEW(d)
- (c) NEW DELHI
- (q) IHLED WEN

- 2. escape
 - (a) epacse
- (p) apeesc
- czeebs (2)
- (q) escape

- 3. 15 AUG 1947
 - (a) 1947 AUG 15
- (p) 15 AUG 1947
- (c) 7491 GUA 51
- (d) 91 9NY 4761

- 4. PERMANENT
 - (a) ANANTPERM
- (p) TNANAMREP
- MKEPTNENA (3)
- (q) PERMANENT

- 5. NEHRU 1964
 - (a) 1964 NEHRU
- NEHBΩ 1804 (d)
- **OBHEN 4691 (2)**
- (q) NEHRU 1964

ANSWERS

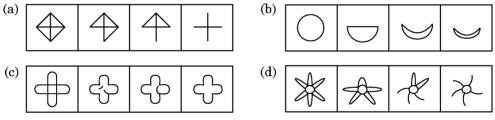
- 1. (c)
- **2.** (d)
- **3.** (b)
- **4.** (d)
- **5.** (d)

Rule Detection

In this chapter, we will solve problems in which a particular rule is given and it is required to select from the given sets of figures, a set of figures which obeys the rule and thus forms a series.

EXAMPLE 1 Which one of the given sets of figures satisfies the following rule?

Rule: Closed figure becomes more and more open.

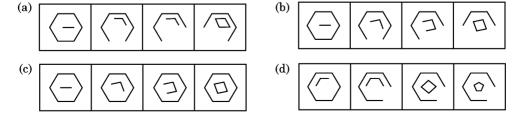


Solution The answer is (a).

EXERCISES

Directions: In each of the following questions, choose the set of figures which follows the given rule.

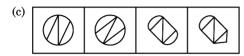
1. Rule: Closed figures losing their sides and open figures gaining their sides.

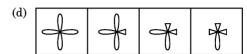


2. Rule: Sectors get converted to triangles one by one.

(a)

(p)

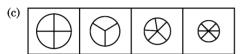




3. Rule: As the circle decreases in size, its sector increases in number.

(a) () () () ()







(ANSWERS)

1. (b) 2. (b) 3. (d)

Model Test Paper 1

1. Find the least number which, upon being divided by 2, 3, 4, 5 and 6 leaves								leaves in each
	case a remainde	er of 1, but when	div	ided by 7 lea	ves	no remainde:	r.	
	(a) 271	(b) 291	(c)	301	(d)	311	(e)	None of these
2.	What is the grea	atest number tha	t wi	ll divide 676	and	942 and will	leav	ve remainders
	4 and 18 respec	tively?						
	(a) 64	(b) 46	(c)	24	(d)	42	(e)	None of these
3.	The sum of two can be formed?	number is 528 an	d th	eir HCF is 33	В. Но	w many pair	s of	such numbers
	(a) 1	(b) 2	(c)	3	(d)	4	(e)	None of these
4.	The product of to pair of numbers		46 a	and their HCl	F is 2	21. Find the r	num	ber of possible
	(a) 1	(b) 2	(c)	3	(d)	4	(e)	None of these $$
5.	Find the value of	of $\frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}}$: 	_				
	(a) $\frac{17}{43}$	(b) $\frac{18}{43}$	(c)	$\frac{19}{43}$	(d)	$\frac{21}{43}$	(e)	None of these

(c) 630

with the person when he met A?

(b) 640

(a) 620

6. A person is having some amount with him. He gave 1/3rd of the amount as loan to A and took back 54 more. He gave 1/2 of the amount with him as loan to B and took back 25 more. When he met C, the amount left was ₹ 262. What was the amount

(d) 650

(e) None of these

and R do not sit together?

has one ace and one jack?

(b) 600

(b) 48

(a) 800

(a) 120

	(a) $\frac{4}{663}$	(b) $\frac{8}{663}$	(c) $\frac{16}{663}$	(d) $\frac{32}{663}$	(e) None of these
10.					a team of 2 boys and
		•		•	w many ways it can
	be done?				
	(a) 3696	(b) 1848	(c) 924	(d) 1386	(e) None of these
11.	Two dices are th	rown. What is t	he probability t	that the sum of n	umbers appeared is
	less than 8?				
	(a) $\frac{3}{12}$	(b) 5	7	(4) 11	(e) None of these
12.					d second and second
	and third are res	=			
	(a) 48	(b) 28	(c) 30	(d) 36	(e) None of these
13.					ys, working 8 hours
		•			6 boys to do another
	piece of work $2\frac{1}{2}$	times as great	t in 30 days wo	rking 6 hrs a day	y?
	(a) 24	(b) 18	(c) 10	(d) 8	(e) None of these
14.				number is incre	eased by 9, the ratio
	becomes 5:6. Fi				
					(e) None of these
15.	-			y 10% and then	decreased by 10%.
	What was the ch	-	•	(3)	() 37 0.1
	(a) 0%				(e) None of these
16.	Tax on a commo Find the effect o		d by 20% and 1	ts consumption i	s decreased by 15%.
			(a) 1.90/	(4) 90/	(e) None of these
17	(a) +5%				
17.	24 km/h. Find th				returns at a speed of
	(a) 24 km/h		•	•	(e) None of these
18	` '	, ,	, ,	, ,	he age of father will
10.	be three times th		•	•	ne age of father will
					(e) None of these
19.	` '	` '	` '	` '	t half more than the
	listed price. Wha			1	
	(a) 25%			(d) 100%	(e) None of these
				•	

7. How many number of five digits can be formed with the digits 0, 1, 2, 4, 6 and 8?

8. There are five students P, Q, R, S, and T. In how many ways can they sit so that Q

9. From a pack of 52 playing cards 2 cards are drawn. What is the probability that it

(d) 400

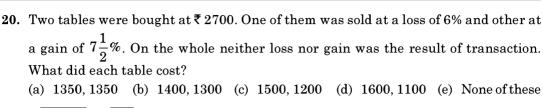
(d) 72

(e) None of these

(e) None of these

(c) 500

(c) 66



21.
$$\frac{\sqrt[3]{456533} \times \sqrt{256}}{\sqrt{64} \times \sqrt[3]{1331}} = ?$$

- (a) 7
- (b) 14
- (c) 21
- (d) 28
- (e) None of these

Directions (22-24): In each of the following questions a number series is given in which one number is wrong. You have to find out that number and have to follow the new series which will be started by that number. Then what will be the third number of the new series?

- **22**. 2 9 5 36 125 648 3861
 - (a) 12
- (b) 11
- (c) 75
- (d) 72
- (e) None of these

- **23.** 3 4 12 190 1005 6066 45
 - (a) 98
- (b) 96
- (c) 384
- (d) 386
- (e) None of these

- 99 209 **24.** 2 7 19 43
 - (a) 181
- (b) 183
- (c) 87
- (d) 85
- (e) None of these

Directions (25-27): In each of the following questions a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given, following the sequence of the given series.

- **25.** 2 10 27 60
 - (b) (a)
- (c) (d) (e)

Which of the following number will come in place of (b)?

- (a) 39
- (b) 13
- (c) 34
- (d) 38
- (e) None of these

- **26.** 15 16 25 50
 - 189 (a)
- (c) (d) (e)

Which of the following numbers will come in place of (e)?

- (a) 354
- (b) 273
- (c) 394
- (d) 426
- (e) None of these

- **27.** 5 149 49 113
 - 146 (a) (b)
 - (c) (d) (e)

Which of the following number will come in place of (d)?

- (a) 290
- (b) 234
- (c) 254
- (d) 218
- (e) None of these

- 28. If $x + \frac{1}{x} = 3$, then find the value of $x^2 + \frac{1}{x^2}$?
 - (a) 3
- (b) 5
- (c) 7
- (d) 9
- (e) None of these

- **29.** If a + b + c = 0, find the value of $\frac{a^2 + b^2 + c^2}{c^2 ab}$
 - (a) 0
- (b) 1
- (d) 3
- (e) None of these

(a) $4 + 7\sqrt{3}$

(a) 4 months

used?

30. Find the value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

(b) $7 - 4\sqrt{3}$

(b) 8 months

	ind the sum ler	патож ре	r annum.						
	(a) ₹5000	(b) ₹4000	0 (c)	₹ 3000	(d)	₹ 2000	(e)	None of the	se
33.	Two equal amou	unts of mo	ney are de	eposited in	ı two ba	ınks, eacl	n at 15%	% per annui	m,
	for $3\frac{1}{2}$ years an	nd 5 years	respective	ely. If the	differe	nce betwe	een the	ir interests	is
	₹ 144, find the s	um.							
	(a) ₹400	(b) ₹300	(c)	₹ 640	(d)	₹ 720	(e)	None of the	se
34.	A certain sum grinterest. Find th	_		2 years an	d upto₹	5324 in	3 years	on compoui	ıd
	(a) 5%	(b) 10%	(c)	15%	(d)	20%	(e)	None of the	se
35.	A certain number the work could b								e,
	(a) 20 men	(b) 30 me	en (c)	40 men	(d)	50 men	(e)	None of the	se
36.	P, Q and R toget Q alone can do share of R for it	it in 6 days				_			
	(a) ₹100	(b) ₹150	(c)	₹ 200	(d)	₹ 250	(e)	None of the	se
37.	A cistern has a admits 6 litres a litres does the c	ı minute in	the cister						
	(a) 8460 L	(b) 4320 l	L (c)	$8640\mathrm{L}$	(d)	$4230~\mathrm{L}$	(e)	None of the	se
38.	Two trains are 64 km/h and 96 l whereas the sectrains to cross e	km/h respe cond train	ctively. The passes the	ne first tra e post in 6	in passe	es a telegr	aph pos	st in 5 secon	ds
	(a) $\frac{28}{11}$ s	(b) $\frac{14}{11}$ s	(c)	$\frac{28}{5}$ s	(d)	$\frac{14}{5}$ s	(e)	None of the	se
39.	A train passes train is moving, Find the length	at the rate	of 5m/s ar	nd 10 m/s in	n 6 secoi				
	(a) 140 m, 10 m	ı/s	(b) 150	m, 20 m/s		(c) 150	m, 30 n	n/s	
	(d) 140 m, 40 m	ı/s	(e) Non	e of these					

(c) $7 + 4\sqrt{3}$

31. A and B enter into a partnership with capitals 4: 5, and at the end of 8 months, A withdraws. If they receive profits in the ratio 8: 15, find how long B's capital was

32. Out of ₹ 7000, some amount was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 1600,

(d) $4 - 7\sqrt{3}$

(c) 10 months (d) 12 months (e) None of these

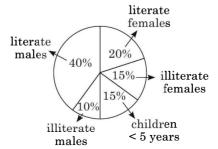
(e) None of these

- **40.** A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of boat in still water is 9 km/h and speed of the current is 3 km/h, find the distance between A and B in km.
 - (a) 6 km
- (b) 9 km
- (c) 12 km
- (d) 18 km
- (e) None of these
- 41. A hall, whose length is 16 metres and breadth twice its height takes 168 metres of paper 2 m wide for sticking on its four walls. Find the area of floor.
 - (a) 384 m^2
- (b) 192 m^2
- (c) 576 m^2
- (d) 288 m^2
- (e) None of these
- **42.** The radius of a sphere is increased by 5%. Find the percentage increase in its surface area.
 - (a) 20
- (b) 15.5
- (c) 10.25
- (d) 5.25
- (e) None of these
- **43.** A man covers a certain distance by car at a speed of 60 km/h and returns on a scooter at a speed of 40 km/h. Find his average speed for the whole journey.
 - (a) 40 km/h
- (b) 48 km/h
- (c) 50 km/h
- (d) 60 km/h
- (e) None of these
- **44.** The upper part of a tree broken by wind makes an angle of 30° with the ground and the distance from the root of the point where the top of the tree touches the ground is 10 m. What is the height of the tree?
 - (a) 30 m
- (b) 40 m
- (c) 50 m
- (d) 60 m
- (e) None of these

Directions (45-46): The questions below consist of a question followed by two statements labelled I and II. You have to decide whether these statements are sufficient to answer the question. Give answer

- (a) if statement I alone is sufficient to answer the question but statement II alone is not sufficient to answer the question;
- (b) if statement II alone is sufficient to answer the question but statement I alone is not sufficient to answer the question.
- (c) if you can get answer from I and II together.
- (d) if statement I alone or II alone is sufficient.
- (e) if both statements taken together are not sufficient.
- **45.** What is the value of a two-digit number?
 - I. The sum of the two digits is 4.
 - II. The difference between the two digits is 2.
- **46.** If x greater than y?
 - I. x is greater than 145.
 - I. v is greater than 140.

Directions (47–49): Study the following pie chart and answer the questions given below.



Total population of a town in 2011 = 10 lakhs.

47. What is the total number of literate males in the town?

- (a) 500000
- (b) 400000
- (c) 200000
- (d) 150000
- (e) None of these

48. What is the ratio of literate females to literate males?

- (a) 1:2
- (b) 2:3
- (c) 1:4
- (d) 1.5:4
- (e) None of these

49. What is the total strength of literate people to total population expressed as percentage?

- (a) 40%
- (b) 30%
- (c) 50%
- (d) 60%
- (e) None of these

50. 6 September 1986 was Saturday. What will be the day on 15 August 2012?

- (a) Monday
- (b) Tuesday
- (c) Wednesday (d) Thursday
- (e) None of these

ANSWERS

1. (c) 2. (d) 3. (d) 4. (b) 5. (c) 6. (c) 7. (b) 8. (d) 9. (b) 10. (b)

11. (c) 12. (c) 13. (d) 14. (c) 15. (c) 16. (c) 17. (c) 18. (b) 19. (d) 20. (c)

21. (b) **22.** (e) **23.** (d) **24.** (b) **25.** (a) **26.** (a) **27.** (d) **28.** (c) **29.** (c) **30.** (c)

31. (d) **32.** (d) **33.** (e) **34.** (b) **35.** (e) **36.** (b) **37.** (e) **38.** (e) **39.** (b) **40.** (e)

41. (b) **42.** (c) **43.** (b) **44.** (e) **45.** (e) **46.** (e) **47.** (b) **48.** (a) **49.** (d) **50.** (c)

Solutions and Hints

- 7. Required number of numbers = $5 \times {}^5P_4 = 5 \times 5! = 5 \times 120 = 600$
- 8. No. of ways in which Q and R sit together = $2 \times 4! = 48$
 - \therefore No. of ways in which Q and R do not sit together = 5! 48 = 120 48 = 72

9. Required probability =
$$\frac{{}^{4}C_{1} \times {}^{4}C_{1}}{{}^{52}C_{2}} = \frac{4 \times 4 \times 2}{52 \times 51} = \frac{8}{13 \times 51} = \frac{8}{663}$$

- 10. Required number of ways = ${}^{8}C_{2} \times {}^{12}C_{2} = \frac{8 \times 7}{1 \times 2} \times \frac{12 \times 11}{1 \times 2} = 28 \times 66 = 1848$
- 11. Desired sum of number are 2, 3, 4, 5, 6 and 7.

$$n(s) = 1 + 2 + 3 + 4 + 5 + 6 = 21$$

$$\therefore$$
 Required Probability = $\frac{21}{36} = \frac{7}{12}$

12. x: y = 2: 3 and y: z = 5: 8

$$x: y: z = 10: 15: 24$$

:. Second number
$$y = \frac{15}{10 + 15 + 24} \times 98 = \frac{15}{49} \times 98 = 30$$

13. 30 days : 12 days : 6 h : 8 h

 $1 \text{ work}: 2\frac{1}{4} \text{ work}$

13. 30 days : 12 days :: 15 men : required no. of men

$$\therefore \text{ Required no. of men} = \frac{15 \times 12 \times 8 \times 2.25}{30 \times 6 \times 1} = 18$$

24 women = 15 men

 \therefore 12 women = 7.5 men

36 boys = 15 men

$$\therefore \qquad 6 \text{ boys} = \frac{15}{6} = 2.5 \text{ men}$$

 \therefore 12 women + 6 boys = 7.5 + 2.5 = 10 men

 \therefore 18 – 10 = 8 men must be associated.

14. Ratio of numbers = 4:5

i.e.
$$4k:5k$$

After increasing, ratio = (4k + 9) : (5k + 9) = 5 : 6

$$\Rightarrow$$
 $k=9$

.. The numbers are 36 and 45

15. Net change =
$$\frac{-x^2}{100}\% = \frac{-10^2}{100} = -1\%$$

16. Net revenue =
$$20\% - 15\% - \frac{20 \times 15\%}{100} = (20 - 15 - 3)\% = 2%$$
 i.e. increase of **2**%.

17. Average speed =
$$\frac{2 \times 48 \times 24}{48 + 24} = \frac{2 \times 48 \times 24}{72} = 2 \times 16 = 32$$
 km/h

18.
$$x + (56 - x) = 56$$

 $\frac{(x+4)}{(60-x)} = 1/3 \text{ or } 60 - x = 3x + 12$

$$\Rightarrow x = 12 \text{ years}.$$

19. Let the listed price of article be ₹ 100.

$$\therefore \text{ Cost price} = \frac{3}{4} \times 100 = ₹75$$

Selling price =
$$\frac{3}{2} \times 100 = ₹150$$

$$Profit = \frac{SP - CP}{CP} \times 100 = \frac{150 - 75}{75} \times 100 = 100\%$$

20. Let the cost of first table be ξx .

$$\therefore x \times \left(\frac{94}{100}\right) + (2700 - x) \frac{107.5}{100} = 2700$$

$$x \left(\frac{107.5 - 94}{100}\right) = 2700 \left(\frac{107.5 - 100}{100}\right)$$

$$\therefore x = \frac{2700 \times 7.5}{107.5 - 94} = \frac{2700 \times 7.5}{13.5} = ₹ 1500$$

∴ Prices of tables are ₹ 1500 and ₹ 1200

21.
$$\frac{77 \times 16}{8 \times 11} = 14$$

22. The series is $\times 1 + 7$, $\times 2 - 11$, $\times 3 + 15$, ...

Here the wrong number in the series is 5. Therefore, the series starting with 5 is 5, $(5 \times 1 + 7)$, $(5 \times 1 + 7) \times 2 - 11$

i.e., 5, 12, 3.

:. The third term of series is 3.

Hence the answer here is (e).

23. The series is $\times 1 + 1^2$, $\times 2 + 2^2$, $\times 3 + 3^2$, ...

Here the wrong number in the series is 190. Therefore, series starting with 190 is $190. (190 \times 1 + 1^2). (190 \times 1 + 1^2) \times 2 + 2^2$

i.e., 190, 191, 386.

:. The third term of series is 386.

Hence the answer is (d).

24. The series is $\times 2 + 3$, $\times 2 + 5$, $\times 2 + 7$, $\times 2 + 9$, ...

Here the wrong number in the series is 43. Therefore, series starting with 43 is 43, $(43 \times 2 + 3)$, $(43 \times 2 + 3) \times 2 + 5$

i.e., 43, 89, 183.

.. The third term of series is 183.

Hence the answer is (b).

25. The series is $\times 2 + 6$, $\times 2 + 7$, $\times 2 + 6$, ...

Here the required term in place of (b)

$$= (5 \times 2 + 6) \times 2 + 7 = 39$$

Hence the answer is (a).

26. The series is $+1^2$, $+3^2$, $+5^2$, $+7^2$, ...

Here the required term in place of (e)

$$= 189 + 1^2 + 3^2 + 5^2 + 7^2 + 9^2 = 354$$

Hence the answer is (a).

27. The series is $+12^2$, -10^2 , $+8^2$, -6^2 , ...

Here the required number in place of (d)

$$= 146 + 12^2 - 10^2 + 8^2 - 6^2 = 146 + 44 + 28 = 218.$$

Hence the answer is (d).

28.
$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$$

29.
$$a+b+c=0 \Rightarrow (a+b+c)^2=0 \Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=0$$

 $\Rightarrow a^2+b^2+c^2=-2(ab+bc+ca)=-2[ab+c(a+b)]$
 $=-2[ab+c(-c)]=2[c^2-ab]$

$$\therefore \frac{a^2+b^2+c^2}{c^2-ab}=2$$

30.
$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4+3+4\sqrt{3}}{4-3} = \mathbf{7} + 4\sqrt{\mathbf{3}}$$

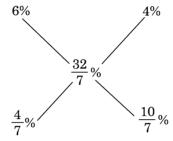
31. Ratio of capitals = 4:5; ratio of time = 8:x

$$\Rightarrow \qquad 4 \times 8 : 5 \times x = 8 : 15$$

$$\Rightarrow \qquad \frac{32}{5x} = \frac{8}{15}$$

$$\Rightarrow \qquad x = \frac{32 \times 15}{5 \times 8} = 4 \times 3 = 12 \text{ months}$$

32. Overall rate of interest = $\frac{1600 \times 100}{5 \times 7000} = \frac{32}{7} \%$



Ratio of two amounts = 2:5

∴ Amount lent at
$$6\% = 7000 \times \frac{2}{7} = ₹ 2000$$

33. Sum =
$$\frac{\text{Difference in interests}}{\text{Rate} \times \text{Difference in period}} \times 100$$

= $\frac{144 \times 100}{15 \times 15}$ = ₹ 640

34. Rate =
$$\frac{\text{Difference in amounts of } n \text{th and } (n+1) \text{ years}}{\text{Amount after } n \text{ years}} \times 100$$
$$= \frac{5324 - 4840}{4840} \times 100 = 10\%$$

35.
$$60 \times x = 50(x + 8)$$

$$\Rightarrow x = \frac{50 \times 8}{60 - 50} = \frac{400}{10} = 40 \text{ men}$$

=42:35:30

36. P's share : Q's share : R's share $= 6 \times 7 : 5 \times 7 : 5 \times 6$

∴ R's share =
$$\frac{30}{42 + 35 + 30} \times 535$$

= $\frac{30}{107} \times 535 = ₹ 150$

- 37. The filler tap can fill the tank in $\frac{12 \times 8}{12 8} = 24$ hours
 - \therefore Capacity of tank = $6 \times 60 \times 24 = 8640$ litres
- **38.** Ratio of speeds = 64:96=2:3

Time to cross telegraph posts = 5 seconds and 6 seconds

Required time to cross each other
$$=\frac{ax + by}{x + y} = \frac{2 \times 5 + 3 \times 6}{2 + 3} = \frac{28}{5}$$
 s

39. Length of train = Difference in speed $\times \frac{\text{time } 1 \times \text{time } 2}{\text{time } 1 \sim \text{time } 2}$

$$=\frac{(10-5)\times 6\times 5}{6-5}=$$
150 m

Speed of train be x m/s

$$(x + 5) \times 6 = (x + 10)5$$

$$\Rightarrow$$
 $6x + 30 = 5x + 50$

$$\Rightarrow$$
 $x = 50 - 30 = 20 \text{ m/s}$

40. u = 9 km/h; v = 3 km/h

Let
$$AB = x km$$

$$x = (u + v) t_1 = (u - v)t_2$$

$$t_1 + t_2 = 3 \text{ hours}$$

$$\therefore$$
 $(9+3)t_1=(9-3)(3-t_1)$

$$\Rightarrow$$
 12 $t_1 = 18 - 6t_1$

$$\Rightarrow t_1 = \frac{18}{12 + 6} = 1 \text{ hour}$$

$$\therefore$$
 $x = (u + v)t_1 = (9 + 3) \times 1 = 12 \text{ km}$

= Total time $\times \frac{(Speed in still water)^2 - (Speed of current)^2}{2 \times Speed in still water}$

$$= 3 \times \frac{9^2 - 3^2}{2 \times 9} = \frac{3 \times 72}{18} = 12 \text{ km}$$

41. Let height be h metres

$$\therefore$$
 $b = 2h$

$$\therefore$$
 2(16 + 2h)h = 168 × 2

$$\Rightarrow$$
 $(16+2h)h = 168$

$$\Rightarrow \qquad h^2 + 8h - 84 = 0$$

$$(h + 14) (h - 6) = 0$$
∴ $h = 6 \text{ m}$
∴ Floor area = $16 \times 2h = 16 \times 12 = 192 \text{ m}^2$

42. Required percentage value = $2x + \frac{x^2}{100}$

$$= 2 \times 5 + \frac{5^2}{100} = 10 + 0.25 = 10.25$$

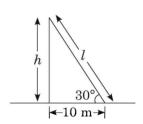
43. Average speed =
$$\frac{2 \times 60 \times 40}{60 + 40} = 48$$
 km/h

44.
$$\tan 30^\circ = \frac{h}{10} = \frac{1}{\sqrt{3}}$$

or
$$h = \frac{10}{\sqrt{3}}$$

$$l\sin 30 = h = \frac{10}{\sqrt{3}} = \frac{l}{2}$$

or
$$l = \frac{20}{\sqrt{3}}$$



:. Total height of tree =
$$l + h = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m} = 17.32 \text{ m}$$

- 45. Even both statements taken together will not give the correct answer.
- **46.** x can take values from 146 and y can take values from 141. y can also take values more than 146 such as 147, 148, So you can't clearly state that x is greater than y.
 - \therefore The answer is (e).
- **47.** 40% of 1000000 = **400000**
- 48. $\frac{20\% \text{ of } 10 \text{ lakh}}{40\% \text{ of } 10 \text{ lakh}} = \frac{1}{2}$
 - .. Required ratio is 1:2
- **49.** Total strength of literates = 40 + 20 = 60%
- 50. 6 September 1986 Saturday.
 - $6 \; September \; 1986-6 \; September \; 2011$

$$= 25 \text{ years} = 6 \times 4 + 1 \text{ years}$$

$$= 6 \times 5 + 1$$
 odd days $= 31$ odd days

31 % 7 = 3 odd days.

 $6\;September\;2011-15\;August\;2012$

= 344 days

- \therefore Total odd days = 3 + 1 = 4 odd day
- \therefore 15th August 2012 is Saturday + 4 = **Wednesday**

Model Test Paper 2

Each question (1-3) given below consists of a statement followed by two arguments numbered I and II. You have to decide which of the arguments is a 'strong' argument and which is a 'weak' argument.

Give answer (a) if only argument I is strong; (b) if only argument II is strong; (c) if either I or II is strong; (d) if neither I nor II is strong and (e) if both I and II are strong.

1. Statement: Should state lotteries be stopped?

Arguments: I. Yes. Government should not promote gambling habits.

II. No. Government will lose a large amount of revenue.

2. Statement: Should the illiterate be debarred from voting?

Arguments: I. Yes. They are easily misguided.

II. No. It is their constitutional right.

 ${f 3.}$ Statement: Is monarchy better than democracy?

Arguments: I. Yes. If the chair has one confirmed ruler, there are no ambitious aspirants fighting for it.

II. No. People are more contended and happy in democracy.

In each question (4-6) below is given a statement followed by two assumptions numbered I and II. Consider the statement and decide which of the given assumptions is implicit.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II is implicit and (e) if both I and II are implicit.

4. Statement : Please consult me before making any decision on exports from the company.

Assumptions: I. You may take a wrong decision if you don't consult me.

II. It is important to take a right decision.

5. Statement : "If you are beautiful, we will catch your beauty. If you are not, we will make you beautiful"—An advertisement of a photo studio.

Assumptions: I. How to look beautiful, is a problem of youngster.

II. Nobody desires to be beautiful.

6. Statement : A warning in a train compartment—"To stop train, pull chain.

Penalty for use without sufficient cause attracts 6 months

imprisonment and/or fine up to ₹ 1000".

Assumptions: I. Some people misuse the alarm chain.

II. On certain occasions, people may want to stop a running train.

In each question (7–9) below is given a statement followed by two courses of action numbered I and II. A course of action is a step or administrative decision to be taken for improvement, follow-up or further action in regard to the problem policy, etc. on the basis of the information given in the statement. You have to assume everything in the statement to be true, then decide which of the two given suggested courses of actions logically follows for pursuing.

Give answer (a) if only I follows; (b) if only II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow:

7. Statement : A shopkeeper was reported to be selling adulterated grains.

Courses of Action: I. He should be fined and his shop sealed.

II. He should be asked to leave the town and open a shop elsewhere.

8. Statement : A train has derailed near a station while moving over a bridge and fell into the river.

Courses of Action: I. The railway authorities should clarify the reason of the accident to the government.

 The government should allocate funds to compensate the destruction caused.

9. Statement : The Courts take too long in deciding important disputes of various departments.

Courses of Action: I. The Courts should be ordered to speed up matters.

II. Special powers should be granted to officers to settle disputes concerning their department.

In each of the following questions (10–12) a statement is given followed by two conclusions I and II. Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows: (d) if neither I nor II follows and (e) if both I and II follow.

10. Statement : Domestic demand has been increasing faster than the production of indegenous crude oil.

Conclusions: I. Crude oil must be imported.

II. Domestic demand should be reduced.

11. Statement : Patients with minor ailments usually do not go to eminent doctors.

Conclusions: I. Eminent doctors remain too busy with patients suffering from serious complications.

II. Their charges are rather high.

12. Statement : Parents are prepared to pay any price for an elite education to their children.

Conclusions: I. All parents these days are very well off.

II. Parents have an obsessive passion for a perfect development of their children through good schooling.

In each of the following questions (13–19) there is a certain relation between two given words on one side of:: and one word is given on the another side of:: while another word is to be found from the given alternatives, having the same relation with this word as the words of the given pair bear. Choose the correct alternative.

13.	Me	lt : Liquid : :	Free	e z e :	?									
	(a)	Ice		(b)	Conden	ise		(c)	Soli	d		(d)	Cry	rstal
14.	Act	ting : Theatre	e : : (Gam	bling:?	•								
	(a)	Casino		(b)	Club			(c)	Bar			(d)	Gyı	m
15.	Du	ngeon : Confi	inem	ient	:: Asylı	ım :	?							
	(a)	Refuge		(b)	Mercy			(c)	Tru	ancy	7	(d)	Rei	norse
16.	Ru	by : Red : : Sa	apph	ire:	?									
	(a)	Blue		(b)	White			(c)	Gre	en		(d)	Silv	ver
17.	Scr	ap : Food : : l	Lees	:?										
	` '	Bread		` '	Tea			` ′	Win	ıe		(d)	Ric	e
18.	Wi	mbledon Trop	phy	: Te	nnis::V	Valk	er's C	up	:?					
	(a)	Hockey		(b)	Polo			(c)	Gol	f		(d)	Wr	estling
19.	Me	at : Vegetaria	an:	: Lic	quor : ?									
	(a)	Insane		(b)	Introve	\mathbf{rt}		(c)	Tee	tota	ller	(d)	Foo	lish
			-					-						ords is given.
		a word from t			n alterna	ative	s, tha	at b	elon	gs to	the sam	ne gi	roup).
20.		as : Gram : Pu	ulses		D 1				ъ			(1)	a	
	` '	Rice	_	` '	Barley			(c)	Bea	ns		(d)	Coc	conut
21.		tato : Carrot :	Rac						~				~	
		Tomato	~		Spinacl	n		(c)	Ses	ame		(d)	Gro	oundnut
22.		rble : Slate :			.				~				~	•
ъ.		Quartzite										, ,		ndstone
									east			er w		in the group.
		Whale			lphin		Shar				Cod		` ′	Starfish
		-					Yello				Pink			Green
	• •	•				٠,					Adrenal		` ′	Testis
		Nerves					Arte	•			Valve			Aorta
	` ′	Metre	, ,		•		Yard				Mile			Acre
28.												rrec	t alt	ternative that
	W1l	l continue the	e sa	me p							paces.			
			a \		3, 7, 2	•			72				, ,	
	` ′	144		166		` ′	121			(d)	112		(e)	None of these
29.	F'ir	nd the wrong	tern	n in										
	, .		a .		2,		2, 24,	30,	42	. •			, .	
	(a)	6	(b)	12		(c)	24			(d)	30		(e)	42

 31. If in a certain language 'TABLE' is coded as 'FMCBU' then which word is coded as 'TOWER' in the same language? (a) SFXPU (b) QDVNS (c) UPXFS (d) SNVDQ (e) None of these 32. If 'book' is called 'pencil', 'pencil' is called 'pen', 'pen' is called 'rubber' and 'rubber' is called 'sky'. Then what is used to draw figure? (a) pencil (b) pen (c) book (d) rubber (e) None of these 33. In a certain language, '45' means 'Good morning', '357' means 'Buy good items'. '268 means 'Man is handsome'. What stands for 'morning' in that code? (a) 3 (b) 4 (c) 5 (d) 6 (e) 8 34. Pointing to a photograph, a woman said—"He is the only son of my mother's father.' Then the relation of the person in the photograph to the woman is 	30.	If BATCH is	coded as ABCTH,	how is S	EVEN co	ded in that co	de?
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suitably using one of the following Venn diagram. The best representation of it is (a) Engineers Doctors (b) Engineers Doctors (c) Engineers Doctors (d) Engineers Doctors Lawyers (e) None of these 37. Dinesh is ranked 4th from top among boys and 19th from top among all students. It the total number of girls in the class is 22, then find the number of girls behind Dinesh in ranking. (a) 3 (b) 7 (c) 18 (d) 15 (e) None of these (38–42) Six persons A, B, C, D, E and F are seated around a round table. (i) C is seated to the left of E and opposite to B. (ii) A is seated opposite to D and left of B. (iii) F is between B and D. 38. Who is seated in between A and F? (a) C (b) D (c) E (d) B 39. Who is seated to the right of C?	9.0	` '	` '	, ,		` '	` '
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 (i) C is seated to the left of E and opposite to B. (ii) A is seated opposite to D and left of B. (iii) F is between B and D. 38. Who is seated in between A and F? (a) C (b) D (c) E (d) B 39. Who is seated to the right of C? 	(28_	, ,	` '	, ,	ented are	` '	, ,
 (ii) A is seated opposite to D and left of B. (iii) F is between B and D. 38. Who is seated in between A and F? (a) C (b) D (c) E (d) B 39. Who is seated to the right of C? 	(00-					una a rouna t	able.
38. Who is seated in between A and F?(a) C(b) D(c) E(d) B39. Who is seated to the right of C?		(ii) A is seate	ed opposite to D a				
(a) C (b) D (c) E (d) B 39. Who is seated to the right of C ?	38.	, ,		d F?			
39. Who is seated to the right of C?				•	(c) E	(d) B
	39.	, ,	, ,	?	(-/ _		· / · -
					(c) E	(d) F

16	Model Test Paper	· 2										
40.	Who is seated o	pposi	te to E?									
	(a) A		(b) B			(c)	D			(d)	\mathbf{F}	
41.	Who is seated to	o the	right of l	E?								
	(a) A		(b) B			(c)	\mathbf{C}			(d)	D	
42 .	Who is seated b	etwe	en D and	E?								
	(a) A		(b) B			(c)	\mathbf{C}			(d)	\mathbf{F}	
43.	How many num as 4 and the nu				een 1	and	100) wh	ich is	having	at l	east one digit
	(a) 3	(b)	5	(c)	7			(d)	9		(e)	None of these
44.	How many days	will t	here be f	rom 15	Augu	ıst 1	988	to 5	June	1989 bo	oth d	lays included?
	(a) 272	(b)	29 3	(c)	294			(d)	295		(e)	None of these
45 .	How many 2 ar	re the	ere in the	e follov	ving	nun	ıber	seq	uence	which	are	immediately
	preceded by 5 b	ut no	t followe	d by 9?	•							
			$1\ 3\ 5\ 2$	8642	273	9 5 2	2 3 8	3 5 2	8			
	(a) 1	(b)	2	(c)	3			(d)	4		(e)	None of these
(46-	48) In the follo	wing	g figure,	recta	ngle	, sq	uar	e an	d tri	angle	rep	resent male,
	l servants and	_		s in a	socie	ety.	On	the	basis	of thi	s fi	gure answer
the	following ques	tions	s.					7				
					_	4						
			1		$2 \mid 3$		5					
			8	6	\							
			_	7	8 \							
46.	Who among the	follo	wing is a	male	and a	pro	fess	siona	.1?			
	(a) 7	(b)	8	(c)	6			(d)	5		(e)	4
47 .	Who among the	follo	wing is a	femal	e civi	l sei	rvan	ıt?				
	(a) 3	(b)	4	(c)	2			(d)	1		(e)	9
48.	Who among the	follo	wing is n	either	a civ	il se	rvai	nt no	or a p	rofessio	nal	?
	(a) 6	(b)	5	(c)	4			(d)	2		(e)	7
Dire	ections (49–53):	Stu	dy the fol	llowing	infor	ma	tion	care	fully a	and ans	wer	the questions

 \mathbf{D} given below it. The following are the criteria to get employment in a pharmaceutical company. The applicant must

- (i) have passed graduation with science subject with at least 50% marks.
- (ii) have completed 21 years (in case of males) and 19 years (in case of females) of age as on 1 August 2010.
- (iii) Pay deposit as follows:

Marks at graduation	If son/daughter of trustee or staff	If not son/daughter of trustee or staff
75% and above	₹ 5000	₹ 10000
61% to 74%	₹ 10000	₹ 20000
60% and below	₹ 20000	₹ 40000

However,

- (iv) If the candidate has secured more than 90% marks at graduation, he should be referred to the Managing Director for consideration for further concession in deposit.
- (v) In the case of SC/ST candidates, deposit payable is 20% less in each of the above cases.
- (vi) If the candidate remits in cash the amount of deposit immediately on the date of first call, the candidate can be selected, provided he fulfils conditions (i) and (ii).
- (vii) If, on the date of first call, the amount of deposit brought by a candidate eligible as per (i) and (ii) is less than the total amount required but more than 3/4th the candidate can be provisionally selected, provided he deposits the balance amount within next ten days.
- (viii) If on the date of first call, the amount of deposit brought is less than 3/4th but more than 1/2, the candidate eligible as per (i) and (ii) can be sent to the Managing Director.
 - (ix) If on the date of first call, the amount of deposit brought is less than 1/2, the candidate though eligible as per (i) and (ii), cannot be selected.

On the basis of the above criteria, decide which of the following courses is described in each question.

Mark answer (a) if the candidate can be selected; (b) if the candidate can be provisionally selected; (c) if the candidate should be referred to the Managing Director; (d) if the candidate cannot be selected; (e) if the data provided is inadequate.

- **49.** Satheesh, son of a trustee of the organisation, obtained 68% marks in B.Sc. He brings a deposit of ₹8200 on the day of the first call, but intends to pay the balance amount within ten days. He is a general candidate and his date of birth is 2nd November 1988.
- **50.** Sujatha is an SC candidate. Her date of birth is 26 January 1990. She passed her graduation in Science with 67% marks. She can deposit ₹ 18000.
- 51. Mahesh, a 22 year old general category candidate, secured 92% marks in graduation with Science. He can pay only ₹ 3000 as deposit. He is not a son of a staff or trustee of the organisation.
- **52.** Ramesh, born on 21 August, 1985 is a son of an employee working in the organisation secured 66% marks in M.Sc. and is in a position to deposit only ₹ 2000. He is an SC category candidate.
- **53.** Suresh Varma, a general category candidate, is a graduate in Science with 57% marks. His date of birth is 13 December 1987.

Directions (54–58): In the following questions, two statements are given namely Assertion (A) and Reason (R). Choose the correct alternative from the following:

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.
- (e) Both are false.
- **54.** Assertion (A): An atom is neutral despite the charged particles in it.

Reason (R) : Neutrons do not have any charge.

55. Assertion (A): Glass tumbler breaks in winter when hot water is poured in it.

Reason (R) : When hot water is poured, the outer surface of glass expands.

56. Assertion (A): Silver is not used to make electric wires.

Reason (R) : Silver is a bad conductor.

57. Assertion (A): Carbon forms the largest number of compounds.

Reason (R) : Carbon has the catenation property.

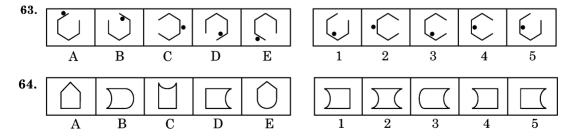
58. Assertion (A): Cut fruits and vegetables should not be kept open for long time.

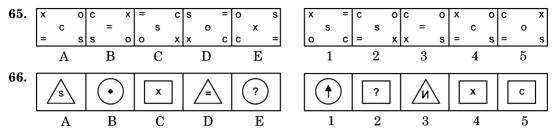
Reason (R) : Their vitamin content is ruined.

Directions (59–60): To each of the following questions four probable answers have been given. Select the most appropriate alternative as the answer.

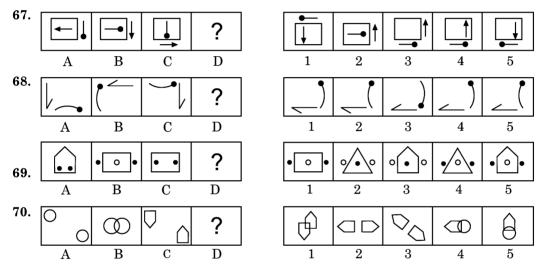
- 59. While firing crackers, a child gets severe burns on the hand. What would you do?
 - (a) Dip the child's hand in cold water till there is no more burning sensation.
 - (b) Wash the hands with dettol.
 - (c) Send some one to call the doctor.
 - (d) Apply some ointment on the affected area.
- **60.** You are moving across the road on a scooter when you observe that two boys on a bike snatch a lady's gold chain and ride away. You would
 - (a) Console the woman.
 - (b) Chose the boys to catch hold of them.
 - (c) Inform the police about the matter.
 - (d) Stand and see what happens next.
- **61.** If '+' means ' \times ', '-' means ' \div ' and ' \times ' means '-', then the value of $6 + 3 \times 5 1$ is
 - (a) 44
- (b) 13
- (c) 20
- (d) 24
- (e) None of these
- **62.** If 'x' means '-', '-' means '+', '+' means '\(\ddot\) and '\(\ddot\) means 'x', then the value of $\frac{12 \times 2 7 \div 5}{1 \times 10^{-100}}$ is
 - (a) 1
- (b) 3
- (c) 5
- (d) 7
- (e) 9

Directions (63-66): Each of the following questions consists of five figures marked A, B, C, D and E called problem figures followed by five other figures marked 1, 2, 3, 4 and 5 called answer figures. Select a figure from amongst the answer figures which will continue the same series as established by the five problem figures.

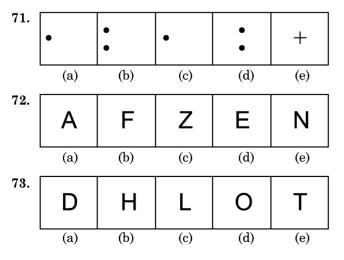




Directions (67–70): Each of the following questions consists of two sets of figures. Figures A, B, C and D constitute problem figures while figures 1, 2, 3, 4 and 5 constitute answer figures. There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by choosing a suitable figure (D) from answer figures.



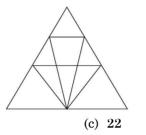
Directions (71–80): Out of the five figures (a), (b), (c), (d) and (e), given in each problem, four are similar in a certain way. However, one figure is not like the other four. Choose the figure which is different from the rest.



74. Ε U O (b) (d) (a) (c) (e)

75. (b) (c) (d) (a) (e)

76. Count the number of triangles in the following figure.

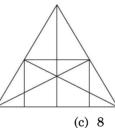


(a) 12

(b) 18

(d) 26

77. How many straight lines are there in the given figure?

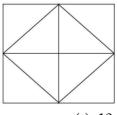


(a) 10

(b) 9

(d) 6

78. Count the number of pentagons in the following figure.



(a) 16

(b) 14

(c) 12

(d) 10

79. A cube in painted blue on all faces is cut into 125 cubes of equal size. How many cubes are there which are painted on one face only?

(a) 8

(b) 16

(c) 36

(d) 54

(e) None of these

80. A cube is painted on all faces is cut into 64 cubes of equal size. How many cubes are there which are painted on at least one face?

(a) 40

(b) 48

(c) 56

(d) 64

(e) None of these

ANSWERS										
1. (d)	2. (b)	3. (d)	4. (e)	5. (d)	6. (e)	7. (a)	8. (d)	9. (e)	10. (c)	
11. (c)	12. (b)	13. (c)	14. (a)	15. (a)	16. (a)	17. (c)	18. (c)	19. (c)	20. (c)	
21. (d)	22. (a)	23. (a)	24. (d)	25. (c)	26. (a)	27. (e)	28. (c)	29. (c)	30. (c)	
31. (b)	32. (b)	33. (b)	34. (b)	35. (c)	36. (b)	37. (b)	38. (d)	39. (c)	40. (d)	
41. (a)	42. (c)	43. (c)	44. (d)	45. (c)	46. (c)	47. (b)	48. (d)	49. (b)	50. (a)	
51. (c)	52. (d)	53. (e)	54. (b)	55. (c)	56. (c)	57. (d)	58. (a)	59. (a)	60. (b)	
61. (b)	62. (e)	63. (4)	64. (4)	65. (4)	66. (5)	67. (4)	68. (4)	69. (4)	70. (1)	
71. (e)	72. (d)	73. (d)	74. (a)	75. (d)	76. (b)	77. (b)	78. (c)	79. (d)	80. (c)	

Solutions and Hints

- 1. Clearly, none of the reasons is strong enough in favour of or against the statement. So none of these holds.
- 2. Argument I is not strong enough because no one can be debarred from their constitutional rights even if they cannot practise it to their benefit. But argument II is strong.
- **3.** The success of a government depends on its outlook and policies. So argument I is not strong enough. Argument II is also vague because a democracy is coveted for the reason that in it, the voice of the people is above all.
- 4. The statement was spoken for fear that the other person may not take a wrong decision. So, assumption I is implicit. The statement confirms that it was important to take the right decision. So assumption II is also implicit.
- 5. Nothing is mentioned in the advertisement about the problem of youngsters. So assumption I is not implicit. The advertisement is meant for persons who desire to look beautiful. So assumption II is also not implicit.
- **6.** Penalty is imposed to prevent people from misusing the alarm chain. So assumption I is implicit. The alarm chain is provided to stop the running train in times of urgency. So assumption II is also implicit.
- 7. If he is allowed to continue without being punished, he would create the problem elsewhere. So only I follows.
- 8. Preventive measures to protect the passengers and pay them adequate compensation are necessary in this case. So none of the courses follows.
- 9. For quick disposal of cases, either the methods in the court should be speeded up or the matters should be cleared up in their respective departments to prevent the delay. So, both the courses follow.
- 10. As given in the statement, domestic demand is increasing and for having a balance between production of crude oil and consumption of oil and oil by-products either of the two conclusions should follow.
- 11. As given in the statement, patients with minor ailments do not go to eminent doctors. This may be either of the two reasons stated in conclusions I and II. Therefore, either I or II follows.

- 12. Most parents wish to give the best education to their children as they can. This can be done only through good schooling. Therefore, only conclusion II follows. But conclusion I does not follow.
- 13. The first is the process of formation of the second.
- 14. The second is the place for performing the first.
- 15. A prisoner is confined within the dungeon, and an unsheltered person takes refuge within the asylum.
- **16.** Ruby is a red precious stone and sapphire is a blue precious stone.
- 17. The first is the left over the second.
- 18. The Wimbledon Trophy is associated with Tennis while the Walker's Cup is associated with Golf.
- 19. A vegetarian never eats meat. Similarly, a teetotaller never drinks liquor.
- 23. All except whale belong to the family of fishes.
- 24. All except pink are colours seen in a rainbow.
- 25. All except Thalamus are hormone secreting glands.
- **26.** All except nerves are parts of the heart.
- 27. All except acre are units for measuring distance, while acre is a unit of area.
- **28.** The series is obtained by multiplying with 3 and subtracting 2, multiplying with 4 and subtracting 3 and so on. So $25 \times 5 4 = 121$ is the missing term.
- **29.** The series obtained is $1^2 + 1$, $2^2 + 2$, $3^2 + 3$, ...

Next term = $4^2 + 4 = 20$

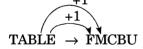
Here it is 24

 \therefore 24 is wrong.

30. BATCH \rightarrow ABCTH

Every pair of letters is reversed in the code. Therefore 'SEVEN' is coded as 'ESEVN'

31.



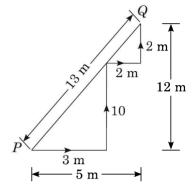
Word is reversed and 1 is added to every character.

Similarly ? \rightarrow 'TOWER'

The required word is obtained by reversing the code 'TOWER' and subtracting 1 from each character.

- .. The answer is 'QDVNS'
- **32.** From the given items, pencil is the one which can be used for drawing a figure. But, here 'pencil' is called 'pen'. Therefore '**pen**' is the answer.
- **33.** Here '45' means 'good morning', '357' means 'Buy good items'. Above two codes consists of '5' as a code for the common word 'good'.
 - .: In '45', '5' stands for 'good' and '4' stands for 'morning'
 - \therefore The answer is **(b)**.
- 34. Only son of my mother's father means brother of my mother.
 - :. The person is brother of woman's mother, i.e. uncle.

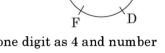
- **35.** $PQ = \sqrt{(3+2)^2 + (10+2)^2}$ = $\sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ m}$
 - $\therefore Q \text{ is } 13 \text{ m NE of } P.$



- **36.** The 3 groups of persons are of 3 different profession and is not having any common property. Therefore, it can be represented by the choice (b).
- 37. Number of girls in front of Dinesh = 19 4 = 15

Total number of girls = 22

- \therefore Number of girls behind Dinesh = 22 15 = 7
- **38.** B is seated between A and F.
- **39.** E is seated to the right of C.
- **40.** F is seated opposite to E.
- 41. A is seated to the right of E.
- 42. C is seated between D and E.



В

- **43.** There are seven such numbers which are having at least one digit as 4 and number fully divisible by 4. They are 4, 24, 40, 44, 48, 64 and 84.
- **44.** Number of days = 17 + 30 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 + 5 =**295 days**
- **45.** 1 3 5 <u>2</u> 8 6 4 2 7 3 9 5 <u>2</u> 3 8 5 <u>2</u> 8

Number '2', which is underlined is given as per the condition given in the question. The total number of such 2s is 3.

- 46. Only 6 is a male and a professional.
- 47. Among the civil servants only one person is male. 4 and 5 are female civil servants∴ The answer is (b).
- **48.** Among the choices given, number '2' is the only person who is neither a civil servant nor a professional.
- 49. As he intends to pay the balance within ten days condition (vii) is fulfilled.
- **50.** The candidate is supposed to deposit ₹ 16000 only as she being an SC candidate gets 20% deduction. So all conditions of eligibility are fulfilled.
- ${f 51.}$ The candidate has secured more than ${f 90\%}$ marks. So, condition (iv) is fulfilled.
- **52.** The candidate does not satisfy condition (iii), as he is supposed to pay ₹ 8000.
- 53. It is not mentioned whether the candidate is a son of a trustee or not.
- **54.** An atom contains positively charged protons and an equal number of negatively charged electrons. So it is neutral. However, it is true that neutrons in the nucleus of an atom are neutral.

- **55.** When hot water is poured in the glass tumbler in winter, its inner surface tends to expand while the outer surface in contact with cold atmosphere does not. This opposite interaction causes the tumbler to break.
- **56.** Silver is a good conductor of electricity but it is not used to make electric wires because it is expensive.
- **57.** Carbon forms a large number of compounds due to its tendency to form chains and rings of varying sizes, called its catenation property. However, the largest number of compounds are formed by Hydrogen.
- 58. When cut fruits and vegetables are kept in open, the vitamins in them get oxidised and remain of no use.
- **61.** $6 + 3 \times 5 1 = ?$

Putting the actual sign in the above expression, we get,

$$6 \times 3 - 5 \div 1 = 18 - 5 = 13$$

62.
$$\frac{12 \times 2 - 7 \div 5}{8 - 2 \times 5} = ?$$

Putting the actual sign in the above expression, we get

$$\frac{12-2+7\times 5}{8+2-5} = \frac{12-2+35}{5} = \frac{45}{5} = \mathbf{9}$$

- **63.** The main figure rotates one step ACW in each turn and dot moves one step CW in each turn and also gets inside and outside the main figure alternately.
- **64.** The figure is rotated 90° CW in each step. Then, two elements, one element, no element, again two elements and one element change their shape.
- 65. The symbols move in the order in each step.
- **66.** A similar figure appears every third step and the symbols inside the figure are replaced by new ones in each step.
- **67.** The pin and the arrow moves to the adjacent side in ACW direction. Out of these two, the one which was inside, comes out and the other which was outside gets in.
- **68.** The half arrow rotates 90° CW, moves to the adjacent side CW and gets inverted. The bent pin rotates 90° ACW and moves to the adjacent side CW.
- **69.** The main figure is replaced by a figure with the number of sides less by one. The black circles inside the figure comes out on either sides of the figure and a white circle is introduced inside it.
- 70. The two figures approach each other and gets overlapped.
- 71. All except figure (e) are punctuation marks.
- 72. It is the only letter having four lines. The rest of all have three lines.
- 73. All other letters are fourth starting from the previous one. So figure (d) should be 'P'.

- 74. All except figure (a) are vowels.
- 75. In all other cases, lines drawn inside the figure divide it into equal parts.
- **76.** Simplest triangles = 8

Triangles with two components = 5

Triangles with three components = 3

Triangles with four components = 1

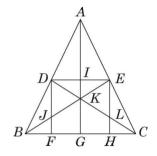
Triangles with nine components = 1

- \therefore Number of triangles = 8 + 5 + 3 + 1 + 1 = 18
- 77. Horizontal lines are DE and BC i.e. 2 Nos.

Vertical lines are DF, EH and AG. i.e. 3 Nos.

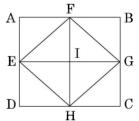
Starting lines are DC, BE, AB and AC. i.e. 4 Nos.

 \therefore Total number of straight lines = 2 + 3 + 4 = 9 Nos.



78. From the figure,

DEFGC, AFGHD, BGHEA, CHEFB are 4 pentagons. AFGHE, BGHEF, CHEFG and DEFGH and 4 pentagons. DAFGC, DEFBC, AFGCD and ABGHD are 4 pentagons. In all, total number of pentagons = 4 + 4 + 4 = 12.



- **79.** $9 + 12 \times 3 + 9 = 54$, such cubes are painted on one face only.
- 80. Number of cubes with at least one face painted

= Total number of cubes - Number of cubes which are not painted

$$= 64 - 8 = 56$$

Model Test Paper 3

1. On a railway route between two places A and B, there are 10 stations on the way. If 4 new stations are to be added, how many types of new tickets will be required if

2. Aryan runs at a speed of 40 m/min. Rahul follows him after an interval of 5 minutes and runs at a speed of 50 m/min. Rahul's dog runs at a speed of 60 m/min and starts along with Rahul. The dog reaches Aryan and than comes back to Rahul, and continues to do so till Rahul reaches Aryan. What is the total distance covered by the dog?

(c) 96

(c) 980 m

(d) 108

each ticket is issued for a one way journey? (b) 48

(b) 750 m

(a) 14

3.	A big rectangular plo	t of area 4	1320 m~ 1s d1v1	ded into 3 squar	e shaped smaller plots				
	by fencing parallel to the smaller side of the plot. However, some area of land was								
	still left as a square could not be formed. So, 3 more square shaped plots were formed								
					that no area of the plot				
	was left surplus. What are the dimensions of original plot?								
	(a) $160 \text{ m} \times 27 \text{ m}$	(b) 240 m	\times 18 m (c)	120 m × 36 m	(d) $135 \text{ m} \times 32 \text{ m}$				
4.	Two men and one wo	man boar	d a bus in whi	ch 5 seats are va	cant. One of these five				
					the seat reserved for				
				•	In how many different				
	ways can the five sea				•				
			_	48	(d) 60				
	` '	, ,	, ,						
5 .	_			-	entical balls are to be				
	placed in these smaller squares such that each of the three rows gets at least one								
	ball (one ball in one square only). In how many different ways can this be done?								
	(a) 27	(b) 36	(c)	54	(d) 81				
6.	There are six persons	s A, B, C,	D, E and F. Tl	ney are to be sea	ted in a row such that				
	B never sits anywher	re ahead (of A, and C ne	ever sits anywhe	ere ahead of B. In how				
	many different ways	can this b	e done?						
	(a) 60	(b) 72	(c)	120	(d) None of these				

(d) Not possible

(d) 296

	men or judges, and 35 are women judges. There are no foreign judges. How many									
	Indian wo	men atte	ended	l the me	eting?					
	(a) 35		` '	45		(c) 55		(d) 6	30	
10.	There are	-			D, E and	d F.				
	A has 3 it	ems mor	e tha	n C.						
	D has 4 it	ems less	than	В.						
	E has 6 it	ems less	than	F.						
	C has 2 it	ems mor	e tha	n F.						
	F has 3 it	ems mor	e tha	n D.						
	Which on	e of the	follov	ving fig	ures can	not be e	qual to tl	he total r	number	of items
	possessed	by all th	1e 6 p	ersons?						
	(a) 41		(b)	47		(c) 53		(d) 8	58	
					Z A NICKE	ED C				
					ANSW	ERS				
1.	(d) 2. ((d) 3. (c)	4. (b)	5. (b)	6. (c)	7. (c)	8. (d)	9. (c)	10. (d)
				G - 1	4.	11124				
				501	utions a	nd Hint				
1.	Including 11 other s				welve st	ations in	all. Fro	m each s	tation t	ickets to
	∴ Total n	umbono	f tials	ota noan	inod for	o one we	u ioumo	12 D -	12!_	$=\frac{12!}{}$
			1 LICK	ets requ	iiieu ioi	a one wa	ty journe,	y – 1 ₂ -	(12-2)	2)! 10!
	$= 12 \times 11$					_				
	When fou way journ	ey from	16 sta	ations ir	ı all is gi	ven by			quired fo	or a one-
	Number o	of tickets	requi	ired = 10	$^{6}P_{2}=\frac{1}{(16)}$	$\frac{16!}{-2)!} = \frac{1}{1}$	$\frac{6!}{4!} = 16 \times$	15 = 240		
	∴ Numbe									
2.	Speed of A									
	Speed of I	-		_						
	r		0							

7. An equilateral triangular plate is to be cut into n number of identical small equilateral

8. There are 10 identical coins and each one of them has 'H' engraved on its one face and 'T' engraved on its other face. These 10 coins are lying on a table and each one

In one attempt, exactly four (neither more nor less) coins can be turned upside down. What is the minimum total number of attempts in which the 'T' faces of all

9. 300 persons are participating in a meeting, out of which 120 are foreigners, and the

(c) 8

(c) 256

triangular plates. Which one of the following can be possible value of *n*?

(b) 216

the 10 coins can be brought to be the upper faces? (b) 7

of them has 'H' face as the upper face.

(a) 196

(a) 4

Distance travelled by Aryan in 5 minutes = $40 \times 5 = 200$ m. Relative speed of Aryan with respect to Rahul = 40 - 50 = 10 m/min or relative speed of Rahul with respect to Aryan = 10 m/min

- \therefore Time taken to reach Rahul with Aryan = $\frac{200}{10}$ = 20 min
- \therefore Distance covered by dog in 20 minutes = $60 \times 20 = 1200$ m
- Short cut: Distance covered by the dog = Speed of the dog × time taken

 = Speed of the dog × $\frac{\text{Relative distance of Aryan w.r.t. Rahul}}{\text{Relative speed of Rahul w.r.t. Aryan}}$ = $60 \times \frac{200}{10} = 1200 \text{ m}$
- 3. Here ABCD is divided into 3 squares by fencing EJ, GH and FI. The remaining rectangle GBCH is divided into 3 squares by fencing KN and LM.

Let AD be x.

$$\therefore$$
 AE = EF = FG = DJ = JI = IH = x
Similarly AD = EJ = FI = GH = BC = x

$$\therefore$$
 BK = KL = LC, BK = $\frac{AD}{3} = \frac{x}{3}$

:. GB = NK = ML = HC =
$$\frac{x}{3}$$
 (:: GBKN is a square)

Total area of rectangle ABCD =
$$3x^2 + 3 \times \left(\frac{x}{3}\right)^2 = 4320 \text{ m}^2$$

i.e.
$$3x^2 + 3 \times \frac{x^2}{9} = 4320 \text{ m}^2$$

i.e.
$$3x^2 + \frac{x^2}{3} = 4320 \text{ m}^2$$

i.e.
$$\frac{10x^2}{3} = 4320 \text{ m}^2$$
or
$$x^2 = 3 \times 432 = 1296 \text{ m}^2$$

or
$$x^2 = 3 \times 432 = 1296$$
: $x = 36$ m

$$\therefore \qquad \text{Size of plot} = \left(3x + \frac{x}{3}\right) \times x$$

 $= 120 \text{ m} \times 36 \text{ m}$

4. If the lady takes the seat reserved for ladies, it can be filled in ${}^{1}C_{1}$ ways = 1 way. Remaining seats can be filled in by the remaining two men using 4 seats available in

$${}^{4}C_{2}$$
 ways = $\frac{4!}{(4-2)!}$ = 4×3 = 12 ways

 \therefore Total = 1 × 12 = 12 ways.

If the lady doesn't take the seat reserved for ladies, that seat can be filled in ${}^{1}C_{0} = 1$ way.

The remaining seats can be filled by the three passengers in ${}^4P_3 = \frac{4!}{(4-3)!}$ $4 \times 3 \times 2 = 24$ ways.

- \therefore Total = $1 \times 24 = 24$ ways
- \therefore Total number of ways = 12 + 24 = 36 ways.
- ${f 5.}$ The square is divided into ${f 9}$ smaller squares as shown in the figure.

The squares can be filled in 6C_1 , 3C_1 , 2C_1 , 1C_1 , 5C_1 , 0C_1 , 0C_0 , 0C_0 and 4C_1 ways respectively as shown in the figure subject to the given condition.

.. Maximum number of ways in which a row can be filled is given by ${}^6C_1 \times {}^3C_1 \times {}^2C_1 = 6 \times 3 \times 2 = 36$ ways.

$^6\!C_1$	${}^3\!C_1$	2C_1
$^1\!C_1$	$^5\!C_1$	${}^{\scriptscriptstyle{0}}\!C_{\scriptscriptstyle{0}}$
${}^{0}C_{0}$	${}^{\scriptscriptstyle{0}}\!C_{\scriptscriptstyle{0}}$	4C_1

So the answer is (b).

- 6. Taking the given condition, take A, B and C are together in the order, then number of ways of arrangement = $\frac{6!}{3!}$ = 120 ways
- 7. An equilateral triangle can be cut into smaller equilateral triangles in 4 numbers.

So here the area of triangle = $\frac{\sqrt{3}}{4} \alpha^2 = \frac{\sqrt{3}}{4} \alpha_1^2 \times n$ where

a = side of larger equilateral triangle.

 a_1 = side of smaller equilateral triangle.

- \therefore n will be a power of 4.
- \therefore The answer is (c).
- 8. Not possible. Because you can't turn 'T' faces of all the coins in an attempt.
- 9. Given total participants = 300

Number of foreigners = 120

Number of Indian participants = 300 - 120 = 180

Number of men or judges = 160

 \therefore Women who are not judges = 180 - 160 = 20

Number of women judges = 35

- \therefore Total number of Indian women attended the meeting = 20 + 35 = 55.
- **10.** Put d = 0; b = d + 4 = 0 + 4 = 4; f = d + 3 = 0 + 3 = 3; c = f + 2 = 3 + 2 = 5; e = f 6 = 3 6 = -3; a = c + 3 = 5 + 3 = 8.

$$\therefore a = 8; b = 4; c = 5; d = 0; e = -3; f = 3$$

$$\therefore a+b+c+d+e+f=8+4+5+0-3+3=17$$

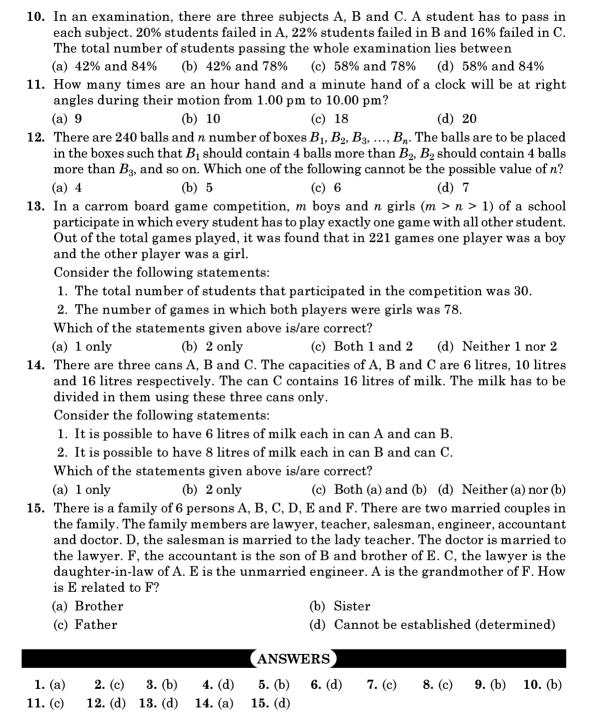
Adding multiples of 6 to 17, we get, $17 + 4 \times 6 = 41$; 41 + 6 = 47; 47 + 6 = 53; $53 + 6 = 59 \neq 58$.

So the answer is (d).

Model Test Paper 4

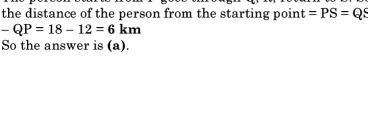
Directions: Four alternatives are given for the following questions. You have to choose the most correct answer from the given alternatives.

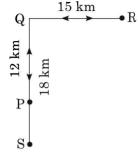
1.	. A person travels 12 km due north, then 15 km due east, after that 15 km due west and then 18 km due south. How far is he from the starting point?							
	(a) 6 km	(b)	12 km	(c)	33 km	(d)	60 km	
2.	Six persons A, B, C, each other alongside standing on the extr	е Е .	B is standing be	_			•	
	(a) A and F	(b)	B and D	(c)	B and F	(d)	None of these	
3.	A person has 4 coins sums of money the p							
	(a) 16	(b)	15	(c)	12	(d)	11	
4.	How many numbers	lie	between 300 and	500	in which 4 come	s on	ly one time?	
	(a) 99	(b)	100	(c)	110	(d)	120	
5.	How many letters of a mirror?	the	English alphabet	t (ca	pitals) appear sa	me	when looked at in	
	(a) 9	(b)	10	(c)	11	(d)	12	
6.	How many three-dig that the digits are in				rated from 1, 2,	3, 4,	5, 6, 7, 8, 9 such	
	(a) 80	(b)	81	(c)	83	(d)	84	
7.	There are four person B and 4 more beside of the coins to D and coins. How many contains the coins of the coi	s. B d 4	gave half of the omore besides. Bo	coin th I	s to C and 4 mor 3 and D end up	e be	sides. C gave half	
	(a) 96	(b)	84	(c)	72	(d)	64	
8.	While adding the first few continuous natural numbers, candidate missed one of the numbers and wrote the answer 177. Which was the number missed?							
	(a) 11	(b)	12	(c)	13	(d)	14	
9.	Four metal rods of lengths 78 cm, 104 cm, 117 cm and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?							
	(a) 27	(b)	36	(c)	43	(d)	480	



Solutions and Hints

1. The person starts from P goes through Q. R. return to S. So the distance of the person from the starting point = PS = QS-QP = 18 - 12 = 6 km





- 2. The arrangement of six persons is given on the right side
 - .. B and F are standing on the extremes So the answer is (c)

- D F
- 3. The number of denominations that can be formed using the given 4 coins
 - (i) When only one coin was used, the required number = 4
 - (ii) When two coins were used, the required number = ${}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$
 - (iii) When three coins were used, the required number = ${}^4C_3 = {}^4C_1 = 4$
 - (iv) When all the four coins were used, the required number = 1
 - \therefore Total number of denominations = 4 + 6 + 4 + 1 = 15
- **4.** Numbers which are having 4 in unit place only = $20 \times 1 = 20$ (once in 10 numbers) Numbers which are having 4 in tenth place = $2 \times 10 = 20$

Numbers which are having 4 in hundredth place = $100 \times 1 = 100$

Number of 4s repeating in one's and tenth's place = 1 number

Number of 4s repeating in one's and hundredth place = 9 numbers

Number of 4s repeating in tenth and hundredth place = 9 numbers

Number of 4s repeating in all three positions = 1 number

- ... Number of numbers with '4' in one place only = (20 + 20 + 100) (1 + 9 + 9 + 1) = 120. So the answer is (d).
- 6. (i) If 1 and 2 are provided in the first two places, then the third place can be filled in ${}^{7}C_{1} = 7$ ways

So the number of numbers so formed = 7

(ii) If 1 and 3 are filled in first two position, then the third place can be filled in 6C_1 = 6 ways and so on. Total number of numbers so formed with '1' as 1st place = 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28.

Similarly with 2 in the first place and 3 in the second place, third place can be filled in ${}^6C_1 = 6$ ways. So total number of numbers with '2' as 1st place = 6 + 5 + 4+3+2+1=21 and so on.

Similarly with 7 in the first place and 8 in the second place, third place can be filled in ${}^{1}C_{1} = 1$ way. So the total number of numbers with '7' as 1st digit and '8' as 2nd digit is 1.

So the total number of three digit numbers thus formed with all the given 9 digits = 28 + 21 + 15 + 10 + 6 + 3 + 1 = 84.

7. Let the number of coins with A, B, C and D be a, b, c and d respectively.

No. of coins with A = 'a'

No. of coins with $B = b = \frac{a}{2} + 4$

No. of coins with $C = c = \frac{b}{2} + 4 = \frac{1}{2} \left(\frac{a}{2} + 4 \right) + 4 = \frac{a}{4} + 2 + 4 = \frac{a}{4} + 6$

No. of coins with $D = d = \frac{c}{2} + 4 = \frac{1}{2} \left(\frac{a}{4} + 6 \right) + 4 = \frac{a}{8} + 3 + 4 = \frac{a}{8} + 7$

Given b = d at the end

i.e.
$$\left(\frac{a}{2}+4\right)-\left(\frac{a}{4}+6\right)=\frac{a}{8}+7$$

or

$$\frac{a}{4} - 2 = \frac{a}{8} + 7$$

$$\frac{a}{8} = 2 + 7 = 9$$

$$a = 72$$

or

Hence the answer is (c).

- 8. The sum of first 19 natural numbers comes to $\frac{19 \times 20}{2} = 190$
 - : Addition of 13 makes it possible to get the correct sum.

Hence the answer is (c)

9. To find the maximum possible length, find HCF of given numbers.

HCF of 78, 104, 117 and 169

HCF of 78 and 104 is 26 (i.e. last divisor)

HCF of 117 and last HCF (26) is 13.

HCF of 169 and 13 is 13.

$$\begin{array}{r}
 13 \\
 13 \overline{\smash{\big)}\, 169} \\
 \underline{169} \\
 0
 \end{array}$$

Maximum number of pieces that can be cut, which are having 13 cm length is given by

required number =
$$\frac{78}{13} + \frac{104}{13} + \frac{117}{13} + \frac{169}{13} = 6 + 8 + 9 + 13 = 36$$

So, the answer is **(b)**.

10. Subjects are A, B and C

Students passed in A = 100 - 20 = 80%

Students passed in B = 100 - 22 = 78%

Students passed in C = 100 - 16 = 84%

Minimum pass percentage = 78% and that can be maximum possible value of whole pass percentage. If students failed in A, B and C independently in the respectively given percentages, total failure percentage can be of maximum possible upto 20 + 22 + 16 = 58% or pass percentage = 42%

.. Pass percentage lies between 42% and 78%

Hence the answer is (b).

11. In every hour there will be two positions of hour hand and minute hand coming at right angles. Therefore in 9 hours there will be 18 such positions.

Hence the answer is (c).

12. If m is the number of balls in the first box, then in the second box, it will be (m-4), in the third box it will be (m-8) and so on.

$$\therefore \text{ Sum of balls in } n \text{ boxes together} = \frac{n}{2} \left[(m-4) + (m-4) + (n-1) - 4 \right]$$

$$= \frac{n}{2} \left[2m - 8 - 4n + 4 \right]$$

$$= \frac{n}{2} \left[2m - 4n - 4 \right] = n[m - 2n - 2] = 240$$

- \therefore Sum of balls obtained above is a multiple of n, value of n which is not possible from the given alternatives is 7.
- n is a factor of 240, the value of n which is not possible from the given alternatives is 7. So the answer is (d).
- 13. It is given that number of boys (m) > Number of girls (n)

Given in 221 games plays, one player was boy and the other was girl

So, minimum number of boys = 221

So, minimum number of girls = 221

However, the number of boys is greater than that of girls.

So neither of the 2 given statements are correct. Hence the answer is (d).

- 14. We are having cans A, B and C with measurements 6 litres, 10 litres and 16 litres respectively. We can take 6 litres each in can A and can B. So statement 1 is correct. We are not having can for measuring 8 litres of milk. So second statement is false. Hence 1 only is correct.
- 15. It is mentioned that F is the brother of E and E is the unmarried engineer. So it is not clear that whether E is male or female. Hence the relation of E to F cannot be established. Hence the answer is (d).

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